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Prevalent fixed point theorems on MIFM-Spaces using the (CLR_{SR}) property and implicit function

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Abstract

The main goal of this study is to use an implicit function to demonstrate the existence of a common fixed point on modified intuitionistic fuzzy metric spaces by using the concept of common limit range property with regard to two self-mappings S and R, i.e., (CLR_{SR}) property. Our primary result is supported by an example that validates the hypotheses of our result. Our findings improve and generalize the findings of Tanveer et al. [M. Tanveer, M. Imdad, D. Gopal, D. K. Patel, Fixed Point Theory Appl., **2012** (2012), 1–12], and other existing results related to this study.

Keywords: Common fixed point, modified intuitionistic fuzzy metric space (MIFM-Space), common property (E-A), common limit in range property (CLR property), implicit function.

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1. Introduction

Zadeh [48] introduced the notion of a fuzzy set. Atanassov [4] introduced the concept of an intuitionistic fuzzy set by generalizing the idea of a fuzzy set introduced in [48]. Coker [12] developed the notion of topology on intuitionistic fuzzy sets after that. The intuitionistic gradation of openness was introduced by Mondal [33]. In 2004, Park [36] suggested the notion of intuitionistic fuzzy metric spaces (IFMS), which is a generalization of George and Veeramani's fuzzy metric space [15]. Many authors have recently proven fixed point theorems in IFMS ([2, 3, 6, 20, 35, 38, 40, 42, 44]).

Gregory et al. [16] went on to show that "the topology induced by fuzzy metric coincides with the topology induced by intuitionistic fuzzy metric". Saadati et al. [37] reframed the definition of intuitionistic fuzzy metric spaces by adding the concept of continuous t-representable and proposed a new concept known as modified IFMS. They also characterized strong (introduced by Jungck [29]) and weak (introduced by Jungck and Rhodes [30]) compatibility to modified IFMS. Pant's [34] research into common fixed points of non-compatible maps is also natural. In the recent past, Tanveer et al. [46] and Imdad et al. [22] proved some results in MIFM-Spaces using the notions of the property (E-A) (defined by Aamri and El-Moutawakil [1]) and the common property (E-A) (originated by Liu et al. [32]). It is worth noting

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that both the property (E-A) and the common property (E-A) demand that the subspace be closed for a common fixed point to existing. More recently, Gupta et al. [19], and Shatanawi et al. [41] proved some common fixed point results on MIFM-Spaces by using integral type contraction, property (E-A), and common property (E-A).

Sintunavarat and Kumam [43] introduced the idea of the common limit in range property, which states that the existence of a common fixed point does not require the subspace to be closed (also see [45]). Many authors have recently proven the superiority of common limit in range property over the property (E-A) and common property (E-A) for maps defined on different spaces such as modified IFMS, Menger spaces, and Metric space through common limit range property (e.g., [5, 7, 9–11, 25–28, 31, 39, 42, 47]).

In this research article, we prove some common fixed point theorems on MIFM-Space by using the common limit range property with regard to two self maps. We use an implicit function defined in [22] and [46] to prove our results.

2. Preliminaries

Lemma 2.1 ([13]). *Let the set* L^* *and* $\leqslant_{L^*}$ *operation defined by*

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2, x_1 + x_2 \leqslant 1\},$$

$(x_1, x_2) \leqslant_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leqslant y_1$ *and* $x_2 \geqslant y_2$, *for every* $(x_1, x_2), (y_1, y_2) \in L^*$. *The lattice* $(L^* \leqslant_{L^*})$ *is then complete.*

Definition 2.2 ([4]). In a universe U, there is an intuitionistic fuzzy set $A_{\zeta,\eta}$ such that

$$A_{\zeta,\eta} = \{(\zeta_A(\nu), \eta_A(\nu) \mid \nu \in U)\},$$

where $\forall\ \nu \in U$, $\zeta_A(\nu) \in [0,1]$, and $\eta_A(\nu) \in [0,1]$ are the membership and the non-membership degree of $\nu \in A_{\zeta,\eta}$, respectively, which also satisfy $\zeta_A(\nu) + \eta_A(\nu) \leqslant 1$. For every $z_i = (x_i, y_i) \in L^*$, if $a_i \in [0,1]$ such that $\Sigma a_j = 1 (1 \leqslant j \leqslant n)$, then it is easy to see that

$$a_1(x_1, y_1) + \cdots + a_n(x_n, y_n) = \sum a_j(x_j, y_j) = \left(\sum a_j x_j, \sum a_j y_j\right) \in L^* \ (\forall\ j \text{ from } j = 1 \text{ to } n).$$

Its units are denoted by $0_{L^*} = (0, 1)$ and $1_{L^*} = (1, 0)$.

Mathematically, a triangular norm $* = T$ on $[0,1]$ is defined as an increasing, associative, commutative mapping $T : [0,1]^2 \to [0,1]$ which satisfies $T(1, x) = 1 * x = x$, $\forall\ x \in [0,1]$. A triangular conorm $S = \diamondsuit$ is defined as an increasing, commutative, associative mapping $S : [0,1]^2 \to [0,1]$ which satisfies $S(0, x) = 0 \diamondsuit x = x$, $\forall\ x \in [0,1]$. By using $(L^* \leqslant_{L^*})$, these definitions can easily be extended.

Definition 2.3 ([14]). A triangular norm (in short t-norm) on L^* is a mapping $T : (L^*)^2 \to L^*$ which satisfies the following four conditions, $\forall\ x, y, x', y' \in L^*$:

(1) $T(x, 1_{L^*}) = x$;
(2) $T(x, y) = T(y, x)$;
(3) $T(x, T(y, z)) = T(T(x, y), z)$;
(4) $x \leqslant_{L^*} x'$ and $y \leqslant_{L^*} y' \Rightarrow T(x, y) \leqslant_{L^*} T(x', y')$.

Definition 2.4 ([13, 14]). A continuous t-norm T on L^* is known as continuous t-representable if and only if $\forall\ x = (x_1, x_2),\ y = (y_1, y_2) \in L^*, T(x, y) = (x_1 * y_1, x_2 \diamondsuit y_2)$. Now, we recursively define a sequence $\{T_n\}$ by $\{T^1 = T\}$ and

$$T^n\left(x^{(1)}, \ldots, x^{(n+1)}\right) = T\left(T^{(n-1)}\left(x^{(1)}, \ldots, x^n\right), x^{(n+1)}\right) \text{ for } n \geqslant 2 \text{ and } x^i \in L^*.$$

Definition 2.5 ([13, 14]). Any decreasing mapping $N : L^* \to L^*$ that satisfies $N(0_{L^*}) = 1_{L^*}$ and $N(1_{L^*}) = 0_{L^*}$ is a negator on L^*. When $N(N(x)) = x$, for all $x \in L^*$, then N is referred to as an involutive negator. A negator on $[0, 1]$ is a decreasing mapping $N : [0, 1] \to [0, 1]$ that satisfies $N(0) = 1$ and $N(1) = 0$. The standard negator N_s on $[0, 1]$ is defined as $N_s(x) = 1–x\ \forall\ x \in [0, 1]$.

Definition 2.6 ([29]). Let M, N be fuzzy sets ranging from $X^2 \times (0,\infty) \to [0, 1]$ with $M(x, y, t)+N(x, y, t) \leqslant 1\ \forall\ x, y \in X$, and $t > 0$. The 3-tuple $(X, F_{M,N}, T)$ is said to be a MIFM-Space if X is an arbitrary nonempty set, T is a continuous t-representable, and $F_{M,N}$ is an intuitionistic fuzzy set from $X^2 \times (0,\infty)$ to L^* that satisfies the following conditions (for all $x, y, z \in X$ and $t, s > 0$):

(1) $F_{M,N}(x, y, t) >_{L^*} 0_{L^*}$;
(2) $F_{M,N}(x, y, t) = 1_{L^*}$ if and only if $x = y$;
(3) $F_{M,N}(x, y, t) = F_{M,N}(y, x, t)$;
(4) $F_{M,N}(x, y, t + s) \geqslant_{L^*} T\Big(F_{M,N}(x, z, t), F_{M,N}(z, y, s)\Big)$;
(5) $F_{M,N}(x, y, \cdot) : (0,\infty) \to L^*$ is continuous.

$F_{M,N}$ is referred to as a modified intuitionistic fuzzy metric in this case.

Noted that, here $F_{M,N}(x, y, t) = \big(M(x, y, t), N(x, y, t)\big)$.

Remark 2.7 ([29]). In an IFMS $(X, F_{M,N}, T), M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing $\forall\ x, y \in X$. As, $F_{M,N}(x, y, t) = \big(M(x, y, t), N(x, y, t)\big)$, hence $F_{M,N}(x, y, t)$ is a non-decreasing function with respect to t, $\forall\ x, y \in X$.

Example 2.8 ([37]). Let (X, d) be a metric space. Define $T(u, v) = \big(u_1v_1, \min\{u_2 + v_2, 1\}\big)$ for all $u = (u_1, u_2)$ and $v = (v_1, v_2) \in L^*$, and let M and N be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows:

$$F_{M,N}(x, y, t) = \big(M(x, y, t), N(x, y, t)\big) = \left(\frac{rt^n}{rt^n + md(x, y)}, \frac{sd(x, y)}{rt^n + sd(x, y)}\right), \quad \forall\ r, s, n, t \in R^+.$$

Then $(X, F_{M,N}, T)$ is a MIFM-Space.

Example 2.9 ([37]). Let $X = \mathbb{N}$. Define $T(u, v) = \big(\max\{0, u_1 + v_1 - 1\}, u_2 + v_2 – u_2v_2\big)$ for all $u = (u_1, u_2)$ and $v = (v_1, v_2) \in L^*$, let M and N be fuzzy sets on $X^2 \times (0,\infty)$. Then $F_{M,N}(x, y, t)$ is defined as follows:

$$F_{M,N}(x, y, t) = \big(M(x, y, t), N(x, y, t)\big),\ \forall\ x, y \in X \text{ and } t > 0 = \begin{cases} \left(\frac{x}{y}, \frac{y-x}{y}\right), & \text{if } x \leqslant y, \\ \left(\frac{y}{x}, \frac{x-y}{x}\right), & \text{if } y \leqslant x. \end{cases}$$

Then $(X, F_{M,N}, T)$ is a MIFM-Space.

Definition 2.10 ([37]). Let $(X, F_{M,N}, T)$ be a MIFM-Space. For $t > 0$, consider $O(x, r, t) = \big\{y \in X : F_{M,N}(x, y, t) >_{L^*} \big(N_s(r), r\big)\big\}$ is an open ball with center $x \in X$ and radius $0 < r < 1$.

If for each $x \in A\ \exists\ t > 0$ and $0 < r < 1$ such that $O(x, r, t) \subseteq A$, a subset A of X is called open. The topology induced by intuitionistic fuzzy metric $F_{M,N}$ is defined as the family of all open subsets of X denoted by $\tau_{F_{M,N}}$.

Definition 2.11 ([37]). A Cauchy sequence $\{x_n\}$ in a MIFM-Space$(X, F_{M,N}, T)$ is one in which for each $0 < \delta < 1$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that $F_{M,N}(x_n, y_m, t) >_{L^*} \big(N_s(\delta), \delta\big)$, for each $n, m \geqslant n_0$.

In the MIFM-Space $(X, F_{M,N}, T)$, the sequence $\{x_n\}$ is said to be convergent to $x \in X$ and is usually denoted by $x_n \to F_{M,N}x$ if $F_{M,N}(x_n, x, t) \to 1_{L^*}$ whenever $n \to \infty$ for every $t > 0$.

A MIFM-Space $(X, F_{M,N}, T)$ is said to be complete if and only if every Cauchy sequence in it is convergent in it.

Lemma 2.12 ([37]). *Let $F_{M,N}$ be an intuitionistic fuzzy metric. Then, for any $t > 0$, $F_{M,N}(x, y, t)$ in $(L^*, \leqslant_{L^*})$, $\forall\ x, y \in X$ is non-decreasing with respect to* t.

Definition 2.13 ([37]). Let $(X, F_{M,N}, T)$ be a MIFM-Space. Then $F_{M,N}$ on $X^2 \times (0, \infty)$ is said to be continuous, if $\lim_{n\to\infty} F_{M,N}(x_n, y_n, t_n) = F_{M,N}(x, y, t)$ whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $\{(x, y, t)\} \in X^2 \times (0, \infty)$; that is, $\lim_{n\to\infty} F_{M,N}(x_n, x, t) = \lim_{n\to\infty} F_{M,N}(y_n, x, t) = 1_{L^*}$, $\lim_{n\to\infty} F_{M,N}(x, y, t_n) = F_{M,N}(x, y, t)$.

Lemma 2.14 ([37]). *Let*$(X, F_{M,N}, T)$ *be a MIFM-Space. Then,*$F_{M,N}$ *on* $X^2 \times (0, \infty)$ *is a continuous function.*

Definition 2.15 ([22, 37]). Let P and Q be two self maps on a MIFM-Space $(X, F_{M,N}, T)$. Then, the pair (P, Q) is said to be

(1) commuting if $PQx = QPx$, $\forall\, x \in X$;
(2) weakly commuting if $F_{M,N}(PQx, QPx, t) \geqslant_{L^*} F_{M,N}(Px, Qx, t)$ $\forall\, x \in X$ and $t > 0$;
(3) compatible if $\lim_{n\to\infty} F_{M,N}(PQx_n, QPx_n, t) = 1_{L^*}$ for all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = x \in X$;
(4) non-compatible if exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = x \in X$, but $\lim_{n\to\infty} F_{M,N}(PQx_n, QPx_n, t) \neq 1_{L^*}$ or non existent for at least one $t > 0$.

Definition 2.16 ([23]). Two families of self-mappings $\{P_i\}(i = 1$ to $m)$ and $\{Q_k\}(k = 1$ to $n)$ are said to be pairwise commuting if

(1) $P_a P_b = P_b P_a$, $\forall\, a, b \in \{1, 2, \dots, m\}$;
(2) $Q_c Q_d = Q_d Q_c$, $\forall\, c, d \in \{1, 2, \dots, n\}$;
(3) $P_a P_c = P_c P_a$, $\forall\, a \in \{1, 2, \dots, m\}$ and $c \in \{1, 2, \dots, n\}$.

Definition 2.17 ([38]). On a MIFM-Space $(X, F_{M,N}, T)$, let P and Q be two self-maps. If a sequence $\{x_n\}$ in X such that $\forall\, t > 0 \lim_{n\to\infty} F_{M,N}(Px_n, z, t) = \lim_{n\to\infty} F_{M,N}(Qx_n, z, t) = 1_{L^*}$, for some $z \in X$, then the pair (P, Q) is said to propitiate the property (E-A).

Definition 2.18 ([46]). Two pairs (P, S) and (Q, R) of self mappings of a MIFM-Space $(X, F_{M,N}, T)$ are said to propitiate the common property (E-A) if exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} F_{M,N}(Px_n, z, t) = \lim_{n\to\infty} F_{M,N}(Sx_n, z, t) = \lim_{n\to\infty} F_{M,N}(Qy_n, z, t) = \lim_{n\to\infty} F_{M,N}(Ry_n, z, t) = 1_{L^*}$, for some $z \in X$ and $t > 0$.

Definition 2.19 ([42]). A pair (P, S) of self-mappings of a MIFM-Space $(X, F_{M,N}, T)$ is said to propitiate the common limit in range property concerning S, denoted by (CLR_s) if $\exists$ a sequence $\{x_n\}$ in X such that $\forall\, t > 0$, $\lim_{n\to\infty} F_{M,N}(Px_n, z, t) = \lim_{n\to\infty} F_{M,N}(Sx_n, z, t) = \lim_{n\to\infty}$,where $z \in S(X)$.

As a result, a pair (P, S) satisfying the property (E-A) along with the closedness of the subspace $S(X)$ always has the property (CLR_s) with regard to the mapping S ([11, 42]).

In modified IFMS $(X, F_{M,N}, T)$, we now extend the common limit in range property for two pairs of self-mappings as follows.

Definition 2.20. If there are two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} F_{M,N}(Px_n, z, t) = \lim_{n\to\infty} F_{M,N}(Sx_n, z, t) = \lim_{n\to\infty} F_{M,N}(Qy_n, z, t) = \lim_{n\to\infty} F_{M,N}(Ry_n, z, t) = 1_{L^*}$, where $z \in S(X) \cap R(X)$ and $t > 0$, then pairs (P, S) and (Q, R) of self-mappings of a MIFM-Space $(X, F_{M,N}, T)$ are said to propitiate the common limit in range property concerning maps S and R, denoted by (CLR_{SR}).

Example on (CLR_{SR}) property: For seeing an example on (CLR_{SR}) property, one can refer to the last part of Example 4.2. By setting $P = Q$ and $S = R$ in Definition 2.20 implies Definition 2.19 (due to Sintunavarat et al. [42]), whereas Definition 2.20 implies Definition 2.18, but not in general. This fact can be shown by the following example.

Example 2.21. Let $(X, F_{M,N}, T)$ be a MIFM-Space, where $X = [4, 21]$ and $F_{M,N}(x, y, t) = \left(\frac{t}{t+|x-y|}, \frac{|x-y|}{t+|x-y|}\right)$, $\forall\, x, y \in X$ and $t > 0$. Define four self-mappings P, Q, S, and R on X as

$$P(x) = \begin{cases} 8, & \text{if } x = 4, \\ 6, & \text{if } 4 < x \leqslant 15, \\ \frac{x+9}{6}, & \text{if } x > 15, \end{cases} \qquad Q(x) = \begin{cases} 5, & \text{if } x = 4, \\ \frac{5x+4}{6}, & \text{if } 4 < x \leqslant 15, \\ 13, & \text{if } x > 14, \end{cases}$$

$$S(x) = \begin{cases} 6, & \text{if } x = 4, \\ 16, & \text{if } 4 < x \leqslant 15, \\ \frac{2x+6}{9}, & \text{if } x > 15, \end{cases} \qquad R(x) = \begin{cases} 7, & \text{if } x = 4, \\ \frac{x+4}{2}, & \text{if } 4 < x \leqslant 15, \\ 18, & \text{if } x > 15. \end{cases}$$

If we take two sequences as $\{x_n\} = \left\{15 + \frac{1}{n}\right\} n \in \mathbb{N}$ and $\{y_n\} = \left\{4 + \frac{1}{n}\right\} n \in \mathbb{N}$, then the pairs (P, S) and (Q, R) satisfy the common property (E-A) $\forall\, t > 0$:

$$\lim_{n\to\infty} F_{M,N}(Px_n, 4, t) = \lim_{n\to\infty} F_{M,N}(Sx_n, 4, t) = \lim_{n\to\infty} F_{M,N}(Qy_n, 4, t) = \lim_{n\to\infty} F_{M,N}(Ry_n, 4, t) = 1_{L^*},$$

where $4 \in X$. Here, it is noticed that $4 \notin S(X) \cap R(X)$. Therefore, the pairs (P, S) and (Q, R) do not propitiate the common limit in range property with regard to the mappings S and R.

Based on the result of Example 2.21, a proposition is given as follows.

Proposition 2.22. *If the pairs* (P, S) *and* (Q, R) *have the common property* (E-A) *and* $S(X)$ *and* $R(X)$*, are closed subsets of* X*, then these pairs satisfy the* (CLR_{SR}) *property as well.*

3. Implicit relations

Motivated by Imdad et al. [22], we adopt a new collection of implicit functions as follows.

Let Ψ be the collection of all upper continuous functions $F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) : L^{*^6} \to L^*$, that satisfy the following conditions $(\forall\, \nu, 0, 1 \in L^*$, where $\nu = (\nu_1, \nu_2), \mathbf{0} = 0_{L^*} = (0, 1)$, and $\mathbf{1} = 1_{L^*} = (1, 0))$:

(F_1) $F(\upsilon, \mathbf{1}, \upsilon, \mathbf{1}, \mathbf{1}, \upsilon) <_{L^*} \mathbf{0}$, $\forall\, \upsilon >_{L^*} \mathbf{0}$;
(F_1) $F(\upsilon, \mathbf{1}, \mathbf{1}, \upsilon, \upsilon, \mathbf{1}) <_{L^*} \mathbf{0}\ \forall\, \upsilon >_{L^*} \mathbf{0}$;
(F_1) $F(\upsilon, \upsilon, \mathbf{1}, \mathbf{1}, \upsilon, \upsilon) <_{L^*} \mathbf{0}\ \forall\, \upsilon >_{L^*} \mathbf{0}$.

The following examples satisfy $(F_1), (F_2)$, and (F_3).

Example 3.1. Define $F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) : L^{*^6} \to L^*$ as

$$F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) = \tau_1 - \beta \min\{\tau_2, \tau_3, \tau_4, \tau_5, \tau_6\}, \text{ where } \beta > 1.$$

Example 3.2. Define $F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) : L^{*^6} \to L^*$, as

$$F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) = \tau_1^2 - \beta_1 \min\{\tau_2^2, \tau_3^2, \tau_4^2\} - \beta_2 \min\{\tau_3\tau_6, \tau_4\tau_5\},$$

where $\beta_1, \beta_2 > 0$, $\beta_1 + \beta_2 > 1$, and $\beta_1 \geqslant 1$.

Example 3.3. Define $F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) : L^{*^6} \to L^*$ as

$$F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) = \tau_1 - \beta_1\tau_2 - \beta_2\tau_3 - \beta_3\tau_4 - \beta_4\tau_5 - \beta_5\tau_6,$$

where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5 > 0$, $\beta_2 + \beta_5 \geqslant 1$, $\beta_3 + \beta_4 \geqslant 1$, and $\beta_1 + \beta_4 + \beta_5 \geqslant 1$.

Motivated by Tanveer et al. [46], let X be the collection of all continuous functions $\chi(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) : L^{*^6} \to L^*$, satisfying $(\forall\, u, v, \mathbf{1} \in L^*$, where $a = (a_1, a_2), b = (b_1, b_2)$, and $\mathbf{1} = 1_{L^*} = (1, 0))$:

(χ_1) for all $a, b >_{L^*} \mathbf{0}$, $\chi(a, b, a, b, b, a) \geqslant_{L^*} \mathbf{0}$ or $\chi(a, b, b, a, a, b) \geqslant_{L^*} \mathbf{0}$ implies that $a \geqslant_{L^*} b$;
(χ_1) $\chi(a, a, \mathbf{1}, \mathbf{1}, b, b) \geqslant_{L^*} \mathbf{0}$ implies that $b \geqslant_{L^*} \mathbf{1}$.

Example 3.4. Define $\chi(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) = 18\tau_1 - 16\tau_2 + 8\tau_3 - 10\tau_4 + \tau_5 - \tau_6$. Then $\chi \in X$.

Example 3.5. Define $\chi(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) = \tau_1 - \left(\frac{1}{2}\right)\tau_2 - \left(\frac{5}{6}\right)\tau_3 + \left(\frac{1}{3}\right)\tau_4 + \tau_5 - \tau_6$. Then $\chi \in X$.

It should be noted that the above classes of functions Ψ and X are completely independent of one another as the implicit function $F(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) = \tau_1 - \beta \min\{\tau_2, \tau_3, \tau_4, \tau_5, \tau_6\}$ (where $\beta > 1$ and $F \in \Psi$) does not belong to X as $F(\upsilon, \upsilon, \mathbf{1}, \mathbf{1}, \upsilon, \upsilon) <_{L^*} \mathbf{0}$, for all $\upsilon >_{L^*} \mathbf{0}$, while the implicit function $\chi(\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6) = 15\tau_1 - 13\tau_2 + 5\tau_3 - 7\tau_4 + \tau_5 - \tau_6$ (where $\chi \in X$) does not belong to Ψ as $\chi(a, b, a, b, b, a) = 0$ implies $a = b$ instead of $a >_{L^*} b$. For the major collection of implicit relations in different settings, one can refer to ([8, 17, 18, 21, 24]).

Lemma 3.6. *Let $P, Q, S,$ and R be self-mappings of a MIFM-Space $(X, F_{M,N}, T)$. Suppose that*

(1) *the pair (P, S) or (Q, R) shares the property $(E\text{-}A)$;*
(2) *$P(X) \subset R(X)$ (or $Q(X) \subset S(X)$);*
(3) *$\{Qy_n\}$ converges for every sequence $\{y_n\}$ in X whenever $\{Ry_n\}$ converges (or $\{Px_n\}$ converges for every sequence $\{x_n\}$ in X whenever $\{Sx_n\}$ converges);*
(4) *for all $x, y \in X$ and $F \in \Psi$*

$$F\Big(F_{M,N}(Px, Qy, t), F_{M,N}(Sx, Ry, t), F_{M,N}(Px, Sx, t), F_{M,N}(Qy, Ry, t), F_{M,N}(Sx, Qy, t), F_{M,N}(Px, Ry, t)\Big) \geqslant_{L^*} \mathbf{0}.$$

Then the pairs (P, S) and (Q, R) share the common property $(E\text{-}A)$.

By using Lemma 3.6, Tanveer et al. [46] proved the following Theorem-A for the common fixed point under the common property $(E\text{-}A)$:

Theorem 3.7. *Let $P, Q, S,$ and R be self-mappings of a MIFM-Space $(X, F_{M,N}, T)$ satisfying condition 4 of Lemma 3.6. Assume that*

(1) *the pairs (P, S) and (Q, R) propitiate the common property $(E\text{-}A)$;*
(2) *$R(X)$ and $S(X)$ are closed subset of X.*

The pairs (P, S) and (Q, R), then have a point of coincidence.Furthermore, if pairs (P, S) and (Q, R) are weakly compatible, then $P, Q, S,$ and R have a unique common fixed point.

The following lemma 3.8, which is a generalization of Lemma 3.6, is required to prove our main result.

Lemma 3.8. *Let $P, Q, R,$ and S be self-mappings of a MIFM-Space $(X, F_{M,N}, T)$. Assume that*

(1) *the pair (P, S) shares the (CLR_S) property (or (Q, R) shares the (CLR_R) property);*
(2) *$P(X) \subset R(X)$ (or $Q(X) \subset S(X)$);*
(3) *$R(X)$ (or $S(X)$) is a closed subset of X;*
(4) *$\{Qy_n\}$ converges for every sequence $\{y_n\}$ in X whenever $\{Ry_n\}$ converges to $R(X)$ (or $\{Px_n\}$ converges for every sequence $\{x_n\}$ in X whenever $\{Sx_n\}$ converges to $S(X)$;*
(5) *for all $x, y \in X$ and $F \in \Psi$*

$$F\Big(F_{M,N}(Px, Qy, t), F_{M,N}(Sx, Ry, t), F_{M,N}(Px, Sx, t), F_{M,N}(Qy, Ry, t), F_{M,N}(Sx, Qy, t), F_{M,N}(Px, Ry, t)\Big) \geqslant_{L^*} \mathbf{0}.$$

Then the pairs (P, S) and (Q, R) share the $(CLR_{SR}$ property.

4. Main results

Results in Theorem 3.7 are being proved by taking common property $(E\text{-}A)$ along with closedness of subspaces of $R(X)$ and $S(X)$, while here in Theorem 4.1 we prove the same results by taking only the (CLR_{SR}) property. In this Theorem 4.1, we consider a different type of implicit function which is defined in Example 3.3.

We improve and generalize Theorem 3.7 as follows.

Theorem 4.1. *Let $P, Q, S,$ and R be four self-mappings of a MIFM-Space $(X, F_{M,N}, T)$ satisfying the following conditions:*

$$\begin{aligned} F_{M,N}(Px, Qy, t) \geqslant_{L^*} \ & \beta_1 F_{M,N}(Sx, Ry, t) + \beta_2 F_{M,N}(Px, Sx, t) + \beta_3 F_{M,N}(Qy, Ry, t) \\ & + \beta_4 F_{M,N}(Sx, Qy, t) + \beta_5 F_{M,N}(Px, Ry, t), \end{aligned} \tag{4.1}$$

where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5 > 0$, $\beta_2 + \beta_5 \geqslant 1$, $\beta_3 + \beta_4 \geqslant 1$, and $\beta_1 + \beta_4 + \beta_5 \geqslant 1$.

Assume that the pairs (P, S), and (Q, R) satisfy the (CLR_{SR}) property, then the pairs (P, S) and (Q, R) have a coincidence point. Furthermore, if pairs (P, S) and (Q, R) are weakly compatible, then $P, Q, S,$ and R have unique common fixed points.

Proof. Since the pairs (P, S), and (Q, R) satisfy the (CLR_{SR}) property, there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} F_{M,N}(Px_n, z, t) = \lim_{n\to\infty} F_{M,N}(Sx_n, z, t) = \lim_{n\to\infty} F_{M,N}(Qy_n, z, t) = \lim_{n\to\infty} F_{M,N}(Ry_n, z, t) = 1_{L^*},$$

where $z \in S(X) \cap R(X)$. Since $z \in S(X)$, there exists a point $w \in X$ such that $Sw = z$.

We show that $Pw = Sw$. If not, then by (4.1) at $x = w$, and $y = y_n$, we get

$$\begin{aligned} F_{M,N}(Px, Qy_n, t) \geqslant_{L^*} \ & \beta_1 F_{M,N}(Sw, Ry_n, t) + \beta_2 F_{M,N}(Pw, Sw, t) + \beta_3 F_{M,N}(Qy, Ry_n, t) \\ & + \beta_4 F_{M,N}(Sw, Qy_n, t) + \beta_5 F_{M,N}(Pw, Ry_n, t). \end{aligned} \tag{4.2}$$

On taking $n \to \infty$, (4.2) reduces to

$$F_{M,N}(Pw, z, t) \geqslant_{L^*} \ \beta_1 F_{M,N}(z, z, t) + \beta_2 F_{M,N}(Pw, z, t) + \beta_3 F_{M,N}(z, z, t) + \beta_4 F_{M,N}(z, z, t) + \beta_5 F_{M,N}(Pw, z, t).$$

So that

$$F_{M,N}(Pw, z, t) \geqslant_{L^*} \ \beta_1 1 + \beta_2 F_{M,N}(Pw, z, t) + \beta_3 1 + \beta_4 1 + \beta_5 F_{M,N}(Pw, z, t).$$

This is a contradiction to (F_1). Hence $F_{M,N}(Pw, z, t) = 1$; that is; $Pw = Sw = z$. Therefore w is a coincidence point of (P, S). Also $z \in R(X)$; there exists a point $v \in X$ such that $Rv = z$. We assert that $Qv = Rv$. If not, then by using (4.1) with $x = w$, and $y = v$, we get

$$\begin{aligned} F_{M,N}(Pw, Qv, t) \geqslant_{L^*} \ & \beta_1 F_{M,N}(Sw, Rv, t) + \beta_2 F_{M,N}(Pw, Sw, t) + \beta_3 F_{M,N}(Qv, Rv, t) \\ & + \beta_4 F_{M,N}(Sw, Qv, t) + \beta_5 F_{M,N}(Pw, Rv, t). \end{aligned}$$

So that

$$\begin{aligned} F_{M,N}(z, Qv, t) \geqslant_{L^*} \ & \beta_1 F_{M,N}(z, Qv, t) + \beta_2 F_{M,N}(z, z, t) + \beta_3 F_{M,N}(Qv, z, t) \\ & + \beta_4 F_{M,N}(z, Qv, t) + \beta_5 F_{M,N}(z, z, t). \end{aligned}$$

or

$$F_{M,N}(z, Qv, t) \geqslant_{L^*} \beta_1 F_{M,N}(z, Qv, t) + \beta_2 1 + \beta_3 F_{M,N}(Qv, z, t) + \beta_4 F_{M,N}(z, Qv, t) + \beta_5 1.$$

This is a contradiction to (F_2). Hence $F_{M,N}(z, Qv, t) = 1$, and so $Qv = Rv = z$, this shows that v is a coincidence point of (Q, R).

Since the pair (P, S) is weakly compatible and $Pw = Sw$, hence $Pz = PSw = SPw = Sz$. Now we show that z is a common fixed point of (P, S). Suppose that $Az \neq z$; by using (4.1) with $x = z$, and $y = v$, we have

$$\begin{aligned} F_{M,N}(Pz, Qv, t) \geqslant_{L^*} \ & \beta_1 F_{M,N}(Sz, Rv, t) + \beta_2 F_{M,N}(Pz, Sz, t) + \beta_3 F_{M,N}(Qv, Rv, t) \\ & + \beta_4 F_{M,N}(Sz, Qv, t) + \beta_5 F_{M,N}(Pz, Rv, t). \end{aligned} \tag{4.3}$$

So that

$$F_{M,N}(Pz,z,t) \geqslant_{L^*} \beta_1 F_{M,N}(z,z,t) + \beta_2 F_{M,N}(Pz,z,t) + \beta_3 F_{M,N}(z,z,t) + \beta_4 F_{M,N}(z,z,t) + \beta_5 F_{M,N}(Pz,z,t).$$

Or

$$F_{M,N}(Pz,z,t) \geqslant_{L^*} \beta_1 1 + \beta_2 F_{M,N}(Pz,z,t) + \beta_3 1 + \beta_4 1 + \beta_5 F_{M,N}(Pz,z,t).$$

This is a contradiction to (F_3). Therefore, $Pz = z = Sz$, shows that z is the common fixed point of (P,S). Also, the pair (Q,R) is weakly compatible, and $Qv = Rv$, therefore, $Qz = QRv = RQv = Rz$. Suppose that $Qz \neq z$; then using (4.1) with $x = w$, and $y = z$ we have

$$\begin{aligned} F_{M,N}(Pw,Qz,t) \geqslant_{L^*} &\beta_1 F_{M,N}(Sw,Rz,t) + \beta_2 F_{M,N}(Pw,Sw,t) + \beta_3 F_{M,N}(Qz,Rz,t) \\ &+ \beta_4 F_{M,N}(Sw,Qz,t) + \beta_5 F_{M,N}(Pw,Rz,t). \end{aligned}$$

So that

$$F_{M,N}(z,Qz,t) \geqslant_{L^*} \beta_1 F_{M,N}(z,z,t) + \beta_2 F_{M,N}(z,z,t) + \beta_3 F_{M,N}(Qz,z,t) + \beta_4 F_{M,N}(z,Qz,t) + \beta_5 F_{M,N}(z,z,t).$$

or

$$F_{M,N}(z,Qz,t) \geqslant_{L^*} \beta_1 1 + \beta_2 1 + \beta_3 F_{M,N}(Qz,z,t) + \beta_4 F_{M,N}(z,Qz,t) + \beta_5 1,$$

which is a contradiction to (F_3). Therefore, $Qz = z = Rz$, which shows that z is the common fixed point of (Q,R). Hence, z is a common fixed point of (P,S) and (Q,R).

The uniqueness of a common fixed point can be shown easily by inequality (4.1) and condition (F_3). Hence, the theorem is proved.

Next, we give an example (Ex. 4.2) in support of Theorem 4.1, which validates the hypotheses and extent of the generality of our result □

Example 4.2. Let $(X, F_{M,N}, T)$ be a MIFM-Space, where $X = [4,20), T(a,b) = (a_1 b_1, \min(a_2+b_2,1))$, $\forall\ a = (a_1,a_2)$ and $b = (b_1,b_2) \in L^*$ with $F_{M,N}(x,y,t) = \left(\frac{t}{t+|x-y|}, \frac{|x-y|}{t+|x-y|}\right)$, $\forall\ x,y \in X$ and $t > 0$. Define four self-mappings P, Q, S, and R by

$$P(x) = \begin{cases} 4, & \text{if } x \in \{4\} \cup (8,20), \\ 19, & \text{if } x \in (4,8], \end{cases} \qquad Q(x) = \begin{cases} 4, & \text{if } x \in \{4\} \cup (8,20), \\ 12, & \text{if } x \in (4,8], \end{cases}$$

and

$$S(x) = \begin{cases} 4, & \text{if } x = 4, \\ 9, & \text{if } x \in (4,8], \\ \frac{x}{2}, & \text{if } x \in (8,20), \end{cases} \qquad R(x) = \begin{cases} 4, & \text{if } x = 4, \\ 17, & \text{if } x \in (4,8], \\ x-4, & \text{if } x \in (8,20). \end{cases}$$

Define an implicit function $F(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6) : {L^*}^6 \to L^*$ as

$$F(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6) = \tau_1 - \beta_1\tau_2 - \beta_2\tau_3 - \beta_3\tau_4 - \beta_4\tau_5 - \beta_5\tau_6, \tag{4.4}$$

where $\beta_1,\beta_2,\beta_3,\beta_4,\beta_5 > 0$, $\beta_2+\beta_5 \geqslant 1$, $\beta_3+\beta_4 \geqslant 1$, and $\beta_1+\beta_4+\beta_5 \geqslant 1$. Hence (4.4) implies

$$\begin{aligned} F_{M,N}(Px,Qy,t) \geqslant_{L^*} &\beta_1 F_{M,N}(Sx,Ry,t) + \beta_2 F_{M,N}(Px,Sx,t) + \beta_3 F_{M,N}(Qy,Ry,t) \\ &+ \beta_4 F_{M,N}(Sx,Qy,t) + \beta_5 F_{M,N}(Px,Ry,t), \end{aligned} \tag{4.5}$$

where $\beta_1,\beta_2,\beta_3,\beta_4,\beta_5 > 0$, $\beta_2+\beta_5 \geqslant 1$, $\beta_3+\beta_4 \geqslant 1$, and $\beta_1+\beta_4+\beta_5 \geqslant 1$.

With two sequences $\{x_n\} = \left\{8+\frac{1}{n}\right\} n \in N$ and $\{y_n\} = \{4\}$ (or $\{x_n\} = \{4\}$, $\{y_n\} = \left\{8+\frac{1}{n}\right\} n \in N$, the pairs (P,S) and (Q,R) satisfy the (CLR_{SR}) property:

$$\lim_{n\to\infty} F_{M,N}(Px_n, 4, t) = \lim_{n\to\infty} F_{M,N}(Sx_n, 4, t) = \lim_{n\to\infty} F_{M,N}(Qy_n, 4, t) = \lim_{n\to\infty} F_{M,N}(Ry_n, 4, t) = 1_{L^*},$$

where $4 \in S(X) \cap R(X)$. Also, $P(X) = \{4, 19\} \nsubseteq [4, 16) \cup \{17\} = R(X)$ and $Q(X) = \{4, 12\} \nsubseteq [4, 10) = S(X)$. By usual enumeration, the inequality (4.5) can be verified easily, $\forall\, x, y \in X$.

As a result, all of Theorem 4.1's conditions have been met, and it has been demonstrated that the pairs (P, S) and (Q, R) have a common fixed point 4, which is also a coincidence point.

The concluding remark on the above proved main result: It should be noted that even at point 4, all the involved mappings are discontinuous. The subspace $S(X)$ and $R(X)$ are not closed subspaces of X, it is also pointed out.

Now we prove the following result (application of Theorem 4.1) which involves four finite families of self-mappings.

Theorem 4.3. *Let $\{P_i\}$ (for $i = 1$ to m), $\{Q_j\}$ (for $j = 1$ to n), $\{S_k\}$ (for $k = 1$ to p), and $\{R_l\}$ (for $l = 1$ to q) be finite families of self-mappings of a MIFM-Space $(X, F_(M, N), T)$ with $P = P_1P_2 \cdots P_m$, $Q = Q_1, Q_2 \cdots Q_n$, $S = S_1S_2 \cdots S_p$, and $R = R_1R_2 \cdots R_q$ satisfying the condition (4.1). Assume that the pairs (P, S) and (Q, R) enjoy the (CLR_{SR}) property; then (P, S) and (Q, R) have a point of coincidence. Furthermore, $\{P_i\}$ (for $i = 1$ to m), $\{Q_j\}$ (for $j = 1$ to n), $\{S_k\}$ (for $k = 1$ to p), and $\{R_l\}$ (for $l = 1$ to q) have a unique common fixed point if the families $(\{P_i\}, \{S_k\})$ and $(\{Q_j\}, \{R_l\})$ commute pairwise, wherein $i \in \{1, 2, \ldots, m\}, k \in \{1, 2, \ldots, p\}, j \in \{1, 2, \ldots, n\}$, and $l \in \{1, 2, \ldots, q\}$.*

Proof. This theorem's proof is like the one presented by Imdad et al. [23]. As a result, the proof of this theorem has been omitted. □

Remark 4.4. Similarly, Theorems 4.1 and 4.3 can be asserted and proved for another group of implicit functions $\chi \in X$ used by Tanveer et al. [46].

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