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Fuzzy Ostrowski type inequalities via ϕ-λ-convex functions

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Abstract

We would like to state well-known Ostrowski inequality via ϕ-λ-convex by using the Fuzzy Reimann integrals. In addition, we establish some Fuzzy Ostrowski type inequalities for the class of functions whose derivatives in absolute values at certain powers are ϕ-λ-convex by Hölder's and power mean inequalities. We are introducing very first time that the class of ϕ-λ-convex function, which is the generalization of many important classes including class of h-convex, Godunova-Levin s-convex, s-convex in the 2^{nd} kind and hence contains convex functions. It also contains class of P-convex and class of Godunova-Levin. In this way we also capture the results with respect to convexity of functions.

Keywords: Ostrowski inequality, convex functions, fuzzy sets.

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1. Introduction

In recent years, the generalization of classical convex function have emerged resulting in applications in the field of Mathematics. From literature, we recall some definitions for different types of convex functions.

Definition 1.1 ([3]). A function $\eta : I \subset \mathbb{R} \to \mathbb{R}$ is said to be convex, if

$$\eta\left(tx + (1-t)y\right) \leqslant t\eta(x) + (1-t)\eta(y),$$

$\forall x, y \in I, t \in [0, 1]$.

Definition 1.2 ([3]). A function $\eta : I \subset \mathbb{R} \to \mathbb{R}$ is said to be MT-convex, if η is a non-negative and

$$\eta\left(tx + (1-t)y\right) \leqslant \frac{\sqrt{t}}{2\sqrt{1-t}}\eta(x) + \frac{\sqrt{1-t}}{2\sqrt{t}}\eta(y),$$

$\forall x, y \in I, t \in [0, 1]$.

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Definition 1.3 ([15]). We say that $\eta : I \subset \mathbb{R} \to \mathbb{R}$ is a P-convex function, if η is a non-negative and $\forall x, y \in I$ and $t \in [0, 1]$ we have

$$\eta(tx + (1-t)y) \leqslant \eta(x) + \eta(y).$$

Definition 1.4 ([18]). We say that $\eta : I \subset \mathbb{R} \to \mathbb{R}$ is a Godunova-Levin convex function, if η is non-negative and $\forall x, y \in I$ and $t \in (0, 1)$ we have

$$\eta(tx + (1-t)y) \leqslant \frac{1}{t}\eta(x) + \frac{1}{1-t}\eta(y).$$

Definition 1.5 ([4]). Let $s \in [0, 1]$. A function $\eta : I \subset [0, \infty) \to \mathbb{R}$ is said to be s-convex in the 2^{nd} kind, if

$$\eta(tx + (1-t)y) \leqslant t^s\eta(x) + (1-t)^s\eta(y),$$

$\forall x, y \in I, t \in [0, 1]$.

Definition 1.6 ([11]). We say that the function $\eta : I \subset \mathbb{R} \to [0, \infty)$ is of Godunova-Levin s-convex function, with $s \in [0, 1]$, if

$$\eta(tx + (1-t)y) \leqslant \frac{1}{t^s}\eta(x) + \frac{1}{(1-t)^s}\eta(y),$$

$\forall t \in (0, 1)$ and $x, y \in I$.

Definition 1.7 ([29]). Let $h : J \subseteq \mathbb{R} \to [0, \infty)$ with h not identical to 0. We say that $\eta : I \subseteq \mathbb{R} \to [0, \infty)$ is an h-convex function if $\forall x, y \in I$, we have

$$\eta(tx + (1-t)y) \leqslant h(t)\eta(x) + h(1-t)\eta(y),$$

$\forall t \in (0, 1)$.

Definition 1.8 ([10]). Let $\phi : (0, 1) \to (0, \infty)$ be a measurable function. We say that the $\eta : I \to [0, \infty)$ is a ϕ-convex function on the interval I, if $\forall x, y \in I$ we have

$$\eta(tx + (1-t)y) \leqslant t\phi(t)\eta(x) + (1-t)\phi(1-t)\eta(y),$$

$\forall t \in (0, 1)$.

Theorem 1.9 ([27]). *Let $\varphi : [\rho_a, \rho_b] \to \mathbb{R}$ be differentiable function on (ρ_a, ρ_b) with the property that $|\varphi'(t)| \leqslant M$ $\forall t \in (\rho_a, \rho_b)$. Then*

$$\left|\varphi(x) - \frac{1}{\rho_b - \rho_a}\int_{\rho_a}^{\rho_b} \varphi(t)dt\right| \leqslant M(\rho_b - \rho_a)\left[\frac{1}{4} + \left(\frac{x - \frac{\rho_a+\rho_b}{2}}{\rho_b - \rho_a}\right)^2\right], \tag{1.1}$$

$\forall x \in (\rho_a, \rho_b)$.

Definition 1.10 ([31]). A fuzzy number is $\phi : \mathbb{R} \to [0, 1]$, can be defined as

1. $[\phi]^0 = \text{closure}(\{r \in \mathbb{R} : \phi(r) > 0\})$ is compact;
2. ϕ is normal (i.e., $\exists\, r_0 \in \mathbb{R}$ such that $\phi(r_0) = 1$);
3. ϕ is fuzzy convex, i.e., $\phi(\eta r_1 + (1-\eta)r_2) \geqslant \min\{\phi(r_1), \phi(r_2)\}$, $\forall r_1, r_2 \in \mathbb{R}, \eta \in [0, 1]$;
4. $\forall r_0 \in R$ and $\epsilon > 0$, $\exists$ neighborhood $V(r_0)$, such that $\phi(r) \leqslant \phi(r_0) + \epsilon$, $\forall r \in \mathbb{R}$.

Definition 1.11 ([30]). For any $\zeta \in [0, 1]$, and ϕ be any fuzzy number, then ζ-level set $[\phi]^\zeta = \{r \in \mathbb{R} : \phi(r) \geqslant \zeta\}$. Moreover $[\phi]^\zeta = \left[\phi_-^{(\zeta)}, \phi_+^{(\zeta)}\right]$, $\forall \zeta \in [0, 1]$.

Proposition 1.12 ([24]). *Let $\phi, \varphi \in F_{\mathbb{R}}$ (set of all Fuzzy numbers) and $\eta \in \mathbb{R}$, then the following properties holds.*

1. $[\phi]^{\zeta_1} \subseteq [\varphi]^{\zeta_2}$, *whenever* $0 \leqslant \zeta_2 \leqslant \zeta_1 \leqslant 1$;
2. $[\phi + \varphi]^{\zeta} = [\phi]^{\zeta} + [\varphi]^{\zeta}$;
3. $[\eta \odot \phi]^{\zeta} = \eta\, [\phi]^{\zeta}$;
4. $\phi \oplus \varphi = \varphi \oplus \phi$;
5. $\eta \odot \phi = \phi \odot \eta$;
6. $\widetilde{1} \odot \phi = \phi$,

$\forall \zeta \in [0, 1]$, *where* $\widetilde{1} \in F_{\mathbb{R}}$, *defined by* $\forall r \in \mathbb{R}, \widetilde{1}(r) = 1$.

Definition 1.13 ([31]). Let $D : F_{\mathbb{R}} \times F_{\mathbb{R}} \to \mathbb{R}_+ \cup \{0\}$, defined as

$$D(\phi, \varphi) = \sup_{\zeta \in [0,1]} \max \left\{ \left| \phi_-^{(\zeta)} - \phi_+^{(\zeta)} \right|, \left| \varphi_-^{(\zeta)} - \varphi_+^{(\zeta)} \right| \right\},$$

$\forall \phi, \varphi \in F_{\mathbb{R}}$. Then D is a metric on $F_{\mathbb{R}}$.

Proposition 1.14 ([31]). *Let $\phi_1, \phi_2, \phi_3, \phi_4 \in F_{\mathbb{R}}$ and $\eta \in F_{\mathbb{R}}$, we have*

1. $(F_{\mathbb{R}}, D)$ *is complete;*
2. $D(\phi_1 \oplus \phi_3, \phi_2 \oplus \phi_3) = D(\phi_1, \phi_2)$;
3. $D(\eta \odot \phi_1, \eta \odot \phi_2) = |\eta| D(\phi_1, \phi_2)$;
4. $D(\phi_1 \oplus \phi_2, \phi_3 \oplus \phi_4) \leqslant D(\phi_1, \phi_3) + D(\phi_2, \phi_4)$;
5. $D(\phi_1 \oplus \phi_2, \widetilde{0}) \leqslant D(\phi_1, \widetilde{0}) + D(\phi_2, \widetilde{0})$;
6. $D(\phi_1 \oplus \phi_2, \phi_3) \leqslant D(\phi_1, \phi_3) + D(\phi_2, \widetilde{0})$,

where $\widetilde{0} \in F_{\mathbb{R}}$, *defined by* $\forall r \in \mathbb{R}, \widetilde{0}(r) = 0$.

Definition 1.15 ([30]). Let $\phi, \varphi \in F_{\mathbb{R}}$, if $\exists\, \theta \in F_{\mathbb{R}}$, such that $\phi = \varphi \oplus \theta$, then θ is H-difference of ϕ and φ, denoted by $\theta = \phi \ominus \varphi$.

Definition 1.16 ([30]). A function $\phi : [r_0, r_0 + \epsilon] \to F_{\mathbb{R}}$ is H-differentiable at r, if $\exists\, \phi'(r) \in F_{\mathbb{R}}$, i.e., both limits

$$\lim_{h \to 0^+} \frac{\phi(r + h) \ominus \phi(r)}{h}, \quad \lim_{h \to 0^+} \frac{\phi(r) \ominus \phi(r - h)}{h},$$

exist and are equal to $\phi'(r)$.

Definition 1.17 ([17]). Let $\phi : [\rho_a, \rho_b] \to F_{\mathbb{R}}$, if $\forall \zeta > 0, \exists \eta > 0$, for any partition $P = \{[u, v] : \delta\}$ of $[\rho_a, \rho_b]$ with norm $\Delta(P) < \eta$, we have

$$D\left(\sum_P^{*} (v - u)\phi(\delta), \varphi \right) < \zeta,$$

then we say that ϕ is Fuzzy-Riemann integrable to $\varphi \in F_{\mathbb{R}}$ and we write it as

$$\varphi = (FR) \int_{\rho_a}^{\rho_b} \phi(x)dx.$$

In order to prove our main results, we need the following lemma that has been obtained in [28].

Lemma 1.18. *Let $\varphi : [\rho_a, \rho_b] \to F_{\mathbb{R}}$ be an absolutely continuous mapping on (ρ_a, ρ_b) with $\rho_a < \rho_b$. If $\varphi' \in C_F[\rho_a, \rho_b] \cap L_F[\rho_a, \rho_b]$, then for $x \in (\rho_a, \rho_b)$ the following identity holds:*

$$\frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt \oplus \frac{(x-\rho_a)^2}{\rho_b - \rho_a} \odot (FR)\int_0^1 t \odot \varphi'(tx + (1-t)\rho_a)dt$$
$$= \varphi(x) \oplus \frac{(\rho_b - x)^2}{\rho_b - \rho_a} \odot (FR)\int_0^1 t \odot \varphi'(tx + (1-t)\rho_b)dt.$$

We make use of the beta function of Euler type, which is for $x, y > 0$ defined as

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)},$$

where $\Gamma(x) = \int_0^\infty e^{-u}u^{x-1}du$.

2. Fuzzy Ostrowski type inequalities via ϕ-λ-convex functions

In this section, we are introducing for very first time the concept of ϕ-λ-convex function, which contains many classes of convex functions in litrature.

Definition 2.1. Let $\lambda \in (0,1], \phi : (0,1) \to (0,\infty)$ be a measurable function. We say that the $\eta : I \to [0,\infty)$ is a ϕ-λ-convex function on the interval I if for all $x, y \in I$ we have

$$\eta\left(tx + (1-t)y\right) \leqslant t^\lambda \phi(t)\eta(x) + (1-t)^\lambda \phi(1-t)\eta(y), \tag{2.1}$$

$\forall t \in (0,1)$.

Remark 2.2. In Definition 2.1, one can see the following.

1. If we put $\lambda = 1$, in (2.1), then we get the concept of ϕ-convex function.
2. If we denote $l(t) = t$, and by taking $\lambda = 1$, $h = l\phi$ in (2.1), we get h-convex function.
3. If we take $\lambda = 1$, $\phi(t) = \frac{1}{t^{s+1}}$ with $s \in [0,1]$ in (2.1), then we get the class of Godunova-Levin s-convex functions.
4. If we put $\lambda = 1$, $\phi(t) = \frac{1}{t^2}$ in (2.1), then we get the concept of Godunova-Levin convex function.
5. If we put $\lambda = 1$, $\phi(t) = t^{s-1}$ with $s \in [0,1]$ in (2.1), then we get the concept of s-convex in 2nd kind.
6. If we put $\lambda = 1$, $\phi(t) = \frac{1}{t}$ in (2.1), then we get the concept of P-convex function.
7. If we put $\lambda = 1$, $\phi(t) = 1$ in (2.1), then we get the concept of ordinary convex function.
8. If we put $\lambda = 1$, $\phi(t) = \frac{1}{2\sqrt{t(1-t)}}$ in (2.1), then we get the concept of MT-convex function.

Theorem 2.3. *Suppose all the assumptions of Lemma 1.18 hold. Additionally, $\lambda \in (0,1], \phi : (0,1) \to (0,\infty)$ be a measurable function with $\phi(t) \neq \frac{1}{t^2}$, $D(\varphi', \widetilde{0})$ be a ϕ-convex function on $[\rho_a, \rho_b]$, and $D(\varphi'(x), \widetilde{0}) \leqslant M$. Then for each $x \in (\rho_a, \rho_b)$ the following inequality holds:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant M\left(\int_0^1 \left(t^{1+\lambda}\phi(t) + t(1-t)^\lambda \phi(1-t)\right)dt\right)\kappa(x), \tag{2.2}$$

where $\kappa(x) = \frac{(x-\rho_a)^2 + (\rho_b - x)^2}{\rho_b - \rho_a}$.

Proof. From the Lemma 1.18,

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$

$$
\begin{aligned}
&\leqslant D\left(\frac{(x-\rho_a)^2}{\rho_b-\rho_a}\odot (FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_a)dt, \frac{(\rho_b-x)^2}{\rho_b-\rho_a}\odot (FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_b)dt\right),\\
&\leqslant D\left(\frac{(x-\rho_a)^2}{\rho_b-\rho_a}\odot (FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_a)dt,\widetilde{0}\right) \qquad (2.3)\\
&\quad + D\left(\frac{(\rho_b-x)^2}{\rho_b-\rho_a}\odot (FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_b)dt,\widetilde{0}\right),\\
&= \frac{(x-\rho_a)^2}{\rho_b-\rho_a}D\left((FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_a)dt,\widetilde{0}\right)+\frac{(\rho_b-x)^2}{\rho_b-\rho_a}D\left((FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_b)dt,\widetilde{0}\right),\\
&\leqslant \frac{(x-\rho_a)^2}{\rho_b-\rho_a}\int_0^1 tD\left(\varphi'(tx+(1-t)\rho_a),\widetilde{0}\right)dt+\frac{(\rho_b-x)^2}{\rho_b-\rho_a}\int_0^1 tD\left(\varphi'(tx+(1-t)\rho_b),\widetilde{0}\right)dt.
\end{aligned}
$$

Since $D(\varphi',\widetilde{0})$ is ϕ-λ-convex function and $D(\varphi'(x),\widetilde{0})\leqslant M$, we have

$$
\begin{aligned}
D\left(\varphi'(tx+(1-t)\rho_a),\widetilde{0}\right) &\leqslant t^\lambda\phi(t)D\left(\varphi'(x),\widetilde{0}\right)+(1-t)^\lambda\phi(1-t)D\left(\varphi'(\rho_a),\widetilde{0}\right)\\
&\leqslant M\left[t^\lambda\phi(t)+(1-t)^\lambda\phi(1-t)\right], \qquad (2.4)\\
D\left(\varphi'(tx+(1-t)\rho_b),\widetilde{0}\right) &\leqslant t^\lambda\phi(t)D\left(\varphi'(x),\widetilde{0}\right)+(1-t)^\lambda\phi(1-t)D\left(\varphi'(\rho_b),\widetilde{0}\right)\\
&\leqslant M\left[t^\lambda\phi(t)+(1-t)^\lambda\phi(1-t)\right]. \qquad (2.5)
\end{aligned}
$$

Now using (2.4) and (2.5) in (2.3) we get (2.2). □

Corollary 2.4. *In Theorem 2.3, one can see the following.*

1. *If $\lambda=1$ in (2.2), then Fuzzy Ostrowski inequality for ϕ-convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot (FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant M\left(\int_0^1\left(t^2\phi(t)+t(1-t)\phi(1-t)\right)dt\right)\kappa(x).$$

2. *If $\lambda=1$, $l(t)=t$, then by taking $h=l\phi$ in (2.2), Fuzzy Ostrowski inequality for h-convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot (FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant M\left(\int_0^1\left(th(t)+th(1-t)\right)dt\right)\kappa(x).$$

3. *If $\lambda=1$, $\phi(t)=t^{-(s+1)}$ in (2.2), then Fuzzy Ostrowski inequality for Godunova-Levin s-convex functions is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot (FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant M\left(\frac{1}{1-s}\right)\kappa(x).$$

4. *If $\lambda=1$, $\phi(t)=t^{s-1}$ where $s\in(0,1]$ in (2.2), then Fuzzy Ostrowski inequality for s-convex functions in 2^{nd} kind is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot (FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant M\left(\frac{1}{1+s}\right)\kappa(x).$$

5. *If $\lambda=1$, $\phi(t)=t^{-1}$ in (2.2), then Fuzzy Ostrowski inequality for P-convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot (FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant M\kappa(x).$$

6. *If* $\phi(t) = \lambda = 1$ *in* (2.2), *then Fuzzy Ostrowski inequality for convex function is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2}\kappa(x).$$

7. *If* $\lambda = 1$, $\phi(t) = \frac{1}{2\sqrt{t(1-t)}}$ *in in* (2.2), *then Fuzzy Ostrowski inequality for* MT*-convex function is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M\pi}{4}\kappa(x).$$

Theorem 2.5. *Suppose all the assumptions of Lemma 1.18 hold. Additionally,* $\lambda \in (0,1]$, $\phi : (0,1) \to (0,\infty)$ *be a measurable function with* $\phi(t) \neq \frac{1}{t^2}$, $[D(\varphi', \widetilde{0})]^q$ *for* $q \geqslant 1$ *be* ϕ-λ*-convex function on* $[\rho_a, \rho_b]$ *and* $D(\varphi'(x), \widetilde{0}) \leqslant M$. *Then* $\forall x \in (\rho_a, \rho_b)$ *the following inequality holds:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2^{1-\frac{1}{q}}}\left(\int_0^1 \left(t^{1+\lambda}\phi(t) + t(1-t)^{\lambda}\phi(1-t)\right) dt\right)^{\frac{1}{q}} \kappa(x). \tag{2.6}$$

Proof. From the inequality (2.3) and power mean inequality [32]

$$\begin{aligned} &D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \\ &\leqslant \frac{(x-\rho_a)^2}{\rho_b - \rho_a}\left(\int_0^1 t dt\right)^{1-\frac{1}{q}}\left(\int_0^1 t\left[D\left(\varphi'(tx + (1-t)\rho_a), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}} \\ &\quad + \frac{(\rho_b - x)^2}{\rho_b - \rho_a}\left(\int_0^1 t dt\right)^{1-\frac{1}{q}}\left(\int_0^1 t\left[D\left(\varphi'(tx + (1-t)\rho_b), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}}. \end{aligned} \tag{2.7}$$

Since $[D(\varphi', \widetilde{0})]^q$ is ϕ-λ-convex function and $D(\varphi'(x), \widetilde{0}) \leqslant M$, we have

$$\begin{aligned} \left[D\left(\varphi'(tx + (1-t)\rho_a), \widetilde{0}\right)\right]^q &\leqslant t^{\lambda}\phi(t)\left[D\left(\varphi'(x), \widetilde{0}\right)\right]^q + (1-t)^{\lambda}\phi(1-t)\left[D\left(\varphi'(\rho_a), \widetilde{0}\right)\right]^q \\ &\leqslant M^q\left[t^{\lambda}\phi(t) + (1-t)^{\lambda}\phi(1-t)\right], \end{aligned} \tag{2.8}$$

$$\begin{aligned} \left[D\left(\varphi'(tx + (1-t)\rho_b), \widetilde{0}\right)\right]^q &\leqslant t^{\lambda}\phi(t)\left[D\left(\varphi'(x), \widetilde{0}\right)\right]^q + (1-t)^{\lambda}\phi(1-t)\left[D\left(\varphi'(\rho_b), \widetilde{0}\right)\right]^q \\ &\leqslant M^q\left[t^{\lambda}\phi(t) + (1-t)^{\lambda}\phi(1-t)\right], \end{aligned} \tag{2.9}$$

Now using (2.8) and (2.9) in (2.7) we get (2.6). □

Corollary 2.6. *In Theorem 2.5, one can see the following.*

1. *If* $q = 1$, *then we have Theorem 2.3.*
2. *If* $\lambda = 1$ *in* (2.6), *then Fuzzy Ostrowski inequality for* ϕ*-convex function is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2^{1-\frac{1}{q}}}\left(\int_0^1 \left(t^2\phi(t) + t(1-t)\phi(1-t)\right) dt\right)^{\frac{1}{q}} \kappa(x).$$

3. *If* $\lambda = 1$, $l(t) = t$, *then by taking* $h = l\phi$ *in* (2.6), *Fuzzy Ostrowski inequality for* h*-convex function is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2^{1-\frac{1}{q}}}\left(\int_0^1 \left(th(t) + th(1-t)\right) dt\right)^{\frac{1}{q}} \kappa(x).$$

4. *If* $\lambda = 1$, $\phi(t) = t^{-(s+1)}$ *in* (2.6), *then one has Fuzzy Ostrowski inequality for Godunova-Levin* s*-convex functions is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2^{1-\frac{1}{q}}}\left(\frac{1}{1-s}\right)^{\frac{1}{q}} \kappa(x).$$

5. *If* $\lambda = 1$, $\phi(t) = t^{s-1}$ *where* $s \in [0,1]$ *in* (2.6), *then one has Fuzzy Ostrowski inequality for* s*-convex functions in* 2^{nd} *kind is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2^{1-\frac{1}{q}}}\left(\frac{1}{1+s}\right)^{\frac{1}{q}} \kappa(x).$$

6. *If* $\lambda = 1$, $\phi(t) = t^{-1}$ *in* (2.6), *then Fuzzy Ostrowski inequality for* P*-convex function is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2^{1-\frac{1}{q}}} \kappa(x).$$

7. *If* $\lambda = 1$, $\phi(t) = 1$ *in* (2.6), *then Fuzzy Ostrowski inequality for convex function is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{2} \kappa(x).$$

8. *If* $\lambda = 1$, $\phi(t) = \frac{1}{2\sqrt{t(1-t)}}$ *in* (2.6), *then Fuzzy Ostrowski inequality for* MT*-convex function is*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M\pi^{\frac{1}{q}}}{2^{1+\frac{1}{q}}} \kappa(x).$$

Theorem 2.7. *Suppose all the assumptions of Lemma 1.18 hold. Additionally, assume that* $\lambda \in (0,1]$, $\phi : (0,1) \to (0,\infty)$ *be a measurable function with* $\phi(t) \neq \frac{1}{t^2}$, $[D(\varphi', \widetilde{0})]^q$ *be a* ϕ*-*λ*-convex function on* $[\rho_a, \rho_b]$, $q > 1$ *and* $D(\varphi'(x), \widetilde{0}) \leqslant M$. *Then for each* $x \in (\rho_a, \rho_b)$, *the following inequality holds:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leqslant \frac{M}{(p+1)^{\frac{1}{p}}}\left(\int_0^1 \left(t^\lambda \phi(t) + (1-t)^\lambda \phi(1-t)\right) dt\right)^{\frac{1}{q}} \kappa(x), \tag{2.10}$$

where $p^{-1} + q^{-1} = 1$.

Proof. From the inequality (2.3) and Hölder's inequality [33]

$$\begin{aligned} &D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \\ &\leqslant \frac{(x-\rho_a)^2}{\rho_b - \rho_a}\left(\int_0^1 t^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left[D\left(\varphi'(tx + (1-t)\rho_a), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}} \\ &\quad + \frac{(\rho_b - x)^2}{\rho_b - \rho_a}\left(\int_0^1 t^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left[D\left(\varphi'(tx + (1-t)\rho_b), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}}. \end{aligned} \tag{2.11}$$

Since $[D(\varphi', \widetilde{0})]^q$ be ϕ-λ-convex function and $D(\varphi'(x), \widetilde{0}) \leqslant M$, we have

$$\begin{aligned} \left[D\left(\varphi'(tx + (1-t)\rho_a), \widetilde{0}\right)\right]^q &\leqslant t^\lambda \phi(t)\left[D\left(\varphi'(x), \widetilde{0}\right)\right]^q + (1-t)^\lambda \phi(1-t)\left[D\left(\varphi'(\rho_a), \widetilde{0}\right)\right]^q \\ &\leqslant M^q\left[t^\lambda \phi(t) + (1-t)^\lambda \phi(1-t)\right], \end{aligned} \tag{2.12}$$

$$\begin{aligned}\left[D\left(\varphi'(tx+(1-t)\rho_b),\widetilde{0}\right)\right]^q &\leqslant t^\lambda\phi(t)\left[D\left(\varphi'(x),\widetilde{0}\right)\right]^q+(1-t)^\lambda\phi(1-t)\left[D\left(\varphi'(\rho_b),\widetilde{0}\right)\right]^q\\ &\leqslant M^q\left[t^\lambda\phi(t)+(1-t)^\lambda\phi(1-t)\right].\end{aligned} \tag{2.13}$$

Now using (2.12) and (2.13) in (2.11) we get (2.10). □

Corollary 2.8. *In Theorem 2.7, one can see the following.*

1. *If* $\lambda=1$ *in* (2.10), *then Fuzzy Ostrowski inequality for* ϕ*-convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant\frac{M}{(p+1)^{\frac{1}{p}}}\left(\int_0^1\left(t\phi(t)+(1-t)\phi(1-t)\right)dt\right)^{\frac{1}{q}}\kappa(x).$$

2. *If* $\lambda=1$, $l(t)=t$, *the identity function, then by taking* $h=l\phi$ *in* (2.10), *then Fuzzy Ostrowski inequality for* h*-convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant\frac{M}{(p+1)^{\frac{1}{p}}}\left(\int_0^1\left(h(t)+h(1-t)\right)dt\right)^{\frac{1}{q}}\kappa(x).$$

3. *If* $\lambda=1$, $\phi(t)=t^{-(s+1)}$, *where* $s\in[0,1)$ *in* (2.10), *then Fuzzy Ostrowski inequality for Godunova-Levin* s*-convex functions is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant\frac{M}{(p+1)^{\frac{1}{p}}}\left(\frac{2}{1-s}\right)^{\frac{1}{q}}\kappa(x).$$

4. *If* $\lambda=1$, $\phi(t)=t^{s-1}$, *where* $s\in(0,1]$ *in* (2.10), *then Fuzzy Ostrowski inequality for* s*-convex functions in* 2^{nd} *kind is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant\frac{M}{(p+1)^{\frac{1}{p}}}\left(\frac{2}{1+s}\right)^{\frac{1}{q}}\kappa(x).$$

5. *If* $\lambda=1$, $\phi(t)=t^{-1}$, *in* (2.10), *then Fuzzy Ostrowski inequality for* P*-convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant\frac{2^{\frac{1}{q}}M}{(p+1)^{\frac{1}{p}}}\kappa(x).$$

6. *If* $\phi(t)=\lambda=1$, *in* (2.10), *then Fuzzy Ostrowski inequality for convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant\frac{M}{(p+1)^{\frac{1}{p}}}\kappa(x).$$

7. *If* $\lambda=1$, $\phi(t)=\frac{1}{2\sqrt{t(1-t)}}$ *in* (2.10), *then Fuzzy Ostrowski inequality for* MT*-convex function is*

$$D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leqslant\frac{M\left(\frac{\pi}{2}\right)^{\frac{1}{q}}}{(1+p)^{\frac{1}{p}}}\kappa(x).$$

2.1. Fuzzy Ostrowski type midpoint inequalities via ϕ-λ-convex functions

Remark 2.9. In Theorem 2.5, one can see the following.

1. If $x = \frac{\rho_a+\rho_b}{2}$ in (2.6), then Fuzzy Ostrowski Midpoint inequality for ϕ-λ-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2^{2-\frac{1}{q}}}\left(\int_0^1\left(t^{1+\lambda}\phi(t)+t(1-t)^{\lambda}\phi(1-t)\right)dt\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
2. If $\lambda = 1$, $x = \frac{\rho_a+\rho_b}{2}$ in (2.6), then Fuzzy Ostrowski Midpoint inequality for ϕ-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2^{2-\frac{1}{q}}}\left(\int_0^1\left(t^{2}\phi(t)+t(1-t)\phi(1-t)\right)dt\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
3. If $\lambda = 1$, $x = \frac{\rho_a+\rho_b}{2}$, $l(t) = t$ and $h = l\phi$ in (2.6), then Fuzzy Ostrowski Midpoint inequality for h-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2^{2-\frac{1}{q}}}\left(\int_0^1\left(th(t)+th(1-t)\right)dt\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
4. If $\lambda = 1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t) = t^{-(s+1)}$ where $s \in [0,1)$ in (2.6), then one has Fuzzy Ostrowski Midpoint inequality for Godunova-Levin s-convex functions as
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2^{2-\frac{1}{q}}}\left(\frac{1}{1-s}\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
5. If $\lambda = 1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t) = t^{s-1}$ where $s \in [0,1]$ in (2.6), then one has Fuzzy Ostrowski Midpoint inequality for s-convex functions in 2^{nd} kind as
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2^{2-\frac{1}{q}}}\left(\frac{1}{1+s}\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
6. If $\lambda = 1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t) = t^{-1}$ in (2.6), then Fuzzy Ostrowski Midpoint inequality for P-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2^{2-\frac{1}{q}}}(\rho_b-\rho_a).$$
7. If $\lambda = 1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t) = 1$ in (2.6), then Fuzzy Ostrowski Midpoint inequality for convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{4}(\rho_b-\rho_a).$$
8. If $\lambda = 1$, $\phi(t) = \frac{1}{2\sqrt{t(1-t)}}$ in (2.6), then Fuzzy Ostrowski inequality for MT-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M\pi^{\frac{1}{q}}}{2^{1+\frac{1}{q}}}(\rho_b-\rho_a).$$

Remark 2.10. In Theorem 2.7, one can see the following.

1. If $x = \frac{\rho_a+\rho_b}{2}$ in (2.10), then Fuzzy Ostrowski Midpoint inequality for ϕ-λ-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2(p+1)^{\frac{1}{p}}}\left(\int_0^1\left(t^{\lambda}\phi(t)+(1-t)^{\lambda}\phi(1-t)\right)dt\right)^{\frac{1}{q}}(\rho_b-\rho_a),$$
2. If $\lambda=1$, $x = \frac{\rho_a+\rho_b}{2}$ in (2.10), then Fuzzy Ostrowski Midpoint inequality for ϕ-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2(p+1)^{\frac{1}{p}}}\left(\int_0^1\left(t\phi(t)+(1-t)\phi(1-t)\right)dt\right)^{\frac{1}{q}}(\rho_b-\rho_a),$$
3. If $\lambda=1$, $x = \frac{\rho_a+\rho_b}{2}$, $l(t)=t$ and $h = l\phi$ in (2.10), then Fuzzy Ostrowski Midpoint inequality for h-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2(p+1)^{\frac{1}{p}}}\left(\int_0^1\left(h(t)+h(1-t)\right)dt\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
4. If $\lambda=1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t)=t^{-(s+1)}$ where $s\in[0,1)$ in (2.10), then Fuzzy Ostrowski Midpoint inequality for Godunova-Levin s-convex functions is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{2^{\frac{1}{q}-1}M}{(p+1)^{\frac{1}{p}}}\left(\frac{1}{1-s}\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
5. If $\lambda=1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t)=t^{s-1}$, where $s\in(0,1]$ in (2.10), then Fuzzy Ostrowski Midpoint inequality for s-convex functions in 2nd kind is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{2^{\frac{1}{q}-1}M}{(p+1)^{\frac{1}{p}}}\left(\frac{1}{1+s}\right)^{\frac{1}{q}}(\rho_b-\rho_a).$$
6. If $\lambda=1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t)=t^{-1}$ in (2.10), then Fuzzy Ostrowski Midpoint inequality for P-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{2^{\frac{1}{q}-1}M}{(p+1)^{\frac{1}{p}}}(\rho_b-\rho_a).$$
7. If $\lambda=1$, $x = \frac{\rho_a+\rho_b}{2}$ and $\phi(t)=1$ in (2.10), then Fuzzy Ostrowski Midpoint inequality for convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M}{2(p+1)^{\frac{1}{p}}}(\rho_b-\rho_a).$$
8. If $\lambda=1$, $\phi(t)=\frac{1}{2\sqrt{t(1-t)}}$ in (2.10), then Fuzzy Ostrowski inequality for MT-convex function is
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leqslant \frac{M\pi^{\frac{1}{q}}}{2^{\frac{1}{q}+1}(1+p)^{\frac{1}{p}}}(\rho_b-\rho_a).$$

3. Conclusion

Ostrowski inequality is one of the most celebrated inequalities, we can find its various generalizations and variants in literature. In this paper, we presented the generalized notion of ϕ-λ-convex function which is the generalization of many important classes including class of h-convex [10], h-convex [29], Godunova-Levin s-convex [11], s-convex in the 2^{nd} kind [4] (and hence contains class of convex functions [3]). It also contains class of P-convex functions [15] and class of Godunova-Levin functions [18]. We would like to state well-known Fuzzy Ostrowski inequality via ϕ-λ-convex function. In addition, we establish some Fuzzy Ostrowski type inequalities for the class of functions whose derivatives in absolute values at certain powers are ϕ-λ-convex functions by using different techniques including Hölder's inequality [33] and power mean inequality [32].

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