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Passivity based on synchronization of T-S fuzzy energy resources system

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Abstract

This paper investigates the use of Takagi-Sugeno fuzzy and passive control techniques to synchronize a four-dimensional energy resource system. A passivity-based fuzzy controller is designed to synchronize the two identical four-dimensional energy resource systems using an effective Lyapunov function and linear matrix inequality approach. Finally, computational and simulation results are presented to show the benefits of the proposed findings.

Keywords: Synchronization, energy resource system, passivity, T-S fuzzy control, linear matrix inequality.

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1. Introduction

When the evolution of a deterministic system is responsive to the initial conditions, it is said to be chaotic. This property states that two trajectories originating from two different nearby initial conditions can separate exponentially over time. In order for a deterministic system to be chaotic, it must be nonlinear and at least three dimensional. Synchronization is the process of using the output of the driving system to monitor the response system so that both systems oscillate at the same time. Readers should refer to [13, 30] for more information. Nowadays, it is a well known fact that chaotic dynamics exist in a large variety of nature systems (e.g., aerodynamics, biological and physical systems) and chaos synchronization has attracted many engineers and scientists of different fields due to its wide variety of applications in physics, engineering including biological systems, chemical reactions, human heart beat regulation, ecological systems, fractional-order systems, secure communication, information processing and so on, for more details see [1, 4, 18, 25, 29, 52, 53]. Many synchronization systems have been devised to date. Several control and synchronization systems, including linear and nonlinear state feedback approaches, have recently been investigated in the literature [2, 9, 26, 27, 51, 54].

Energy is now essential for human life, economic growth, and social change. Furthermore, energy resources are not the same as forms of energy; they refer to methods of obtaining energy in order to

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produce electrical power. Energy resources can be classified as renewable energy and non-renewable energy according to the capability of sustainable utilization. Renewable energy sources include solar, wind, hydropower, geothermal, biomass energy, and so on, while non-renewable energy sources include coal, petroleum, lignite, nuclear power, and so on. Energy resource systems are a type of complex nonlinear system that has a wide range of applications in science and engineering. Energy resources demand-supply has become a hot topic for economic growth around the world, and scientists and researchers have discussed it from different perspectives, see [16, 33–41, 47, 48] and references therein.

Sun et al. [35] developed a three-dimensional energy resource demand-supply structure without taking renewable energy resources into account. Sun et al. have also created a four-dimensional energy resource system by introducing a new variable to the three-dimensional energy resource demand-supply system, as stated in [34]. As a result of changing China's energy resource utilisation strategy, the real energy resources demand-supply mechanism will either be in a state of steady growth or in the context of periodic vibrations, see [34, 39]. Authors in [37] have considered the Hopf Bifurcation analysis of the energy resource chaotic system using analytic methods. [33, 36] have experimented with synchronization for the four-dimensional energy resources system using adaptive control techniques. In [16, 36, 38], the robust synchronization issue for a four-dimensional energy resource system with mismatched uncertainties and parameters was considered. Furthermore, authors in [41, 47, 48] investigated the synchronization of energy resource systems using linear feedback control techniques. A systemic method for synchronising the nonlinear energy resources system has been proposed in [40] using the fuzzy interpolation method.

The passive control technique has been widely used in nonlinear control systems [8, 15] in recent years. Passivity theory has its roots in circuit theory and is used extensively in electrical networks and control systems [11]. Since the 1970s, it has taken a big hit from the control community. Passivity is one of the most important aspects of dissipation; the energy supply rate is defined as the product of input and output, and passivity represents the attenuation characteristic of the systems under bounded input conditions. In fact, passivity is a more advanced abstract of stability because it can lead to general conclusions about stability using only input-output characteristics [24]. As a result of this, many authors have studied the chaos synchronization passivity theory. For example, the chaos synchronization of the Rikitake system, hyperchaotic Lü system, and delayed neural networks have been studied using the passive control principle in [2], [51], and [54], respectively. In [49], a linear feedback controller has been designed to use passive control to stabilise the Lorenz system to any desired equilibrium points. In [3], stability results such as asymptotic stability, input-to-state stability, and bounded input-bounded output stability based on the passive learning law have been presented for switched delayed Hopfield neural networks.

Fuzzy control, on the other hand, has recently been demonstrated to be a powerful method for the control problem of complex nonlinear systems. Many fuzzy controllers, in general, can be expressed as nonlinear controllers with a bounded continuous input-output mapping and some symmetric properties [8]. The concept of fuzzy system theory allows us to build a mathematical model for a system using qualitative, linguistic information. For many real-world systems that are highly complex and inherently nonlinear, conventional modelling approaches are frequently inapplicable, whereas the fuzzy approach may be the only viable option. The control technique based on the so-called Takagi-Sugeno (T-S) fuzzy model, in particular, has received a lot of attention. The T-S fuzzy model's main purpose is to represent or approximate a complex nonlinear system. This fuzzy model is described by a set of fuzzy IF-THEN rules that correspond to the system's local linear input-output relationships. Through the membership functions, the overall fuzzy model of the system is blended into these local linear models. Then, based on this fuzzy model, a control design is created to achieve the system's stability and performance. The T-S fuzzy model approach will provide a powerful method for nonlinear system analysis [10, 12, 23, 44]. Following that, this fuzzy control was successfully applied in many areas, including truck-trailer control [43], spark ignition engine [19], servo control design [50], industries [31, 32], medicine [5], speed wind turbine [6], wind energy conservation [17], vehicle cruise control system [28], and so on. Furthermore, chaos synchronization using T-S fuzzy control approaches has been widely discussed in a variety of applications [20–22, 46].

Many fuzzy controllers can be thought of as passive dynamic nonlinear controllers with a single input and a single output. It is demonstrated in [8] that passivity is a result of some characteristics of the fuzzy controller's input-output mapping. Furthermore, the passivity theory deals with control systems whose controller characteristics can be poorly defined, and it produces more appropriate solutions for the existence of such systems' stability. This demonstrates the importance of the passivity theory framework for the stability analysis of control systems based on fuzzy logics or neural networks [8]. Exact linearization, in particular, will play an important role in synchronization and chaotic models via fuzzy controllers [44]. Furthermore, for more general nonlinear systems, formulating the passivity-based synchronization conditions in terms of linear matrix inequalities may be difficult (LMIs). The energy resources system, which is a type of complex nonlinear system, is converted into linear sub models in this paper using the T-S fuzzy control technique. Besides that, Lyapunov stability theory has been used to ensure the synchronization of this energy resources system, and the corresponding sufficient conditions are expressed in terms of LMIs, which can be efficiently solved by resorting to some standard algorithms [7]. Consequently, fuzzy modelling techniques have been used to model four-dimensional chaotic energy resource systems in such a way that the fuzzy energy resource systems can be chaotic. Unlike previous research, this paper focuses on the passive synchronization of energy resources systems using fuzzy state-feedback controllers based on LMI techniques. Finally, numerical results and analysis are presented to demonstrate the efficacy of the derived results.

The detailed layout is as follows. The four-dimensional energy resource system and its corresponding parameters are described in Section 2. The fundamental concepts of the passivity theory are presented in Section 3. The fuzzy modelling theory for four-dimensional energy resources systems is introduced in Section 4. Section 5 presents a passivity-based T-S fuzzy synchronization scheme for energy resource systems, and Section 6 presents numerical simulations and analysis to validate the results. Section 7 comes to some conclusions.

2. Description of energy resource system

In this paper, we look at Sun et al. [34] four-dimensional energy resource system. It is a component of the energy system because it is used to describe actual energy demand-supply and is instructive for energy resource demand-supply in some regions. It consists of four ordinary differential equations that are dependent on some positive real parameters. The dynamics of this system can be described by the nonlinear ordinary differential equations listed below.

$$\begin{cases} \dot{x}_1 = a_1x_1(1-\frac{x_1}{M}) - a_2(y_1+z_1) - d_3w_1, \\ \dot{y}_1 = -b_1y_1 - b_2z_1 + b_3x_1[N-(x_1-z_1)], \\ \dot{z}_1 = c_1z_1(c_2x_1 - c_3), \\ \dot{w}_1 = d_1x_1 - d_2w_1, \end{cases}$$

while $x_1(t)$ is the energy resource shortage in region R_1; $y_1(t)$ is the energy resource supply increment in region R_2; $z_1(t)$ is the energy resource import in region R_1; $w_1(t)$ is renewable energy resources in region R_1. $M, N, a_i, b_j, c_j, d_j, (i = 1, 2; j = 1, 2, 3)$ are all positive real parameters. This system has three equilibria: $O(0, 0, 0, 0)$, $S_1(1.75, -1.52, 0, 2.91)$, $S_2(0.8, 0.669, -1.11, 1.33)$. When $a_1 = 0.09$, $a_2 = 0.15$, $b_1 = 0.06$, $b_2 = 0.083$, $b_3 = 0.07$, $c_1 = 0.2$, $c_2 = 0.5$, $c_3 = 0.4$, $d_1 = 0.1$, $d_2 = 0.06$, $d_3 = 0.08$, $M = 1.8$, $N = 1.0$, a chaotic attractor is observed as shown in Figure 1 (a)-(c), the time series of $(x_1(t), y_1(t), z_1(t), w_1(t))$ as shown in Figure 1 (d), with $O(0, 0, 0, 0)$ as an unstable saddle focus and $[0.82, 0.29, 0.48, 0.1]$ as the initial conditions. Moreover S_1 and S_2 are two saddle points. Choosing $a_3 = \frac{a_1}{M}$, $b_4 = b_3N$, $r_1 = c_1c_2$, and $r_2 = c_1c_3$, the above system can be rewritten as

$$\begin{cases} \dot{x}_1 = a_1x_1 - a_2(y_1 + z_1) - a_3x_1^2 - d_3w_1, \\ \dot{y}_1 = -b_1y_1 - b_2z_1 - b_3x_1(x_1 - z_1) + b_4x_1, \\ \dot{z}_1 = r_1x_1z_1 - r_2z_1, \\ \dot{w}_1 = d_1x_1 - d_2w_1. \end{cases} \tag{2.1}$$

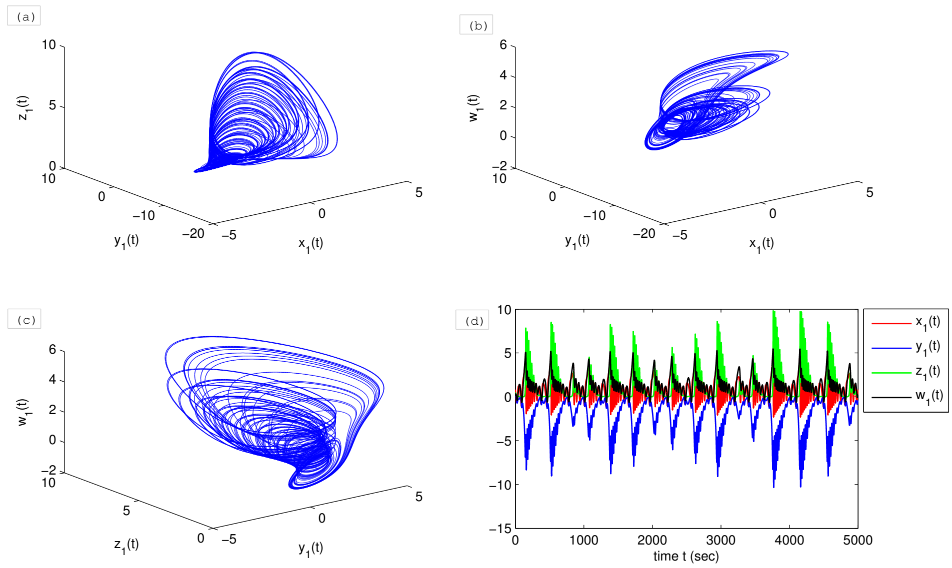

Figure 1: A four-dimensional energy resources chaotic attractor: (a) Chaotic attractor in $(x_1 - y_1 - z_1)$-space; (b) Chaotic attractor in $(x_1 - y_1 - w_1)$-plane; (c) Chaotic attractor in $(x_1 - z_1 - w_1)$-plane; (d) Time series of (x_1, y_1, z_1, w_1)-plane.

Further, the four-dimensional energy resource chaotic system (2.1) will be suppressed to its unique unstable equilibrium $O(0, 0, 0, 0)$ by applying the passivity theory. For this purpose, we consider the following controlled system

$$\begin{cases} \dot{x}_1 = a_1x_1 - a_2(y_1 + z_1) - a_3x_1^2 - d_3w_1 + u_1, \\ \dot{y}_1 = -b_1y_1 - b_2z_1 - b_3x_1(x_1 - z_1) + b_4x_1 + u_2, \\ \dot{z}_1 = r_1x_1z_1 - r_2z_1 + u_3, \\ \dot{w}_1 = d_1x_1 - d_2w_1 + u_4. \end{cases} \tag{2.2}$$

where $u_k,\ k = 1, 2, 3, 4$ are the controllers to be designed.

3. Passivity control technique for general nonlinear system

Consider the following differential equations:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \quad y(t) = h(x(t)), \tag{3.1}$$

where $x(t) \in \mathcal{R}^n$ is the state variable, $u(t) \in \mathcal{R}^m$ is the external input, and $y(t) \in \mathcal{R}^m$ is the output. f and g are smooth vector fields and h is a smooth mapping. Without loss of generality, we suppose that the vector field f has at least one equilibrium point. The notion of passivity can be described by the following definition.

Definition 3.1 ([2]). If there exists a nonnegative constant β and a positive semi-definite function $V(x(t))$ such that

$$\int_0^t u^T(s)y(s)ds + \beta \geqslant \int_0^t V(x(s))ds, \ \forall t \geqslant 0, \tag{3.2}$$

then the systems (3.1) is said to be passive from the external input $u(t)$ to the output $y(t)$.

Remark 3.2. The concept of a passive system can be interpreted as the fact that the energy of nonlinear systems (3.1) can only be increased by supplying it from an external source. A passive system, in general, cannot store more energy than it receives. Passive systems are a subset of dissipative systems. The energy dissipated within a dynamic system is less than the energy supplied from an external source in dissipative systems. Stability issues are frequently linked to the theory of dissipative systems in many engineering problems. Passive systems are inherently stable. To analyse stability properties, a passive system uses the input-output relationship based on energy-related considerations. The main idea of passivity theory is that the passive properties of system can keep the system internally stable.

4. Fuzzy modelling of energy resource system

It is not easy to find an appropriate Lyapunov function V for the general nonlinear energy resource system (2.2) that meets the passivity condition (3.2). T-S fuzzy control logic, on the other hand, has been proposed as a potential tool for approximating nonlinear energy resource systems, resulting in an easy way to apply Lyapunov stability theory in terms of LMIs. The system (2.2) can be conventionally written in state-space matrix form for this purpose as

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{4.1}$$

where $\dot{x}(t) = [\dot{x}_1, \dot{y}_1, \dot{z}_1, \dot{w}_1]^T$; $A = \begin{bmatrix} a_1 - a_3x_1 & -a_2 & -a_2 & -d_3 \\ -b_3x_1 + b_4 & -b_1 & -b_2 + b_3x_1 & 0 \\ 0 & 0 & r_1x_1 - r_2 & 0 \\ d_1 & 0 & 0 & -d_2 \end{bmatrix}$, $x = [x_1, y_1, z_1, w_1]^T$.

Let the premise variable $\gamma_1(t) = x_1(t)$, then the membership functions can be chosen as $M_1^1(\gamma_1(t)) = \frac{1}{2}(1 + x_1(t))$, $M_1^2(\gamma_1(t)) = \frac{1}{2}(1 - x_1(t))$. By using M_1^1 and M_1^2, the controlled energy resource system (4.1) with control input $u(t)$ can be expressed by the following T-S fuzzy model:

Plant Rule 1:
IF $\gamma_1(t)$ **is** M_1^1 **THEN** $\dot{x}(t) = A_1x + Bu$,
Plant Rule 2:
IF $\gamma_1(t)$ **is** M_1^2 **THEN** $\dot{x}(t) = A_2x + Bu$,
where $x = [x_1, y_1, z_1, w_1]^T$ and

$$A_1 = \begin{bmatrix} a_1 - a_3 & -a_2 & -a_2 & -d_3 \\ -b_3 + b_4 & -b_1 & -b_2 + b_3 & 0 \\ 0 & 0 & r_1 - r_2 & 0 \\ d_1 & 0 & 0 & -d_2 \end{bmatrix}, \ A_2 = \begin{bmatrix} a_1 + a_3 & -a_2 & -a_2 & -d_3 \\ b_3 + b_4 & -b_1 & -b_2 - b_3 & 0 \\ 0 & 0 & -r_1 - r_2 & 0 \\ d_1 & 0 & 0 & -d_2 \end{bmatrix}, \ B = [1 \quad 1 \quad 1 \quad 1]^T.$$

Hence the nonlinear energy resource system (4.1) can be described by a T-S fuzzy system:

$$\dot{x}(t) = A_ix(t) + Bu(t), \tag{4.2}$$

where $i = 1, 2$. By using a center average defuzzifier, the dynamical model of the T-S fuzzy system (4.2) can be reformulated as

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\gamma(t))\Big\{A_ix(t) + Bu(t)\Big\}, \tag{4.3}$$

with $h_i(\gamma(t)) = H_i(\gamma(t))/\sum_{i=1}^{2} H_i(\gamma(t))$, where $\gamma = [\gamma_1, \gamma_2]$, $H_i : \mathcal{R}^2 \to [0, 1]$, is the membership degree of the consequent of each system rule with respect to plant rule i. The normalized fuzzy weighting functions $h_i(\gamma(t))$ satisfy $h_i(\gamma(t)) \geqslant 0$, $\sum_{i=1}^{2} h_i(\gamma(t)) = 1$.

5. Passivity based synchronization of energy resource systems

In this section, the LMI problem for achieving the passivity based synchronization for two identical energy resource system is presented. By using drive-response concept, the system (4.3) of T-S fuzzy energy resource system is given as

$$\dot{x}(t)=\sum_{i=1}^{2}h_i(\gamma(t))\Big\{A_ix(t)\Big\}. \tag{5.1}$$

Then, the controlled response system is given by

$$\dot{\hat{x}}(t)=\sum_{i=1}^{2}h_i(\gamma(t))\Big\{A_i\hat{x}(t)+Bu(t)\Big\}, \tag{5.2}$$

where $\hat{x}(t)\in\mathcal{R}^4$ is the state vector, and $B\in\mathcal{R}^{4\times1}$ is the known constant matrix.

Define the synchronization error $e(t)=\hat{x}(t)-x(t)$. Then we obtain the synchronization error system as follows,

$$\dot{e}(t)=\sum_{i=1}^{2}h_i(\gamma(t))\Big\{A_ie(t)+Bu(t)\Big\}. \tag{5.3}$$

In order to verify the passive synchronization of the error system (5.3), the following Lemma and Theorem are very useful.

Lemma 5.1 ([45])**.** *If the following conditions hold:*

$$N_{ii}<0,\ 1\leqslant i\leqslant r,\qquad \frac{1}{r-1}N_{ii}+\frac{1}{2}(N_{ij}+N_{ji})<0,\ 1\leqslant i\neq j\leqslant r,$$

then the following inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_jN_{ij}<0,$$

where $h_i,\ 1\leqslant i\leqslant r$, *satisfy* $0\leqslant h_i\leqslant 1,\ \sum_{i=1}^{r}h_i=1$; *and* r *denotes the number of IF-THEN rules.*

Theorem 5.2. *If there exist matrices* $X=X^T>0,\ Z=Z^T>0$ *and* Y_j *such that*

$$\begin{aligned}\begin{bmatrix}\Theta_{ii} & X\\ * & -Z\end{bmatrix}&<0,\ 1\leqslant i\leqslant r,\\ \frac{1}{r-1}\begin{bmatrix}\Theta_{ii} & X\\ * & -Z\end{bmatrix}+\frac{1}{2}\begin{bmatrix}\Theta_{ij} & X\\ * & -Z\end{bmatrix}+\frac{1}{2}\begin{bmatrix}\Theta_{ji} & X\\ * & -Z\end{bmatrix}&<0,\ 1\leqslant\ i\neq j\leqslant r,\end{aligned} \tag{5.4}$$

where $\Theta_{ij}=A_iX+XA_i^T+BY_j+Y_j^TB^T$, *then the synchronization error system* (5.3), *under the control input* $u(t)=Y_jX^{-1}e(t)+\mu(t)$, *where* $\mu(t)$ *is an external input signal, is passive from the external input signal* $\mu(t)$ *to the output* $y(t)$ *which is defined as:* $y(t):=2B^TPe(t)$.

Proof. The closed-loop error system with the control input $u(t)=\sum_{j=1}^{2}h_j(\gamma(t))K_je(t)+\mu(t)$, where $K_j\in\mathcal{R}^{1\times4}$ is the gain matrix of the control input $u(t)$, can be written as

$$\dot{e}(t)=\sum_{i=1}^{2}\sum_{j=1}^{2}h_i(\gamma(t))h_j(\gamma(t))\Big\{[A_i+BK_j]e(t)+B\mu(t)\Big\}. \tag{5.5}$$

Consider the following Lyapunov function

$$V(e(t)) = e^T(t)Pe(t), \ (P = P^T > 0).$$

The time derivative of $V(e(t))$ along the trajectory of (5.5) is

$$\begin{aligned}
\dot{V}(e(t)) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t),\\
&= \Big\{\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\gamma(t))h_j(\gamma(t))[A_i e(t) + BK_j e(t) + B\mu(t)]\Big\}^T Pe(t)\\
&\quad + e^T(t)P\Big\{\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\gamma(t))h_j(\gamma(t))[A_i e(t) + BK_j e(t) + B\mu(t)]\Big\},\\
&= \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\gamma(t))h_j(\gamma(t))\Big\{e^T(t)A_i^T Pe(t) + e^T(t)K_j^T B^T Pe(t) + \mu^T(t)B^T Pe(t)\\
&\quad + e^T(t)PA_i e(t) + e^T(t)PBK_j e(t) + e^T(t)PB\mu(t)\Big\},\\
&= \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\gamma(t))h_j(\gamma(t))e^T(t)[PA_i + A_i^T P + PBK_j + K_j^T B^T P]e(t) + y^T(t)\mu(t),\\
&= e^T(t)\Big\{\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\gamma(t))h_j(\gamma(t))[PA_i + A_i^T P + PBK_j + K_j^T B^T P + Q]\Big\}e(t)\\
&\quad - e^T(t)Qe(t) + y^T(t)\mu(t).
\end{aligned}$$

If the following matrix inequality is satisfied,

$$\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\gamma(t))h_j(\gamma(t))\Big\{PA_i + A_i^T P + PBK_j + K_j^T B^T P + Q\Big\} < 0, \tag{5.6}$$

then we have

$$\dot{V}(e(t)) < -e^T(t)Qe(t) + y^T(t)\mu(t). \tag{5.7}$$

Integrating (5.7) both sides from 0 to t, we get

$$\begin{aligned}
\int_0^t \dot{V}(e(s))ds &< -\int_0^t e^T(s)Qe(s)ds + \int_0^t y^T(s)\mu(s)ds,\\
[V(e(t))]_0^t &< -\int_0^t e^T(s)Qe(s)ds + \int_0^t y^T(s)\mu(s)ds,\\
V(e(t)) - V(e(0)) &< -\int_0^t e^T(s)Qe(s)ds + \int_0^t y^T(s)\mu(s)ds.
\end{aligned}$$

Let $\beta = V(e(0))$. Since $V(e(t)) \geqslant 0$,

$$\int_0^t y^T(s)\mu(s)ds + \beta > \int_0^t e^T(s)Qe(s)ds + V(e(t)) \geqslant \int_0^t e^T(s)Qe(s)ds.$$

This implies that the system (5.5) is passive from the external input signal $\mu(t)$ to the output $y(t)$ under the feedback control input $u(t) = \sum_{j=1}^{2} h_j(\gamma(t))K_j e(t) + \mu(t)$ in the sense of Definition 3.1.

By using Lemma 5.1 and Schur complement lemma [14], matrix inequality (5.6) is equivalent to

$$\begin{aligned}
&\begin{bmatrix} \Omega_{ii} & I \\ * & -Q^{-1} \end{bmatrix} < 0, \ 1 \leqslant i \leqslant r,\\
\frac{1}{r-1}\begin{bmatrix} \Omega_{ii} & I \\ * & -Q^{-1} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \Omega_{ij} & I \\ * & -Q^{-1} \end{bmatrix} + \frac{1}{2}&\begin{bmatrix} \Omega_{ji} & I \\ * & -Q^{-1} \end{bmatrix} < 0, \ 1 \leqslant \ i \neq j \leqslant r,
\end{aligned} \tag{5.8}$$

where $\Omega_{ij} = PA_i + A_i^T P + PBK_j + K_j^T B^T P$. Pre- and Post- multiplying by diag$\{P^{-1}, I\}$ and introducingchange of variables $X = P^{-1}$, $Z = Q^{-1}$, $Y_j = K_j P^{-1}$, LMI (5.8) are equivalent to (5.4). Then, the gain matrix of the control input $u(t)$ is given by $K_j = Y_j X^{-1}$. This completes the proof. □

Remark 5.3. When the external input $\mu(t)$ is zero, the passive control law $u(t)$ makes the synchronization error system (5.5) asymptotically stable, a phenomenon known as **zero-input state response**. In this case, the synchronization error system (5.5) considered in this paper is reduced to the system discussed in [40], so our results are finer than those previously reported. If the external input $\mu(t)$ is nonzero, that is, $\mu(t) = -\nu y(t)$, where $\nu > 0$ is the gain parameter, the passive control law $u(t)$ also makes the synchronization error system asymptotically stable, which is known as **nonzero-input state response**. The simulation results in Figures 7 and 14 shows synchronization error trajectories for different values of the gain parameter ν in the external input signal $\mu(t)$.

Remark 5.4. Sun et al. [40], investigated the robust stabilisation and synchronization problems for non-linear energy resource systems using the T-S fuzzy model. Unlike [40], different parameterized LMI characterizations for fuzzy control systems subject to passivity techniques are proposed in this paper. These parametrized LMI characterizations are then relaxed into pure LMI programmes, making the LMI formulation more feasible [45]. Furthermore, it offers tractable and effective methods for designing sub-optimal fuzzy control systems. Further in [40], numerical results show that the receiver has full access to all drive system states (B is full rank), transforming the synchronization problem into a stabilisation problem with only the difference between initial conditions to overcome. In contrast to [40], we considered synchronization with less access to the drive system in this paper, which means B may not have full rank (refer Section 6).

Remark 5.5. We presented the T-S fuzzy model representation for the chaotic four-dimensional energy resources system subject to passive control techniques in this paper. Furthermore, linear state feedback control methods are used in synchronization control schemes. Sliding-mode control has now become an important component of control theory. Sliding-mode control is highly resistant to parameter uncertainties and external noise disturbances in the controlled system, and it has been used successfully to control chaos [9]. In the future, we will use the novel sliding-mode control to design a sliding mode controller, along with fuzzy modelling techniques, to improve the control performance of a four-dimensional energy resources system. In addition, we will take into account the uncertain model of the four-dimensional energy resources system and design a robust sliding mode controller. These works will be released in the near future.

6. Numerical results and analysis

In this section, numerical simulations are given to check the validity of the obtained theoretical results. The parameters of the energy resource system are selected as in Section 2. The initial conditions of the drive and response system are chosen to be $(x_1(0), y_1(0), z_1(0), w_1(0)) = (0.82, 0.29, 0.48, 0.1)$ and $(x_2(0), y_2(0), z_2(0), w_2(0)) = (0.78, 0.35, 0.4, 0.15)$, respectively. In the numerical simulations, the Euler method is employed to solve the systems of differential equations with step size 0.1.

Example 6.1 (Four linear feedback controllers)**.** In this example, we have considered, four linear fuzzy state controllers in order to synchronize the two identical energy resource systems (5.1) and (5.2). From Theorem 5.2, by using the MATLAB LMI Solver, the following feasible solutions are obtained.

$$X = \begin{bmatrix} 1.4848 & 1.1844 & 1.1975 & 1.0614 \\ 1.1844 & 1.2511 & 0.8968 & 0.6279 \\ 1.1975 & 0.8968 & 1.0517 & 0.7839 \\ 1.0614 & 0.6279 & 0.7839 & 1.7221 \end{bmatrix}, \qquad Z = \begin{bmatrix} 8.3988 & 0.7908 & 2.1682 & 0.9181 \\ 0.7908 & 10.0482 & -0.6344 & -1.0331 \\ 2.1682 & -0.6344 & 9.3479 & -0.0926 \\ 0.9181 & -1.0331 & -0.0926 & 13.4170 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 12.4915 & -4.4779 & -9.6064 & -2.2529 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 5.6990 & -2.5565 & -4.0171 & -1.3138 \end{bmatrix}.$$

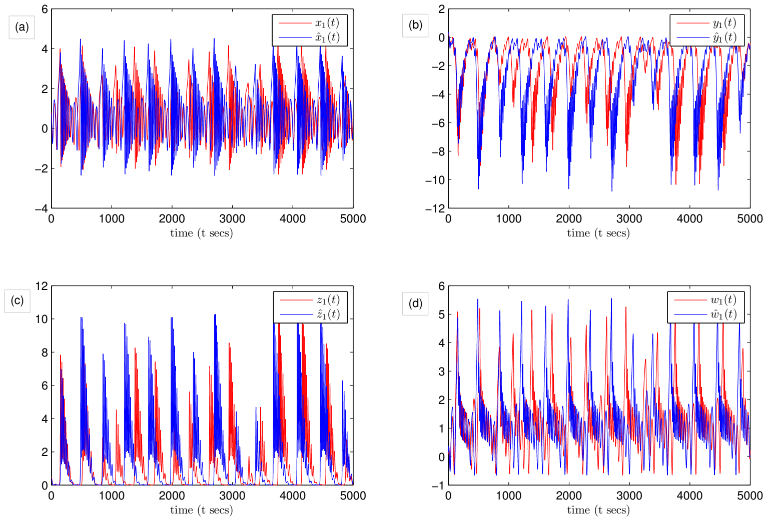

Figure 2: No synchronization between drive and response systems when control input $u(t) = 0$ in Example 6.1.

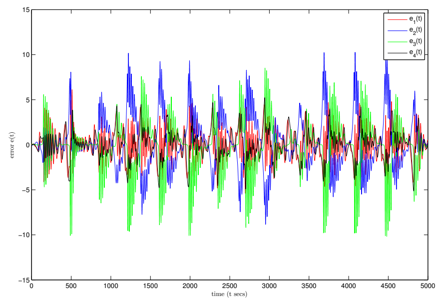

Figure 3: Synchronization errors without control input $u(t)$ in Example 6.1.

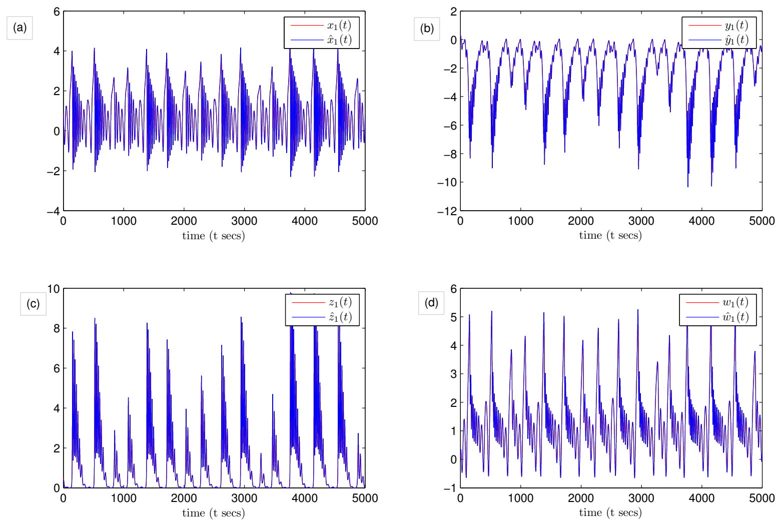

Figure 4: Drive and response systems are synchronized when control input $u(t)$ is presented in Example 6.1.

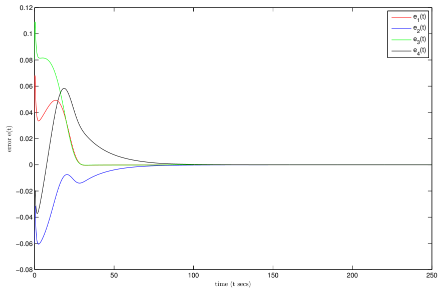

Figure 5: Synchronization errors with control input $u(t)$ in Example 6.1.

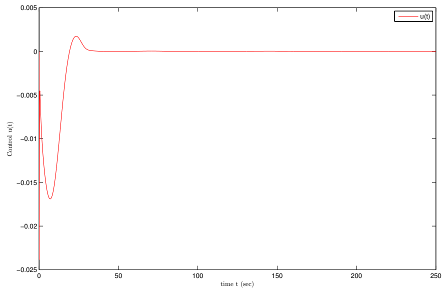

Figure 6: The graph of the control actions that are being applied to the slave system in Example 6.1.

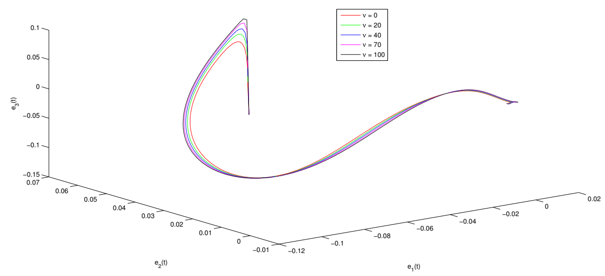

Figure 7: Synchronization error trajectories (ν: the gain parameter of the external input signal) in Example 6.1.

Figure 2 displays that the time evolution curves of drive system and response system without the control input $u(t)$. Synchronization error of the system (5.3) without the passive fuzzy control law $u(t)$ is shown in Figure 3. Figures 2 and 3 show that in the absence of control $u(t)$, no synchronization occurs between drive and response systems. On the other hand, when the control $u(t)$ is taken into account, Figure 4 displays that the time evolution curves of drive system and response system. By taking passive fuzzy control law as $u(t) = \sum_{j=1}^{2} h_j(\gamma(t))K_j e(t) + \mu(t)$, synchronization errors of the system (5.3) are shown in Figure 5. The graphs of the control actions that are being applied to the slave system is shown in Figure 6. The simulation results imply that the two identical energy resource systems (5.1) and (5.2) are synchronized with each other when the control is applied, and validate the effectiveness of the proposed method. The simulation result in Figure 7 shows synchronization error trajectories for different values of the gain parameter ν in the external input signal $\mu(t)$.

Example 6.2 (Three linear feedback controller)**.** If the the energy resource shortage, renewable energy resources in region R_1, and the energy resource supply increment in region R_2 to R_1 should be controlled, then in this case the matrix B in the response system (5.2) takes the form $B = [1 \quad 1 \quad 1 \quad 0]^T$. In this situation, the synchronization of the two identical energy systems can be obtained via three simple linear fuzzy state feedback controller. The initial values of the two systems and the values of parameters are same as in Example 6.1. From Theorem 5.2, by using the MATLAB LMI Solver, the following feasible solutions are obtained.

$$X = \begin{bmatrix} 126.0262 & 110.4277 & 108.5965 & -5.3546 \\ 110.4277 & 116.2437 & 89.0137 & -26.4472 \\ 108.5965 & 89.0137 & 103.3678 & -9.3799 \\ -5.3546 & -26.4472 & -9.3799 & 58.7662 \end{bmatrix},$$

$$Z = \begin{bmatrix} 654.4252 & 132.1603 & 157.9680 & 14.9148 \\ 132.1603 & 794.0547 & -33.5719 & -129.4233 \\ 157.9680 & -33.5719 & 750.4484 & -36.8554 \\ 14.9148 & -129.4233 & -36.8554 & 882.4416 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 21.3141 & -11.8540 & -13.5918 & -5.7060 \end{bmatrix}, \ Y_2 = \begin{bmatrix} 7.5387 & -4.9820 & -4.5362 & -2.4330 \end{bmatrix}.$$

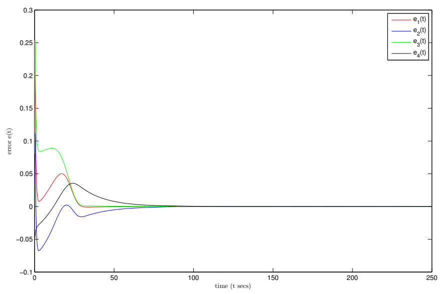

Figure 8: Synchronization errors with control input $u(t)$ in Example 6.2.

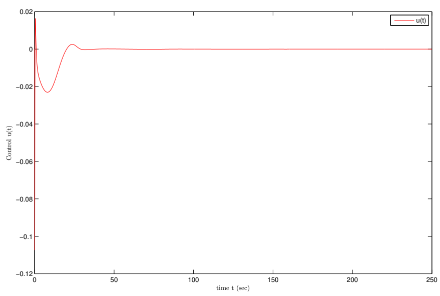

Figure 9: The graph of the control actions that are being applied to the slave system in Example 6.2.

Synchronization errors of the system (5.3) via the passive fuzzy control law $u(t)$ are shown in Figure 8. The graphs of the control actions that are being applied to the slave system is shown in Figure 9. From Figure 8, it is clear that the synchronization errors converge asymptotically to zero and two different systems are indeed achieved with synchronization.

Example 6.3 (Two linear feedback controller)**.** If the energy resource supply increment in region R_2 to R_1 and the energy resource import in R_1 of the response system is controlled, then B in the response system (5.2) takes the form $B = [0 \quad 1 \quad 1 \quad 0]^T$. In such case, two identical energy resource systems can be synchronized via two simple linear fuzzy state feedback controller. The initial values of the two systems and the values of parameters are same as in Example 6.1. From Theorem 5.2, by using the MATLAB LMI Solver, the following feasible solutions are obtained.

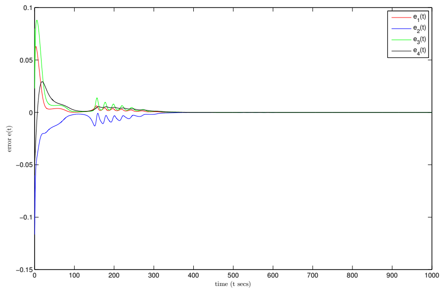

Figure 10: Synchronization errors with control input $u(t)$ in Example 6.3.

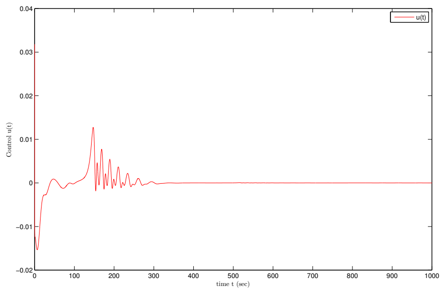

Figure 11: The graph of the control actions that are being applied to the slave system in Example 6.3.

$$X = \begin{bmatrix} 3.3183 & 5.5579 & 5.6944 & -1.0746 \\ 5.5579 & 19.9207 & 15.7135 & -2.4852 \\ 5.6944 & 15.7135 & 16.1621 & 0.0928 \\ -1.0746 & -2.4852 & 0.0928 & 6.0043 \end{bmatrix}, \quad Z = \begin{bmatrix} 57.5094 & -0.6462 & 17.0236 & -2.0583 \\ -0.6462 & 89.8797 & -1.4139 & -24.7411 \\ 17.0236 & -1.4139 & 77.2367 & 7.3368 \\ -2.0583 & -24.7411 & 7.3368 & 85.6005 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 7.2240 & 1.0696 & -4.4862 & 1.7483 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 6.1084 & 0.4465 & -3.2855 & 1.2870 \end{bmatrix}.$$

Figure 10 indicates that synchronization errors of the system (5.3) via the passive fuzzy control law $u(t)$ is converges asymptotically to zero. In Figure 11, the trajectory of the control input $u(t)$ being applied to the slave system is depicted.

Example 6.4 (Single linear feedback controller)**.** If the energy resource import variable in region R_1 of the response system should be controlled, then B in the response system (5.2) takes the form $B = [0 \quad 0 \quad 1 \quad 0]^T$. By applying a single fuzzy state feedback controller, the synchronization of the following two identical energy resource systems are obtained. The initial values of the two systems and the values of parameters are same as in Example 6.1. From Theorem 5.2, by using the MATLAB LMI Solver, the following feasible solutions are obtained.

$$X = \begin{bmatrix} 23.7863 & 4.1694 & 32.0932 & -2.0039 \\ 4.1694 & 15.6965 & 10.2929 & -28.5421 \\ 32.0932 & 10.2929 & 193.1906 & -0.3998 \\ -2.0039 & -28.5421 & -0.3998 & 69.5972 \end{bmatrix}, \quad Z = \begin{bmatrix} 636.0044 & -30.9036 & 223.4416 & 45.8446 \\ -30.9036 & 534.3771 & 1.5410 & -213.2350 \\ 223.4416 & 1.5410 & 754.7407 & 68.2632 \\ 45.8446 & -213.2350 & 68.2632 & 938.3907 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 3.0090 & 1.6059 & -1.7402 & 0.7081 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 2.2101 & 8.6092 & -1.7836 & 3.5573 \end{bmatrix}.$$

Synchronization errors of the system (5.3) via the passive fuzzy control law $u(t)$ converges asymptotically to zero as shown in Figure 12. The trajectory of the control input $u(t)$ being applied to the slave system is shown in Figure 13. The simulation result in Figure 14 shows synchronization error in phase trajectories for different values of the gain parameter ν in the external input signal $\mu(t)$.

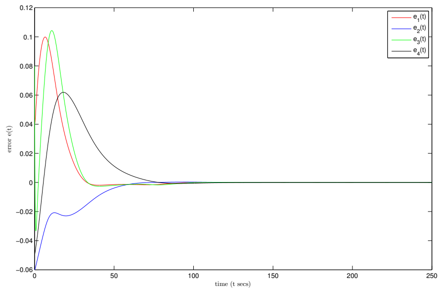

Figure 12: Synchronization errors with control input $u(t)$ in Example 6.4.

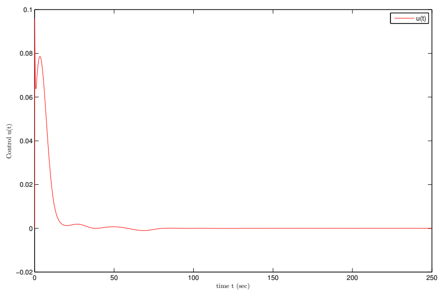

Figure 13: The graph of the control actions that are being applied to the slave system in Example 6.4.

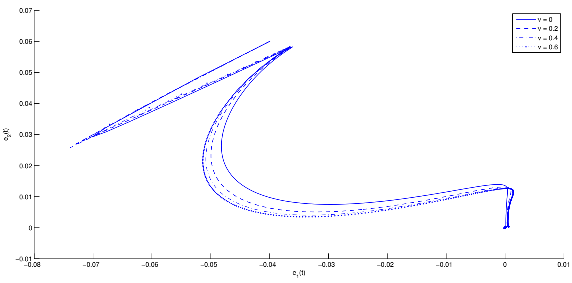

Figure 14: Synchronization error trajectories (ν: the gain parameter of the external input signal) in Example 6.4.

Remark 6.5. When there is no control input $u(t)$ in the two identical energy resource chaotic system, the trajectories will quickly separate and synchronization does not exist on the condition that the initial values are different. However, with appropriate fuzzy state feedback control schemes $u(t)$, the two identical energy resource systems will approach synchronization for any initial value. Further, from Examples 6.1, 6.2, 6.3, the sufficient conditions for the synchronization are obtained analytically when $B = [1\ 1\ 1\ 1]^T$, $B = [1\ 1\ 1\ 0]^T$, and $B = [0\ 1\ 1\ 0]^T$, respectively. By Example 6.4, the synchronization condition can be obtained when $B = [0\ 0\ 1\ 0]^T$, in which only one state feedback controller is contained, which is of important significance in synchronization. Therefore controlling the demand-supply energy resources system is of significant important for steady state development.

7. Conclusions

In this study, we looked at four techniques for synchronizing two identical energy resource systems utilising only the simplest linear controllers, as well as passivity-based synchronization of a nonlinear energy resource system when it exhibits chaotic behaviour. Using the Lyapunov function and building a fuzzy state-feedback controller, the necessary conditions for passive synchronization of the energy resource system have been developed. Furthermore, using numerical simulations based on the LMI approach, synchronization strategies have been obtained and shown.

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References

[1] O. Ababneh, *Adaptive synchronization and anti-synchronization of fractional order chaotic optical systems with uncertain parameters*, J. Math. Computer Sci., **23** (2021), 302–314. 1

[2] C. K. Ahn, *A passivity approach to synchronization for time-delayed chaotic systems*, Mod. Phys. Lett. B, **23** (2009), 3531–3541. 1, 3.1

[3] C. K. Ahn, *Passive learning and input-to-state stability of switched Hopfield neural networks with time-delay*, Inf. Sci., **180** (2010), 4582–4594. 1

[4] A. Arenas, A. D. Guilera, J. Kurths, Y. Moreno, C. Zhou, *Synchronization in complex networks*, Phys. Rep., **469** (2008), 93–153. 1

[5] W. Assawinchaichote, S. Junhom, H_∞ *fuzzy controller design for HIV/AIDS infection system with dual drug dosages via an LMI approach*, Int. J. Energy, **5** (2011), 27–33. 1

[6] S. Bououden, M. Chadli, S. Filali, A. E. Hajjaji, *Fuzzy model based multivariable predictive control of a variable speed wind turbine: LMI approach*, Renew. Energy, **37** (2012), 434–439. 1

[7] S. Boyd, L. E. Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM, Philadelphia, (1994). 1

[8] G. Calcev, R. Gorez, M. D. Neyer, *Passivity approach to fuzzy control systems*, Automatica, **34** (1998), 339–344. 1

[9] C. K. Chen, J. J. Yan, T. L. Liao, *Sliding mode control for synchronization of Rössler systems with time delays and its application to secure communication*, Phys. Scr., **76** (2007), 12 pages. 1, 5.5

[10] T. S. Chiang, P. Liu, *Robust output tracking control for discrete-time nonlinear systems with time-varying delay: Virtual fuzzy model LMI-based approach*, Expert Syst. Appl., **39** (2012), 8239–8247. 1

[11] L. O. Chua, *Passivity and complexity*, IEEE Trans. Circuits Syst. I. Fundam. Theory Appl., **46** (1999), 71–82. 1

[12] G. Feng, *A survey on analysis and design of model-based fuzzy control systems*, IEEE Trans. Fuzzy Syst., **14** (2006), 676–697. 1

[13] J. M. González-Miranda, *Synchronization and control of chaos: an introduction for scientists and engineers*, Imperial College Press, London, (2004). 1

[14] K. Gu, V. L. Kharitonov, J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Boston, (2003). 5

[15] D. J. Hill, P. J. Moylan, *Stability results for nonlinear feedback systems*, Automatica, **13** (1977), 377–382. 1

[16] C. F. Huang, K. H. Cheng, J. J. Yan, *Robust chaos synchronization of four-dimensional energy resource systems subject to unmatched uncertainties*, Commun. Nonlinear Sci. Numer. Simul., **14** (2009), 2784–2792. 1

[17] E. Kamal, A. Aitouche, R. Ghorbani, M. Bayart, *Robust nonlinear control of wind energy conversion systems*, Int. J. Elec. Power Energy Syst., **44** (2013), 202–209. 1

[18] M. A. Khan, M. Abid, N. Ahmed, A. Wadood, H. Park, *Nonlinear control design of a half-car model using feedback linearization and an LQR controller*, Appl. Sci., **10** (2020), 17 pages. 1

[19] D. Khiar, J. Lauber, T. Floquet, G. Colin, T. M. Guerra, Y. Chamaillard, *Robust Takagi-Sugeno fuzzy control of a spark ignition engine*, Control Eng. Pract., **15** (2007), 1446–1456. 1

[20] K. Y. Lian, T. S. Chiang, C. S. Chiu, P. Liu, *Synthesis of fuzzy model-based designs to synchronization and secure communications for chaotic systems*, IEEE Trans. Fuzzy Syst., **31** (2001), 66–83. 1

[21] K. Y. Lian, C. S. Chiu, T. S. Chiang, P. Liu, *LMI-based fuzzy chaotic synchronization and communications*, IEEE Trans. Fuzzy Syst., **9** (2001), 539–553.

[22] K. Y. Lian, P. Liu, T. C. Wu, W. C. Lin, *Chaos control using fuzzy model-based methods*, Int. J. Bifurcat. Chaos, **12** (2002), 1827–1841. 1

[23] P. Liu, W. T. Yang, C. E. Yang, *Robust observer-based output feedback control for fuzzy descriptor systems*, Expert Syst. Appl., **40** (2013), 4503–4510. 1

[24] R. Lozano, B. Brogliato, O. Egeland, B. Maschke, *Dissipative Systems Analysis and Control*, Springer, New York, (2000). 1

[25] J. L. M. Machuca, R. M. Guerra, R. A. Lopez, C. A. Ibanez, *A chaotic system in synchronization and secure communications*, Commun. Nonlinear Sci. Numer. Simul., **17** (2012), 1706–1713. 1

[26] M. Manikandan, K. Ratnavelu, P. Balasubramaniam, S. H. Ong, *Synchronization of BAM Cohen-Grossberg FCNNs with mixed time delays*, Iran. J. Fuzzy Syst., **18** (2021), 159–173. 1

[27] M. Manikandan, K. Ratnavelu, P. Balasubramaniam, S. H. Ong, *Synchronization of Cohen-Grossberg fuzzy cellular neural networks with time-varying delays*, Int. J. Nonlinear Sci. Numer. Simul., **22** (2021), 45–58. 1

[28] E. Onieva, J. Godoy, J. Villagra, V. Milanes, J. Perez, *On-line learning of a fuzzy controller for a precise vehicle cruise control system*, Expert Syst. Appl., **40** (2013) 1046–1053. 1

[29] A. Othman Almatroud, O. Ababneh, M. Mossa Al-sawalha, *Modify adaptive combined synchronization of fractional order chaotic systems with fully unknown parameters*, J. Math. Computer Sci., **21** (2020), 99–112. 1

[30] L. M. Pecora, T. L. Carroll, *Synchronization in chaotic systems*, Phys. Rev. Lett., **64** (1990), 821–825. 1

[31] R. E. Precup, H. Hellendoorn, *A survey on industrial applications of fuzzy control*, Comput. Ind., **62** (2011), 213–226. 1

[32] R. E. Precup, M. L. Tomescu, M. B. Radac, E. M. Petriu, S. Preitl, C. A. Dragos, *Iterative performance improvement of fuzzy control systems for three tank systems*, Expert Syst. Appl., **39** (2012), 8288–8299. 1

[33] X. Shi, Z. Wang, *Robust chaos synchronization of four-dimensional energy resource system via adaptive feedback control*, Nonlinear Dynam., **60** (2010), 631–637. 1

[34] M. Sun, Q. Jia, L. Tian, *A new four-dimensional energy resources system and its linear feedback control*, Chaos Solitons Fract., **39** (2009), 101–108. 1, 2

[35] M. Sun, L. Tian, Y. Fu, *An energy resources demand-supply system and its dynamical analysis*, Chaos Solitons Fract., **32** (2007), 168–180. 1

[36] M. Sun, L. Tian, Q. Jia, *Adaptive control and synchronization of a four-dimensional energy resources system with unknown parameters*, Chaos Solitons Fract., **39** (2009), 1943–1949. 1

[37] M. Sun, L. Tian, J. Yin, *Hopf bifurcation analysis of the energy resource chaotic system*, Int. J. Nonlinear Sci., **1** (2006), 49–53. 1

[38] M. Sun, L. Tian, C. Zeng, *The energy resources system with parametric perturbations and its hyperchaos control*, Nonlinear Anal. Real World Appl., **10** (2009), 2620–2626. 1

[39] M. Sun, X. Wang, Y. Chen, L. Tian, *Energy resources demand-supply system analysis and empirical research based on non-linear approach*, Energy, **36** (2011), 5460–5465. 1

[40] Y. Sun, Z. Wei, G. Sun, P. Ju, S. Huang, *Robust stabilization and synchronization of nonlinear energy resource system via fuzzy control approach*, Int. J. Fuzzy Syst., **14** (2012), 337–343. 1, 5.3, 5.4

[41] Y. Sun, Z. Wei, G. Sun, P. Ju, Y. Wei, *Stochastic synchronization of nonlinear energy resource system via partial feedback control*, Nonlinear Dyn., **70** (2012), 2269–2278. 1

[42] T. Takagi, M. Sugeno, *Fuzzy identification of systems and its applications to modelling and control*, IEEE Trans. Syst. Man Cybern. Syst., **15** (1995), 116–132.

[43] K. Tanaka, M. Sano, *A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer*, IEEE Trans. Fuzzy Syst., **2** (1994), 119–134. 1

[44] T. Tanaka, H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, Wiley, New York, (2001). 1

[45] H. D. Tuan, P. Apkarian, T. Narikiyo, Y. Yamamoto, *Parameterized linear matrix inequality techniques in fuzzy control system design*, IEEE Trans. Fuzzy Syst., **9** (2001), 324–332. 5.1, 5.4

[46] V. Vembarasan, P. Balasubramaniam, *Chaotic synchronization of Rikitake system based on T-S fuzzy control techniques*, Nonlinear Dynam., **74** (2013), 31–44. 1

[47] Z. Wang, *Chaos synchronization of an energy resource system based on linear control*, Nonlinear Anal. Real World Appl., **11** (2010), 3336–3343. 1

[48] Z. Wang, X. Shi, *Synchronization of a four-dimensional energy resource system via linear control*, Commun. Nonlinear Sci. Numer. Simul., **16** (2011), 463–474. 1

[49] Y. Wen, *Passive equivalence of chaos in Lorenz system*, IEEE Trans. Circuits Syst. I. Fundam. Theory Appl., **46** (1999), 876–878. 1

[50] S. J. Wu, *Affine TS-model-based fuzzy regulating/servo control design*, Fuzzy Sets Syst., **158** (2007), 2288–2305. 1

[51] X. J. Wu, J. S. Liu, G. R. Chen, *Chaos synchronization of Rikitake chaotic attractor using the passive control technique*, Nonlinear Dyn., **53** (2008), 45–53. 1

[52] K. Wu, C. Ren, *Control and stability analysis of double time-delay active suspension based on particle swarm optimization*, Shock Vib., **216** (2010), 1605–1613. 1

[53] A. A. Zaher, A. A. Rezq, *On the design of chaos-based secure communication systems*, Commun. Nonlinear Sci. Numer. Simul., **16** (2011), 3721–3737. 1

[54] X. Zhou, B. Kong, H. Ding, *Synchronization and anti-synchronization of a new hyperchaotic Lu system with uncertain parameters via the passive control technique*, Phys. Scr., **85** (2012), 11 pages. 1