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Interval-valued picture fuzzy hypersoft TOPSIS method based on correlation coefficient

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Abstract

In multi-criteria decision-making problems, we may have to deal with numbers that are in interval forms, like those of positive, negative, and neutral grades representing different attributes of elements. When decision-makers come across such an environment, the decisions are harder to make and the most significant factor is that we need to combine these interval numbers to generate a single real number, which can be done using aggregation operators or score functions. To overcome this hindrance, we introduce the notion of interval-valued picture fuzzy hypersoft set. This eventually helps the decision-maker to collect the data without any misconceptions. We present some properties of the correlation coefficient and aggregation operators on it. Also, we propose an algorithm for the technique of order of preference by similarity to ideal solution (TOPSIS) method based on correlation coefficients to choose a suitable employee among the available alternative using Leipzig leadership model in an organization for an upcoming new project. Finally, we present a comparative study with existing similarity measures to show the effectiveness of the proposed method.

Keywords: Picture fuzzy set, intuitionistic set, hypersoft set.

2020 MSC: 03E72, 03F55.

1. Introduction

The concept of fuzzy set (FS) theory was introduced by Zadeh [29] and since then it has been extensively used in various fields. The idea of intuitionistic FS (IFS) was presented by Atanassov [3], an extension of FS. Bui Cong Cuong [8] defined the concepts of picture FS (PFS) and interval-valued PFS (IVPFS) characterized by the values of positive, neutral, and negative grades for each element of the set. Molodtsov [21] introduced the idea of soft set (SS) to deal with uncertainties. Smarandache [25] presented the notion of hypersoft set (HSS) to overcome the restriction faced in SS.

Esi and Hazarika [11] established and studied the properties of convergent double interval numbers. Esi [9] presented the concept of convergence of interval numbers. Esi and Braha [10] defined the notion

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of statistical equivalent sequences of interval numbers. Hazarika and Esi [15] introduced the concept of convergent interval valued by using Orlicz function. Khalil et al. [17] defined some operations and relations over IVPFS. Wei [28] presented a method to measure the similarity between PFS. Kamaci et al. [16] introduced Einstein operators to aggregate the information of interval-valued picture hesitant fuzzy set. Ashraf et al. [2] proposed a method to aggregate values for cubic PFS. Garg and Kumar [14] defined the concept of linguistic interval-valued Atanassov intuitionistic fuzzy set, whose positive and negative degrees are in linguistic terms. Son et al. [26] introduced the concept of inference system on PFS to improve the traditional fuzzy method. Wei [27] investigated the multiple-criteria decision-making (MCDM) problems with PFS. Khan et al. [19] studied some of the basic properties of picture fuzzy soft sets. Garg [12] discussed some picture fuzzy aggregation operators (PFAO) and used them to solve MCDM problems. Qiyas et al. [23] utilized PFAO to solve MCDM problems which are given in linguistic terms. Chinnadurai et al. [6] proposed the idea of finding unique ranking among the alternatives by using parameters in MCDM problems. Khan et al. [18] developed neutrosophic cubic generalized unified aggregation and neutrosophic cubic quasi-generalized unified aggregation operators to solve MCDM problems. Ayele et al. [4] presented a method for traffic signal control using interval-valued neutrosophic soft set. Musa and Asaad [22] studied the properties of bipolar hypersoft set by combining HSSs and bipolarity. Ajay et al. [1] studied the properties of neutrosophic semiopen and closed HSSs. Zulqarnain et al. [30] developed a new score function to solve MCDM problems. Rahman et al. [24] presented the notion of neutrosophic parameterized soft set for dealing with MCDM problems. Christianto and Smarandache [7] proposed the idea of a third-way leadership model, a blend of hard-style and soft-style leadership. Garg et al. [13] used a combination of TOPSIS and Choquet integral method in hesistant FS to solve MCDM problems. Zulqarnain et al. [31] introduced the concept of intuitionistic fuzzy HSS and used the TOPSIS method based on correlation coefficient (CC). Chinnadurai and Bobin [5] discussed the concept of Pythagorean fuzzy hypersoft set and used the aggregation operators to solve MCDM problems.

The main aim of the present study is to rank the alternatives based on interval-valued picture fuzzy hypersoft set (IVPFHSS) data using aggregation operators and also making use of TOPSIS method based on CC. Also, we show the significance of CC method by comparing it with similarity measures. To the best of our knowledge, research on IVPFHSS is confined to its theory and related development and applications. Therefore, the new method proposed in this paper can examine and provide a suitable solution to the decision-makers in ranking the alternatives. We present a MCDM approach based on TOPSIS, and the effectiveness of this method is demonstrated through the selection of a suitable employee who can lead the project successfully. To prove the efficacy of the proposed method, a comparative analysis between the proposed and existing similarity measures (SMs) is given. Thus, IVPFHSS turns out to be a robust tool to predict uncertainties when the grades are in interval form for all positive, neutral and negative grades for all the attributes.

The manuscript consists of the following sections. Section 2 briefs on existing definitions. Section 3 introduces the concept of IVPFHSS and discusses some properties of CC and weighted CC of IVPFHSS. Section 4 deals with interval-valued picture fuzzy hypersoft weighted average operator (IVPFHSWAO) and interval-valued picture fuzzy hypersoft weighted geometric operator (IVPFHSWGO). Section 5 highlights the combination of CC with TOPSIS method. Section 6 shows the significance of the proposed method with comparative analysis. Section 7 ends with a conclusion.

2. Preliminaries

We present some of the basic definitions required for this study. Let us consider the following notations throughout this study unless otherwise specified. Let $\mathcal{V}$ be the universe and $\nu_i \in \mathcal{V}$, $P(\mathcal{V})$ be the power set of $\mathcal{V}$, $\mathbb{N}$ represents natural numbers, $C[0,1]$ denotes the set of all closed sub intervals of [0,1] and $\mathcal{N}^{\mathcal{U}}$ represent the collection of interval-valued picture fuzzy set (IVPFS) over $\mathcal{V}$.

Definition 2.1 ([29])**.** A fuzzy set (FS) is a set of the form $\mathcal{F} = \{(\nu, \mathcal{P}_{\mathcal{F}}(\nu)) : \nu \in \mathcal{V}\}$, where $\mathcal{P}_{\mathcal{F}}(\nu) : \mathcal{V} \to [0,1]$ defines the degree of membership of the element $\nu \in \mathcal{V}$.

Definition 2.2 ([3]). An intuitionistic FS (IFS) is an object of the form $\mathcal{C} = \{(\nu, \mathcal{P}_{\mathcal{C}}(\nu), \mathcal{N}_{\mathcal{C}}(\nu)) : \nu \in \mathcal{V}\}$, where $\mathcal{P}_{\mathcal{C}}(\nu) : \mathcal{V} \to [0,1]$ and $\mathcal{N}_{\mathcal{C}}(\nu) : \mathcal{V} \to [0,1]$ define the degree of membership and degree of non-membership of the element $\nu \in \mathcal{V}$, respectively and for every $\nu \in \mathcal{V}$, $0 \leqslant \mathcal{P}_{\mathcal{C}}(\nu) + \mathcal{N}_{\mathcal{C}}(\nu) \leqslant 1$, where $\pi_{\mathcal{C}}(\nu) = 1 - \mathcal{P}_{\mathcal{C}}(\nu) - \mathcal{N}_{\mathcal{C}}(\nu)$.

Definition 2.3 ([8]). A PFS in $\mathcal{V}$ is an object of the form $\Omega = \{\langle \nu, \mathcal{P}_{\Omega}(\nu), \mathcal{E}_{\Omega}(\nu), \mathcal{N}_{\Omega}(\nu)\rangle\}$, where $\mathcal{P}_{\Omega}(\nu)$, $\mathcal{E}_{\Omega}(\nu), \mathcal{N}_{\Omega}(\nu) : \mathcal{V} \to [0,1]$, are the membership values of positive, neutral and negative of the element $\nu \in \mathcal{V}$, respectively, such that $0 \leqslant \mathcal{P}_{\Omega}(\nu) + \mathcal{E}_{\Omega}(\nu) + \mathcal{N}_{\Omega}(\nu) \leqslant 1$ and the degree of refusal membership is $1 - (\mathcal{P}_{\Omega}(\nu) + \mathcal{E}_{\Omega}(\nu) + \mathcal{N}_{\Omega}(\nu))\ \forall\, \nu \in \mathcal{V}$.

Definition 2.4 ([21]). A pair $(\mathcal{O}, \mathcal{E})$ is called a soft set (SS) over $\mathcal{V}$, if $\mathcal{O} : \mathcal{E} \to \mathcal{P}(\mathcal{V})$. Then for any $p \in \mathcal{E}$, $\mathcal{O}(p) = 1$ is equivalent to $\nu \in \mathcal{O}(p)$ and $\mathcal{O}(p) = 0$ is equivalent to $\nu \notin \mathcal{O}(p)$. Thus a SS is not a set, but a parameterized family of subsets of $\mathcal{V}$.

Definition 2.5 ([8]). An IVPFS is an object of the form $\Omega = \{\langle \nu, \mathcal{P}_{\tilde{\Omega}}(\nu), \mathcal{E}_{\tilde{\Omega}}(\nu), \mathcal{N}_{\tilde{\Omega}}(\nu)\rangle\}$, where $\mathcal{P}_{\tilde{\Omega}}(\nu) : \mathcal{V} \to C[0,1]$, $\mathcal{E}_{\tilde{\Omega}}(\nu) : \mathcal{V} \to C[0,1]$ and $\mathcal{N}_{\tilde{\Omega}}(\nu) : \mathcal{V} \to C[0,1]$. $\mathcal{P}_{\tilde{\Omega}}(\nu)$, $\mathcal{E}_{\tilde{\Omega}}(\nu)$ and $\mathcal{N}_{\tilde{\Omega}}(\nu)$ are closed sub intervals of [0,1], representing the membership grades of positive, neutral and negative of the element $\nu \in \mathcal{V}$. The lower and upper ends of $\mathcal{P}_{\tilde{\Omega}}(\nu)$, $\mathcal{E}_{\tilde{\Omega}}(\nu)$ and $\mathcal{N}_{\tilde{\Omega}}(\nu)$ are denoted, respectively by $\underline{\mathcal{P}}_{\Omega}(\nu)$, $\overline{\mathcal{P}}_{\Omega}(\nu)$, $\underline{\mathcal{E}}_{\Omega}(\nu)$, $\overline{\mathcal{E}}_{\Omega}(\nu)$, and $\underline{\mathcal{N}}_{\Omega}(\nu)$, $\overline{\mathcal{N}}_{\Omega}(\nu)$, where $0 \leqslant \overline{\mathcal{P}}_{\Omega}(\nu) + \overline{\mathcal{E}}_{\Omega}(\nu) + \overline{\mathcal{N}}_{\Omega}(\nu) \leqslant 1$. Similarly, the lower and upper ends of refusal grades are $\underline{\mathcal{R}}_{\Omega}(\nu) = 1 - (\underline{\mathcal{P}}_{\Omega}(\nu) + \underline{\mathcal{E}}_{\Omega}(\nu) + \underline{\mathcal{N}}_{\Omega}(\nu))$ and $\overline{\mathcal{R}}_{\Omega}(\nu) = 1 - (\overline{\mathcal{P}}_{\Omega}(\nu) + \overline{\mathcal{E}}_{\Omega}(\nu) + \overline{\mathcal{N}}_{\Omega}(\nu))$, $\forall\ \nu \in \mathcal{V}$.

Definition 2.6 ([25]). Let $\Delta_1, \Delta_2, \ldots, \Delta_k$, be distinct attribute sets, whose corresponding sub-attributes are $\Delta_1 = \{\lambda_{11}, \lambda_{12}, \ldots, \lambda_{1f}\}, \Delta_2 = \{\lambda_{21}, \lambda_{22}, \ldots, \lambda_{2g}\}, \ldots, \Delta_k = \{\lambda_{k1}, \lambda_{k2}, \ldots, \lambda_{kh}\}$, where $1 \leqslant f \leqslant p$, $1 \leqslant g \leqslant q$, $1 \leqslant h \leqslant r$ and $p,\ q,\ r \in \mathbb{N}$, such that $\Delta_i \cap \Delta_j = \emptyset$, for each $i, j \in \{1, 2, \ldots, k\}$ and $i \neq j$. Then the Cartesian product of the distinct attribute sets $\Delta_1 \times \Delta_2 \times \cdots \times \Delta_k = \tilde{\Delta} = \{\lambda_{1f} \times \lambda_{2g} \times \cdots \times \lambda_{kh}\}$, represent a collection of multi- attributes. The pair $(\Omega, \tilde{\Delta})$ is called a hypersoft set (HSS) over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to P(\mathcal{V})$. HSS can be represented as $(\Omega, \tilde{\Delta}) = \{(\tilde{\lambda}, \Omega(\tilde{\lambda})) | \tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in P(\mathcal{V})\}$.

3. Interval-valued picture fuzzy hypersoft set

We present the notion of interval-valued picture fuzzy hypersoft set (IVPFHSS). Also, we discuss some basic properties of correlation coefficient (CC) and weighted CC (WCC) on IVPFHSS.

Definition 3.1. A pair $(\Omega, \tilde{\Delta})$ is called an IVPFHSS over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to \mathcal{N}^{u}$. IVPFHSS can be represented as $(\Omega, \tilde{\Delta}) = \{(\tilde{\lambda}, \Omega(\tilde{\lambda})) | \tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in \mathcal{N}^{u} \in C[0,1]\}$, where $\Omega(\tilde{\lambda}) = \{\langle \nu, \mathcal{P}_{\Omega(\tilde{\lambda})}(\nu), \mathcal{E}_{\Omega(\tilde{\lambda})}(\nu), \mathcal{N}_{\Omega(\tilde{\lambda})}(\nu)\rangle | \nu \in \mathcal{V}\}$, where $\mathcal{P}_{\Omega(\tilde{\lambda})}(\nu) : \mathcal{V} \to C[0,1]$, $\mathcal{E}_{\Omega(\tilde{\lambda})}(\nu) : \mathcal{V} \to C[0,1]$ and $\mathcal{N}_{\Omega(\tilde{\lambda})}(\nu) : \mathcal{V} \to C[0,1]$. $\mathcal{P}_{\Omega(\tilde{\lambda})}(\nu)$, $\mathcal{E}_{\Omega(\tilde{\lambda})}(\nu)$ and $\mathcal{N}_{\Omega(\tilde{\lambda})}(\nu)$ are closed sub intervals of [0,1], representing the membership grades of positive, neutral and negative. The lower and upper ends of $\mathcal{P}_{\Omega(\tilde{\lambda})}(\nu)$, $\mathcal{E}_{\Omega(\tilde{\lambda})}(\nu)$ and $\mathcal{N}_{\Omega(\tilde{\lambda})}(\nu)$ are denoted, respectively by $\underline{\mathcal{P}}_{\Omega(\tilde{\lambda})}(\nu)$, $\overline{\mathcal{P}}_{\Omega(\tilde{\lambda})}(\nu)$, $\underline{\mathcal{E}}_{\Omega(\tilde{\lambda})}(\nu)$, $\overline{\mathcal{E}}_{\Omega(\tilde{\lambda})}(\nu)$, and $\underline{\mathcal{N}}_{\Omega(\tilde{\lambda})}(\nu)$, $\overline{\mathcal{N}}_{\Omega(\tilde{\lambda})}(\nu)$, where $0 \leqslant \overline{\mathcal{P}}_{\Omega(\tilde{\lambda})}(\nu) + \overline{\mathcal{E}}_{\Omega(\tilde{\lambda})}(\nu) + \overline{\mathcal{N}}_{\Omega(\tilde{\lambda})}(\nu) \leqslant 1$. Similarly, the lower and upper ends of refusal grades are $\underline{\mathcal{R}}_{\Omega(\tilde{\lambda})}(\nu) = 1 - (\underline{\mathcal{P}}_{\Omega(\tilde{\lambda})}(\nu) + \underline{\mathcal{E}}_{\Omega(\tilde{\lambda})}(\nu) + \underline{\mathcal{N}}_{\Omega(\tilde{\lambda})}(\nu))$ and $\overline{\mathcal{R}}_{\Omega(\tilde{\lambda})}(\nu) = 1 - (\overline{\mathcal{P}}_{\Omega(\tilde{\lambda})}(\nu) + \overline{\mathcal{E}}_{\Omega(\tilde{\lambda})}(\nu) + \overline{\mathcal{N}}_{\Omega(\tilde{\lambda})}(\nu))$.

Example 3.2. Let $\mathcal{V} = \{m_1, m_2, m_3\}$ be a set of managers who evaluate an employee based on the Leipzig leadership model for an upcoming project. Let Δ_1, Δ_2, Δ_3 and Δ_4 be distinct attribute sets whose corresponding sub-attributes are represented as Δ_1 = purpose = $\{\lambda_{11}$ = achieve goals$\}$, Δ_2 = entrepreneurial spirit = $\{\lambda_{21}$ = quick decision, λ_{22} = logical decision$\}$, Δ_3 = responsibility = $\{\lambda_{31}$ = inspire and motivate, λ_{32} = time management$\}$ and Δ_4 = effectiveness = $\{\lambda_{41}$ = successful accomplishment$\}$. Then $\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4$ is the distinct attribute set given by

$$\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4 = \{\lambda_{11}\} \times \{\lambda_{21}, \lambda_{22}\} \times \{\lambda_{31}, \lambda_{32}\} \times \{\lambda_{41}\}$$

$$=\left\{(\lambda_{11},\lambda_{21},\lambda_{31},\lambda_{41}),(\lambda_{11},\lambda_{21},\lambda_{32},\lambda_{41}),(\lambda_{11},\lambda_{22},\lambda_{31},\lambda_{41}),(\lambda_{11},\lambda_{22},\lambda_{32},\lambda_{41})\right\}$$
$$=\left\{\tilde{\lambda_1},\tilde{\lambda_2},\tilde{\lambda_3},\tilde{\lambda_4}\right\}.$$

An IVPFHSS $(\Omega,\tilde{\Delta})$ is a collection of subsets of $\mathcal{V}$, given by the managers for each employee is presented in Table 1.

Table 1: Leadership skills of an employee in IVPFHSS $(\Omega,\tilde{\Delta})$ form.

$\mathcal{V}$	$\tilde{\lambda_1}$	$\tilde{\lambda_2}$
m_1	$\langle[0.05,0.08],[0.25,0.35],[0.23,0.28]\rangle$	$\langle[0.16,0.18],[0.21,0.25],[0.32,0.37]\rangle$
m_2	$\langle[0.32,0.42],[0.21,0.22],[0.02,0.05]\rangle$	$\langle[0.43,0.44],[0.23,0.24],[0.07,0.12]\rangle$
m_3	$\langle[0.13,0.17],[0.19,0.29],[0.39,0.49]\rangle$	$\langle[0.26,0.36],[0.24,0.34],[0.16,0.23]\rangle$

$\mathcal{V}$	$\tilde{\lambda_3}$	$\tilde{\lambda_4}$
m_1	$\langle[0.25,0.32],[0.42,0.45],[0.12,0.13]\rangle$	$\langle[0.23,0.31],[0.31,0.33],[0.32,0.33]\rangle$
m_2	$\langle[0.39,0.41],[0.32,0.34],[0.21,0.23]\rangle$	$\langle[0.24,0.32],[0.41,0.43],[0.23,0.25]\rangle$
m_3	$\langle[0.31,0.34],[0.21,0.25],[0.26,0.29]\rangle$	$\langle[0.31,0.33],[0.23,0.26],[0.31,0.37]\rangle$

3.1. Correlation coefficient for IVPFHSS

Let the two IVPFHSS over $\mathcal{V}$ be as given below.

$$(\Omega_1,\tilde{\Delta}_1)=\left\{(\nu_i,[\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i),\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)],[\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i),\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)],[\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i),\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)]\right\},$$
$$(\Omega_2,\tilde{\Delta}_2)=\left\{(\nu_i,[\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i),\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)],[\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i),\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)],[\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i),\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)]\right\}.$$

Definition 3.3. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVPFHSS. Then the interval-valued picture fuzzy informational energies of $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ are represented as

$$\begin{aligned}\Phi(\Omega_1,\tilde{\Delta}_1)=\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[&(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\\&+(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\Big],\\ \Phi(\Omega_2,\tilde{\Delta}_2)=\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[&(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\\&+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big].\end{aligned}$$

Definition 3.4. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVPFHSS. Then the correlation measure between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ is defined as

$$\begin{aligned}\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[&(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))*(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))+(\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))*(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))\\&+(\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))*(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))+(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))*(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))\\&+(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))*(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))+(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))*(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))\Big].\end{aligned}$$

Proposition 3.5. *Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then,*

(i) $\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_1, \tilde{\Delta}_1)) = \Phi(\Omega_1, \tilde{\Delta}_1)$;
(ii) $\mathcal{C}_{\mathcal{M}}((\Omega_2, \tilde{\Delta}_2), (\Omega_2, \tilde{\Delta}_2)) = \Phi(\Omega_2, \tilde{\Delta}_2)$.

Proof. Straight forward □

Definition 3.6. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then, the CC between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ is given by

$$\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1, \tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2, \tilde{\Delta}_2)}}. \tag{3.1}$$

Proposition 3.7. *Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then, the following CC properties hold:*

(i) $0 \leqslant \mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leqslant 1$;
(ii) $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \mathcal{C}_C((\Omega_2, \tilde{\Delta}_2), (\Omega_1, \tilde{\Delta}_1))$;
(iii) *if* $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$, *then* $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1$.

Proof.

(i) Obviously, $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \geqslant 0$. Now, we present the proof of $\mathcal{C}_C\big((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)\big) \leqslant 1$.

$$\begin{aligned}
&\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \\
&= \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\
&\quad + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\
&\quad + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))\Big] \\
&= \sum_{k=1}^{m}\Bigg[\bigg((\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) \\
&\quad * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) \\
&\quad + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))\bigg) + \bigg((\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) \\
&\quad * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) \\
&\quad + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))\bigg) + \cdots \\
&\quad + \bigg((\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) \\
&\quad * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) \\
&\quad + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))\bigg)\Bigg].
\end{aligned}$$

By applying Cauchy-Schwarz inequality, we get

$$\begin{aligned}
\mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))^2 \leqslant \sum_{k=1}^{m}\Bigg[&\Big\{(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
&+ \Big\{(\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
&+ \Big\{(\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\}
\end{aligned}$$

$$+\left\{(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2+(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\right\}$$
$$+\left\{(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2+(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\right\}$$
$$+\left\{(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2+(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\right\}\Bigg]$$
$$\times\sum_{k=1}^{m}\Bigg[\left\{(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2+(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\right\}$$
$$+\left\{(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\right\}$$
$$+\left\{(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\right\}$$
$$+\left\{(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\right\}$$
$$+\left\{(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\right\}$$
$$+\left\{(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2+\cdots+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\right\}\Bigg],$$
$$\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2\leqslant\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2$$
$$+(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\Big]\times\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2$$
$$+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big],$$
$$\Rightarrow\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2\leqslant\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)$$
$$\Rightarrow\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leqslant\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\times\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}$$
$$\Rightarrow\frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\times\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}}\leqslant 1.$$

By using Definition 3.6, we get $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leqslant 1$. Hence, $0\leqslant\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leqslant 1$.

(ii) Straight forward.

(iii) $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\times\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}}$. Since, $(\Omega_1,\tilde{\Delta}_1)=(\Omega_2,\tilde{\Delta}_2)$,

$$\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\frac{\begin{array}{c}\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\\+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\end{array}}{\begin{array}{c}\sqrt{\Big\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Big\}}\\\times\sqrt{\Big\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Big\}}\end{array}}.$$
$$\Rightarrow\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=1.$$
□

Definition 3.8. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then, the CC between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2 \tilde{\Delta}_2)$ is defined as

$$\tilde{\mathcal{C}}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\max\ \left\{\Phi(\Omega_1, \tilde{\Delta}_1), \Phi(\Omega_2, \tilde{\Delta}_2)\right\}}, \tag{3.2}$$

$$\begin{aligned}
&\tilde{\mathcal{C}}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \\
&= \frac{\begin{aligned}\sum_{k=1}^m \sum_{i=1}^n \Big[&(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\ &+ (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\ &+ (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))\Big]\end{aligned}}{\begin{aligned}\max \Big\{ &\sum_{k=1}^m \sum_{i=1}^n \Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 \\ &+ (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\Big], \\ &\sum_{k=1}^m \sum_{i=1}^n \Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 \\ &+ (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Big\}.\end{aligned}}
\end{aligned}$$

Proposition 3.9. *Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then, the following CC properties hold:*

(i) $0 \leqslant \tilde{\mathcal{C}}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leqslant 1$;
(ii) $\tilde{\mathcal{C}}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \tilde{\mathcal{C}}_C((\Omega_2, \tilde{\Delta}_2), (\Omega_1, \tilde{\Delta}_1))$;
(iii) *if* $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$, *then* $\tilde{\mathcal{C}}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1$.

Proof.

(i) Obviously, $\tilde{\mathcal{C}}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \geqslant 0$. Now, we present the proof of $\tilde{\mathcal{C}}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leqslant 1$.

$$\begin{aligned}
\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \sum_{k=1}^m \sum_{i=1}^n \Big[&(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\
&+ (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\
&+ (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))\Big] \\
= \sum_{k=1}^m \Big[\Big(&(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) \\
&+ (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) \\
&+ (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))\Big) \\
&+ \Big((\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) \\
&+ (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) \\
&+ (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))\Big) + \cdots \\
&+ \Big((\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))
\end{aligned}$$

$$+ (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))$$
$$+ (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))\Big)\Big].$$

By applying Cauchy-Schwarz inequality, we get

$$\begin{aligned}
\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leqslant \Bigg\{ \sum_{k=1}^{m} \Bigg[& \Big\{(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_1))^2 + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_n))^2\Big\}\Bigg] \\
\times \sum_{k=1}^{m} \Bigg[& \Big\{(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2 + (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2 + (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2 + (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\Big\} \\
& + \Big\{(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_1))^2 + (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_2))^2 + \cdots + (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_n))^2\Big\}\Bigg]\Bigg\}^{\frac{1}{2}},
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leqslant \Bigg\{ & \sum_{k=1}^{m}\sum_{i=1}^{n} \Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 \\
& + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\Big] \\
& \times \sum_{k=1}^{m}\sum_{i=1}^{n} \Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 \\
& + (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Bigg\}^{\frac{1}{2}} \\
\leqslant \Bigg\{\Bigg(\max\Bigg\{ & \sum_{k=1}^{m}\sum_{i=1}^{n} \Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 \\
& + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\Big]
\end{aligned}$$

$$\times \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2$$

$$+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Big\}\Big)^2\Big\}^{\frac{1}{2}}$$

$$=\max\Big\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2$$

$$+(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\Big]\times\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2$$

$$+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Big\}$$

$$\Rightarrow \mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leqslant\max\Big\{\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)\Big\}$$

$$\Rightarrow \frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\Big\{\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)\Big\}}\leqslant 1.$$

By using Definition 3.8, we get $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leqslant 1$. Hence, $0\leqslant\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leqslant 1$.

Proofs of (ii) and (iii) are same as in Proposition 3.7. □

3.2. *Weighted correlation coefficient for IVPFHSS*

We present the concept of weighted correlation coefficient (WCC) for IVPFHSS. WCC facilitates decision-makers (DMs) to provide different weights for each alternative. Consider $\mathcal{D}=\{\mathcal{D}_1,\mathcal{D}_2,\dots,\mathcal{D}_m\}$ and $\mathcal{W}=\{\mathcal{W}_1,\mathcal{W}_2,\dots,\mathcal{W}_n\}$ as weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k,\mathcal{W}_i>0$ and $\sum_{k=1}^{m}\mathcal{D}_k=1$, $\sum_{i=1}^{n}\mathcal{W}_i=1$.

Definition 3.10. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVPFHSS. Then, the WCC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}},\tag{3.3}$$

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\frac{\begin{array}{l}\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)*\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)+\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)*\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)\\ \quad+\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)*\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)+\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)*\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)\\ \quad+\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)*\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)+\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)*\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)\Big]\Big)\end{array}}{\begin{array}{l}\sqrt{\Big\{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2}\\ \quad\overline{+(\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Big)\Big\}}\\ \quad\times\sqrt{\Big\{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2}\\ \quad\overline{+(\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2\Big]\Big)\Big\}}\end{array}}.$$

If $\mathcal{D}=\{\frac{1}{m},\frac{1}{m},\dots,\frac{1}{m},\}$ and $\mathcal{W}=\{\frac{1}{n},\frac{1}{n},\dots,\frac{1}{n},\}$, then WCC given in Eq. (3.3) reduces to CC as in Eq. (3.1).

Proposition 3.11. *Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then, the following WCC properties hold:*

(i) $0 \leqslant \mathcal{C}_{C_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leqslant 1$;
(ii) $\mathcal{C}_{C_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \mathcal{C}_{C_W}((\Omega_2, \tilde{\Delta}_2), (\Omega_1, \tilde{\Delta}_1))$;
(iii) *if* $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$, *then* $\mathcal{C}_{C_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1$.

Proof. Similar to Proposition 3.7. □

Definition 3.12. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then, the WCC between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2 \tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{\tilde{C}_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\max\ \{\Phi(\Omega_1, \tilde{\Delta}_1), \Phi(\Omega_2, \tilde{\Delta}_2)\}}, \tag{3.4}$$

$$\mathcal{C}_{\tilde{C}_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\begin{array}{l}\sum_{k=1}^{m} \mathcal{D}_k \Big(\sum_{i=1}^{n} \mathcal{W}_i \Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\ \quad + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \\ \quad + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i)) * (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i)) \Big] \Big)\end{array}}{\begin{array}{l}\max \Big\{ \sum_{k=1}^{m} \mathcal{D}_k \Big(\sum_{i=1}^{n} \mathcal{W}_i \Big[(\underline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 \\ \quad + (\overline{\mathcal{P}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{E}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{N}}_{\Omega_1(\tilde{\lambda}_k)}(\nu_i))^2 \Big] \Big), \\ \sum_{k=1}^{m} \mathcal{D}_k \Big(\sum_{i=1}^{n} \mathcal{W}_i \Big[(\underline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\underline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 \\ \quad + (\overline{\mathcal{P}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{E}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 + (\overline{\mathcal{N}}_{\Omega_2(\tilde{\lambda}_k)}(\nu_i))^2 \Big] \Big) \Big\}.\end{array}}$$

If $\mathcal{D} = \{\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}, \}$ and $\mathcal{W} = \{\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}, \}$, then WCC given in Eq. (3.4) reduces to CC as in Eq. (3.2).

Proposition 3.13. *Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVPFHSS. Then, the following WCC properties hold:*

(i) $0 \leqslant \mathcal{C}_{\tilde{C}_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leqslant 1$;
(ii) $\mathcal{C}_{\tilde{C}_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \mathcal{C}_{\tilde{C}_W}((\Omega_2, \tilde{\Delta}_2), (\Omega_1, \tilde{\Delta}_1))$;
(iii) *if* $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$, *then* $\mathcal{C}_{\tilde{C}_W}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1$.

Proof. Similiar to Proposition 3.7. □

4. Aggregation operators for IVPFHSS

We now present the concept of interval-valued picture fuzzy hypersoft weighted average operator(IVPFHSWAO) and interval-valued picture fuzzy hypersoft weighted geometric operator (IVPFHSWGO) by using operational laws. Let κ represent the collection of interval-valued picture fuzzy hypersoft numbers (IVPFHSNs).

4.1. Operational laws for IVPFHSS

Definition 4.1. Let $\Omega_{e_{11}} = \langle [\underline{\mathcal{P}}_{11}, \overline{\mathcal{P}}_{11}], [\underline{\mathcal{E}}_{11}, \overline{\mathcal{E}}_{11}], [\underline{\mathcal{N}}_{11}, \overline{\mathcal{N}}_{11}] \rangle$ and $\Omega_{e_{12}} = \langle [\underline{\mathcal{P}}_{12}, \overline{\mathcal{P}}_{12}], [\underline{\mathcal{E}}_{12}, \overline{\mathcal{E}}_{12}], [\underline{\mathcal{N}}_{12}, \overline{\mathcal{N}}_{12}] \rangle$ be two IVPFHSS and δ a positive integer. Then,

(i) $\Omega_{e_{11}} \oplus \Omega_{e_{12}} = \langle [\underline{\mathcal{P}}_{11} + \underline{\mathcal{P}}_{12} - \underline{\mathcal{P}}_{11}\underline{\mathcal{P}}_{12}, \overline{\mathcal{P}}_{11} + \overline{\mathcal{P}}_{12} - \overline{\mathcal{P}}_{11}\overline{\mathcal{P}}_{12}], [\underline{\mathcal{E}}_{11} + \underline{\mathcal{E}}_{12} - \underline{\mathcal{E}}_{11}\underline{\mathcal{E}}_{12}, \overline{\mathcal{E}}_{11} + \overline{\mathcal{E}}_{12} - \overline{\mathcal{E}}_{11}\overline{\mathcal{E}}_{12}], [\underline{\mathcal{N}}_{11}\underline{\mathcal{N}}_{12}, \overline{\mathcal{N}}_{11}\overline{\mathcal{N}}_{12}] \rangle$;
(ii) $\Omega_{e_{11}} \otimes \Omega_{e_{12}} = \langle [\underline{\mathcal{P}}_{11}\underline{\mathcal{P}}_{12}, \overline{\mathcal{P}}_{11}\overline{\mathcal{P}}_{12}], [\underline{\mathcal{E}}_{11}\underline{\mathcal{E}}_{12}, \overline{\mathcal{E}}_{11}\overline{\mathcal{E}}_{12}], [\underline{\mathcal{N}}_{11} + \underline{\mathcal{N}}_{12} - \underline{\mathcal{N}}_{11}\underline{\mathcal{N}}_{12}, \overline{\mathcal{N}}_{11} + \overline{\mathcal{N}}_{12} - \overline{\mathcal{N}}_{11}\overline{\mathcal{N}}_{12}] \rangle$;
(iii) $\delta\Omega_{e_{11}} = \langle [(1 - (1 - \underline{\mathcal{P}}_{11})^{\delta}, (1 - (1 - \overline{\mathcal{P}}_{11})^{\delta}], [(1 - (1 - \underline{\mathcal{E}}_{11})^{\delta}, (1 - (1 - \overline{\mathcal{E}}_{11})^{\delta}], [(\underline{\mathcal{N}}_{11})^{\delta}, (\overline{\mathcal{N}}_{11})^{\delta}] \rangle$;
(iv) $(\Omega_{e_{11}})^{\delta} = \langle [(\underline{\mathcal{P}}_{11})^{\delta}, (\overline{\mathcal{P}}_{11})^{\delta}], [(\underline{\mathcal{E}}_{11})^{\delta}, (\overline{\mathcal{E}}_{11})^{\delta}], [(1 - (1 - \underline{\mathcal{N}}_{11})^{\delta}, (1 - (1 - \overline{\mathcal{N}}_{11})^{\delta}] \rangle$.

4.2. Interval-valued picture fuzzy hypersoft weighted average operator

Definition 4.2. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1$, $\sum_{i=1}^{n} \mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = \langle [\underline{\mathcal{P}}_{ik}, \overline{\mathcal{P}}_{ik}], [\underline{\mathcal{E}}_{ik}, \overline{\mathcal{E}}_{ik}], [\underline{\mathcal{N}}_{ik}, \overline{\mathcal{N}}_{ik}] \rangle$ be an IVPFHSN, where $i = \{1, 2, \dots, n\}$, $k = \{1, 2, \dots, m\}$. Then, $\mathcal{A} : \kappa^n \to \kappa$, IVPFHSWAO is represented as

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{nm}}) = \bigoplus_{k=1}^{m} \mathcal{D}_k \left(\bigoplus_{i=1}^{n} \mathcal{W}_i \Omega_{e_{ik}} \right).$$

Theorem 4.3. *Let $\Omega_{e_{ik}} = \langle [\underline{\mathcal{P}}_{ik}, \overline{\mathcal{P}}_{ik}], [\underline{\mathcal{E}}_{ik}, \overline{\mathcal{E}}_{ik}], [\underline{\mathcal{N}}_{ik}, \overline{\mathcal{N}}_{ik}] \rangle$ be an IVPFHSN, where $i = \{1, 2, \dots, n\}$, $k = \{1, 2, \dots, m\}$. Then, the aggregated value of IVPFHSWAO is also an IVPFHSN, which is given by*

$$\begin{aligned}
\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{nm}}) = \Bigg\langle & \left[1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}}_{i} \right)^{\mathcal{D}_k} \right], \\
& \left[1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\
& \left[\prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(\underline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(\overline{\mathcal{N}}_{ik}\right)^{\mathcal{W}}_{i} \right)^{\mathcal{D}_k}, \right] \Bigg\rangle.
\end{aligned}$$

Proof. If $n = 1$, then $\mathcal{W}_1 = 1$. By using Definition 4.1, we get

$$\begin{aligned}
\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{1m}}) &= \bigoplus_{k=1}^{m} \mathcal{D}_k \Omega_{e_{1k}} \\
&= \Bigg\langle \left[1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{1} \left(1 - \underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{1} \left(1 - \overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}}_{i} \right)^{\mathcal{D}_k} \right], \\
&\quad \left[1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{1} \left(1 - \underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{1} \left(1 - \overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\
&\quad \left[\prod_{k=1}^{m} \left(\prod_{i=1}^{1} \left(\underline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{m} \left(\prod_{i=1}^{1} \left(\overline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right] \Bigg\rangle.
\end{aligned}$$

If $m = 1$, then $\mathcal{D}_1 = 1$. By using Definition 4.2, we get

$$\begin{aligned}
\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{21}}, \dots, \Omega_{e_{n1}}) &= \bigoplus_{i=1}^{n} \mathcal{W}_i \Omega_{e_{i1}} \\
&= \Bigg\langle \left[1 - \prod_{k=1}^{1} \left(\prod_{i=1}^{n} \left(1 - \underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{1} \left(\prod_{i=1}^{n} \left(1 - \overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\
&\quad \left[1 - \prod_{k=1}^{1} \left(\prod_{i=1}^{n} \left(1 - \underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{1} \left(\prod_{i=1}^{n} \left(1 - \overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\
&\quad \left[\prod_{k=1}^{1} \left(\prod_{i=1}^{n} \left(\underline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{1} \left(\prod_{i=1}^{n} \left(\overline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right] \Bigg\rangle.
\end{aligned}$$

Hence, the results hold for $n = 1$ and $m = 1$. Now, if $m = l_1 + 1$ and $n = l_2$, then,

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{l_2(l_1+1)}}) = \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \left(\bigoplus_{i=1}^{l_2} \mathcal{W}_i \Omega_{e_{ik}} \right)$$

$$
\begin{aligned}
&= \Big\langle \Big[1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\underline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\overline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big],\\
&\quad \Big[1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\underline{\mathcal{E}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\overline{\mathcal{E}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big],\\
&\quad \Big[\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(\underline{\mathcal{N}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(\overline{\mathcal{N}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big]\Big\rangle.
\end{aligned}
$$

Similarly, if $m = l_1$, $n = l_2 + 1$, then,

$$
\begin{aligned}
\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{(l_2+1)l_1}}) &= \bigoplus_{k=1}^{l_1}\mathcal{D}_k\Big(\bigoplus_{i=1}^{l_2+1}\mathcal{W}_i\Omega_{e_{ik}}\Big)\\
&= \Big\langle \Big[1-\prod_{k=1}^{l_1}\Big(\prod_{i=1}^{l_2+1}\Big(1-\underline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1}\Big(\prod_{i=1}^{l_2+1}\Big(1-\overline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big],\\
&\quad \Big[1-\prod_{k=1}^{l_1}\Big(\prod_{i=1}^{l_2+1}\Big(1-\underline{\mathcal{E}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1}\Big(\prod_{i=1}^{l_2+1}\Big(1-\overline{\mathcal{E}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big],\\
&\quad \Big[\prod_{k=1}^{l_1}\Big(\prod_{i=1}^{l_2+1}\Big(\underline{\mathcal{N}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1}\Big(\prod_{i=1}^{l_2+1}\Big(\overline{\mathcal{N}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big]\Big\rangle.
\end{aligned}
$$

Now, if $m = l_1 + 1$, $n = l_2 + 1$, then,

$$
\begin{aligned}
&\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{(l_2+1)(l_1+1)}})\\
&= \bigoplus_{k=1}^{l_1+1}\mathcal{D}_k\Big(\bigoplus_{i=1}^{l_2+1}\mathcal{W}_i\Omega_{e_{ik}}\Big) = \bigoplus_{k=1}^{l_1+1}\mathcal{D}_k\Big(\bigoplus_{i=1}^{l_2}\mathcal{W}_i\Omega_{e_{ik}}\Big)\bigoplus_{k=1}^{l_1+1}\mathcal{D}_k\Big(\mathcal{W}_{l_2+1}\Omega_{e_{(l_2+1)k}}\Big),\\
&\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{(l_2+1)(l_1+1)}})\\
&= \Big\langle \Big[1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\underline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\overline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big]\\
&\quad \oplus \Big[1-\prod_{k=1}^{l_1+1}\Big(\Big(1-\underline{\mathcal{P}}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\Big(\Big(1-\overline{\mathcal{P}}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}}\Big)^{\mathcal{D}_k}\Big],\\
&\quad \Big[1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\underline{\mathcal{E}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(1-\overline{\mathcal{E}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big]\\
&\quad \oplus \Big[1-\prod_{k=1}^{l_1+1}\Big(\Big(1-\underline{\mathcal{E}}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\Big(\Big(1-\overline{\mathcal{E}}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}}\Big)^{\mathcal{D}_k}\Big],\\
&\quad \Big[\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(\underline{\mathcal{N}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2}\Big(\overline{\mathcal{N}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big]\\
&\quad \oplus \Big[\prod_{k=1}^{l_1+1}\Big(\Big(\underline{\mathcal{N}}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}}\Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1}\Big(\Big(\overline{\mathcal{N}}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}}\Big)^{\mathcal{D}_k}\Big\rangle\\
&= \Big\langle \Big[1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2+1}\Big(1-\underline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\Big(\prod_{i=1}^{l_2+1}\Big(1-\overline{\mathcal{P}}_{ik}\Big)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big],
\end{aligned}
$$

$$\left[1-\prod_{k=1}^{l_1+1}\left(\prod_{i=1}^{l_2+1}\left(1-\underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}\left(\prod_{i=1}^{l_2+1}\left(1-\overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right],$$
$$\left[\prod_{k=1}^{l_1+1}\left(\prod_{i=1}^{l_2+1}\left(\underline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1}\left(\prod_{i=1}^{l_2+1}\left(\overline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right]\Big\rangle.$$

Hence, the results hold for $n = l_2 + 1$ and $m = l_1 + 1$. Therefore, by induction method, the result is true $\forall$ $m, n \geqslant 1$. Since $0 \leqslant \overline{\mathcal{P}}_{ik} + \overline{\mathcal{N}}_{ik} \leqslant 1$ and $0 \leqslant \overline{\mathcal{E}}_{ik} \leqslant 1$,

$$\begin{aligned}
\Leftrightarrow & 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \\
& + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\underline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\overline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leqslant 1 \text{ and} \\
& 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leqslant 1 \\
\Leftrightarrow & 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \\
& + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leqslant 1 \text{ and} \\
& 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leqslant 1 \\
\Leftrightarrow & 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \\
& + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \\
& + 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1-\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leqslant 2.
\end{aligned}$$

Therefore, the aggregated value given by IVPFHSWAO is also an IVPFHSN. □

Example 4.4. Let us consider the same values mentioned in Example 3.2. Also, let $\mathcal{W}_i = \{0.25, 0.35, 0.40\}$ and $\mathcal{D}_k = \{0.30, 0.20, 0.40, 0.10\}$ be the weight of managers and attributes, respectively. Then,

$$\begin{aligned}
\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{34}}) = & \Big\langle \left[1-\prod_{k=1}^{4}\left(\prod_{i=1}^{3}\left(1-\underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \left[1-\prod_{k=1}^{4}\left(\prod_{i=1}^{3}\left(1-\overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right],\right. \\
& \left[1-\prod_{k=1}^{4}\left(\prod_{i=1}^{3}\left(1-\underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \left[1-\prod_{k=1}^{4}\left(\prod_{i=1}^{3}\left(1-\overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right],\right. \\
& \left[\prod_{k=1}^{4}\left(\prod_{i=1}^{3}\left(\underline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \left[\prod_{k=1}^{4}\left(\prod_{i=1}^{3}\left(\overline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right]\right\rangle \\
= & \langle [0.27, 0.32], [0.27, 0.31], [0.17, 0.22] \rangle .
\end{aligned}$$

4.3. Interval-valued picture fuzzy hypersoft weighted geometric operator

Definition 4.5. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1$, $\sum_{i=1}^{n} \mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = (\mathcal{P}_{ik}, \mathcal{E}_{ik}, \mathcal{N}_{ik})$ be an IVPFHSN, where $i = \{1, 2, \dots, n\}$, $k = \{1, 2, \dots, m\}$. Then, $\mathcal{G} : \kappa^n \to \kappa$, IVPFHSWGO is defined as

$$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{nm}}) = \bigotimes_{k=1}^{m} \left(\bigotimes_{i=1}^{n} \left(\Omega_{e_{ik}} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}.$$

Theorem 4.6. *Let* $\Omega_{e_{ik}} = \left\langle [\underline{\mathcal{P}}_{ik}, \overline{\mathcal{P}}_{ik}], [\underline{\mathcal{E}}_{ik}, \overline{\mathcal{E}}_{ik}], [\underline{\mathcal{N}}_{ik}, \overline{\mathcal{N}}_{ik}] \right\rangle$ *be an IVPFHSN, where* $i = \{1, 2, \dots, n\}$, $k = \{1, 2, \dots, m\}$. *Then, the aggregated value of IVPFHSWGO is also an IVPFHSN, which is given by*

$$\begin{aligned}\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{nm}}) = \Bigg\langle & \left[\prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(\underline{\mathcal{P}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(\overline{\mathcal{P}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\ & \left[\prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(\underline{\mathcal{E}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(\overline{\mathcal{E}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\ & \left[1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \underline{\mathcal{N}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \overline{\mathcal{N}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right] \Bigg\rangle.\end{aligned}$$

Proof. Similar to Theorem 4.3. □

Example 4.7. Let us consider the same values mentioned in Example 3.2 and the weight of managers and attributes be as in Example 4.4. Then,

$$\begin{aligned}\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, \dots, \Omega_{e_{34}}) = \Bigg\langle & \left[\prod_{k=1}^{4} \left(\prod_{i=1}^{3} \left(\underline{\mathcal{P}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{4} \left(\prod_{i=1}^{3} \left(\overline{\mathcal{P}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\ & \left[\prod_{k=1}^{4} \left(\prod_{i=1}^{3} \left(\underline{\mathcal{E}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{4} \left(\prod_{i=1}^{3} \left(\overline{\mathcal{E}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right], \\ & \left[1 - \prod_{k=1}^{4} \left(\prod_{i=1}^{3} \left(1 - \underline{\mathcal{N}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{4} \left(\prod_{i=1}^{3} \left(1 - \overline{\mathcal{N}}_{ik} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right] \Bigg\rangle \\ = & \left\langle [0.24, 0.29], [0.25, 0.30], [0.22, 0.26] \right\rangle.\end{aligned}$$

5. MCDM problems based on TOPSIS and CC method

TOPSIS method helps to find the best alternative based on minimum and maximum distance from the interval-valued picture fuzzy positive ideal solution (IVPFPIS) and interval-valued picture fuzzy negative ideal solution (IVPFNIS). Also, when TOPSIS method is combined with CC instead of SMs, it provides reliable results for predicting the closeness coefficients. We present an algorithm and a case study to illustrate the IVPFHSS TOPSIS method based on CC.

5.1. Algorithm to solve MCDM problems with IVPFHSS data based on TOPSIS and CC method

Let $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \dots, \mathcal{A}^x\}$ be a set of selected employees and $\mathcal{V} = \{m_1, m_2, \dots, m_n\}$ be a set of managers responsible to evaluate the employees with weights $\mathcal{W}_i = (\mathcal{W}_1, \mathcal{W}_2, \dots, \mathcal{W}_n)$, such that $\mathcal{W}_i > 0$ and $\sum_{i=1}^{n} \mathcal{W}_i = 1$. Let $\tilde{\Delta} = \{\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_m\}$ be a set of multi-valued sub-attributes with weights $\mathcal{D}_k =$

$(\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_m)$, such that $\mathcal{D}_k > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1$. The evaluation of employees $\mathcal{A}^t$, $(t = 1, 2, \dots, x)$ performed by the managers ν_i, $(i = 1, 2, \dots, n)$ based on the multi-valued sub-attributes $\tilde{\lambda}_k$, $(k = 1, 2, \dots, m)$ are given in IVPFHSS form and represented as $\Omega^t_{ik} = \langle [\underline{\mathcal{P}}_{ik}, \overline{\mathcal{P}}_{ik}], [\underline{\mathcal{E}}_{ik}, \overline{\mathcal{E}}_{ik}], [\underline{\mathcal{N}}_{ik}, \overline{\mathcal{N}}_{ik}] \rangle$, such that $0 \leqslant \overline{\mathcal{P}}^t_{ik} + \overline{\mathcal{E}}^t_{ik} + \overline{\mathcal{N}}^t_{ik} \leqslant 1,\ \forall\, i, k$.

Step 1. Construct the matrix for each multi-valued sub-attributes in IVPFHSS form as below:

$$[\mathcal{A}^t, \tilde{\Delta}]_{n\times m} = [\mathcal{A}^t]_{n\times m} = \begin{array}{c} \\ m_1 \\ m_2 \\ \vdots \\ m_n \end{array} \begin{array}{c} \begin{array}{cccc} \tilde{\lambda}_1 & \tilde{\lambda}_2 & \dots & \tilde{\lambda}_m \end{array} \\ \begin{bmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1m} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{n1} & \mu_{n2} & \dots & \mu_{nm} \end{bmatrix} \end{array},$$

such that $[\mathcal{A}^t]_{n\times m} = \mu_{ik} = \left\langle [\underline{\mathcal{P}}^t_{ik}, \overline{\mathcal{P}}^t_{ik}], [\underline{\mathcal{E}}^t_{ik}, \overline{\mathcal{E}}^t_{ik}], [\underline{\mathcal{N}}^t_{ik}, \overline{\mathcal{N}}^t_{11}] \right\rangle, i = 1, 2, \dots, n$ and $k = 1, 2, \dots, m$.

Step 2. Obtain the weighted decision matrix for each multi-valued sub-attributes,

$$\begin{aligned}
[\tilde{A}^t_{ik}]_{n\times m} = \Bigg\langle & \left[1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \underline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \overline{\mathcal{P}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right], \\
& \left[1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \underline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \overline{\mathcal{E}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right], \\
& \left[\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\underline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\overline{\mathcal{N}}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right]\Bigg\rangle \\
= \Bigg\langle & \left[\underline{\mathcal{P}}^{\sim}_{ik}, \overline{\mathcal{P}}^{\sim}_{ik}\right], \left[\underline{\mathcal{E}}^{\sim}_{ik}, \overline{\mathcal{E}}^{\sim}_{ik}\right], \left[\underline{\mathcal{N}}^{\sim}_{ik}, \overline{\mathcal{N}}^{\sim}_{ik}\right]\Bigg\rangle.
\end{aligned}$$

Step 3. Determine the IVPFPIS and IVPFNIS for weighted IVPFHSS as below:

$$\begin{aligned}
\tilde{\mathcal{A}}^{+} &= \left\langle \left[\underline{\mathcal{P}}^{\tilde{+}}, \overline{\mathcal{P}}^{\tilde{+}}\right], \left[\underline{\mathcal{E}}^{\tilde{+}}, \overline{\mathcal{E}}^{\tilde{+}}\right], \left[\underline{\mathcal{N}}^{\tilde{+}}, \overline{\mathcal{N}}^{\tilde{+}}\right] \right\rangle_{n\times m} = \left\langle \left[\underline{\mathcal{P}}^{(\tilde{\vee}_{ij})}, \overline{\mathcal{P}}^{(\tilde{\vee}_{ij})}\right], \left[\underline{\mathcal{E}}^{(\tilde{\wedge}_{ij})}, \overline{\mathcal{E}}^{(\tilde{\wedge}_{ij})}\right], \left[\underline{\mathcal{N}}^{(\tilde{\wedge}_{ij})}, \overline{\mathcal{N}}^{(\tilde{\wedge}_{ij})}\right] \right\rangle, \\
\tilde{\mathcal{A}}^{-} &= \left\langle \left[\underline{\mathcal{P}}^{\tilde{-}}, \overline{\mathcal{P}}^{\tilde{-}}\right], \left[\underline{\mathcal{E}}^{\tilde{-}}, \overline{\mathcal{E}}^{\tilde{-}}\right], \left[\underline{\mathcal{N}}^{\tilde{-}}, \overline{\mathcal{N}}^{\tilde{-}}\right] \right\rangle_{n\times m} = \left\langle \left[\underline{\mathcal{P}}^{(\tilde{\wedge}_{ij})}, \overline{\mathcal{P}}^{(\tilde{\wedge}_{ij})}\right], \left[\underline{\mathcal{E}}^{(\tilde{\wedge}_{ij})}, \overline{\mathcal{E}}^{(\tilde{\wedge}_{ij})}\right], \left[\underline{\mathcal{N}}^{(\tilde{\vee}_{ij})}, \overline{\mathcal{N}}^{(\tilde{\vee}_{ij})}\right] \right\rangle,
\end{aligned}$$

where $\vee_{ij} = \arg\max_t \left\{\varphi^t_{ij}\right\}$ and $\wedge_{ij} = \arg\min_t \left\{\varphi^t_{ij}\right\}$.

Step 4. Determine the CC for each alternative from IVPFPIS and IVPFNIS.

$$\chi^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^{+}) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^{+})}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{(\Phi\tilde{A}^{+})}} \quad \text{and} \quad \lambda^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^{-}) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^{-})}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{\Phi(\tilde{A}^{-})}}.$$

Step 5. Compute the closeness coefficient of interval-valued picture fuzzy ideal solution as below:

$$\epsilon^t = \frac{1 - \lambda^t}{2 - \chi^t - \lambda^t}.$$

Step 6. Arrange the ϵ^t values in descending order and determine the rank of the alternatives $\mathcal{A}^t$, $(t =$

$1, 2, \ldots, x$). The one with the maximum value is the suitable employee to lead the new project.

5.2. Application based on TOPSIS and CC method

Let $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \mathcal{A}^3, \mathcal{A}^4\}$ be a set of employees and let $\mathcal{V} = \{m_1, m_2, m_3, m_4\}$ be a set of managers who evaluate the employees based on the Leipzig leadership model for an upcoming project with weights $\mathcal{W}_i = (0.35, 0.15, 0.30, 0.20)$. Let Δ_1, Δ_2, Δ_3 and Δ_4 be distinct attribute sets whose corresponding sub-attributes are represented as $\Delta_1 = \text{purpose} = \{\lambda_{11} = \text{achieve goals}\}$, $\Delta_2 = \text{entrepreneurial spirit} = \{\lambda_{21} = \text{quick decision}, \lambda_{22} = \text{logical decision}\}$, $\Delta_3 = \text{responsibility} = \{\lambda_{31} = \text{inspire and motivate}, \lambda_{32} = \text{time management}\}$ and $\Delta_4 = \text{effectiveness} = \{\lambda_{41} = \text{successful accomplishment}\}$. Then $\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4$ is the distinct attribute set given by

$$\begin{aligned}\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4 &= \{\lambda_{11}\} \times \{\lambda_{21}, \lambda_{22}\} \times \{\lambda_{31}, \lambda_{32}\} \times \{\lambda_{41}\} \\ &= \left\{(\lambda_{11}, \lambda_{21}, \lambda_{31}, \lambda_{41}), (\lambda_{11}, \lambda_{21}, \lambda_{32}, \lambda_{41}), (\lambda_{11}, \lambda_{22}, \lambda_{31}, \lambda_{41}), (\lambda_{11}, \lambda_{22}, \lambda_{32}, \lambda_{41})\right\} \\ &= \left\{\tilde{\lambda_1}, \tilde{\lambda_2}, \tilde{\lambda_3}, \tilde{\lambda_4}\right\} \text{ with weights } \mathcal{D}_k = (0.20, 0.25, 0.30, 0.25).\end{aligned}$$

This study aims to find an employee who can successfully lead the project.

Step 1. Construct $\mathcal{A}^1$, $\mathcal{A}^2$, $\mathcal{A}^3$, and $\mathcal{A}^4$ matrices for each multi-valued sub-attributes in IVPFHSS form.

Table 2: Representation of values in IVPFHSS form for $\mathcal{A}^1$.

$\mathcal{A}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	$\langle[0.12, 0.15], [0.34, 0.36], [0.21, 0.23]\rangle$	$\langle[0.23, 0.33], [0.48, 0.49], [0.12, 0.13]\rangle$
m_2	$\langle[0.32, 0.45], [0.41, 0.42], [0.12, 0.13]\rangle$	$\langle[0.54, 0.56], [0.34, 0.36], [0.04, 0.07]\rangle$
m_3	$\langle[0.21, 0.26], [0.33, 0.36], [0.31, 0.37]\rangle$	$\langle[0.37, 0.38], [0.23, 0.26], [0.29, 0.31]\rangle$
m_4	$\langle[0.21, 0.22], [0.47, 0.49], [0.25, 0.28]\rangle$	$\langle[0.47, 0.48], [0.29, 0.31], [0.19, 0.21]\rangle$
$\mathcal{A}^1$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	$\langle[0.16, 0.17], [0.21, 0.31], [0.19, 0.29]\rangle$	$\langle[0.15, 0.19], [0.21, 0.25], [0.32, 0.34]\rangle$
m_2	$\langle[0.31, 0.32], [0.21, 0.24], [0.25, 0.35]\rangle$	$\langle[0.24, 0.29], [0.31, 0.34], [0.23, 0.25]\rangle$
m_3	$\langle[0.32, 0.34], [0.42, 0.43], [0.16, 0.17]\rangle$	$\langle[0.33, 0.39], [0.12, 0.14], [0.31, 0.34]\rangle$
m_4	$\langle[0.21, 0.22], [0.14, 0.15], [0.39, 0.45]\rangle$	$\langle[0.48, 0.49], [0.11, 0.12], [0.26, 0.27]\rangle$

Table 3: Representation of values in IVPFHSS form for $\mathcal{A}^2$.

$\mathcal{A}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	$\langle[0.21, 0.25], [0.25, 0.32], [0.22, 0.31]\rangle$	$\langle[0.32, 0.33], [0.34, 0.35], [0.11, 0.12]\rangle$
m_2	$\langle[0.32, 0.39], [0.21, 0.28], [0.25, 0.31]\rangle$	$\langle[0.21, 0.22], [0.25, 0.26], [0.21, 0.26]\rangle$
m_3	$\langle[0.24, 0.26], [0.36, 0.37], [0.29, 0.31]\rangle$	$\langle[0.48, 0.49], [0.21, 0.22], [0.12, 0.15]\rangle$
m_4	$\langle[0.34, 0.36], [0.44, 0.45], [0.12, 0.15]\rangle$	$\langle[0.58, 0.59], [0.12, 0.13], [0.19, 0.21]\rangle$
$\mathcal{A}^2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	$\langle[0.44, 0.45], [0.42, 0.43], [0.11, 0.12]\rangle$	$\langle[0.21, 0.28], [0.15, 0.17], [0.41, 0.43]\rangle$
m_2	$\langle[0.54, 0.55], [0.12, 0.13], [0.14, 0.15]\rangle$	$\langle[0.28, 0.31], [0.12, 0.13], [0.31, 0.33]\rangle$
m_3	$\langle[0.49, 0.51], [0.15, 0.17], [0.25, 0.29]\rangle$	$\langle[0.41, 0.46], [0.16, 0.19], [0.23, 0.29]\rangle$
m_4	$\langle[0.58, 0.59], [0.14, 0.15], [0.16, 0.18]\rangle$	$\langle[0.21, 0.29], [0.17, 0.21], [0.44, 0.49]\rangle$

Step 2. Obtain $\tilde{\mathcal{A}}^1$, $\tilde{\mathcal{A}}^2$, $\tilde{\mathcal{A}}^3$ and $\tilde{\mathcal{A}}^4$, the weighted matrices for each multi-valued sub-attributes.

Table 4: Representation of values in IVPFHSS form for $\mathcal{A}^3$.

$\mathcal{A}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	$\langle[0.05, 0.12], [0.15, 0.18], [0.32, 0.37]\rangle$	$\langle[0.23, 0.25], [0.31, 0.35], [0.11, 0.28]\rangle$
m_2	$\langle[0.25, 0.31], [0.25, 0.35], [0.25, 0.33]\rangle$	$\langle[0.39, 0.45], [0.04, 0.07], [0.29, 0.31]\rangle$
m_3	$\langle[0.15, 0.21], [0.15, 0.22], [0.15, 0.24]\rangle$	$\langle[0.34, 0.35], [0.21, 0.22], [0.24, 0.29]\rangle$
m_4	$\langle[0.31, 0.32], [0.35, 0.36], [0.14, 0.29]\rangle$	$\langle[0.35, 0.41], [0.21, 0.25], [0.31, 0.32]\rangle$
$\mathcal{A}^3$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	$\langle[0.51, 0.52], [0.12, 0.19], [0.26, 0.29]\rangle$	$\langle[0.51, 0.53], [0.22, 0.23], [0.21, 0.24]\rangle$
m_2	$\langle[0.47, 0.49], [0.16, 0.19], [0.21, 0.25]\rangle$	$\langle[0.42, 0.43], [0.31, 0.33], [0.13, 0.14]\rangle$
m_3	$\langle[0.32, 0.34], [0.31, 0.35], [0.21, 0.31]\rangle$	$\langle[0.05, 0.12], [0.14, 0.19], [0.45, 0.49]\rangle$
m_4	$\langle[0.63, 0.64], [0.12, 0.14], [0.13, 0.14]\rangle$	$\langle[0.21, 0.26], [0.16, 0.19], [0.22, 0.23]\rangle$

Table 5: Representation of values in IVPFHSS form for $\mathcal{A}^4$.

$\mathcal{A}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	$\langle[0.21, 0.23], [0.31, 0.32], [0.09, 0.21]\rangle$	$\langle[0.31, 0.36], [0.31, 0.35], [0.27, 0.28]\rangle$
m_2	$\langle[0.44, 0.45], [0.36, 0.42], [0.12, 0.13]\rangle$	$\langle[0.04, 0.06], [0.77, 0.79], [0.12, 0.15]\rangle$
m_3	$\langle[0.21, 0.22], [0.45, 0.47], [0.12, 0.14]\rangle$	$\langle[0.53, 0.55], [0.03, 0.04], [0.21, 0.28]\rangle$
m_4	$\langle[0.41, 0.42], [0.31, 0.32], [0.14, 0.25]\rangle$	$\langle[0.24, 0.25], [0.32, 0.35], [0.27, 0.34]\rangle$
$\mathcal{A}^4$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	$\langle[0.27, 0.34], [0.17, 0.27], [0.32, 0.35]\rangle$	$\langle[0.37, 0.39], [0.21, 0.24], [0.12, 0.15]\rangle$
m_2	$\langle[0.39, 0.41], [0.14, 0.15], [0.17, 0.18]\rangle$	$\langle[0.14, 0.18], [0.38, 0.42], [0.29, 0.31]\rangle$
m_3	$\langle[0.14, 0.15], [0.49, 0.59], [0.16, 0.19]\rangle$	$\langle[0.45, 0.47], [0.12, 0.17], [0.23, 0.29]\rangle$
m_4	$\langle[0.25, 0.29], [0.23, 0.31], [0.31, 0.34]\rangle$	$\langle[0.31, 0.36], [0.42, 0.44], [0.12, 0.15]\rangle$

Table 6: Representation of weighted values in IVPFHSS form for $\tilde{\mathcal{A}}^1$.

$\tilde{\mathcal{A}}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	$\langle[0.0090, 0.0003], [0.0287, 0.0009], [0.8966, 0.9974]\rangle$	$\langle[0.0227, 0.0010], [0.0557, 0.0049], [0.8307, 0.0308]\rangle$
m_2	$\langle[0.0116, 0.0014], [0.0158, 0.0013], [0.9384, 0.9953]\rangle$	$\langle[0.0287, 0.0024], [0.0155, 0.0018], [0.8863, 0.0169]\rangle$
m_3	$\langle[0.0141, 0.0009], [0.0238, 0.0013], [0.9322, 0.9973]\rangle$	$\langle[0.0341, 0.0017], [0.0195, 0.0018], [0.9114, 0.0829]\rangle$
m_4	$\langle[0.0094, 0.0005], [0.0251, 0.0013], [0.9461, 0.9977]\rangle$	$\langle[0.0313, 0.0016], [0.0170, 0.0024], [0.9204, 0.0543]\rangle$
$\tilde{\mathcal{A}}^1$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	$\langle[0.0182, 0.0006], [0.0245, 0.0032], [0.8400, 0.0881]\rangle$	$\langle[0.0142, 0.0005], [0.0205, 0.0021], [0.9052, 0.0890]\rangle$
m_2	$\langle[0.0166, 0.0014], [0.0106, 0.0013], [0.9396, 0.1143]\rangle$	$\langle[0.0103, 0.0010], [0.0139, 0.0017], [0.9464, 0.0653]\rangle$
m_3	$\langle[0.0342, 0.0018], [0.0479, 0.0041], [0.8480, 0.0508]\rangle$	$\langle[0.0296, 0.0018], [0.0096, 0.0009], [0.9160, 0.0923]\rangle$
m_4	$\langle[0.0141, 0.0008], [0.0091, 0.0013], [0.9451, 0.1561]\rangle$	$\langle[0.0322, 0.0016], [0.0059, 0.0009], [0.9349, 0.0718]\rangle$

Table 7: Representation of weighted values in IVPFHSS form for $\tilde{\mathcal{A}}^2$.

$\tilde{\mathcal{A}}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	$\langle[0.0164, 0.0010], [0.0200, 0.0013], [0.8995, 0.9962]\rangle$	$\langle[0.0332, 0.0017], [0.0358, 0.0022], [0.8244, 0.0284]\rangle$
m_2	$\langle[0.0116, 0.0012], [0.0071, 0.0008], [0.9593, 0.9973]\rangle$	$\langle[0.0089, 0.0008], [0.0108, 0.0006], [0.9432, 0.0697]\rangle$
m_3	$\langle[0.0164, 0.0010], [0.0265, 0.0016], [0.9285, 0.9962]\rangle$	$\langle[0.0479, 0.0028], [0.0176, 0.0017], [0.8530, 0.0371]\rangle$
m_4	$\langle[0.0165, 0.0015], [0.0230, 0.0020], [0.9187, 0.9938]\rangle$	$\langle[0.0425, 0.0037], [0.0064, 0.0009], [0.9204, 0.0528]\rangle$
$\tilde{\mathcal{A}}^2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	$\langle[0.0591, 0.0030], [0.0556, 0.0034], [0.7932, 0.0340]\rangle$	$\langle[0.0205, 0.0014], [0.0142, 0.0010], [0.9250, 0.1188]\rangle$
m_2	$\langle[0.0344, 0.0028], [0.0058, 0.0003], [0.9154, 0.0457]\rangle$	$\langle[0.0123, 0.0011], [0.0048, 0.0003], [0.9571, 0.0916]\rangle$
m_3	$\langle[0.0589, 0.0036], [0.0146, 0.0015], [0.8828, 0.0910]\rangle$	$\langle[0.0388, 0.0026], [0.0130, 0.0014], [0.8957, 0.0765]\rangle$
m_4	$\langle[0.0508, 0.0045], [0.0091, 0.0012], [0.8959, 0.0533]\rangle$	$\langle[0.0118, 0.0015], [0.0093, 0.0014], [0.9598, 0.1433]\rangle$

Table 8: Representation of weighted values in IVPFHSS form for $\tilde{\mathcal{A}}^3$.

$\tilde{\mathcal{A}}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	⟨[0.0036, 0.0001], [0.0114, 0.0002], [0.9234, 0.9993]⟩	⟨[0.0227, 0.0003], [0.0320, 0.0013], [0.8244, 0.0731]⟩
m_2	⟨[0.0086, 0.0007], [0.0086, 0.0008], [0.9593, 0.9981]⟩	⟨[0.0184, 0.0013], [0.0016, 0.0002], [0.9547, 0.0852]⟩
m_3	⟨[0.0098, 0.0005], [0.0098, 0.0005], [0.8925, 0.9973]⟩	⟨[0.0307, 0.0011], [0.0176, 0.0007], [0.8985, 0.0736]⟩
m_4	⟨[0.0148, 0.0012], [0.0171, 0.0014], [0.9244, 0.9964]⟩	⟨[0.0214, 0.0020], [0.0118, 0.0013], [0.9432, 0.0853]⟩
$\tilde{\mathcal{A}}^3$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	⟨[0.0722, 0.0008], [0.0134, 0.0008], [0.8682, 0.0906]⟩	⟨[0.0606, 0.0007], [0.0216, 0.0008], [0.8724, 0.0614]⟩
m_2	⟨[0.0282, 0.0018], [0.0079, 0.0006], [0.9322, 0.0795]⟩	⟨[0.0203, 0.0013], [0.0139, 0.0009], [0.9264, 0.0356]⟩
m_3	⟨[0.0342, 0.0013], [0.0329, 0.0013], [0.8690, 0.0946]⟩	⟨[0.0039, 0.0004], [0.0113, 0.0006], [0.9419, 0.1395]⟩
m_4	⟨[0.0580, 0.0046], [0.0077, 0.0008], [0.8848, 0.0410]⟩	⟨[0.0118, 0.0012], [0.0087, 0.0010], [0.9271, 0.0587]⟩

Table 9: Representation of weighted values in IVPFHSS form for $\tilde{\mathcal{A}}^4$.

$\tilde{\mathcal{A}}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	⟨[0.0164, 0.0009], [0.0257, 0.0013], [0.8449, 0.9949]⟩	⟨[0.0320, 0.0019], [0.0320, 0.0028], [0.8918, 0.0671]⟩
m_2	⟨[0.0173, 0.0021], [0.0133, 0.0019], [0.9384, 0.9930]⟩	⟨[0.0016, 0.0003], [0.0537, 0.0052], [0.9236, 0.0375]⟩
m_3	⟨[0.0141, 0.0008], [0.0353, 0.0018], [0.8806, 0.9945]⟩	⟨[0.0551, 0.0029], [0.0023, 0.0004], [0.8896, 0.0698]⟩
m_4	⟨[0.0209, 0.0023], [0.0148, 0.0017], [0.9244, 0.9943]⟩	⟨[0.0137, 0.0016], [0.0191, 0.0016], [0.9367, 0.0916]⟩
$\tilde{\mathcal{A}}^4$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	⟨[0.0326, 0.0021], [0.0194, 0.0025], [0.8873, 0.1035]⟩	⟨[0.0397, 0.0021], [0.0205, 0.0018], [0.8307, 0.0338]⟩
m_2	⟨[0.0220, 0.0028], [0.0068, 0.0007], [0.9234, 0.0544]⟩	⟨[0.0057, 0.0009], [0.0178, 0.0019], [0.9547, 0.0834]⟩
m_3	⟨[0.0135, 0.0007], [0.0589, 0.0094], [0.8480, 0.0542]⟩	⟨[0.0439, 0.0023], [0.0096, 0.0017], [0.8957, 0.0727]⟩
m_4	⟨[0.0172, 0.0022], [0.0156, 0.0017], [0.9322, 0.1089]⟩	⟨[0.0184, 0.0024], [0.0269, 0.0022], [0.8995, 0.0369]⟩

Step 3. Determine the IVPFPIS and IVPFNIS from the weighted matrices, $\tilde{\mathcal{A}}^1$, $\tilde{\mathcal{A}}^2$, $\tilde{\mathcal{A}}^3$, and $\tilde{\mathcal{A}}^4$.

Table 10: Representation of IVPFPIS ($\tilde{\mathcal{A}}^+$) from the weighted matrices.

$\tilde{\mathcal{A}}^+$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	⟨[0.0164, 0.0010], [0.0114, 0.0002], [0.8449, 0.9949]⟩	⟨[0.0332, 0.0019], [0.0320, 0.0013], [0.8244, 0.0284]⟩
m_2	⟨[0.0173, 0.0021], [0.0071, 0.0008], [0.9384, 0.9930]⟩	⟨[0.0287, 0.0024], [0.0016, 0.0002], [0.8863, 0.0169]⟩
m_3	⟨[0.0164, 0.0010], [0.0098, 0.0005], [0.8806, 0.9945]⟩	⟨[0.0551, 0.0029], [0.0023, 0.0004], [0.8530, 0.0371]⟩
m_4	⟨[0.0209, 0.0023], [0.0148, 0.0013], [0.9187, 0.9938]⟩	⟨[0.0425, 0.0037], [0.0064, 0.0009], [0.9204, 0.0528]⟩
$\tilde{\mathcal{A}}^+$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	⟨[0.0722, 0.0030], [0.0134, 0.0008], [0.7932, 0.0340]⟩	⟨[0.0606, 0.0021], [0.0142, 0.0008], [0.8307, 0.0338]⟩
m_2	⟨[0.0344, 0.0028], [0.0058, 0.0003], [0.9154, 0.0457]⟩	⟨[0.0203, 0.0013], [0.0048, 0.0003], [0.9264, 0.0356]⟩
m_3	⟨[0.0589, 0.0036], [0.0146, 0.0013], [0.8480, 0.0508]⟩	⟨[0.0439, 0.0026], [0.0096, 0.0006], [0.8957, 0.0727]⟩
m_4	⟨[0.0580, 0.0046], [0.0077, 0.0008], [0.8848, 0.0410]⟩	⟨[0.0322, 0.0024], [0.0059, 0.0009], [0.8995, 0.0369]⟩

Table 11: Representation of IVPFPIS ($\tilde{\mathcal{A}}^-$) from the weighted matrices.

$\tilde{\mathcal{A}}^-$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
m_1	⟨[0.0036, 0.0001], [0.0114, 0.0002], [0.9234, 0.9993]⟩	⟨[0.0227, 0.0003], [0.0320, 0.0013], [0.8918, 0.0731]⟩
m_2	⟨[0.0086, 0.0007], [0.0071, 0.0002], [0.9593, 0.9981]⟩	⟨[0.0016, 0.0003], [0.0016, 0.0002], [0.9547, 0.0852]⟩
m_3	⟨[0.0098, 0.0005], [0.0071, 0.0005], [0.9322, 0.9973]⟩	⟨[0.0307, 0.0011], [0.0016, 0.0002], [0.9114, 0.0829]⟩
m_4	⟨[0.0094, 0.0005], [0.0098, 0.0005], [0.9461, 0.9977]⟩	⟨[0.0137, 0.0016], [0.0023, 0.0004], [0.9432, 0.0916]⟩
$\tilde{\mathcal{A}}^-$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
m_1	⟨[0.0182, 0.0006], [0.0134, 0.0008], [0.8873, 0.1035]⟩	⟨[0.0142, 0.0005], [0.01420.0008], [0.9250, 0.1188]⟩
m_2	⟨[0.0166, 0.0014], [0.0058, 0.0003], [0.9396, 0.1143]⟩	⟨[0.0057, 0.0009], [0.00480.0003], [0.9571, 0.0916]⟩
m_3	⟨[0.0135, 0.0007], [0.0058, 0.0003], [0.8828, 0.0946]⟩	⟨[0.0039, 0.0004], [0.00480.0003], [0.9419, 0.1395]⟩
m_4	⟨[0.0141, 0.0008], [0.0077, 0.0008], [0.9451, 0.1561]⟩	⟨[0.0118, 0.0012], [0.00590.0006], [0.9598, 0.1433]⟩

Step 4. Determine the CC for the alternatives by using the values of IVPFPIS and IVPFNIS,

$$\chi^1 = 0.9985, \chi^2 = 0.9986, \chi^3 = 0.9989, \text{ and } \chi^4 = 0.9987,$$
$$\lambda^1 = 0.9990, \lambda^2 = 0.9984, \lambda^3 = 0.9987, \text{ and } \lambda^4 = 0.9984.$$

Step 5. Compute the closeness coefficient of interval-valued picture fuzzy ideal solution as

$$\epsilon^1 = 0.4000, \epsilon^2 = 0.5355, \epsilon^3 = 0.5417, \text{ and } \epsilon^4 = 0.5517.$$

Step 6. Arrange the values in descending order.

$$\epsilon^4 > \epsilon^3 > \epsilon^2 > \epsilon^1 \Rightarrow \mathcal{A}^4 > \mathcal{A}^3 > \mathcal{A}^2 > \mathcal{A}^1.$$

Hence, $\mathcal{A}^4$ is the best among the group who can lead the project successfully.

6. Comparative Analysis

We compare the proposed interval-valued picture fuzzy TOPSIS method with existing SMs to show the reliability, validity and effectiveness of the proposed TOPSIS method based on CC.

Consider the same IVPFHSS values and weights mentioned in Section 5.2. We now combine the proposed TOPSIS method, with the SMs given below to rank the alternatives.

(i)

$$\mathcal{S}_1(\Omega_1, \Omega_2) = \frac{1}{n}\sum_{i=1}^{n} \cos\Bigg[\frac{\pi}{2}\Bigg(|\underline{\mathcal{P}}_{\Omega_1}(\nu_i) - \underline{\mathcal{P}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{P}}_{\Omega_1}(\nu_i) - \overline{\mathcal{P}}_{\Omega_2}(\nu_i)| \vee |\underline{\mathcal{E}}_{\Omega_1}(\nu_i) - \underline{\mathcal{E}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{E}}_{\Omega_1}(\nu_i) - \overline{\mathcal{E}}_{\Omega_2}(\nu_i)| \vee |\underline{\mathcal{N}}_{\Omega_1}(\nu_i) - \underline{\mathcal{N}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{N}}_{\Omega_1}(\nu_i) - \overline{\mathcal{N}}_{\Omega_2}(\nu_i)|\Bigg)\Bigg], \text{ [20]}.$$

(ii)

$$\mathcal{S}_2(\Omega_1, \Omega_2) = \frac{1}{n}\sum_{i=1}^{n} \cos\Bigg[\frac{\pi}{2}\Bigg(|\underline{\mathcal{P}}_{\Omega_1}(\nu_i) - \underline{\mathcal{P}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{P}}_{\Omega_1}(\nu_i) - \overline{\mathcal{P}}_{\Omega_2}(\nu_i)| \vee |\underline{\mathcal{E}}_{\Omega_1}(\nu_i) - \underline{\mathcal{E}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{E}}_{\Omega_1}(\nu_i) - \overline{\mathcal{E}}_{\Omega_2}(\nu_i)| \vee |\underline{\mathcal{N}}_{\Omega_1}(\nu_i) - \underline{\mathcal{N}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{N}}_{\Omega_1}(\nu_i) - \overline{\mathcal{N}}_{\Omega_2}(\nu_i)| \vee |\underline{\mathcal{R}}_{\Omega_1}(\nu_i) - \underline{\mathcal{R}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{R}}_{\Omega_1}(\nu_i) - \overline{\mathcal{R}}_{\Omega_2}(\nu_i)|\Bigg)\Bigg], \text{ [20]}.$$

(iii)

$$\mathcal{S}_3(\Omega_1, \Omega_2) = \frac{1}{n}\sum_{i=1}^{n} \cos\Bigg[\frac{\pi}{4}\Bigg(|\underline{\mathcal{P}}_{\Omega_1}(\nu_i) - \underline{\mathcal{P}}_{\Omega_2}(\nu_i)| + |\overline{\mathcal{P}}_{\Omega_1}(\nu_i) - \overline{\mathcal{P}}_{\Omega_2}(\nu_i)| + |\underline{\mathcal{E}}_{\Omega_1}(\nu_i) - \underline{\mathcal{E}}_{\Omega_2}(\nu_i)| + |\overline{\mathcal{E}}_{\Omega_1}(\nu_i) - \overline{\mathcal{E}}_{\Omega_2}(\nu_i)| + |\underline{\mathcal{N}}_{\Omega_1}(\nu_i) - \underline{\mathcal{N}}_{\Omega_2}(\nu_i)| + |\overline{\mathcal{N}}_{\Omega_1}(\nu_i) - \overline{\mathcal{N}}_{\Omega_2}(\nu_i)|\Bigg)\Bigg], \text{ [20]}.$$

(iv)

$$\mathcal{S}_4(\Omega_1, \Omega_2) = \frac{1}{n}\sum_{i=1}^{n} \cos\Bigg[\frac{\pi}{4}\Bigg(|\underline{\mathcal{P}}_{\Omega_1}(\nu_i) - \underline{\mathcal{P}}_{\Omega_2}(\nu_i)| \vee |\overline{\mathcal{P}}_{\Omega_1}(\nu_i) - \overline{\mathcal{P}}_{\Omega_2}(\nu_i)| + |\underline{\mathcal{E}}_{\Omega_1}(\nu_i) - \underline{\mathcal{E}}_{\Omega_2}(\nu_i)| + |\overline{\mathcal{E}}_{\Omega_1}(\nu_i) - \overline{\mathcal{E}}_{\Omega_2}(\nu_i)| + |\underline{\mathcal{N}}_{\Omega_1}(\nu_i) - \underline{\mathcal{N}}_{\Omega_2}(\nu_i)| + |\overline{\mathcal{N}}_{\Omega_1}(\nu_i) - \overline{\mathcal{N}}_{\Omega_2}(\nu_i)| + |\underline{\mathcal{R}}_{\Omega_1}(\nu_i) - \underline{\mathcal{R}}_{\Omega_2}(\nu_i)| + |\overline{\mathcal{R}}_{\Omega_1}(\nu_i) - \overline{\mathcal{R}}_{\Omega_2}(\nu_i)|\Bigg)\Bigg], \text{ [20]}.$$

(v)

$$\mathcal{S}_5(\Omega_1,\Omega_2)=\frac{1}{n}\sum_{i=1}^{n}\cot\Big[\frac{\pi}{4}+\frac{\pi}{4}\Big(|\underline{\mathcal{P}}_{\Omega_1}(\nu_i)-\underline{\mathcal{P}}_{\Omega_2}(\nu_i)|\vee|\overline{\mathcal{P}}_{\Omega_1}(\nu_i)-\overline{\mathcal{P}}_{\Omega_2}(\nu_i)|\vee|\underline{\mathcal{E}}_{\Omega_1}(\nu_i)-\underline{\mathcal{E}}_{\Omega_2}(\nu_i)|\vee|\overline{\mathcal{E}}_{\Omega_1}(\nu_i)-\overline{\mathcal{E}}_{\Omega_2}(\nu_i)|\vee|\underline{\mathcal{N}}_{\Omega_1}(\nu_i)-\underline{\mathcal{N}}_{\Omega_2}(\nu_i)|\vee|\overline{\mathcal{N}}_{\Omega_1}(\nu_i)-\overline{\mathcal{N}}_{\Omega_2}(\nu_i)|\Big)\Big],\ [20].$$

(vi)

$$\mathcal{S}_6(\Omega_1,\Omega_2)=\frac{1}{n}\sum_{i=1}^{n}\cot\Big[\frac{\pi}{4}+\frac{\pi}{4}\Big(|\underline{\mathcal{P}}_{\Omega_1}(\nu_i)-\underline{\mathcal{P}}_{\Omega_2}(\nu_i)|\vee|\overline{\mathcal{P}}_{\Omega_1}(\nu_i)-\overline{\mathcal{P}}_{\Omega_2}(\nu_i)|\vee|\underline{\mathcal{E}}_{\Omega_1}(\nu_i)-\underline{\mathcal{E}}_{\Omega_2}(\nu_i)|\vee|\overline{\mathcal{E}}_{\Omega_1}(\nu_i)-\overline{\mathcal{E}}_{\Omega_2}(\nu_i)|\vee|\underline{\mathcal{N}}_{\Omega_1}(\nu_i)-\underline{\mathcal{N}}_{\Omega_2}(\nu_i)|\vee|\overline{\mathcal{N}}_{\Omega_1}(\nu_i)-\overline{\mathcal{N}}_{\Omega_2}(\nu_i)|\vee|\underline{\mathcal{R}}_{\Omega_1}(\nu_i)-\underline{\mathcal{R}}_{\Omega_2}(\nu_i)|\vee|\overline{\mathcal{R}}_{\Omega_1}(\nu_i)-\overline{\mathcal{R}}_{\Omega_2}(\nu_i)|\Big)\Big],\ [20].$$

(vii)

$$\mathcal{S}_7(\Omega_1,\Omega_2)=\frac{1}{n}\sum_{i=1}^{n}\frac{\Big[(\underline{\mathcal{P}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{P}}_{\Omega_2}(\nu_i))+(\underline{\mathcal{E}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{E}}_{\Omega_2}(\nu_i))+(\underline{\mathcal{N}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{N}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{P}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{P}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{E}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{E}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{N}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{N}}_{\Omega_2}(\nu_i))\Big]}{\max\Big\{\Big[(\underline{\mathcal{P}}_{\Omega_1}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_1}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_1}(\nu_i))^2\Big],\Big[(\underline{\mathcal{P}}_{\Omega_2}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2}(\nu_i))^2\Big]\Big\}},\ [20].$$

(viii)

$$\mathcal{S}_8(\Omega_1,\Omega_2)=\frac{1}{n}\sum_{i=1}^{n}\frac{2\Big[(\underline{\mathcal{P}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{P}}_{\Omega_2}(\nu_i))+(\underline{\mathcal{E}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{E}}_{\Omega_2}(\nu_i))+(\underline{\mathcal{N}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{N}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{P}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{P}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{E}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{E}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{N}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{N}}_{\Omega_2}(\nu_i))\Big]}{\Big[(\underline{\mathcal{P}}_{\Omega_1}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_1}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_1}(\nu_i))^2\Big]+\Big[(\underline{\mathcal{P}}_{\Omega_2}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2}(\nu_i))^2\Big]},\ [20].$$

(ix)

$$\mathcal{S}_9(\Omega_1,\Omega_2)=\frac{1}{n}\sum_{i=1}^{n}\frac{2\Big[(\underline{\mathcal{P}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{P}}_{\Omega_2}(\nu_i))+(\underline{\mathcal{E}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{E}}_{\Omega_2}(\nu_i))+(\underline{\mathcal{N}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{N}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{P}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{P}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{E}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{E}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{N}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{N}}_{\Omega_2}(\nu_i))+(\underline{\mathcal{R}}_{\Omega_1}(\nu_i))*(\underline{\mathcal{R}}_{\Omega_2}(\nu_i))+(\overline{\mathcal{R}}_{\Omega_1}(\nu_i))*(\overline{\mathcal{R}}_{\Omega_2}(\nu_i))\Big]}{\Big[(\underline{\mathcal{P}}_{\Omega_1}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_1}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_1}(\nu_i))^2+(\underline{\mathcal{R}}_{\Omega_1}(\nu_i))^2+(\overline{\mathcal{R}}_{\Omega_1}(\nu_i))^2\Big]+\Big[(\underline{\mathcal{P}}_{\Omega_2}(\nu_i))^2+(\underline{\mathcal{E}}_{\Omega_2}(\nu_i))^2+(\underline{\mathcal{N}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{P}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{E}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{N}}_{\Omega_2}(\nu_i))^2+(\underline{\mathcal{R}}_{\Omega_2}(\nu_i))^2+(\overline{\mathcal{R}}_{\Omega_2}(\nu_i))^2\Big]},\ [20].$$

Analysis : From Table 12, it is evident that, when existing SMs are used in the proposed TOPSIS method instead of CC, it fails to identify the best alternative. However, the best alternative is identified for all the cases when CC is used. Hence, CC performs better than SMs.

Table 12: Comparison of existing SMs with proposed method.

Unable to rank using existing SMs	Able to rank using Proposed method
$\mathcal{S}_1(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_2(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_3(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = \mathcal{A}^3 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_4(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = \mathcal{A}^3 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_5(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = \mathcal{A}^3 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_6(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_7(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = \mathcal{A}^3 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_8(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$
$\mathcal{S}_9(\Omega_1, \Omega_2) \Rightarrow \mathcal{A}^1 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.49$	$\mathcal{A}^4 = 0.55, > \mathcal{A}^3 = 0.54, > \mathcal{A}^2 = 0.53, > \mathcal{A}^1 = 0.40.$

7. Conclusions

In this work, we have introduced the notion of IVPFHSS and established some of its properties. We have proposed aggregation operators and an application based on the TOPSIS method to identify a suitable employee, who can handle the project successfully using the Leipzig leadership model. This study ranks the alternatives whose information is in IVPFHSS form. By using aggregation operators and also by applying the TOPSIS method based on CC, we examine the relevance of IVPFHSS. Also, we show the influence of the CC method by comparing the proposed method with similarity measures. To study the closeness coefficients, we have applied CC instead of SMs in the proposed TOPSIS method. We have presented a comparative study between the proposed method and the existing SMS to prove the reliability of the proposed model.

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