Journal of Mathematics and Computer Science

Impulsive effects on stabilization of stochastic nonlinear reaction-diffusion systems with time delays and boundary feedback control

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Abstract

In this paper, we investigate the stabilization of stochastic nonlinear impulsive reaction-diffusion systems (SNIRDSs) with time delays and boundary feedback control via average impulsive interval approach. Boundary feedback control strategy are designed to stabilization of SNIRDSs. By constructing a Lyapunov-Krasovskii functional (LKF), and using Wirtinger's inequality, Gronwall inequality, average impulsive interval approach, sufficient conditions are derived to guarantee the finite-time stability (FTS) of proposed systems. We investigate the stabilization results by designing the control gain matrices for boundary feedback controller. The criterions are expressed in terms of linear matrix inequalities (LMIs) that can be verified by Matlab LMI toolbox. Finally, numerical example are given to verify the efficiency and superiority of proposed stabilization criterions.

Keywords: Stochastic nonlinear systems, reaction-diffusion terms, impulsive effects, boundary feedback control, average impulsive interval approach.

2020 MSC: 93C10, 93E03, 93E15.

1. Introduction

In recent decades, partial differential systems (PDSs) have been extensively studied by many scholars because they can be adeptly applied to wide range of fields, including science and engineering. The complete structure and non-linear dynamical behaviors of nonlinear systems depend not only on the temporal evolution and space position of all variable, but also on its connections stemming from the space distributed structure of whole networks. However, because of the spatially inhomogeneous environment, diffusion effects are usually unavoidable. As a representative model, reaction-diffusion terms are used to investigate a wide range of issues such as secure communication [18], chemical reaction process [28], oncolytic M1-virotherapy model [8], virus transmission [21], and food web model [46]. As a result, PDSs with reaction-diffusion terms have received a lot of attention [7, 10, 20, 25, 27, 36, 48].

In real-world systems such as nonlinear circuits, biological systems, power systems, chemical industry systems, and reaction-diffusion systems (RDSs), time delays are mostly inevitable. Oscillation or instability in RDSs can be characterized by the presence of time delays. The LKF are useful when dealing with

time delays in RDSs. The work [24] used the LKF functional technique to deal with the effect of time delays on the RDSs. The work [50] studied the stability of the RDSs with time delays by employing the LKF. The LKF was also used to solve the stabilization problems for RDSs with time-varying delays [1, 6]. As a result, time delays are considered for RDSs [3, 26, 32, 33, 47, 54].

For whatever reason, the state of a system may undergo abrupt changes or disturbances in a very short period of time, altering the original trajectory, which is known as the impulse phenomenon. As a result, the RDSs with impulsive effects need to be more realistic and accurate the structure process of evolution. The dynamic behaviors of impulsive systems has attracted much interest recently [4, 5, 17, 19, 31, 34, 35, 37–39, 49, 55]. In general, the study of the stability and stabilization of impulsive nonlinear systems can be classified into two types based on impulsive effects: impulsive perturbation and impulsive control. Basically, impulsive perturbation takes into account the robustness of a system in which destabilizing impulses are typically present. Even as impulsive control is concerned with the stabilization of a systems, stabilizing impulses are often included.

Boundary controllers, as a particular control technique for PDSs, can be efforts to achieve the required dynamic behaviors of such PDSs while saving cost and making it simple to implement [9, 16, 22]. The back stepping method has been used to explore boundary control for RDSs. In [15, 52], the authors designed the boundary controller for RDSs. The authors of [23, 51] utilized the Lyapunov functional techniques for dealing with stabilization problems in RDSs using a boundary controller.

Motivated by preceding discussions, the aim of this paper is to obtain the FTS and stabilization of SNIRDSs with time delays and boundary feedback control via average impulsive interval approach. This paper contains the following major contributions: (i) a boundary feedback controller was designed to FTS and stabilization for SNIRDSs with time delays; (ii) by construct a LKF, and using Wirtinger's inequality, Gronwall inequality, LMIs, sufficient criterions are derived to ensure that the FTS of proposed systems; (iii) our main results reflect the effects of impulsive phenomenon, boundary feedback control, and reaction-diffusion terms on the FTS.

Notations: $\mathbb{N}$: set of natural numbers; $\mathbb{Z}_+$: set of positive integers; $\mathbb{R}$: set of real numbers; $\mathbb{R}_+$: set of positive real numbers; $\mathbb{R}^n$: Euclidean space of n-dimensions; $\mathbb{R}^{m \times n}$: Euclidean space of $(m \times n)$-dimensions; $A < 0$: real symmetric negative definite matrix; $A > 0$: real symmetric positive definite matrix; $A^\top$: transpose of A; $\lambda_{\min}(A)$: minimum eigen value of A; $\lambda_{\max}(A)$: maximum eigenvalue of A; $*$: the entries are implied by symmetric; $He\{A\} = (A + A^\top)$; $\|\cdot\|$: Euclidean norms; $\mathbb{E}(X)$: mathematical expectation of X; $\mathcal{W}^{1,2}([0, \Omega]; \mathbb{R}^n)$: Soblev n-dimensional space of continuous functions; $\int_0^1 \mathfrak{I}^\top(x, t) \mathfrak{I}(x, t) dx = \|\mathfrak{I}(x, t)\|^2$.

2. System description and preliminaries

Consider the following stochastic nonlinear impulsive reaction-diffusion systems (SNIRDSs) with time delays and boundary feedback control

$$
\begin{cases}
d\mathfrak{I}(x, t) = \left[\mathcal{D} \frac{\partial^2 \mathfrak{I}(x,t)}{\partial x^2} + \mathcal{A}\mathfrak{I}(x, t) + \mathcal{B}\mathfrak{I}(x, t - \sigma) + f(t, \mathfrak{I}(x, t)) + g(t, \mathfrak{I}(x, t - \sigma)) \right] dt \\
\qquad + h(t, \mathfrak{I}(x, t), \mathfrak{I}(x, t - \sigma)) d\omega(t), \ t \neq t_k, \\
\mathfrak{I}(x, t_k) = \Upsilon \mathfrak{I}(x, t_k^-), \ t = t_k, \ k \in \mathbb{N},
\end{cases}
\tag{2.1}
$$

with initial and Neumann boundary conditions as follows:

$$
\mathfrak{I}(x, s) = \phi(x, s), \ x \in (0, 1), \ s \in [-\sigma, 0], \ \frac{\partial \mathfrak{I}(x, t)}{\partial x}\bigg|_{x=0} = 0, \ \frac{\partial \mathfrak{I}(x, t)}{\partial x}\bigg|_{x=1} = u(t),
\tag{2.2}
$$

where $\mathfrak{I}(x, t) = [\mathfrak{I}_1(x, t), \mathfrak{I}_2(x, t), \dots, \mathfrak{I}_n(x, t)]^\top \in \mathbb{R}^n$ is a state vector, $t > 0$ is a time variable, $x \in (0, 1)$ is a space variable, and $\phi(x, s) = [\phi_1(x, s), \phi_2(x, s), \dots, \phi_n(x, s)]^\top \in \mathbb{R}^n$ is a continuous initial function. $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^\top \in \mathbb{R}^n$ denotes a boundary feedback control which will be designed later. $\mathcal{D}$ is a positive definite diffusion matrix. $f, g : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ and $h : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times m}$ are the continuous nonlinear functions. $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_n(t)]^\top \in \mathbb{R}^m$ denotes the Brownian motions.

σ is a time delay. $\mathcal{A}, \mathcal{B}$ are constant matrices with suitable dimensions. t_k is a impulsive instant time and satisfying the conditions $0 < t_0 < t_1 < \cdots < t_k < \cdots$, with $\lim_{k \to \infty} t_k = \infty$. Υ is impulsive gain matrix. In this paper, we take $\mathfrak{I}(x, t_k) = \mathfrak{I}(x, t_k^+)$.

Assumption 2.1 ([24]).

$(\mathcal{H}_1)$ There exist nonnegative constants α_1 and α_2 such that

$$(f(\eta_1) - f(\eta_2))^\top (f(\eta_1) - f(\eta_2)) \leqslant \alpha_1 (\eta_1 - \eta_2)^\top (\eta_1 - \eta_2),$$
$$(g(\gamma_1) - g(\gamma_2))^\top (g(\gamma_1) - g(\gamma_2)) \leqslant \alpha_2 (\gamma_1 - \gamma_2)^\top (\gamma_1 - \gamma_2), \quad \forall \eta_1, \eta_2, \gamma_1, \gamma_2 \in \mathbb{R}^n.$$

Assumption 2.2 ([4]).

$(\mathcal{H}_2)$ There exist nonnegative constants β_1 and β_2 such that

$$\text{trace}[h^\top(t, l, m) h(t, l, m)] \leqslant \beta_1 l^\top l + \beta_2 m^\top m, \quad \forall l, m \in \mathbb{R}^n.$$

Lemma 2.3 ([54]). *The following matrix inequality applies to any real matrices $\mathcal{M}, \mathcal{N}$ and a positive definite matrix $\mathcal{R}$:*

$$\mathcal{M}^\top \mathcal{N} + \mathcal{N}^\top \mathcal{M} \leqslant \mathcal{M}^\top \mathcal{R}^{-1} \mathcal{M} + \mathcal{N}^\top \mathcal{R} \mathcal{N}.$$

Lemma 2.4 ([52]). *For a state vector $x(t) \in \mathcal{W}^{1,2}([0, \Omega]; \mathbb{R}^n)$ with $x(0) = 0$ or $x(\Omega) = 0$ and matrix $\mathcal{M} > 0$, we get*

$$\int_0^\Omega x^\top(s) \mathcal{M} x(s) ds \leqslant \frac{4\Omega^2}{\pi^2} \int_0^\Omega \left(\frac{dx(s)}{ds}\right)^\top \mathcal{M} \left(\frac{dx(s)}{ds}\right) ds.$$

Lemma 2.5 ([4]). *Let $\mu \in \mathbb{R}$ and $\rho \in \mathbb{R}_+$ be a constants. If there is a function $y(t)$ that meets the criteria*

$$y(t) \leqslant \mu + \int_b^t \rho y(s) ds, \quad b \leqslant t \leqslant c,$$

then one has

$$y(t) \leqslant \mu e^{\rho(t-b)}.$$

Lemma 2.6 ([39]). *Let $\mathcal{U}, \mathcal{V}, \mathcal{W}$ be given matrices such that $\mathcal{U}^\top = \mathcal{U}$ and $\mathcal{V}^\top = \mathcal{V} > 0$, then*

$$\mathcal{U} + \mathcal{W}^\top \mathcal{V}^{-1} \mathcal{W} < 0 \Leftrightarrow \begin{bmatrix} \mathcal{U} & \mathcal{W}^\top \\ * & -\mathcal{V} \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\mathcal{V} & \mathcal{W} \\ * & \mathcal{U} \end{bmatrix} < 0.$$

Definition 2.7 ([4]). An impulsive sequence $\varpi = \{t_1, t_2, \ldots\}$ is said to have average impulsive interval T_ϵ if there exist constants $T_\epsilon > 0$ and $N_\epsilon \in \mathbb{Z}_+$ such that

$$\frac{T - t}{T_\epsilon} - N_\epsilon \leqslant N_\varpi(T, t) \leqslant N_\epsilon + \frac{T - t}{T_\epsilon}, \quad \forall \, T > t \geqslant 0,$$

where $N_\varpi(T, t)$ is a number of impulsive times of $\varpi = \{t_1, t_2, \ldots\}$.

Definition 2.8 ([15]). Given three constants $\kappa_1, \kappa_2,$ and T with $\kappa_1 < \kappa_2$, the SNIRDSs (2.1) is called finite-time stable (FTS) with respect to (κ_1, κ_2, T) if

$$\mathbb{E}\left\{ \sup_{s \in [-\sigma, 0]} \|\mathfrak{I}(x, s)\|^2 \right\} \leqslant \kappa_1 \Rightarrow \mathbb{E}\|\mathfrak{I}(x, t)\|^2 < \kappa_2, \forall \, t \in [0, T].$$

Definition 2.9 ([54]). The SNIRDSs (2.1) is said to be stabilizable if there exist control gain matrices for boundary feedback controller such that the SNIRDSs (2.1) is FTS with respect to given constants (κ_1, κ_2, T).

Remark 2.10. In this paper, we are interested in studying the dynamic characteristics of the SNIRDSs (2.1) within a finite-time interval $[0, T]$. Therefore, it is supposed that there exists a scalar $N_\varpi(T, 0) \in \mathbb{Z}_+$ such that $0 < t_1 < t_2 < \cdots < t_{N_\varpi(T, 0)} \leqslant T$.

Remark 2.11. In this paper, FTS condition is derived for the class of SNIRDSs (2.1), with the boundary feedback controller is designed. Secondly, stabilization for the class of SNIRDSs (2.1), with the control gain matrix is designed. In the analysis process, Lyapunov-function method and average impulsive interval technique are used to achieve our main results.

3. Main results

In this section, we obtain the FTS and stabilization for SNIRDSs (2.1) by using boundary feedback controller. The boundary feedback controller is proposed as

$$u(t) = \Phi \int_0^1 \Im(x,t)\,dx,$$

where Φ is a control gain matrix will be designed later.

Theorem 3.1. *Let* T_ϵ *be the average impulsive interval of* $\varpi = \{t_1, t_2, \ldots\}$. *Under Assumptions* ($\mathcal{H}_1$) *and* ($\mathcal{H}_2$), *the SNIRDSs* (2.1) *is said to be FTS with respect to given constants* (κ_1, κ_2, T) *if there exist constants* $N_\epsilon, \rho > 0, \epsilon \geqslant 1$ *and symmetric positive definite matrices* $\mathcal{P}, \mathcal{Q}, \mathcal{R}_1, \mathcal{R}_2$ *such that the following LMIs holds:*

$$\text{(i)} \quad \Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Pi_{33} \end{bmatrix} < 0, \tag{3.1}$$

$$\text{(ii)} \quad \Upsilon^{\mathsf{T}} \mathcal{P} \Upsilon \leqslant \epsilon \mathcal{P}, \tag{3.2}$$

$$\text{(iii)} \quad \frac{\kappa_1}{\lambda_{\min}(\mathcal{P})} \epsilon^{N_\epsilon} e^{\left(\frac{\ln(\epsilon)}{T_\epsilon} + \rho\right)\mathsf{T}} \left[\lambda_{\max}(\mathcal{P}) + e^{\rho\sigma}\lambda_{\max}(\mathcal{Q})\right] < \kappa_2, \tag{3.3}$$

where

$$\Pi_{11} = \text{He}(\mathcal{P}\mathcal{A} + \mathcal{D}\mathcal{P}\Phi) + \mathcal{Q} + \mathcal{P}\mathcal{R}_1^{-1}\mathcal{P} + \alpha_1\mathcal{R}_1 + \mathcal{P}\mathcal{R}_2^{-1}\mathcal{P} + \lambda_{\max}(\mathcal{P})\beta_1 - \rho\mathcal{P}, \quad \Pi_{12} = -\Phi^{\mathsf{T}}\mathcal{P}^{\mathsf{T}}\mathcal{D}^{\mathsf{T}},$$

$$\Pi_{13} = \mathcal{P}\mathcal{B}, \quad \Pi_{22} = -\frac{1}{2}\pi^2\mathcal{P}\mathcal{D}, \quad \Pi_{23} = 0, \quad \Pi_{33} = -e^{\rho\sigma}\mathcal{Q} + \alpha_2\mathcal{R}_2 + \lambda_{\max}(\mathcal{P})\beta_2.$$

Proof. Let us construct the following Lyapunov-Krasovskii functional (LKF) candidate,

$$V(t, \Im(x,t)) = \sum_{p=1}^2 V_p(t, \Im(x,t)),$$

where

$$V_1(t, \Im(x,t)) = \int_0^1 \Im^{\mathsf{T}}(x,t)\mathcal{P}\Im(x,t)\,dx, \quad V_2(t, \Im(x,t)) = \int_0^1 \int_{t-\sigma}^t e^{\rho(t-s)}\Im^{\mathsf{T}}(x,s)\mathcal{Q}\Im(x,s)\,ds\,dx.$$

For $t \in [t_k, t_{k+1}), k \in \mathbb{N}$, calculating $\mathcal{L}V(t, \Im(x,t))$ along the trajectories of SNIRDSs (2.1) by Ito's differential formula, we get

$$\mathcal{L}V(t, \Im(x,t)) = \mathcal{L}V_1(t, \Im(x,t)) + \mathcal{L}V_2(t, \Im(x,t)). \tag{3.4}$$

Further, we have

$$\mathcal{L}V_1(t, \Im(x,t)) = 2\int_0^1 \Im^{\mathsf{T}}(x,t)\mathcal{P}\left[\mathcal{D}\frac{\partial^2 \Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-\sigma) + f(t,\Im(x,t)) + g(t,\Im(x,t-\sigma))\right]dx$$

$$+ \int_0^1 \text{trace}\left[h^{\mathsf{T}}(t)\mathcal{P}h(t)\right]dx - \rho\int_0^1 \Im^{\mathsf{T}}(x,t)\mathcal{P}\Im(x,t)\,dx + \rho V_1(t,\Im(x,t)), \tag{3.5}$$

where $h(t) = h(t, \Im(x,t), \Im(x,t-\sigma))$,

$$\mathcal{L}V_2(t, \Im(x,t)) = \rho\int_0^1 \int_{t-\sigma}^t e^{\rho(t-s)}\Im^{\mathsf{T}}(x,s)\mathcal{Q}\Im(x,s)\,ds\,dx + \int_0^1 \Im^{\mathsf{T}}(x,t)\mathcal{Q}\Im(x,t)\,dx$$

$$- e^{\rho\sigma}\int_0^1 \Im^{\mathsf{T}}(x,t-\sigma)\mathcal{Q}\Im(x,t-\sigma)\,dx \tag{3.6}$$

$$\leqslant \rho V_2(t, \Im(x,t)) + \int_0^1 \Im^{\mathsf{T}}(x,t)\mathcal{Q}\Im(x,t)\,dx - e^{\rho\sigma}\int_0^1 \Im^{\mathsf{T}}(x,t-\sigma)\mathcal{Q}\Im(x,t-\sigma)\,dx.$$

Based on Lemma 2.3 and Assumption $(\mathcal{H}_1)$, we have

$$
\begin{aligned}
2\mathfrak{I}^\mathsf{T}(x,t)\mathcal{P}f(t,\mathfrak{I}(x,t)) &\leqslant \mathfrak{I}^\mathsf{T}(x,t)\mathcal{P}\mathcal{R}_1^{-1}\mathcal{P}\mathfrak{I}(x,t) + f^\mathsf{T}(t,\mathfrak{I}(x,t))\mathcal{R}_1 f(t,\mathfrak{I}(x,t)) \\
&\leqslant \mathfrak{I}^\mathsf{T}(x,t)\mathcal{P}\mathcal{R}_1^{-1}\mathcal{P}\mathfrak{I}(x,t) + \mathfrak{I}^\mathsf{T}(x,t)\alpha_1\mathcal{R}_1\mathfrak{I}(x,t),
\end{aligned}
\tag{3.7}
$$

similarly

$$
2\mathfrak{I}^\mathsf{T}(x,t)\mathcal{P}g(t,\mathfrak{I}(x,t-\sigma)) \leqslant \mathfrak{I}^\mathsf{T}(x,t)\mathcal{P}\mathcal{R}_2^{-1}\mathcal{P}\mathfrak{I}(x,t) + \mathfrak{I}^\mathsf{T}(x,t-\sigma)\alpha_2\mathcal{R}_2\mathfrak{I}(x,t-\sigma).
\tag{3.8}
$$

Based on Assumption $(\mathcal{H}_2)$, we get

$$
\operatorname{trace}\Big[\rho^\mathsf{T}(t)\mathcal{P}\rho(t)\Big] \leqslant \lambda_{\max}(\mathcal{P})\Big[\beta_1\mathfrak{I}^\mathsf{T}(x,t)\mathfrak{I}(x,t) + \beta_2\mathfrak{I}^\mathsf{T}(x,t-\sigma)\mathfrak{I}(x,t-\sigma)\Big].
\tag{3.9}
$$

By using integration by parts and Neumann boundary condition (2.2), we obtain that

$$
\begin{aligned}
\int_0^1 \mathfrak{I}^\mathsf{T}(x,t)\mathcal{D}\frac{\partial^2\mathfrak{I}(x,t)}{\partial x^2}dx &= \Big[\mathfrak{I}^\mathsf{T}(x,t)\mathcal{D}\frac{\partial\mathfrak{I}(x,t)}{\partial x}\Big]_{x=0}^{x=1} - \int_0^1 \frac{\partial\mathfrak{I}^\mathsf{T}(x,t)}{\partial x}\mathcal{D}\frac{\partial\mathfrak{I}(x,t)}{\partial x}dx \\
&= \int_0^1 \mathfrak{I}^\mathsf{T}(1,t)\mathcal{D}\Phi\mathfrak{I}(x,t)dx - \int_0^1 \frac{\partial\mathfrak{I}^\mathsf{T}(x,t)}{\partial x}\mathcal{D}\frac{\partial\mathfrak{I}(x,t)}{\partial x}dx.
\end{aligned}
\tag{3.10}
$$

To obtain $\bar{\mathfrak{I}}(x,t) = 0$, create a new state variable $\bar{\mathfrak{I}}(x,t) = \mathfrak{I}(x,t) - \mathfrak{I}(1,t)$ that satisfies the following condition,

$$
\frac{\partial\mathfrak{I}^\mathsf{T}(x,t)}{\partial x}\mathcal{D}\frac{\partial\mathfrak{I}(x,t)}{\partial x} = \frac{\partial\bar{\mathfrak{I}}^\mathsf{T}(x,t)}{\partial x}\mathcal{D}\frac{\partial\bar{\mathfrak{I}}(x,t)}{\partial x}.
\tag{3.11}
$$

Applying Lemma 2.4, we have

$$
\begin{aligned}
\int_0^1 \mathfrak{I}^\mathsf{T}(x,t)\mathcal{D}\frac{\partial^2\mathfrak{I}(x,t)}{\partial x^2}dx &\leqslant \int_0^1 \mathfrak{I}^\mathsf{T}(1,t)\mathcal{D}\Phi\mathfrak{I}(x,t)dx - \frac{1}{4}\pi^2\int_0^1 \bar{\mathfrak{I}}^\mathsf{T}(x,t)\mathcal{D}\bar{\mathfrak{I}}(x,t)dx \\
&\leqslant \int_0^1 \mathfrak{I}^\mathsf{T}(x,t)\mathcal{D}\Phi\mathfrak{I}(x,t)dx - \int_0^1 \bar{\mathfrak{I}}^\mathsf{T}(x,t)\mathcal{D}\Phi\mathfrak{I}(x,t)dx - \frac{1}{4}\pi^2\int_0^1 \bar{\mathfrak{I}}^\mathsf{T}(x,t)\mathcal{D}\bar{\mathfrak{I}}(x,t)dx.
\end{aligned}
\tag{3.12}
$$

Combining the inequalities (3.4)-(3.12), we get

$$
\mathcal{L}V(t,\mathfrak{I}(x,t)) \leqslant \int_0^1 \xi^\mathsf{T}(x,t)\Pi\xi(x,t)dx + \rho V(t,\mathfrak{I}(x,t)),
$$

where

$$
\xi(x,t) = \Big[\mathfrak{I}^\mathsf{T}(x,t)\ \ \bar{\mathfrak{I}}^\mathsf{T}(x,t)\ \ \mathfrak{I}^\mathsf{T}(x,t-\sigma)\Big]^\mathsf{T}.
$$

Based on the inequality (3.1), we get

$$
\mathcal{L}V(t,\mathfrak{I}(x,t)) \leqslant \rho V(t,\mathfrak{I}(x,t)).
$$

Then by taking mathematical expectations on both sides, we have

$$
D^+\mathbb{E}V(t,\mathfrak{I}(x,t)) = \mathbb{E}\mathcal{L}V(t,\mathfrak{I}(x,t)) \leqslant \rho V(t,\mathfrak{I}(x,t)).
$$

Integrating on both sides from t_k to t and using Lemma 2.5, we have

$$
\mathbb{E}V(t,\mathfrak{I}(x,t)) \leqslant \mathbb{E}V(t_k,\mathfrak{I}(x,t_k)) + \rho\int_{t_k}^t \mathbb{E}V(s,\mathfrak{I}(x,s))ds \leqslant \mathbb{E}e^{\rho(t-t_k)}\mathbb{E}V(t_k,\mathfrak{I}(x,t_k)).
$$

Besides, for any $k \in \{1,2,\ldots,t_{N_\omega(T,0)}\}$, it follows from inequality (3.2) that,

$$
V_1(t_k,\mathfrak{I}(x,t_k)) = \int_0^1 \mathfrak{I}^\mathsf{T}(x,t_k)\mathcal{P}\mathfrak{I}(x,t_k)dx = \int_0^1 \mathfrak{I}^\mathsf{T}(x,t_k^-)\Upsilon^\mathsf{T}\mathcal{P}\Upsilon\mathfrak{I}(x,t_k^-)dx \leqslant \varepsilon V_1(t_k,\mathfrak{I}(x,t_k^-)).
\tag{3.13}
$$

Note that $V_2(t,\mathfrak{I}(x,t))$ is continuous on t, then we have

$$V_2(t_k, \Im(x, t_k)) = V_2(t_k^-, \Im(x, t_k^-)) \leqslant \varepsilon V_2(t_k^-, \Im(x, t_k^-)). \tag{3.14}$$

Combining the inequalities (3.13) and (3.14), we have

$$V(t_k, \Im(x, t_k)) \leqslant \varepsilon V(t_k^-, \Im(x, t_k^-)).$$

By using iterative operation, when $t \in [t_k, t_{k+1})$,

$$\begin{aligned}
\mathbb{E}V(t, \Im(x, t)) &\leqslant e^{\rho(t-t_k)}\mathbb{E}V(t_k, \Im(x, t_k)) \\
&\leqslant \varepsilon e^{\rho(t-t_k)}\mathbb{E}V(t_k^-, \Im(x, t_k^-)) \\
&\vdots \\
&\leqslant \varepsilon^k e^{\rho(t-t_0)}\mathbb{E}V(t_0, \Im(x, t_0)),
\end{aligned}$$

and also we obtain for $t \in [t_{N_\varpi(T,0)}, T]$,

$$\begin{aligned}
\mathbb{E}V(t, \Im(x, t)) &\leqslant e^{\rho(t-t_{N_\varpi(T,0)})}\mathbb{E}V(t_{N_\varpi(T,0)}, \Im(x, t_{N_\varpi(T,0)})) \\
&\leqslant \varepsilon e^{\rho(t-t_{N_\varpi(T,0)})}\mathbb{E}V(t_{N_\varpi(T,0)}, \Im(x, t_{N_\varpi(T,0)})) \\
&\vdots \\
&\leqslant \varepsilon^{N_\varpi(T,0)} e^{\rho T}\mathbb{E}V(t_0, \Im(x, t_0)).
\end{aligned}$$

Let $N_\varpi[T, 0]$ be the impulsive time of ϖ on $[0, T)$. According to Definition 2.7, for $\varepsilon > 1$, we get

$$\begin{aligned}
\mathbb{E}V(t, \Im(x, t)) &\leqslant \varepsilon^{N_\varpi(T,0)} e^{\rho T}\mathbb{E}V(t_0, \Im(x, t_0)) \\
&\leqslant \varepsilon^{\left(\frac{T-0}{T_\epsilon} + N_\epsilon\right)} e^{\rho T}\mathbb{E}V(t_0, \Im(x, t_0)) \leqslant \varepsilon^{N_\epsilon} e^{\left(\frac{\ln(\varepsilon)}{T_\epsilon} + \rho\right)T}\mathbb{E}V(t_0, \Im(x, t_0)).
\end{aligned} \tag{3.15}$$

From the definition of $V(t, \Im(x, t))$, we obtain that

$$\mathbb{E}V_1(t_0, \Im(x, t_0)) \leqslant \lambda_{\max}(\mathcal{P})\mathbb{E}\left\{\sup_{s \in [-\sigma, 0]} \|\Im(x, s)\|^2\right\}, \tag{3.16}$$

$$\mathbb{E}V_2(t_0, \Im(x, t_0)) \leqslant e^{\rho\sigma}\lambda_{\max}(\mathcal{Q})\mathbb{E}\left\{\sup_{s \in [-\sigma, 0]} \|\Im(x, s)\|^2\right\}. \tag{3.17}$$

From the inequalities (3.16) and (3.17), we have

$$\mathbb{E}V(t_0, \Im(x, t_0)) \leqslant \left[\lambda_{\max}(\mathcal{P}) + e^{\rho\sigma}\lambda_{\max}(\mathcal{Q})\right]\mathbb{E}\left\{\sup_{s \in [-\sigma, 0]} \|\Im(x, s)\|^2\right\}. \tag{3.18}$$

Also, we have

$$\mathbb{E}V(t, \Im(x, t)) \geqslant \lambda_{\min}(\mathcal{P})\mathbb{E}\left\{\int_0^1 \Im^T(x, t)\Im(x, t)dx\right\} = \lambda_{\min}(\mathcal{P})\mathbb{E}\|\Im(x, t)\|^2. \tag{3.19}$$

Combining the inequalities (3.15), (3.18), and (3.19), we have

$$\mathbb{E}\|\Im(x, t)\|^2 \leqslant \frac{1}{\lambda_{\min}(\mathcal{P})}\varepsilon^{N_\epsilon} e^{\left(\frac{\ln(\varepsilon)}{T_\epsilon} + \rho\right)\mathcal{T}}\left[\lambda_{\max}(\mathcal{P}) + e^{\rho\sigma}\lambda_{\max}(\mathcal{Q})\right]\mathbb{E}\left\{\sup_{s \in [-\sigma, 0]} \|\Im(x, s)\|^2\right\}, \ \forall \ t \in [0, T].$$

Considering the inequality (3.3), when the following initial condition holds:

$$\mathbb{E}\left\{\sup_{s \in [-\sigma, 0]} \|\Im(x, s)\|^2\right\} < \kappa_1,$$

it implies immediately that $\mathbb{E}\|\Im(x, t)\|^2 < \kappa_2, \ \forall \ t \in [0, T]$. According to the Definition 2.8, the SNIRDSs (2.1) is FTS with respect to constants (κ_1, κ_2, T). The proof is completed. $\qquad \square$

The next theorem states that the control gain matrix can be designed to obtain the stabilization for SNIRDSs (2.1).

Theorem 3.2. *Let* T_ϵ *be the average impulsive interval of* $\varpi = \{t_1, t_2, \ldots\}$. *Under Assumptions* $(\mathcal{H}_1)$ *and* $(\mathcal{H}_2)$, *the SNIRDSs* (2.1) *is stabilizable if there exist constants* $N_\epsilon, \rho > 0, \epsilon \geqslant 1$, *symmetric positive definite matrices* $\mathcal{P}, \mathcal{Q}, \mathcal{R}_1, \mathcal{R}_2$ *and constant matrix* Ψ *such that* (3.3) *holds, and*

$$\text{(iv)} \quad \begin{bmatrix} -\epsilon\mathcal{P} & \Upsilon^\mathsf{T}\mathcal{P} \\ * & -\mathcal{P} \end{bmatrix} \leqslant 0, \tag{3.20}$$

$$\text{(v)} \quad \Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} \\ * & * & * & \Xi_{44} & \Xi_{45} \\ * & * & * & * & \Xi_{55} \end{bmatrix} < 0, \tag{3.21}$$

where

$$\Xi_{11} = \mathrm{He}(\mathcal{P}\mathcal{A} + \mathcal{D}\Psi) + \mathcal{Q} + \alpha_1\mathcal{R}_1 + \lambda_{\max}(\mathcal{P})\beta_1 - \rho\mathcal{P}, \; \Xi_{12} = \mathcal{P}, \; \Xi_{13} = \mathcal{P},$$

$$\Xi_{14} = -\Psi^\mathsf{T}\mathcal{D}^\mathsf{T}, \; \Xi_{15} = \mathcal{P}\mathcal{B}, \; \Xi_{22} = -\mathcal{R}_1, \; \Xi_{23} = 0, \; \Xi_{24} = 0, \; \Xi_{25} = 0, \; \Xi_{33} = -\mathcal{R}_2,$$

$$\Xi_{34} = 0, \; \Xi_{35} = 0, \; \Xi_{44} = -\frac{1}{2}\pi^2\mathcal{P}\mathcal{D}, \; \Xi_{45} = 0, \; \Xi_{55} = -e^{\rho\sigma}\mathcal{Q} + \alpha_2\mathcal{R}_2 + \lambda_{\max}(\mathcal{P})\beta_2,$$

are satisfied. Furthermore, the control gain matrix is designed by

$$\text{(vi)} \quad \Phi = \Psi\mathcal{P}^{-1}. \tag{3.22}$$

Proof. Clearly, the proof of the Theorem 3.2 follows from Lemma 2.6 and Theorem 3.1. □

Remark 3.3. The impulsive control approach was utilized to stabilize the RDSs [37–39, 49] and the boundary control technique was also utilized to stabilize the RDSs [9, 15, 16, 22, 23, 51, 52]. To best of our knowledge, there are few works that investigate the FTS and stabilization of SNIRDSs with time delays and the boundary feedback control. That is the issue we addressed in this paper.

Remark 3.4. The obtained results in this paper extended and improved the results in [15]. In [15], the author discussed the FTS for SRDSs with boundary control. In this paper, we discussed the FTS for SNIRDSs with time delays and boundary feedback control.

Remark 3.5. In [22, 49, 51, 52], the authors obtained the FTS and stabilization of RDSs without stochastic terms. In fact, noise presented a fundamental issue in the transmission of information impacting all facets of the neuron systems operating within the neuron systems. It is worth noting that, introduction of stochastic terms into the systems, it is suitable to addressing a practical situations.

Remark 3.6. In this paper, Theorem 3.2 presents a sufficient condition to guarantee the stabilization for a class of SNIRDSs with time delays and boundary feedback control. In [15, 23, 51, 52], the authors investigated the stabilization for a class of RDSs. It's a pity that the impulsive effects are not considered. Thus, our results are improved than those reported in [15, 23, 51, 52].

Remark 3.7. In this paper, we investigated the FTS and stabilization for a class of SNIRDSs with time delays and boundary feedback control. In [11–14, 29, 30, 45], the authors discussed the stability analysis for neural networks and complex dynamical networks. It's a pity that the reaction-diffusion terms are not considered. Thus, our results are improved than those reported in [11–14, 29, 30, 45].

Remark 3.8. In [15, 23, 51, 52], the authors discussed the FTS and stabilization for RDSs by using boundary control. However, for the FTS and stabilization of SNIRDSs with time delays and boundary feedback control related results have not been found in previous works. To shorten this gap, we discussed the FTS and stabilization for SNIRDSs with time delays via boundary feedback control.

Remark 3.9. From SNIRDSs (2.1), the impulsive effects can be ignored. Then, the SNIRDSs (2.1) can be rewritten as:

$$d\Im(x,t) = \left[\mathcal{D}\frac{\partial^2 \Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-\sigma) + f(t,\Im(x,t)) + g(t,\Im(x,t-\sigma)) \right] dt$$
$$+ h(t,\Im(x,t),\Im(x,t-\sigma))d\omega(t).$$

The following corollary states that the control gain matrix can be designed to obtain the stabilization for SNIRDSs (2.1) without impulsive effects.

Corollary 3.10. *Let* T_ϵ *be the average impulsive interval of* $\varpi = \{t_1, t_2, \ldots\}$. *Under Assumptions* $(\mathcal{H}_1)$ *and* $(\mathcal{H}_2)$, *the SNIRDSs* (2.1) *without impulsive effects is stabilizable if there exist constant* $\rho > 0$, *symmetric positive definite matrices* $\mathcal{P}, \mathcal{Q}, \mathcal{R}_1, \mathcal{R}_2$ *and constant matrix* Ψ *such that* (3.21) *holds, and*

$$(vii) \quad \frac{\kappa_1 e^{\rho T}}{\lambda_{\min}(\mathcal{P})} \left[\lambda_{\max}(\mathcal{P}) + e^{\rho \sigma} \lambda_{\max}(\mathcal{Q}) \right] < \kappa_2,$$

are satisfied. Moreover, the control gain matrix is designed by (3.22).

Remark 3.11. From Neumann boundary condition (2.2), let the boundary feedback control $u(t) = 0$. Then, the Neumann boundary condition (2.2) can be rewritten as:

$$\frac{\partial \Im(x,t)}{\partial x}|_{x=0} = 0, \quad \frac{\partial \Im(x,t)}{\partial x}|_{x=1} = 0.$$

The next corollary is to investigate the FTS for SNIRDSs (2.1) without boundary feedback control.

Corollary 3.12. *Let* T_ϵ *be the average impulsive interval of* $\varpi = \{t_1, t_2, \ldots\}$. *Under Assumptions* $(\mathcal{H}_1)$ *and* $(\mathcal{H}_2)$, *the SNIRDSs* (2.1) *without boundary feedback control is said to be FTS with respect to given constants* (κ_1, κ_2, T) *if there exist constants* $N_\epsilon, \rho > 0, \epsilon \geqslant 1$ *and symmetric positive definite matrices* $\mathcal{P}, \mathcal{Q}, \mathcal{R}_1, \mathcal{R}_2$ *such that* (3.3) *and* (3.20) *hold, and*

$$(viii) \quad \Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} \\ * & * & * & \Gamma_{44} & \Gamma_{45} \\ * & * & * & * & \Gamma_{55} \end{bmatrix} < 0,$$

where

$$\Gamma_{11} = He(P\mathcal{A}) + Q + \alpha_1 R_1 + \lambda_{\max}(P)\beta_1 - \rho P, \ \Gamma_{12} = P, \ \Gamma_{13} = P, \ \Gamma_{14} = 0,$$
$$\Gamma_{15} = P\mathcal{B}, \ \Gamma_{22} = -R_1, \ \Gamma_{23} = 0, \ \Gamma_{24} = 0, \ \Gamma_{25} = 0, \ \Gamma_{33} = -R_2, \ \Gamma_{34} = 0,$$
$$\Gamma_{35} = 0, \ \Gamma_{44} = -\frac{1}{2}\pi^2 P\mathcal{D}, \ \Gamma_{45} = 0, \ \Gamma_{55} = -e^{\rho \sigma}Q + \alpha_2 R_2 + \lambda_{\max}(P)\beta_2,$$

are satisfied.

4. Numerical example

In this section, numerical example is given to illustrate the our boundary feedback controller and impulsive phenomenon are effective.

Consider the following 2-dimensional SNIRDSs with time delays and boundary feedback control

$$\begin{cases} d\Im(x,t) = \left[\mathcal{D}\frac{\partial^2 \Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-0.5) + f(t,\Im(x,t)) + g(t,\Im(x,t-0.5)) \right] dt \\ \qquad + h(t,\Im(x,t),\Im(x,t-0.5))d\omega(t), \ t \neq t_k, \\ \Im(x,t_k) = \Upsilon\Im(x,t_k^-), \ t = t_k, \ k \in \mathbb{N}, \end{cases} \quad (4.1)$$

where

$$\mathcal{D} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \ \mathcal{A} = \begin{bmatrix} 0.1 & -0.1 \\ 0.2 & 0.1 \end{bmatrix}, \ \mathcal{B} = \begin{bmatrix} 0.2 & -0.2 \\ 0.5 & 0.1 \end{bmatrix}, \ \Upsilon = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix},$$

$$f(t, \mathfrak{I}(x, t)) = 0.1(1 + \sin(t))\mathfrak{I}(x, t),$$

$$g(t, \mathfrak{I}(x, t - 0.5)) = 0.1(1 + \cos(t))\mathfrak{I}(x, t - 0.5),$$

$$h(t, \mathfrak{I}(x, t), \mathfrak{I}(x, t - 0.5)) = 0.2\mathfrak{I}(x, t) + 0.5\mathfrak{I}(x, t - 0.5).$$

The initial and Neumann boundary conditions of SNIRDSs (4.1) are

$$\begin{cases} \mathfrak{I}_1(x, s) = & 0.01(1 - \sin(0.5\pi x))\text{In}(50(s - 0.5)), \\ \mathfrak{I}_2(x, s) = & 0.01(1 - \cos(0.5\pi x))\text{In}(50(s - 0.5)), \end{cases}$$

and

$$\frac{\partial \mathfrak{I}(x, t)}{\partial x}|_{x=0} = 0, \ \frac{\partial \mathfrak{I}(x, t)}{\partial x}|_{x=1} = u(t).$$

The boundary feedback controller is

$$u(t) = \Phi \int_0^1 \mathfrak{I}(x, t)dx.$$
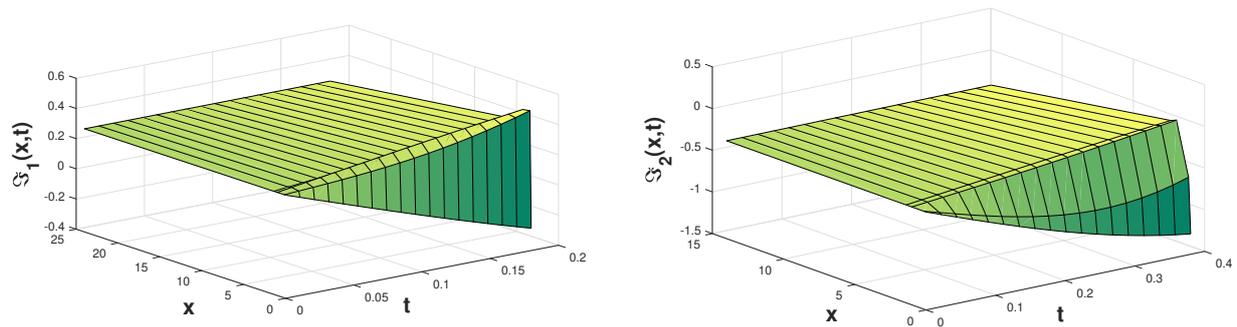
Figure 1: Trajectories for system (4.1) without boundary feedback control.
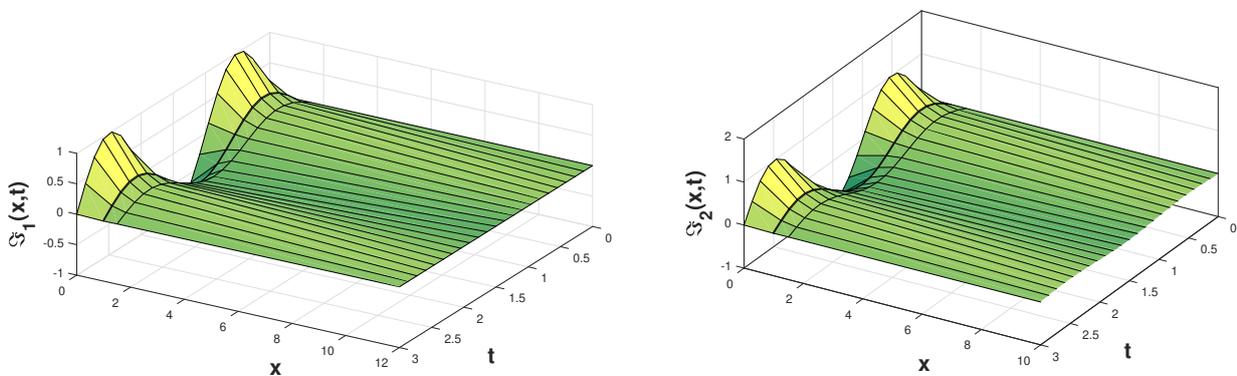
Figure 2: Trajectories for system (4.1) with boundary feedback control.
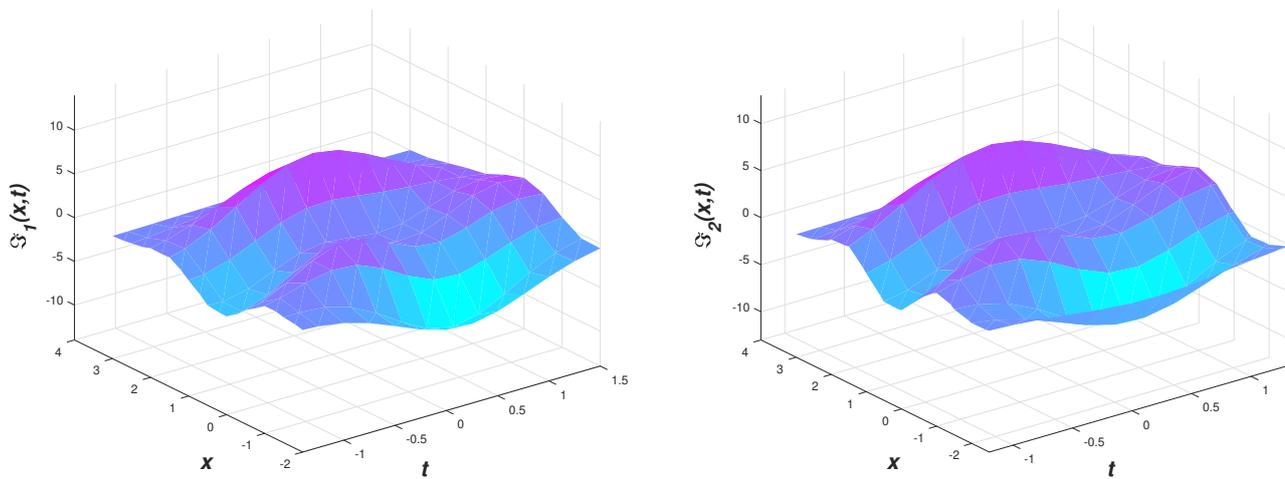
Figure 3: Trajectories for system (4.1) without impulsive effects.
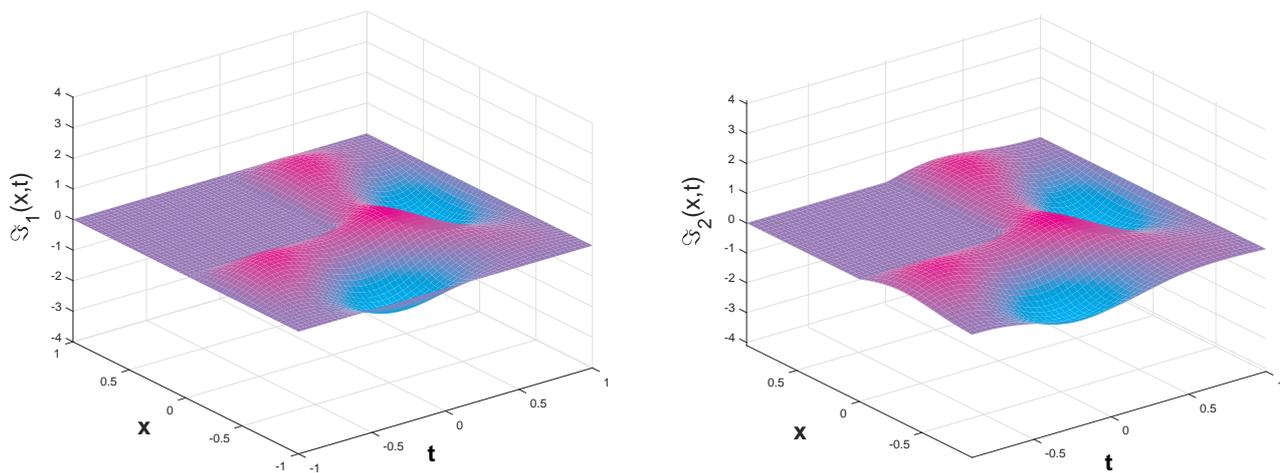
Figure 4: Trajectories for system (4.1) with impulsive effects.

To stabilize the SNIRDSs (4.1), let, $\rho = 2.1$, $T_\epsilon = 0.3$, $N_\epsilon = 5$, $\varepsilon = 1$, $\kappa_1 = 1$, $\kappa_2 = 5$, and $T = 10$. Solving the LMIs in Theorem 3.2 by Matlab LMI toolbox, we obtain the following feasible solutions as:

$$\mathcal{P} = 10^{-4} \times \begin{bmatrix} 0.5539 & -0.0029 \\ -0.0029 & 0.6013 \end{bmatrix}, \qquad \mathcal{Q} = 10^{-4} \times \begin{bmatrix} -0.2030 & 0.0010 \\ 0.0010 & -0.2196 \end{bmatrix},$$

$$\mathcal{R}_1 = \begin{bmatrix} 0.0258 & -0.0002 \\ -0.0002 & 0.0260 \end{bmatrix}, \qquad \mathcal{R}_2 = \begin{bmatrix} 0.0059 & 0.0000 \\ 0.0000 & 0.0057 \end{bmatrix},$$

$$\Psi = 10^{-3} \times \begin{bmatrix} -0.9197 & 0.0028 \\ 0.0028 & -0.9393 \end{bmatrix}, \qquad \Phi = \begin{bmatrix} -16.6057 & -0.0329 \\ -0.0305 & -15.6207 \end{bmatrix}.$$

Therefore, based on Theorem 3.2, the SNIRDSs (4.1) is FTS with respect to constants (κ_1, κ_2, T).

Remark 4.1. Under boundary feedback controller and impulsive phenomenon, the systems $\mathfrak{I}_q(x, t)(q = 1, 2)$ are shown in Figures 2 and 4, and we see that they achieve stabilization for SNIRDSs (4.1). To show the efficiency of boundary feedback controller and impulsive phenomenon, consider $u(t) = 0$ and

$\mathfrak{I}(x, t_k) = 0$, that is, SNIRDSs (4.1) has no boundary feedback controller and impulsive effects. Figures 1 and 3 are display the systems $\mathfrak{I}_q(x, t)(q = 1, 2)$, which means that, no boundary feedback controller and impulsive effects, system (4.1) does not realize the FTS. This illustrates that the boundary feedback controller and impulsive phenomenon are effective.

5. Conclusions

In this paper, boundary feedback controller on stabilization of SNIRDSs with time delays are discussed. By utilizing LKF, Wirtinger's inequality, Gronwall inequality, average impulsive interval approach, and LMIs, sufficient conditions are obtained to ensure the FTS for SNIRDSs. Furthermore, the control gain matrices are designed for the boundary feedback controller with delay-dependent results for stabilization of proposed systems. At last, numerical example is given to show that the efficiency and superiority of obtained theoretical results. Our future study will focus on the stabilization problems for fractional-order SNIRDSs using boundary feedback controller.

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