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Characterization of almost ternary subsemigroups and their fuzzifications

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Abstract

In this paper, we define almost ternary subsemigroups of ternary semigroups. Almost ternary subsemigroups is a one of generalizations of ternary subsemigroups. We show that the union of two almost ternary subsemigroups is also an almost ternary subsemigroup. However, the intersection of two almost ternary subsemigroups need not be an almost ternary subsemigroup. Moreover, we define their fuzzifications and provide the relationships between almost ternary subsemigroups and their fuzzifications.

Keywords: Almost ternary subsemigroups, fuzzy almost ternary subsemigroup, minimal, prime, semiprime.

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1. Introduction

Almost ideals of semigroups were first considered to investigate by Grošek and Satko [3] in 1980. Later, almost ideals were extended to study in other algebraic structures, for example, LA-semihypergroups [4, 10, 17, 18] , Γ-semigroups [11], semihypergroups [9, 12]. In 1965, Zadeh [19] introduced the concept of fuzzy sets. Fuzzy sets were applied to examine fuzzy almost ideals of semigroups [14]. A ternary semigroup is a nonempty set equipped with an associative ternary operation. The existence of ternary operations originated from the study of a ternary analogue of an abelian group in 1932 by Lehmer [6]. Every semigroup can be considered to be a ternary semigroup but some ternary semigroup cannot be considered to be a semigroup. The notion of ternary semigroups came from the problem of Banach who created an example of a ternary semigroup which is not reducible to a semigroup. In addition, he conjectured that every ternary semigroup may be extended to reducible to a semigroup ([7]). Loś [7] exposed Banach's problem and showed that the operation in the ternary semigroup is an extension of the binary operation satisfying associative law on some nonempty set. Then many properties of ternary semigroups

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have been extensively studied by many authors. Almost ideals and fuzzy almost ideals of ternary semigroups were studied in [13]. Recently, almost subsemigroups of semigroups and their fuzzifications were defined and studied in [5]. These are motivations to research this paper. Our purpose of this paper is to define the notion of almost ternary subsemigroups of ternary semigroups by using the concepts of almost subsemigroups of semigroups and the structure of ternary semigroups. Furthermore, we define the notion of fuzzy almost ternary subsemigroups of ternary semigroups and give relationships between almost ternary subsemigroups and fuzzy almost ternary subsemigroups of ternary semigroups. In Section 2, we begin to recall some definitions and examples which are used throughout the next sections. In Section 3, we give the main results of this paper. This section 3 is divided into two subsections. In subsection 3.1, we study on almost ternary subsemigroups and in subsection 3.2, we study on almost fuzzy ternary subsemigroups and give the relationships between almost ternary subsemigroups and almost fuzzy ternary subsemigroups.

2. Preliminaries

In this section, we provide some definitions and examples which are used throughout this paper.

2.1. *Ternary semigroups*

Definition 2.1. A ternary semigroup is a nonempty set T together with a ternary operation $(a, b, c) \mapsto [abc]$ satisfying the associative law

$$[[abc]uv] = [a[bcu]v] = [ab[cuv]], \quad \forall a, b, c, u, v \in T.$$

The following examples show that a ternary semigroup does not necessarily reduce an ordinary semigroup.

Example 2.2 (Banach's Example)**.** Let $T = \{-i, 0, i\}$. It is easy to see that T is a ternary semigroup under multiplication over complex numbers. Moreover, T is not a binary semigroup under multiplication over complex numbers.

Example 2.3. Let $\mathbb{Z}^-$ be the set of all negative integers. We have that $\mathbb{Z}^-$ is a ternary semigroup under multiplication over integer numbers. It is easy to see that $\mathbb{Z}^-$ is not a binary semigroup under multiplication over integer numbers.

Definition 2.4. An element e of a ternary semigroup T is called a selfpotent if $[eee] = e$.

For three nonempty subsets A, B and C of a ternary semigroup T, let

$$[ABC] = \{[abc] \mid a \in A, b \in B, c \in C\}.$$

Definition 2.5. A nonempty subset A of a ternary semigroup T is called a ternary subsemigroup of T if $[AAA] \subseteq A$.

2.2. *Fuzzy subsets*

A fuzzy subset of a set S is a membership function from S into $[0, 1]$. Many interesting results in fuzzy semigroups can be seen in [8]. For any two fuzzy subsets f and g of S,

1. $f \cap g$ is a fuzzy subset of S defined by

$$(f \cap g)(x) = \min\{f(x), g(x)\} = f(x) \wedge g(x),$$

for all $x \in S$;

2. $f \cup g$ is a fuzzy subset of S defined by

$$(f \cup g)(x) = \max\{f(x), g(x)\} = f(x) \vee g(x),$$

for all $x \in S$; and

3. $f \subseteq g$ if $f(x) \leqslant g(x)$, for all $x \in S$.

For a fuzzy subset f of S, the support of f is defined by

$$\mathrm{supp}(f) = \{x \in S \mid f(x) \neq 0\}.$$

The characteristic mapping of a subset A of S is a fuzzy subset of S defined by

$$C_A(x) = \begin{cases} 1, & x \in A, \\ 0, & x \notin A. \end{cases}$$

The concept of fuzzy semigroups were extended to many algebraic structures, one of those are the concept of fuzzy ternary semigroups [1, 2, 15, 16] which were defined as follows.

Definition 2.6. A fuzzy subset f of a ternary semigroup T is called a fuzzy ternary subsemigroup of T if $f([xyz]) \geqslant \min\{f(x), f(y), f(z)\}$ for all $x, y, z \in T$.

Definition 2.7. For any three fuzzy subsets f_1, f_2 and f_3 of a ternary semigroup T. The product $[f_1 \circ f_2 \circ f_3]$ of f_1, f_2 and f_3 is defined by

$$[f_1 \circ f_2 \circ f_3](y) = \begin{cases} \sup_{y=[y_1y_2y_3]} \min\{f_1(y_1), f_2(y_2), f_3(y_3)\}, & \text{if } y \in [TTT], \\ 0, & \text{otherwise.} \end{cases}$$

Then a fuzzy subset f of a ternary semigroup T is a fuzzy ternary subsemigroup of T if and only if $[f \circ f \circ f] \subseteq f$.

3. Main results

3.1. Almost ternary subsemigroups

In this subsection, we define almost ternary subsemigroups of ternary subsemigroups and give their remarkable properties.

Definition 3.1. A nonempty set A of a ternary semigroup T is called an almost ternary subsemigroup of T if $[AAA] \cap A \neq \emptyset$.

Let A be any ternary subsemigroup of a ternary semigroup T. Hence $[AAA] \subseteq A$. This implies that $[AAA] \cap A \neq \emptyset$. Thus A is an almost ternary subsemigroup of T. We can conclude that every ternary subsemigroup of a ternary semigroup T is an almost ternary subsemigroup of T.

Example 3.2. Consider a ternary semigroup $(\mathbb{Z}^-, \cdot)$, let $A = \{-2, -8, -9\}$ and $B = \{-8, -9, -512\}$. Clearly, A and B are almost ternary subsemigroups but are not ternary subsemigroups of $\mathbb{Z}^-$. However, $A \cap B = \{-8, -9\}$ is not an almost ternary subsemigroup of $\mathbb{Z}^-$.

By Example 3.2, we have the following conclusions.

(1) Any almost ternary subsemigroup of a ternary semigroup T need not be a ternary subsemigroup of T.

(2) The intersection of almost ternary subsemigroups of a ternary semigroup T need not be an almost ternary subsemigroup of T.

Theorem 3.3. *Let A and B be nonempty subsets of a ternary semigroup T such that $A \subseteq B$. If A is an almost ternary subsemigroup of T, then B is also an almost ternary subsemigroup of T.*

Proof. Assume that A is an almost ternary subsemigroup of T. Thus $[AAA] \cap A \neq \emptyset$. Since $A \subseteq B$, $[AAA] \cap A \subseteq [BBB] \cap B$. This implies that $[BBB] \cap B \neq \emptyset$. Hence, B is an almost ternary subsemigroup of T. □

The following corollary follows directly from Theorem 3.3.

Corollary 3.4. *The union of almost ternary subsemigroups of a ternary semigroup T is also an almost ternary subsemigroup of T.*

Proposition 3.5. *Let a be any element of a ternary semigroup T.*

(1) *If a is a selfpotent, then $\{a\}$ is an almost ternary subsemigroup of T.*

(2) *If a is not a selfpotent, then $\{a, [aaa]\}$ is an almost ternary subsemigroup of T.*

3.2. Fuzzy almost ternary subsemigroups

In this subsection, we define fuzzy almost ternary subsemigroups of ternary subsemigroups and give their remarkable properties.

Definition 3.6. A nonzero fuzzy subset f of a ternary semigroup T is called a fuzzy almost ternary subsemigroup of T if $[f \circ f \circ f] \cap f \neq 0$.

Let f be any fuzzy ternary subsemigroup of a ternary semigroup T. Thus $[f \circ f \circ f] \subseteq f$. This implies that $[f \circ f \circ f] \cap f \neq 0$, that is, f is a fuzzy almost ternary subsemigroup of T. We can conclude that every fuzzy ternary subsemigroup of a ternary semigroup T is a fuzzy almost ternary subsemigroup of T.

Example 3.7. Consider a ternary semigroup $(\mathbb{Z}^-, \cdot)$, let f and g be fuzzy subsets of $\mathbb{Z}^-$ as follows:

$$f(x) = \begin{cases} 0.5, & \text{if } x = -1, -8, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$g(x) = \begin{cases} 0.4, & \text{if } x = -8, -64, \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, f and g are fuzzy almost ternary subsemigroups but are not fuzzy ternary subsemigroups of $\mathbb{Z}^-$. However, $f \cap g$ is not a fuzzy almost ternary subsemigroup of $\mathbb{Z}^-$.

By Example 3.7, we obtain the following conclusions.

(1) Any fuzzy almost ternary subsemigroup of a ternary semigroup T need not be a fuzzy ternary subsemigroup of T.

(2) The intersection of fuzzy almost ternary subsemigroups of a ternary semigroup T need not be a fuzzy almost ternary subsemigroup of T.

Theorem 3.8. *Let f and g be fuzzy subsets of a ternary semigroup T such that $f \subseteq g$. If f is a fuzzy almost ternary subsemigroup of T, then g is also a fuzzy almost ternary subsemigroup of T.*

Proof. Assume that f is a fuzzy almost ternary subsemigroup of T. Then $[f \circ f \circ f] \cap f \neq 0$. Since $f \subseteq g$, $[f \circ f \circ f] \cap f \subseteq [g \circ g \circ g] \cap g$. This implies that $[g \circ g \circ g] \cap g \neq 0$. So the proof is completed. □

The following corollary follows directly from Theorem 3.8.

Corollary 3.9. *The union of fuzzy almost ternary subsemigroups of a ternary semigroup T is also a fuzzy almost ternary subsemigroup of T.*

3.3. *The relationships between almost ternary subsemigroups and their fuzzifications*

In this subsection, we show the relationships between almost ternary subsemigroups and fuzzy almost ternary subsemigroups of ternary semigroups.

Theorem 3.10. *Let A be a nonempty subset of a ternary semigroup T. Then A is an almost ternary subsemigroup of T if and only if C_A is a fuzzy almost ternary subsemigroup of T.*

Proof. Assume that A is an almost ternary subsemigroup of T. Then $[AAA] \cap A \neq \emptyset$. Thus there exists $x \in [AAA] \cap A$. So $x = [abc]$ for some $a, b, c \in A$ and $x \in A$. So $[C_A \circ C_A \circ C_A](x) = 1$ and $C_A(x) = 1$. Thus $([C_A \circ C_A \circ C_A] \cap C_A)(x) = 1 \neq 0$. Therefore $[C_A \circ C_A \circ C_A] \cap C_A \neq 0$. Hence, C_A is a fuzzy almost ternary subsemigroup of T.

Conversely, assume that C_A is a fuzzy almost ternary subsemigroup of T. Thus

$$[C_A \circ C_A \circ C_A] \cap C_A \neq 0.$$

Then there exists $x \in T$ such that $([C_A \circ C_A \circ C_A] \cap C_A)(x) \neq 0$. So $[C_A \circ C_A \circ C_A](x) \neq 0$ and $C_A(x) \neq 0$. This implies that $[C_A \circ C_A \circ C_A](x) = 1$ and $C_A(x) = 1$. Hence, $x \in [AAA]$ and $x \in A$. Eventually, $[AAA] \cap A \neq \emptyset$. Hence, A is an almost ternary subsemigroup of T. □

Theorem 3.11. *Let f be a fuzzy subset of a ternary semigroup T. Then f is a fuzzy almost ternary subsemigroup of T if and only if $supp(f)$ is an almost ternary subsemigroup of T.*

Proof. Assume that f is a fuzzy almost ternary subsemigroup of T. We have $[f \circ f \circ f] \cap f \neq 0$. Thus, there exists $x \in T$ such that $([f \circ f \circ f] \cap f)(x) \neq 0$. So, $f(x) \neq 0$ and $[f \circ f \circ f](x) \neq 0$. So $x = [abc]$ for some $a, b, c \in T$ such that $f(a) \neq 0, f(b) \neq 0$ and $f(c) \neq 0$. This implies that $a, b, c \in \text{supp}(f)$. Thus,

$$[C_{\text{supp}(f)} \circ C_{\text{supp}(f)} \circ C_{\text{supp}(f)}](x) \neq 0,$$

and

$$C_{\text{supp}(f)}(x) \neq 0.$$

Hence, $[C_{\text{supp}(f)} \circ C_{\text{supp}(f)} \circ C_{\text{supp}(f)}] \cap C_{\text{supp}(f)}](x) \neq 0$. So, $C_{\text{supp}(f)}$ is a fuzzy almost ternary subsemigroup of T. By Theorem 3.10, $\text{supp}(f)$ is an almost ternary subsemigroup of T.

Conversely, assume that $\text{supp}(f)$ is an almost ternary subsemigroup of T. By Theorem 3.10, we have $C_{\text{supp}(f)}$ is a fuzzy almost ternary subsemigroup of T. Then $[C_{\text{supp}(f)} \circ C_{\text{supp}(f)} \circ C_{\text{supp}(f)}] \cap C_{\text{supp}(f)} \neq 0$. Then there exists $x \in T$ such that $[C_{\text{supp}(f)} \circ C_{\text{supp}(f)} \circ C_{\text{supp}(f)}] \cap C_{\text{supp}(f)}](x) \neq 0$. Hence,

$$[C_{\text{supp}(f)} \circ C_{\text{supp}(f)} \circ C_{\text{supp}(f)}](x) \neq 0,$$

and

$$C_{\text{supp}(f)}(x) \neq 0.$$

Then there exist $a, b, c \in \text{supp}(f)$ and $x = [abc]$. Thus $f(a) \neq 0, f(b) \neq 0$ and $f(c) \neq 0$. Therefore,

$$[f \circ f \circ f](x) \neq 0.$$

This implies that $([f \circ f \circ f] \cap f)(x) \neq 0$. Consequently, f is a fuzzy almost ternary subsemigroup of T. □

Definition 3.12. An almost ternary subsemigroup A of a ternary semigroup T is minimal if for any almost ternary subsemigroup B of T such that $B \subseteq A$, we must have $B = A$.

Example 3.13. We have $\{-2, -3, -5, -30\}$ is a minimal almost ternary subsemigroup of a ternary semigroup $(\mathbb{Z}^-, \cdot)$.

Next, we examine the minimality of fuzzy almost ternary subsemigroups.

Definition 3.14. A fuzzy almost ternary subsemigroup f of a ternary semigroup T is called minimal if for any fuzzy almost ternary subsemigroup g of T contained in f, we must have $supp(g) = supp(f)$.

Now, we provide the relationship between minimal almost ternary subsemigroup and their fuzzifications.

Theorem 3.15. *A nonempty subset A of a ternary semigroup T is a minimal almost ternary subsemigroup of T if and only if C_A is a minimal fuzzy almost ternary subsemigroup of T.*

Proof. Let A be a minimal almost ternary subsemigroup of a ternary semigroup T. By Theorem 3.10, we have that C_A is a fuzzy almost ternary subsemigroup of T. Assume that g is a fuzzy almost ternary subsemigroup of T contained in C_A. Thus, $supp(g) \subseteq supp(C_A) = A$. Because of $g \subseteq C_{supp(g)}$, we have $[g \circ g \circ g] \cap g \subseteq (C_{supp(g)} \circ C_{supp(g)} \circ C_{supp(g)}) \cap C_{supp(g)}$. Thus $C_{supp(g)}$ is a fuzzy almost ternary subsemigroup of T. By Theorem 3.10, $supp(g)$ is an almost ternary subsemigroup of T. Because of A is minimal, then $supp(g) = A = supp(C_A)$. Therefore, C_A is minimal.

To prove the converse, assume that C_A is a minimal fuzzy almost ternary subsemigroup of T and B is an almost ternary subsemigroup of T contained in A. Then C_B is a fuzzy almost ternary subsemigroup of T and $C_B \subseteq C_A$. Thus, $B = supp(C_B) = supp(C_A) = A$. We conclude that A is minimal. □

Corollary 3.16. *A ternary semigroup T has no proper almost ternary subsemigroup if and only if for all fuzzy almost ternary subsemigroup f of T, $supp(f) = T$.*

Proof. Assume that T has no proper almost ternary subsemigroup and let f be a fuzzy almost ternary subsemigroup of T. By Theorem 3.11, we have $supp(f)$ is an almost ternary subsemigroup of T. Therefore $supp(f) = T$.

To prove the converse, we let B be any almost ternary subsemigroup of T. Follow by Theorem 3.10, we have that C_B is a fuzzy almost ternary subsemigroup of T. By assumption, we get $B = supp(C_B) = T$. This implies that T has no proper almost ternary subsemigroup. □

Next, we investigate a relationship between prime almost ternary subsemigroups and their fuzzifications.

Definition 3.17. Let T be a ternary semigroup.

(1) An almost ternary subsemigroup A of T is called prime if

$$[xyz] \in A \Rightarrow x \in A \text{ or } y \in A \text{ or } z \in A,$$

for any $x, y, z \in T$.

(2) A fuzzy almost ternary subsemigroup f of T is called prime if

$$f([xyz]) \leqslant \max\{f(x), f(y), f(z)\},$$

for any $x, y, z \in T$.

Example 3.4. We have $\{-2, -3, -5, -8, -27, -30\}$ is a prime almost ternary subsemigroup of a ternary semigroup $(\mathbb{Z}^-, \cdot)$.

Theorem 3.18. *A nonempty subset A of a ternary semigroup T is a prime almost ternary subsemigroup of T if and only if C_A is a prime fuzzy almost ternary subsemigroup of T.*

Proof. Let A be any prime almost ternary subsemigroup of T. Then C_A is a fuzzy almost ternary subsemigroup of T by Theorem 3.10. Let x, y, z be any three elements in T. If $[xyz] \in A$, then $x \in A$ or $y \in A$

or $z \in A$. This implies that

$$C_A([xyz]) = 1 \leqslant \max\{C_A(x), C_A(y), C_A(z)\}.$$

If $[xyz] \notin A$, then

$$C_A([xyz]) = 0 \leqslant \max\{C_A(x), C_A(y), C_A(z)\}.$$

We conclude that $C_A([xyz]) \leqslant \max\{C_A(x), C_A(y), C_A(z)\}$ for all $x, y, z \in T$. Therefore, C_A is a prime fuzzy almost ternary semigroup of T.

Conversely, suppose that C_A is a prime fuzzy almost ternary semigroup of T. By Theorem 3.10, we have that A is an almost ternary semigroup of T. Let x, y, z be elements in T such that $[xyz] \in A$. Thus, $C_A([xyz]) = 1$. By assumption, we have that $C_A([xyz]) \leqslant \max\{C_A(x), C_A(y), C_A(z)\}$. Therefore, $\max\{C_A(x), C_A(y), C_A(z)\} = 1$. We can conclude that $x \in A$ or $y \in A$ or $z \in A$. Hence, A is a prime almost ternary subsemigroup of T. □

Definition 3.19. Let T be a ternary semigroup.

(1) An almost ternary subsemigroup A of T is called semiprime if

$$[xxx] \in A \Rightarrow x \in A,$$

for all $x \in T$.

(2) A fuzzy almost subsemigroup f of T is called semiprime if

$$f([xxx]) \leqslant f(x),$$

for all $x \in T$.

It is obviously every prime almost ternary subsemigroup is semiprime.

Example 3.20. We have $I = \{-2, -8, -45\}$ is a semiprime almost ternary subsemigroup of a ternary semigroup $(\mathbb{Z}^-, \cdot)$ but not a prime almost ternary subsemigroup of $(\mathbb{Z}^-, \cdot)$ because $[(-3)(-3)(-5)] = -45 \in I$ but $-3, -5 \notin I$.

Finally, we give a relationship between semiprime almost ternary subsemigroups and their fuzzifications.

Theorem 3.21. *A nonempty subset A of a ternary semigroup T is a semiprime almost ternary subsemigroup of T if and only if C_A is a semiprime fuzzy almost ternary subsemigroup of T.*

Proof. Let A be a semiprime almost ternary subsemigroup of T. By Theorem 3.10, C_A is a fuzzy almost ternary subsemigroup of T. Let $x \in T$. If $[xxx] \in A$, then $x \in A$. So, $C_A(x) = 1$. Hence, $C_A([xxx]) \leqslant C_A(x)$. If $[xxx] \notin A$, then $C_A([xxx]) = 0 \leqslant C_A(x)$. By both cases, we conclude that $C_A([xxx]) \leqslant C_A(x)$ for all $x \in T$. Thus, C_A is a semiprime fuzzy almost ternary subsemigroup of T.

Conversely, assume that C_A is a semiprime fuzzy ternary subsemigroup of T. By Theorem 3.10, we have that A is an almost ternary subsemigroup of T. Let $x \in T$ be such that $[xxx] \in A$. Thus $C_A([xxx]) = 1$. By assumption, we have that $C_A([xxx]) \leqslant C_A(x)$. Since $C_A([xxx]) = 1$, it follows that $C_A(x) = 1$. Therefore, $x \in A$. Consequently, A is a semiprime almost ternary subsemigroup of T. □

4. Conclusion

In this paper, we define almost ternary subsemigroups and their fuzzifications of ternary semigroups. Every ternary semigroup is also an almost ternary semigroup but the converse is not true in general. We show that the union of two almost ternary subsemigroups is also an almost ternary subsemigroup.

However, it is not generally true in case the intersection. Similarly, we have that the union of two fuzzy almost ternary subsemigroup is also a fuzzy almost ternary subsemigroup but it is not generally true in case the intersection. Moreover, the relationships between almost ternary subsemigroups and their fuzzifications were shown.

In the future work, we would like to study other kinds of subalgebras and their fuzzifications in other algebraic structures.

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