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The extremal iteration solution to a coupled system of nonlinear conformable fractional differential equations

Suli Liu[a,b], Han Wang[a], Xiaoping Li[c], Huilai Li[a,*]

[a]Department of Mathematics, Jilin University, Changchun 130012, P. R. China.

[b]Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, T6G 2G1, Canada.

[c]Institute of Mathematics and Finance, Xiangnan University, Chenzhou 423000, P. R. China.

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Abstract

In this paper, we consider a coupled system of nonlinear conformable fractional differential equations by using the comparison principle and the monotone iterative technique combined with the method of upper and lower solutions:

$$\begin{cases} x^{(\alpha)}(t) = f(t, x(t), y(t)), & t \in [a, b], \\ y^{(\alpha)}(t) = g(t, y(t), x(t)), & t \in [a, b], \\ x(a) = x_0^*, \quad y(a) = y_0^*, \end{cases}$$

where $f, g \in C([a, b] \times \mathbb{R} \times \mathbb{R}, \mathbb{R})$, $x_0^*, y_0^* \in \mathbb{R}$, $x_0^* \leqslant y_0^*$, $x^{(\alpha)}$, $y^{(\alpha)}$ are the conformable fractional derivatives with $0 < \alpha \leqslant 1$. We obtain the existence of extremal iteration solution to the system, and the main results are examined by the help of an example.

Keywords: Nonlinear conformable fractional differential equations, extremal system of solutions, monotone iterative method, comparison principle, upper and lower solutions.
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1. Introduction

Fractional calculus is a discipline of mathematical analysis as old as integer orders. In 1695, L'Hospital asked what does it mean $\frac{d^n f}{dx^n}$ if $n = \frac{1}{2}$, since then, the mathematicians tried to answer this question. The concepts of fractional derivative and integral appear associated with considerable operators: the Riemann-Liouville integral, basing on iterating the integral operator n times and replaced $n!$ by Gamma function, was used to define Riemann-Liouville and Liouville-Caputo fractional derivatives, see [13, 16–18]; the Grünwald-Letnikov derivative which based on iterating the derivative n times and fractionalizing by using the Gamma function in the binomial coefficients.

*Corresponding author
Email addresses: liusl15@mails.jlu.edu.cn (Suli Liu), hanwang15@mails.jlu.ed.cn (Han Wang), lxp418@126.com (Xiaoping Li), lihuilai@jlu.edu.cn (Huilai Li)

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Recently, Khalil et al. in [11] gave a new well-behaved definition called "the conformable fractional derivative" depending on the basic limit definition of the derivative and distincting from other definitions, and it satisfies the familiar product rule, quotient rule. Then, the conformable fractional versions of chain rule, exponential functions, Gronwall's inequality, integration by parts, Taylor power series expansions, and Laplace transforms are proved in [1]. The new definition is getting an increasing of interest, see [2, 5, 7–9, 14]. Katugampola in [10] introduced a new fractional derivative based on the conformable fractional derivative by replacing $t+\epsilon t^{1-\alpha}$ by $te^{\epsilon t^{-\alpha}}$, unfortunately, the proof of chain rule was wrong. It should be noted that the idea of Katugampola is interesting and more general. Asawasamrit et al. [3] obtained the existence of solutions for periodic boundary value problems for impulsive conformable fractional integro-differential equations. Ünal et al. [15] presented the particular solution for non-homogeneous sequential linear local (conformable) fractional differential equations by the help of the operator method. In [6], Bayour et al. proved the existence of solution to a conformable fractional nonlinear differential equation with initial condition using the notion of tube solution and Schauder's fixed-point theorem.

Motivated by the works mentioned above, we consider the following coupled system of conformable nonlinear fractional differential equations:

$$\begin{cases} x^{(\alpha)}(t) = f(t, x(t), y(t)), & t \in [a, b], \\ y^{(\alpha)}(t) = g(t, y(t), x(t)), & t \in [a, b], \\ x(a) = x_0^*, \quad y(a) = y_0^*, \end{cases} \tag{1.1}$$

where $f,\ g \in C([a,b]\times\mathbb{R}\times\mathbb{R},\mathbb{R})$, $x_0^*,\ y_0^* \in \mathbb{R}$, $x_0^* \leqslant y_0^*$, $x^{(\alpha)}$, $y^{(\alpha)}$ are the conformable fractional derivatives with $0 < \alpha \leqslant 1$. We obtain the existence of extremal iteration solution to the system by using the comparison principle and the monotone iterative technique combined with the method of upper and lower solutions. To the best of our knowledge, this is the first paper establishing the system of (1.1) via the conformable fractional calculus developed by [11]. For applications of monotone iterative technique, one can refer to literatures [4, 12, 19, 20].

This paper is organized as follows. In Section 2, we give the basic definitions of conformable fractional calculus and prove some lemmas which will play important roles in the next section. The main results are listed in Section 3. At last, an example is given in Section 4.

2. Preliminaries

Definition 2.1 ([1, 11])**.** The conformable fractional derivative starting from a point a of a function $f : [a,\infty) \to \mathbb{R}$ of order $\alpha \in (0, 1]$ is defined by

$${}_aT^{(\alpha)}(f)(t) = \lim_{\epsilon\to 0} \frac{f\big(t+\epsilon(t-a)^{1-\alpha}\big) - f(t)}{\epsilon},$$

provided that the limit exists.

Definition 2.2 ([1, 11])**.** Let $\alpha \in (0, 1]$. The conformable fractional integral starting from a point a of a function $f : [a,\infty) \to \mathbb{R}$ of order α is defined by

$${}_aI^{\alpha}(f)(t) = \int_a^t (s-a)^{\alpha-1} f(s)ds.$$

We will, sometimes, write $f^{(\alpha)}(t)$ for ${}_aT_\alpha(f)(t)$, and $I^\alpha(f)(t)$ for ${}_aI^\alpha(f)(t)$, to denote α-order conformable fractional derivative and integral of $f(t)$, respectively.

Lemma 2.3. *Let* $0 < \alpha \leqslant 1$, $\sigma(t) \in C([a,b],\mathbb{R})$, $M \in \mathbb{R}$. *The linear problem:*

$$\begin{cases} x^{(\alpha)}(t) + Mx(t) = \sigma(t), & t \in [a, b], \\ x(a) = x_0^*, \end{cases} \tag{2.1}$$

has a unique solution.

Proof. We first consider the associated linear homogeneous equation:

$$\begin{cases} x^{(\alpha)}(t) + Mx(t) = 0, \quad t \in [a, b], \\ x(a) = x_0^*. \end{cases} \tag{2.2}$$

The auxiliary equation is $\alpha r + M = 0$, then $r = -\frac{M}{\alpha}$. We can easily get that the solution to (2.2) is

$$x(t) = Ce^{-\frac{M}{\alpha}t^\alpha},$$

where C is constant. By $x(a) = x_0^*$, we have $Ce^{-\frac{M}{\alpha}a^\alpha} = x_0^*$, i.e., $C = x_0^* e^{\frac{M}{\alpha}a^\alpha}$. Hence, the solution to (2.2) is

$$x(t) = x_0^* e^{\frac{M}{\alpha}a^\alpha} e^{-\frac{M}{\alpha}t^\alpha}.$$

Let $x(t) = C(t)e^{-\frac{M}{\alpha}t^\alpha}$ is the general solution to problem (2.1), and then we can get

$$t^{1-\alpha}C'(t)e^{-\frac{M}{\alpha}t^\alpha} + t^{1-\alpha}e^{-\frac{M}{\alpha}t^\alpha}(-Mt^{\alpha-1})C(t) + MC(t)e^{-\frac{M}{\alpha}t^\alpha} = \sigma(t).$$

By simple calculation,

$$C(t) = x_0^* e^{\frac{M}{\alpha}a^\alpha} + \int_a^t s^{\alpha-1} e^{\frac{M}{\alpha}s^\alpha} \sigma(s)ds.$$

Hence, the solution to (2.1) is

$$x(t) = e^{-\frac{M}{\alpha}t^\alpha}\left(x_0^* e^{\frac{M}{\alpha}a^\alpha} + \int_a^t s^{\alpha-1} e^{\frac{M}{\alpha}s^\alpha} \sigma(s)ds\right).$$

The proof is finished. □

To study the nonlinear system (1.1), we first consider the associated linear system:

$$\begin{cases} x^{(\alpha)}(t) = \sigma_1(t) - Mx(t) - Ny(t), \quad t \in [a, b], \\ y^{(\alpha)}(t) = \sigma_2(t) - My(t) - Nx(t), \quad t \in [a, b], \\ x(a) = x_0^*, \quad y(a) = y_0^*, \end{cases} \tag{2.3}$$

where $0 < \alpha \leqslant 1,\ \sigma_1(t),\ \sigma_2(t) \in C([a, b], \mathbb{R}),\ M \in \mathbb{R},\ N \geqslant 0$.

Lemma 2.4. *The linear system* (2.3) *has a unique system of solutions in* $C^\alpha([a, b], \mathbb{R}) \times C^\alpha([a, b], \mathbb{R})$.

Proof. The pair $(x, y) \in C^\alpha([a, b], \mathbb{R}) \times C^\alpha([a, b], \mathbb{R})$ is a solution to system (2.3) if and only if

$$x(t) = \frac{p(t) + q(t)}{2},\ y(t) = \frac{p(t) - q(t)}{2},\ t \in [a, b],$$

where $p(t)$ and $q(t)$ are the solutions to the following problems:

$$\begin{cases} p^{(\alpha)}(t) = (\sigma_1 + \sigma_2)(t) - (M + N)p(t), \quad t \in [a, b], \\ p(a) = x_0^* + y_0^*, \end{cases}$$

$$\begin{cases} q^{(\alpha)}(t) = (\sigma_1 - \sigma_2)(t) - (M - N)q(t), \quad t \in [a, b], \\ q(a) = x_0^* - y_0^*, \end{cases}$$

with

$$p(t) = e^{-\frac{M+N}{\alpha}t^{\alpha}}\left((x_0^* + y_0^*)e^{\frac{M+N}{\alpha}a^{\alpha}} + \int_a^t s^{\alpha-1}e^{\frac{M+N}{\alpha}s^{\alpha}}(\sigma_1+\sigma_2)(s)ds\right),$$
$$q(t) = e^{-\frac{M-N}{\alpha}t^{\alpha}}\left((x_0^* - y_0^*)e^{\frac{M-N}{\alpha}a^{\alpha}} + \int_a^t s^{\alpha-1}e^{\frac{M-N}{\alpha}s^{\alpha}}(\sigma_1-\sigma_2)(s)ds\right).$$

The proof is finished. □

Lemma 2.5. *Let* $u(t) \in C^{\alpha}([a,b],\mathbb{R})$ *satisfy*

$$\begin{cases} u^{(\alpha)}(t) + Mu(t) \geqslant 0, & t \in [a,b], \\ u(a) \geqslant 0, \end{cases}$$

where $0 < \alpha \leqslant 1,\ M \in \mathbb{R}$, *then* $u(t) \geqslant 0$ *for all* $t \in [a,b]$.

Proof. We prove the lemma by two cases:

Case 1: If $M = 0$. Suppose the contrary. If there exists $t \in [a,b]$ s.t. $u(t) < 0$, then there exists $t_0 \in [a,b]$ s.t. $u(t_0) = \min_{t\in[a,b]} u(t) < 0$. (i) If $t_0 > a$, hence, there exists an interval $[t_1,t_0] \subseteq [a,t_0]$ such that $u(t) < 0$ for all $t \in [t_1,t_0]$. Then, $u(t_0) - u(t_1) =_{t_1} I^{\alpha}(u^{(\alpha)})(t_0) \geqslant 0$, which contradicts the fact that $u(t_0) = \min_{t\in[a,b]} u(t)$. (ii) If $t_0 = a$, then it contradicts to $u(a) \geqslant 0$.

Case 2: If $M \neq 0$. Choose $\sigma(t) \in C([a,b],\mathbb{R})$, $\sigma(t) \geqslant 0$, and $u_0 \geqslant 0$, such that $u^{(\alpha)}(t) + Mu(t) = \sigma(t)$, $u(0) = u_0$. By Lemma 2.3, the expression of the function $u(t)$ is given by

$$u(t) = e^{-\frac{M}{\alpha}t^{\alpha}}\left(u_0 e^{\frac{M}{\alpha}a^{\alpha}} + \int_a^t s^{\alpha-1}e^{\frac{M}{\alpha}s^{\alpha}}\sigma(s)ds\right),$$

which is nonnegative. The proof is finished. □

Lemma 2.6 (Comparison principle)**.** *Let* $x(t), y(t) \in C^{\alpha}([a,b],\mathbb{R})$ *satisfy*

$$\begin{cases} x^{(\alpha)}(t) \geqslant -Mx(t) + Ny(t), & t \in [a,b], \\ y^{(\alpha)}(t) \geqslant -My(t) + Nx(t), & t \in [a,b], \\ x(a) \geqslant 0, \quad y(a) \geqslant 0, \end{cases} \tag{2.4}$$

where $0 < \alpha \leqslant 1$, M, $N \in \mathbb{R}$ *with* $N \geqslant 0$, *then* $x(t) \geqslant 0,\ y(t) \geqslant 0$ *for all* $t \in [a,b]$.

Proof. Let $u(t) = x(t) + y(t)$, then (2.4) is equivalent to the following:

$$\begin{cases} u^{(\alpha)}(t) \geqslant -(M-N)u(t), & t \in [a,b], \\ u(a) \geqslant 0, \end{cases}$$

i.e.,

$$\begin{cases} u^{(\alpha)}(t) + \overline{M}u(t) \geqslant 0, & t \in [a,b], \\ u(a) \geqslant 0. \end{cases}$$

By Lemma 2.5, we know that

$$u(t) \geqslant 0,\ \forall t \in [a,b], \qquad \text{i.e.,} \quad x(t) + y(t) \geqslant 0,\ \forall t \in [a,b]. \tag{2.5}$$

By (2.5), we have

$$\begin{cases} x^{(\alpha)}(t) + (M+N)x(t) \geqslant 0, & t \in [a,b], \\ y^{(\alpha)}(t) + (M+N)y(t) \geqslant 0, & t \in [a,b], \\ x(a) \geqslant 0, \quad y(a) \geqslant 0. \end{cases}$$

By Lemma 2.5, we have $x(t) \geqslant 0,\ y(t) \geqslant 0$ for all $t \in [a,b]$. The proof is completed. □

3. Main results

Now we list the following assumptions.

(H_1) Assume that $f, g \in C([a,b]\times\mathbb{R}\times\mathbb{R},\mathbb{R})$ and there exist $x_0(t)$, $y_0(t) \in C^\alpha([a,b],\mathbb{R})$ satisfying $x_0(t) \leqslant y_0(t)$ for all $t \in [a,b]$ such that

$$\begin{cases} x_0^{(\alpha)}(t) \leqslant f(t, x_0(t), y_0(t)), & x_0(a) \leqslant x_0^*, \\ y_0^{(\alpha)}(t) \geqslant g(t, y_0(t), x_0(t)), & y_0(a) \geqslant y_0^*. \end{cases}$$

(H_2) There exist constants $M \in \mathbb{R}$ and $N \geqslant 0$ such that

$$f(t,x,y) - f(t,\overline{x},\overline{y}) \geqslant -M(x-\overline{x}) - N(y-\overline{y}), \quad g(t,\overline{y},\overline{x}) - g(t,y,x) \geqslant -M(\overline{y}-y) - N(\overline{x}-x),$$

where $x_0 \leqslant \overline{x} \leqslant x \leqslant y_0$, $x_0 \leqslant y \leqslant \overline{y} \leqslant y_0$ for all $t \in [a,b]$, and

$$g(t,y,x) - f(t,x,y) \geqslant -M(y-x) - N(x-y),$$

where $x_0 \leqslant x \leqslant y \leqslant y_0$ for all $t \in [a,b]$.

Theorem 3.1. *Assume that* (H_1) *and* (H_2) *hold. Then* (1.1) *has an extremal system of solutions* $(x^*(t), y^*(t)) \in [x_0(t), y_0(t)] \times [x_0(t), y_0(t)]$*, and there exist monotone iterative sequences* $\{x_n\}_{n=0}^\infty, \{y_n\}_{n=0}^\infty$ *converging uniformly to* x^*, y^**, respectively, where* $x_n(t), y_n(t) \in [x_0(t), y_0(t)]$*,* $\{x_n\}_{n=0}^\infty, \{y_n\}_{n=0}^\infty$ *are defined by* (3.2)*,* (3.3)*,* (3.4)*, and*

$$x_0 \leqslant x_1 \leqslant \cdots \leqslant x_n \leqslant \cdots \leqslant x^* \leqslant y^* \leqslant \cdots \leqslant y_n \leqslant \cdots \leqslant y_1 \leqslant y_0\,.$$

Proof. Firstly, for all $x_n, y_n \in C^\alpha([a,b],\mathbb{R})$, we consider the linear system:

$$\begin{cases} x_{n+1}^{(\alpha)}(t) = f(t, x_n(t), y_n(t)) + M(x_n(t) - x_{n+1}(t)) + N(y_n(t) - y_{n+1}(t)), & t \in [a,b], \\ y_{n+1}^{(\alpha)}(t) = g(t, y_n(t), x_n(t)) + M(y_n(t) - y_{n+1}(t)) + N(x_n(t) - x_{n+1}(t)), & t \in [a,b], \\ x_{n+1}(a) = x_0^*, \quad y_{n+1}(a) = y_0^*. \end{cases} \tag{3.1}$$

By Lemma 2.4, the linear system (3.1) has a unique system of solutions in $C^\alpha([a,b],\mathbb{R}) \times C^\alpha([a,b],\mathbb{R})$, which is defined by

$$x_{n+1} = \frac{p_{n+1} + q_{n+1}}{2}, \quad y_{n+1} = \frac{p_{n+1} - q_{n+1}}{2}, \tag{3.2}$$

$$\begin{aligned} p_{n+1} = e^{-\frac{M+N}{\alpha}t^\alpha} \Bigg\{ (x_0^* + y_0^*) e^{\frac{M+N}{\alpha}a^\alpha} + \int_a^t s^{\alpha-1} e^{\frac{M+N}{\alpha}s^\alpha} \Big[f\big(s, x_n(s), y_n(s)\big) \\ + g\big(s, y_n(s), x_n(s)\big) + (M+N)\big(x_n(s) + y_n(s)\big) \Big] ds \Bigg\}, \end{aligned} \tag{3.3}$$

$$\begin{aligned} q_{n+1} = e^{-\frac{M-N}{\alpha}t^\alpha} \Bigg\{ (x_0^* - y_0^*) e^{\frac{M-N}{\alpha}a^\alpha} + \int_a^t s^{\alpha-1} e^{\frac{M-N}{\alpha}s^\alpha} \Big[f\big(s, x_n(s), y_n(s)\big) \\ - g\big(s, y_n(s), x_n(s)\big) + (M-N)\big(x_n(s) - y_n(s)\big) \Big] ds \Bigg\}. \end{aligned} \tag{3.4}$$

Secondly, we shall prove that

$$x_n \leqslant x_{n+1} \leqslant y_{n+1} \leqslant y_n, \quad n = 0, 1, 2, \ldots.$$

Let $p = x_1 - x_0,\ q = y_0 - y_1$. According to (3.1) and (H_1), we have

$$\begin{cases} p^{(\alpha)} \geqslant M(x_0(t) - x_1(t)) + N(y_0(t) - y_1(t)), & p(a) \geqslant x_0^* - x_0^* = 0, \\ q^{(\alpha)} \geqslant -M(y_0(t) - y_1(t)) - N(x_0(t) - x_1(t)), & q(a) \geqslant y_0^* - y_0^* = 0, \end{cases}$$

i.e.,

$$\begin{cases} p^{(\alpha)} \geqslant -Mp + Nq, & p(a) \geqslant 0, \\ q^{(\alpha)} \geqslant -Mq + Np, & q(a) \geqslant 0. \end{cases}$$

Then, by Lemma 2.6, we have $p(t) \geqslant 0,\ q(t) \geqslant 0$, i.e., $x_1 \geqslant x_0,\ y_1 \leqslant y_0$. Let $\omega = y_1 - x_1$. According to (3.1) and (H_2), we have

$$\begin{cases} \omega^{(\alpha)} = y_1^{(\alpha)} - x_1^{(\alpha)} \\ = g(t, y_0(t), x_0(t)) + M(y_0(t) - y_1(t)) + N(x_0(t) - x_1(t)) \\ \qquad - f(t, x_0(t), y_0(t)) - M(x_0(t) - x_1(t)) - N(y_0(t) - y_1(t)) \\ \quad \geqslant -M(y_1 - x_1) + N(y_1 - x_1) = -(M - N)\omega, \\ \omega(a) = y_0^* - x_0^* \geqslant 0. \end{cases}$$

By Lemma 2.5, we have $\omega(t) \geqslant 0$, i.e., $y_1(t) \geqslant x_1(t)$ for all $t \in [a, b]$. By mathematical induction, we can prove that

$$x_n \leqslant x_{n+1} \leqslant y_{n+1} \leqslant y_n, \quad n = 0, 1, 2, \ldots.$$

Thirdly, the sequences $\{x_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$ are monotone and bounded, hence

$$\lim_{n\to\infty} x_n = x^*, \quad \lim_{n\to\infty} y_n = y^*,$$

(x^*, y^*) is an extremal system of solutions to (1.1).

Finally, we prove that (1.1) has at most one extremal system of solutions. Assume that $(x, y) \in [x_0, y_0] \times [x_0, y_0]$ is the system of solutions to (1.1), then $x_0 \leqslant x$, $y \leqslant y_0$. For some $k \in \mathbb{N}$, assume that the following relation holds

$$x_k(t) \leqslant x(t),\ y(t) \leqslant y_k(t), \quad t \in [a, b].$$

Let $u(t) = x(t) - x_{k+1}(t),\ v(t) = y_{k+1}(t) - y(t)$, we can get

$$\begin{cases} u^{(\alpha)} \geqslant -Mu + Nv, & u(a) \geqslant 0, \\ v^{(\alpha)} \geqslant -Mv + Nu, & v(a) \geqslant 0. \end{cases}$$

By Lemma 2.6, we have $u(t) \geqslant 0$, $v(t) \geqslant 0$, i.e., $x_{k+1}(t) \leqslant x(t)$, $y(t) \leqslant y_{k+1}(t)$, $t \in [a, b]$. By the induction arguments, the following relation holds

$$x_n(t) \leqslant x(t),\ y(t) \leqslant y_n(t), \quad n = 0, 1, 2, \ldots.$$

Taking the limit as $n \to \infty$, we get that $x^* \leqslant x$, $y \leqslant y^*$. Hence, $(x^*, y^*) \in [x_0, y_0] \times [x_0, y_0]$ is the extremal system of solutions to (1.1). So the proof is finished. □

Remark 3.2. Assume that f, g admit a decomposition of form $h = f + g$ when $u = x + y$ in system (1.1), let $u_0^* = x_0^* + y_0^*$, the special case is

$$\begin{cases} u^{(\alpha)}(t) = h(t, u(t)), & t \in [a, b], \\ u(a) = u_0^*, \end{cases} \tag{3.5}$$

which has been investigated in [6].

Corollary 3.3. *When assumptions* (H_1) *and* (H_2) *hold, the special case* (3.5) *has an extremal solution.*

4. An example

Consider the system of nonlinear conformable fractional differential equations:

$$\begin{cases} x^{(0.5)}(t) = \frac{1}{2}\big(2 - x(t)\big)^3 - \frac{\sqrt{t}}{1+t}y^2(t), & t \in [1,2], \\ y^{(0.5)}(t) = \frac{1}{2}\big(2 - y(t)\big)^3 - \frac{\sin^2 t}{2}x^2(t), & t \in [1,2], \\ x(1) = 0, \quad y(1) = 0. \end{cases} \tag{4.1}$$

Choose $x_0(t) = 0$, $y_0(t) = t$, then assumption (H_1) holds. The assumption (H_2) holds when $M = 6$, $N = 0$. By Theorem 3.1, the system (4.1) has an extremal iterative solutions (x^*, y^*) and there exist monotone iterative sequences $\{x_n\}_{n=0}^{\infty}, \{y_n\}_{n=0}^{\infty}$ converging uniformly to x^*, y^*, respectively, moreover

$$x_{n+1}(t) = e^{-12t^{0.5}} \int_1^t s^{-0.5} e^{12s^{0.5}} \Big[\frac{1}{2}\big(2 - x_n(s)\big)^3 - \frac{\sqrt{t}}{1+t}y_n^2(s) + 6x_n(s)\Big] ds,$$
$$y_{n+1}(t) = e^{-12t^{0.5}} \int_1^t s^{-0.5} e^{12s^{0.5}} \Big[\frac{1}{2}\big(2 - y_n(s)\big)^3 - \frac{\sin^2 t}{2}x_n^2(s) + 6y_n(s)\Big] ds.$$

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