J. Math. Computer Sci., 29 (2023), 156–166

Online: ISSN 2008-949X

Journal of Mathematics and Computer Science

Journal Homepage: www.isr-publications.com/jmcs

Normal spaces associated with fuzzy nano M-open sets and its application

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Abstract

In this paper, we introduce some new spaces called fuzzy nano M normal spaces and strongly fuzzy nano M normal spaces with the help of fuzzy nano M open sets in fuzzy nano topological space. Also, find their relations among themselves and with already existing spaces. Also, we study some basic properties and the characterizations of these normal spaces. The stated properties are quantified with numerical data. Furthermore, an algorithm for Multiple Attribute Decision-Making (MADM) with an application regarding candidates choose their works by using fuzzy nano topological spaces is developed.

Keywords: Fuzzy nano open set, fuzzy nano M-open set, fuzzy nano M-closed set, fuzzy nano M normal space, strongly fuzzy nano M normal space, fuzzy score function.

2020 MSC: 03E72, 54A05, 54A40.

1. Introduction

In 1965, Zadeh [29] made his significant theory on fuzzy sets. Later, fuzzy topology was introduced by Chang [6]. Pawlak [15] introduced Rough set theory by handling vagueness and uncertainty. This can be often defined by means of topological operations, interior and closure, called approximations. In 2014, Broumi et al. [5] introduced rough neutrosophic sets. In 2013, Lellis Thivagar [11] introduced an extension of rough set theory called Nano topology and defined its topological spaces in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it.

Saha [18] defined δ-open sets in fuzzy topological spaces, nano topological space by Pankajam et al. [14] and neutrosophic topological space by Vadivel et al. [23, 25, 26]. Also, δ-pre open sets in neutrosophic topological spaces was found by Vadivel et al. [27, 28]. Recently, Lellis Thivagar et al. [21] explored a new concept of neutrosophic nano topology, intuitionistic nano topology and fuzzy nano topology. Ramachandran and Stephan Antony Raj [16, 17] proposed intuitionistic fuzzy nano topological spaces and

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doi: 10.22436/jmcs.029.02.05

Received: 2022-04-20 *Revised:* 2022-05-31 *Accepted:* 2022-07-11

extended to generalized closed sets. Ajay and Joseline [1, 2] introduced pythagorean nano topological spaces and discussed their properties of pythagorean nanogeneralized closed sets with application in decision-making problem. El-Maghrabi and Al-Juhani [10] proposed the concept of M-open sets in topological spaces in 2011 and examined some of their features. Padma et al. [13] also found M-open sets in nano topological spaces. Thangammal et al. [19, 20] introduced fuzzy nano Z-open sets in fuzzy nano topological spaces and their applications.

Multiple Attribute Decision-Making (MADM) is a decision-making process that takes into account the best possible options. Decisions were taken in mediaeval times without taking into account data uncertainties, which could lead to a potential outcome. Inadequate outcomes have real-life consequences. Circumstances in the workplace If we did this, the outcomes would be ambiguous, undefined, or incorrect. Without hesitation, I determined the result of the obtained data. MADM had a significant impact. Management, disease diagnosis, economics, and industry are examples of real-world problems. Each decision maker makes hundreds of decisions each time to carry out the key component. It should be a logical assessment of his or her job. MADM is a programme that helps you tackle difficult problems. For this, there are complex problems with a variety of parameters. The problem must be identified in MADM by defining viable alternatives, assessing each alternative against the criteria established by the decision-maker or community of decision-makers, and finally selecting the optimal alternative. To deal with the complications and complexity of MADM problems, a range of useful mathematical methods such as fuzzy sets, neutrosophic sets, and soft sets were developed.

Zafer et al. [30] introduced and developed the MADM method based on rough fuzzy information. Several mathematicians have worked on correlation coefficients, similarity measurements, aggregation operators, topological spaces, and decision-making applications in this area. These structures feature better decision-making solutions and provide distinct formulas for diverse sets. It has a wide range of applications in domains such as medical diagnosis, pattern identification, social sciences, artificial intelligence, business, and multi-attribute decision making. Zhang et al. [31–33] discussed an application using rough sets on merger and acquisition target selection, TOPSIS method and multi-attribute group decision-making. Al Shumrani et al. [4] applied covering-based rough fuzzy, intuitionistic fuzzy and neutrosophic nano topology. Li et al. [12] applied generalized fuzzy rough approximation operators based on fuzzy coverings. The problems associated with these cases are interesting, and developing a hypothesis for them has prompted many scholars [3, 7–9, 22, 24] to pay attention to them.

Research Gap: No investigation on some new spaces such as fuzzy nano $\mathcal{M}$ normal space and strongly fuzzy nano $\mathcal{M}$ normal space on fuzzy nano topological space has been reported in the fuzzy literature. As a numerical example, we established a method for the solution of MADM problem related to selection committee using $\mathcal{FN}ts$.

In this paper we introduce fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$, and $\mathcal{M}$) normal space and strongly fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$, and $\mathcal{M}$) normal space and discuss their properties in $\mathcal{FN}ts$'s. Also, MADM problem related to selection committee using $\mathcal{FN}ts$ is also developed.

2. Preliminaries

The basic definitions of fuzzy sets and their properties are defined in [29]. The definitions of fuzzy nano lower approximation (briefly, $\underline{\mathcal{FN}}(F)$), fuzzy nano upper approximation (briefly, $\overline{\mathcal{FN}}(F)$), fuzzy nano boundary (briefly, $B_{\mathcal{FN}}(F)$), fuzzy nano topological space (briefly, $\mathcal{FN}ts$), fuzzy nano open (briefly, $\mathcal{FN}o$) sets and fuzzy nano closed (briefly, $\mathcal{FN}c$) sets are defined in [21].

3. Fuzzy nano $\mathcal{M}$ normal spaces

In this section, we introduce fuzzy nano $\mathcal{M}$ normal spaces and study their properties.

Definition 3.1. Let $(U, \tau_F(F))$ be a $\mathcal{FN}ts$ with respect to F where F is a fuzzy subset of U. Then a fuzzy subset S in U is said to be a fuzzy nano

(i) interior of S [20] (briefly, $FNint(S)$) is defined by $FNint(S) = \bigvee\{I : I \leqslant S \;\&\; I$ is a FNo set in $U\}$;
(ii) closure of S [20] (briefly, $FNcl(S)$) is defined by $FNcl(S) = \bigwedge\{A : S \leqslant A \;\&\; A$ is a FNc set in $U\}$;
(iii) regular open [20] (briefly, FNro) set if $S = FNint(FNcl(S))$;
(iv) regular closed [20] (briefly, FNrc) set if $S = FNcl(FNint(S))$;
(v) θ interior of S (briefly, $FN\theta int(S)$) is defined by $FN\theta int(S) = \bigvee\{FNint(I) : I \leqslant S \;\&\; I$ is a FNc set in $U\}$;
(vi) θ closure of S (briefly, $FN\theta cl(S)$) is defined by $FN\theta cl(S) = \bigwedge\{FNcl(A) : S \leqslant A \;\&\; A$ is a FNo set in $U\}$;
(vii) δ interior of S [20] (briefly, $FN\delta int(S)$) is defined by $FN\delta int(S) = \bigvee\{I : I \leqslant S \;\&\; I$ is a FNro set in $U\}$;
(viii) δ closure of S [20] (briefly, $FN\delta cl(S)$) is defined by $FN\delta cl(S) = \bigwedge\{A : S \leqslant A \;\&\; A$ is a FNrc set in $U\}$;
(ix) θ open (briefly, FNθo) set if $S = FN\theta int(S)$;
(x) semi open [20] (briefly, FN$\mathcal{S}$o) set if $S \leqslant FNcl(FNint(S))$;
(xi) θ semi open (briefly, FN$\theta\mathcal{S}$o) set if $S \leqslant FNcl(FN\theta int(S))$;
(xii) pre open [20] (briefly, FN$\mathcal{P}$o) set if $S \leqslant FNint(FNcl(S))$;
(xiii) θ pre open (briefly, FN$\theta\mathcal{P}$o) set if $S \leqslant FNint(FN\theta cl(S))$;
(xiv) δ pre open (briefly, FN$\delta\mathcal{P}$o) set if $S \leqslant FNint(FN\delta cl(S))$;
(xv) θ semi interior of S (briefly, $FN\theta\mathcal{S}int(S)$) is defined by $FN\theta\mathcal{S}int(S) = \bigvee\{I : I \leqslant S \;\&\; I$ is a FN$\theta\mathcal{S}$o set in $U\}$;
(xvi) θ semi closure of S (briefly, $FN\theta\mathcal{S}cl(S)$) is defined by $FN\theta\mathcal{S}cl(S) = \bigwedge\{A : S \leqslant A \;\&\; A$ is a FN$\theta\mathcal{S}$c set in $U\}$;
(xvii) pre interior of S [20] (briefly, $FN\mathcal{P}int(S)$) is defined by $FN\mathcal{P}int(S) = \bigvee\{I : I \leqslant S \;\&\; I$ is a FN$\mathcal{P}$o set in $U\}$;
(xviii) pre closure of S [20] (briefly, $FN\mathcal{P}cl(S)$) is defined by $FN\mathcal{P}cl(S) = \bigwedge\{A : S \leqslant A \;\&\; A$ is a FN$\mathcal{P}$c set in $U\}$;
(xix) θ pre interior of S (briefly, $FN\theta\mathcal{P}int(S)$) is defined by $FN\theta\mathcal{P}int(S) = \bigvee\{I : I \leqslant S \;\&\; I$ is a FN$\theta\mathcal{P}$o set in $U\}$;
(xx) θ pre closure of S (briefly, $FN\theta\mathcal{P}cl(S)$) is defined by $FN\theta\mathcal{P}cl(S) = \bigwedge\{A : S \leqslant A \;\&\; A$ is a FN$\theta\mathcal{P}$c set in $U\}$;
(xxi) δ pre interior of S (briefly, $FN\delta\mathcal{P}int(S)$) is defined by $FN\delta\mathcal{P}int(S) = \bigvee\{I : I \leqslant S \;\&\; I$ is a FN$\delta\mathcal{P}$o set in $U\}$;
(xxii) δ pre closure of S (briefly, $FN\delta\mathcal{P}cl(S)$) is defined by $FN\delta\mathcal{P}cl(S) = \bigwedge\{A : S \leqslant A \;\&\; A$ is a FN$\delta\mathcal{P}$c set in $U\}$;
(xxiii) M-open (briefly, FNMo) set if $S \leqslant FNcl(FN\theta int(S)) \vee FNint(FN\,\delta cl(S))$;
(xxiv) M-closed (briefly, FNMc) set if $FNint(FN\theta cl(S)) \wedge FNcl(FN\delta int(S)) \leqslant S$;
(xxv) M interior of S (briefly, $FNMint(S)$) is defined by $FNMint(S) = \bigvee\{I : I \leqslant S \;\&\; I$ is a FNMo set in $U\}$;
(xxvi) M closure of S (briefly, $FNMcl(S)$) is defined by $FNMcl(S) = \bigwedge\{A : S \leqslant A \;\&\; A$ is a FNMc set in $U\}$.

The complement of the respective fuzzy nano open sets are called as fuzzy nano closed sets.

The family of all FNMo (resp. FNMc) sets of a space $(U, \tau_F(F))$ will be as always denoted by $FNMO(U, A)$ (resp. $FNMC(U, A)$).

Definition 3.2. A function $h : (U_1, \tau_F(F_1)) \to (U_2, \tau_F(F_2))$ is said to be fuzzy nano

(i) continuous (briefly, FNCts), if $\forall$ FNo set M of U_2, the set $h^{-1}(M)$ is FNo set of U_1;
(ii) θ continuous (briefly, FNθCts), if $\forall$ FNo set M of U_2, the set $h^{-1}(M)$ is FNθo set of U_1;
(iii) θ semi continuous (briefly, FN$\theta\mathcal{S}$Cts), if $\forall$ FNo set M of U_2, the set $h^{-1}(M)$ is FN$\theta\mathcal{S}$o set of U_1;
(iv) δ pre continuous (briefly, FN$\delta\mathcal{P}$Cts), if $\forall$ FNo set M of U_2, the set $h^{-1}(M)$ is FN $\delta\mathcal{P}$o set of U_1;
(v) M continuous (briefly, FNMCts), if $\forall$ FNo set M of U_2, the set $h^{-1}(M)$ is FNMo set of U_1.

Theorem 3.3. A function $h : (U_1, \tau_F(F_1)) \to (U_2, \tau_F(F_2))$ is $FNMCts$ iff the inverse image of every FNc set in U_2 is $FNMc$ in U_1.

Definition 3.4. A function $h : (U_1, \tau_F(F_1)) \to (U_2, \tau_F(F_2))$ is called fuzzy nano

(i) irresolute (briefly, $FNIrr$) function, if $\forall$ $FN\mathcal{S}o$ subset M of U_2, the set $h^{-1}(M)$ is $FN\mathcal{S}o$ subset of U_1;
(ii) θ semi irresolute (briefly, $FN\theta\mathcal{S}Irr$) function, if $\forall$ $FN\theta\mathcal{S}o$ subset M of U_2, the set $h^{-1}(M)$ is $FN\theta\mathcal{S}o$ subset of U_1;
(iii) δ pre irresolute (briefly, $FN\delta\mathcal{P}Irr$) function, if $\forall$ $FN\delta\mathcal{P}o$ subset M of U_2, the set $h^{-1}(M)$ is $FN\delta\mathcal{P}o$ subset of U_1;
(iv) M irresolute (briefly, $FNMIrr$) function, if $\forall$ $FNMo$ subset M of U_2, the set $h^{-1}(M)$ is $FNMo$ subset of U_1.

Definition 3.5. Let $(U_1, \tau_F(F_1))$ and $(U_2, \tau_F(F_2))$ be two $FNts$. A function $h : (U_1, \tau_F(F_1)) \to (U_2, \tau_F(F_2))$ is said to be fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$, and M) open map (briefly, FNO (resp. $FN\theta O$, $FN\theta\mathcal{S}O$, $FN\delta\mathcal{P}O$, and $FNMO$)) if the image of each FNo set in U_1 is FNo (resp. $FN\theta o$, $FN\theta\mathcal{S}o$, $FN\delta\mathcal{P}o$, and $FNMo$) in U_2.

Definition 3.6. Let $(U_1, \tau_F(F_1))$ and $(U_2, \tau_F(F_2))$ be two $FNts$. A function $h : (U_1, \tau_F(F_1)) \to (U_2, \tau_F(F_2))$ is said to be fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) closed map (briefly, FNC (resp. $FN\theta C$, $FN\theta\mathcal{S}C$, $FN\delta\mathcal{P}C$ and $FNMC$)) if the image of each FNc set in U_1 is FNc (resp. $FN\theta c$, $FN\theta\mathcal{S}c$, $FN\delta\mathcal{P}c$ and $FNMc$) in U_2.

Definition 3.7. Let A and B be any two fuzzy subsets of a $FNts$'s. Then A is fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) q-neighbourhood (briefly, $FNq\text{-}nbhd$ (resp. $FN\theta q\text{-}nbhd$, $FN\theta\mathcal{S}q\text{-}nbhd$, $FN\delta\mathcal{P}q\text{-}nbhd$ and $FNMq\text{-}nbhd$)) with B if there exists a FNo (resp. $FN\theta o$, $FN\theta\mathcal{S}o$, $FN\delta\mathcal{P}o$ and $FNMo$) set O with $AqO \leqslant B$.

Definition 3.8. Let $(U, \tau_F(F))$ be a $FNts$ is said to be fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) normal (briefly, $FNNor$ (resp. $FN\theta Nor$, $FN\theta\mathcal{S}Nor$, $FN\delta\mathcal{P}Nor$ and $FNMNor$)) normal if for any two disjoint FNc (resp. $FN\theta c$, $FN\theta\mathcal{S}c$, $FN\delta\mathcal{P}c$ and $FNMc$) sets S and O, there exist disjoint FNo (resp. $FN\theta o$, $FN\theta\mathcal{S}o$, $FN\delta\mathcal{P}o$ and $FNMo$) sets L and M such that $S \leqslant L$ and $O \leqslant M$.

Theorem 3.9. In a $FNts$ $(U, \tau_F(F))$, the following are equivalent.

(i) U is $FNMNor$.
(ii) For every $FNMc$ set S in U and every $FNMo$ set L containing S, there exists a $FNMo$ set M containing $S \ni FNMcl(M) \leqslant L$.
(iii) For each pair of disjoint $FNMc$ sets S and O in U, there exists a $FNMo$ set L containing $S \ni FNMcl(L) \wedge O = 0_N$.
(iv) For each pair of disjoint $FNMc$ sets S and O in U, there exist $FNMo$ sets L and M containing S and O, respectively $\ni FNMcl(L) \wedge FNMcl(M) = 0_N$.

Proof.

(i) $\Rightarrow$ (ii): Let L be a $FNMo$ set containing the $FNMc$ set S. Then $O = L^c$ is a $FNMc$ set disjoint from S. Since U is $FNMNor$, there exist disjoint $FNMo$ sets M and W containing S and O, respectively. Then $FNMcl(M)$ is disjoint from O. Since if $y_\beta \in O$, the set W is a $FNMo$ set containing y_β disjoint from M. Hence $FNMcl(M) \leqslant L$.

(ii) $\Rightarrow$ (iii): Let S and O be disjoint $FNMc$ sets in U. Then O^c is a $FNMo$ set containing S. By (ii), there exists a $FNMo$ set L containing $S \ni FNMcl(L) \leqslant O^c$. Hence $FNMcl(L) \wedge O = 0_N$. This proves (iii).

(iii) $\Rightarrow$ (iv): Let S and O be disjoint $FNMc$ sets in U. Then, by (iii), there exists a $FNMo$ set L containing S $\ni FNMcl(L) \wedge O = 0_N$. Since $FNMcl(L)$ is $FNMc$, O and $FN\ Mcl(L)$ are disjoint $FNMc$ sets in U. Again by (iii), there exists a $FNMo$ set M containing $O \ni FNMcl(L) \wedge FNMcl(M) = 0_N$. This proves (iv).

(iv) $\Rightarrow$ (i): Let S and O be the disjoint $FNMc$ sets in U. By (iv), there exist $FNMo$ sets L and M containing S and O, respectively $\ni FNMcl(L) \wedge FNMcl(M) = 0_N$. Since $L \wedge M \leqslant FNMcl(L) \wedge FNMcl(M)$, L and M are disjoint $FNMo$ sets containing S and O, respectively. Thus U is $FNMNor$. $\square$

Theorem 3.10. Let $(\mathrm{U}, \tau_F(F))$ be a FNts is FNMNor if and only if for every FNMc set F and FNMo set G containing F, there exists a FNMo set $M \ni F \leqslant M \leqslant FNMcl(M) \leqslant G$.

Proof. Let $(\mathrm{U}, \tau_F(F))$ be FNMNor. Let F be a FNMc set and let G be a FNMo set containing F. Then F and G^c are disjoint FNMc sets. Since U is FNMNor, there exist disjoint FNMo sets M_1 and $M_2 \ni F \leqslant M_1$ and $G^c \leqslant M_2$. Thus $F \leqslant M_1 \leqslant M_2^c \leqslant G$. Since M_2^c is FNMc, so $FNMcl(M) \leqslant FNMcl(M_2^c) = M_2^c \leqslant G$. Take $M = M_1$. This implies that $F \leqslant M \leqslant FNMcl(M) \leqslant G$.

Conversely, suppose the condition holds. Let H_1 and H_2 be two disjoint FNMc sets in U. Then H_2^c is a FNMo set containing H_1. By assumption, there exists a FNMo set $M \ni H_1 \leqslant M \leqslant FNMcl(M) \leqslant H_2^c$, since M is FNMo and FNMcl(M) is FNMc. Then $(FNMcl(M))^c$ is FNMo. Now $FNMcl(M) \leqslant H_2^c$ implies that $H_2 \leqslant (FN\ Mcl(M))^c$. Also $M \wedge (FNMcl(M))^c \leqslant FNMcl(M) \wedge (FNMcl(M))^c = 0_N$. That is M and $(FNMcl(M))^c$ are disjoint FNMo sets containing H_1 and H_2, respectively. This shows that $(\mathrm{U}, \tau_F(F))$ is FN MNor. □

Theorem 3.11. For a FNts $(\mathrm{U}, \tau_F(F))$, then the following are equivalent.

(i) U is FNMNor.
(ii) For any two FNMo sets L and M whose union is 1_N, there exist FNMc subsets S of L and O of M whose union is also U.

Proof.

(i) ⇒ (ii): Let L and M be two FNMo sets in a FNMNor space $\mathrm{U} \ni 1_N = L \vee M$. Then L^c, M^c are disjoint FNMc sets. Since U is FNMNor, then there exist disjoint FNMo sets G_1 and $G_2 \ni L^c \leqslant G_1$ and $M^c \leqslant G_2$. Let $S = G_1^c$ and $O = G_2^c$. Then S and O are FNMc subsets of L and M, respectively $\ni S \vee O = 1_N$. This proves (ii).

(ii) ⇒ (i): Let S and O be disjoint FNMc sets in U. Then S^c and O^c are FNMo sets whose union is 1_N. By (ii), there exists FNMc sets F_1 and $F_2 \ni F_1 \leqslant S^c, F_2 \leqslant O^c$ and $F_1 \vee F_2 = 1_N$. Then F_1^c and F_2^c are disjoint FNMo sets containing S and O, respectively. Therefore U is FNMNor. □

Theorem 3.12. Let $h : (\mathrm{U}_1, \tau_F(F_1)) \to (\mathrm{U}_2, \tau_F(F_2))$ be a function.

(i) If f is injective, FNMIrr, FNMO, and U_1 is FNMNor, then U_2 is FNMNor.
(ii) If f is FNMIrr, FNMC, and U_2 is FNMNor, then U_1 is FNMNor.

Proof.

(i) Suppose U_1 is FNMNor. Let S and O be disjoint FNMc sets in U_2. Since f is FNMIrr, $h^{-1}(S)$ and $h^{-1}(O)$ are FNMc in U_1. Since U_1 is FNMNor, there exist disjoint FNMo sets L and M in $\mathrm{U}_1 \ni h^{-1}(S) \leqslant L$ and $h^{-1}(O) \leqslant M$. Now $h^{-1}(S) \leqslant L \Rightarrow S \leqslant h(L)$ and $h^{-1}(O) \leqslant M \Rightarrow O \leqslant h(M)$. Since f is a FNMO map, h(L) and h(M) are FNMo in U_2. Also $L \wedge M = 0_N \Rightarrow h(L \wedge M) = 0_N$ and f is injective, then $h(L) \wedge h(M) = 0_N$. Thus h(L) and h(M) are disjoint FNMo sets in U_2 containing S and O, respectively. Thus, U_2 is FNMNor.

(ii) Suppose U_2 is FNMNor. Let S and O be disjoint FNMc sets in U_1. Since f is FNMIrr and FNMC, h(S) and h(O) are FNMc in U_2. Since U_2 is FNMNor, there exist disjoint FNMo sets L and M in $\mathrm{U}_2 \ni h(S) \leqslant L$ and $h(O) \leqslant M$. That is $S \leqslant h^{-1}(L)$ and $O \leqslant h^{-1}(M)$. Since f is FNMIrr, $h^{-1}(L)$ and $h^{-1}(M)$ are disjoint FNMo $\ni S \leqslant h^{-1}(L)$ and $O \leqslant h^{-1}(M)$. Thus U_1 is FNMNor. □

Theorem 3.13. If given a pair of disjoint FNMc sets S, O of $(\mathrm{U}, \tau_F(F))$, there is FNMCts function f $\ni h(S) = 0_N$ and $h(O) = 1_N$, then $(\mathrm{U}, \tau_F(F))$ is FNMNor.

Proof. Let $(\mathrm{U}, \tau_F(F))$ be a FNts. Suppose for any pair of disjoint FNMc sets S, O in U, there exists a FNMCts map f $\ni h(S) = 0_N$ and $h(O) = 1_N$. Let E and F be disjoint FNMc sets in U. Let G and H be disjoint FNMo sets. Since f is FNMCts, $h^{-1}(G)$ and $h^{-1}(H)$ are FNMo in U. By our assumption, $h(E) =$

0_N and $h(F) = 1_N$. Now $h(E) = 0_N$ implies $h^{-1}(h(E)) \leqslant h^{-1}(0_N) \Rightarrow E \leqslant h^{-1}(h(E)) \leqslant h^{-1}(0_N) \Rightarrow E \leqslant h^{-1}(0_N)$. Similarly $F \leqslant h^{-1}(1_N)$. This implies that $E \leqslant h^{-1}(0_N) \leqslant h^{-1}(G)$. Then $F \leqslant h^{-1}(1_N) \leqslant h^{-1}(H)$. Further, $h^{-1}(G) \wedge h^{-1}(H) = h^{-1}(G \wedge H) = h^{-1}(0_N) = 0_N$. So, we have a pair of disjoint FNMo sets, $h^{-1}(G), h^{-1}(H) \leqslant 1_N \ni E \leqslant h^{-1}(G)$ and $F \leqslant h^{-1}(H)$. This proves that $(U, \tau_F(F))$ is FNMNor. □

Theorem 3.14. Let $h : (U_1, \tau_F(F_1)) \to (U_2, \tau_F(F_2))$ be a function. If f is a FNCts, FNMO bijection of a FNNor space U_1 into a space U_2 and if every FNMc set in U_2 is FNc, then U_2 is FNMReg.

Proof. Let S and O be FNMc sets in U_2. Then by assumption, O is FNc in U_2. Since f is a FNCts bijection, $h^{-1}(S)$ and $h^{-1}(O)$ is a FNc set in U_1. Since U_1 is FNNor, there exist disjoint FNo sets L_1 and L_2 in $U_1 \ni h^{-1}(S) \leqslant L_1$ and $h^{-1}(O) \leqslant L_2$. Since f is FNMO, $h(L_1)$ and $h(L_2)$ are disjoint FNMo sets in U_2 containing S and O, respectively. Hence U_2 is FNMNor. □

Remark 3.15. Theorems 3.9, 3.10, 3.11, 3.12, 3.13, 3.14 are also hold for FN$\theta\mathcal{S}$o and FN$\delta\mathcal{P}$o sets.

4. Strongly fuzzy nano M normal spaces

In this section, we introduce strongly fuzzy nano M normal spaces and study their properties.

Definition 4.1. A FNts $(U, \tau_F(F))$ is said to be strongly fuzzy nano M (resp. $\theta\mathcal{S}$ and $\delta\mathcal{P}$) normal (briefly, StFNMNor (resp. StFN$\theta\mathcal{S}$Nor and StFN$\delta\mathcal{P}$Nor)) if for every pair of disjoint FNc sets S and O in U, there are disjoint FNMo (resp. FN$\theta\mathcal{S}$o and FN$\delta\mathcal{P}$o) sets L and M in U containing S and O, respectively.

Theorem 4.2. Let $(U, \tau_F(F))$ be a FNts. Every FNMNor space is StFNMNor.

Proof. Suppose U is FNMNor. Let S and O be disjoint FNc sets in U. Then S and O are FNMc in U. Since U is FNMNor, there exist disjoint FNo sets L and M containing S and O, respectively. Since, every FNo is FNMo, L and M are FNMo in U. This implies that U is StFNMNor. □

Theorem 4.3. In a FNts $(U, \tau_F(F))$, the following are equivalent.

(i) U is StFNMNor.
(ii) For every FNc set F in U and every FNo set L containing F, there exists a FNMo set M containing F $\ni$ FNMcl(M) $\leqslant$ L.
(iii) For each pair of disjoint FNc sets S and O in U, there exists a FNMo set L containing S $\ni$ FNMcl(L) $\wedge$ O $= 0_N$.

Proof.

(i) $\Rightarrow$ (ii): Let L be a FNo set containing the FNc set F. Then $H = L^c$ is a FNc set disjoint from F. Since U is StFNMNor, there exist disjoint FNMo sets M and W containing F and H, respectively. Then FNMcl(M) is disjoint from H, since if $y_\beta \in H$, the set W is a FNMo set containing y_β disjoint from M. Hence FNMcl(M) $\leqslant$ L.

(ii) $\Rightarrow$ (iii): Let S and O be disjoint FNc sets in U. Then O^c is a FNo set containing S. By (ii), there exists a FNMo set L containing S $\ni$ FNMcl(L) $\leqslant O^c$. Hence FNMcl(L) $\wedge$ O $= 0_N$. This proves (iii).

(iii) $\Rightarrow$ (i): Let S and O be the disjoint FNMc sets in U. By (iii), there exists a FNMo set L containing S $\ni$ FNMcl(L) $\wedge$ O $= 0_N$. Take M = FNMcl$(L)^c$. Then L and M are disjoint FNMo sets containing S and O, respectively. Thus U is StFNMNor. □

Theorem 4.4. For a FNts $(U, \tau_F(F))$, then the following are equivalent.

(i) U is StFNMNor.
(ii) For any two FNo sets L and M whose union is 1_N, there exist FNMc subsets S of L and O of M whose union is also 1_N.

Proof.

(i) ⇒ (ii): Let L and M be two FNo sets in a $StFNMNor$ space $U \ni 1_N = L \vee M$. Then L^c, M^c are disjoint FNc sets. Since U is $StFNMNor$, then there exist disjoint $FNMo$ sets G_1 and $G_2 \ni L^c \leqslant G_1$ and $M^c \leqslant G_2$. Let $S = G_1^c$ and $O = G_2^c$. Then S and O are $FNMc$ subsets of L and M, respectively $\ni S \vee O = 1_N$. This proves (ii).

(ii) ⇒ (i): Let S and O be disjoint FNc sets in U. Then S^c and O^c are FNo sets whose union is U. By (ii), there exists $FNMc$ sets F_1 and $F_2 \ni F_1 \leqslant S^c, F_2 \leqslant O^c$ and $F_1 \vee F_2 = 1_N$. Then F_1^c and F_2^c are disjoint $FNMo$ sets containing S and O, respectively. Therefore U is $StFNMNor$. □

Theorem 4.5. Let $h : (U_1, \tau_F(F_1)) \to (U_2, \tau_F(F_2))$ be a function.

(i) If f is injective, $FNCts$, $FNMO$, and U_1 is $StFNMNor$, then U_2 is $StFN\ MNor$.
(ii) If f is $FNMIrr$, $FNMC$, and U_2 is $StFNMNor$, then U_1 is $StFNMNor$.

Proof.

(i) Suppose U_1 is $StFNMNor$. Let S and O be disjoint FNc sets in U_2. Since f is $FNCts$, $h^{-1}(S)$ and $h^{-1}(O)$ are FNc in U_1. Since U_1 is $StFNMNor$, there exist disjoint $FNMo$ sets L and M in $U_1 \ni h^{-1}(S) \leqslant L$ and $h^{-1}(O) \leqslant M$. Now $h^{-1}(S) \leqslant L \Rightarrow S \leqslant h(L)$ and $h^{-1}(O) \leqslant M \Rightarrow O \leqslant h(M)$. Since f is a $FNMO$ map, $h(L)$ and $h(M)$ are $FNMo$ in U_2. Also $L \wedge M = 0_N \Rightarrow h(L \wedge M) = 0_N$ and f is injective, then $h(L) \wedge h(M) = 0_N$. Thus $h(L)$ and $h(M)$ are disjoint $FNMo$ sets in U_2 containing S and O, respectively. Thus, U_2 is $StFNMNor$.

(ii) Suppose U_2 is $FNMNor$. Let S and O be disjoint FNc sets in U_1. Since f is $FN\ MIrr$ and $FNMC$, $h(S)$ and $h(O)$ are $FNMc$ in U_2. Since U_2 is $FNMNor$, there exist disjoint $FNMo$ sets L and M in $U_2 \ni h(S) \leqslant L$ and $h(O) \leqslant M$. That is $S \leqslant h^{-1}(L)$ and $O \leqslant h^{-1}(M)$. Since f is $FNMIrr$, $h^{-1}(L)$ and $h^{-1}(M)$ are disjoint $FNMo \ni S \leqslant h^{-1}(L)$ and $O \leqslant h^{-1}(M)$. Thus U_1 is $FNMNor$. □

Remark 4.6. Theorems 4.2, 4.3, 4.4, 4.5 are also hold for $FN\theta\mathcal{S}o$ and $FN\delta\mathcal{P}o$ sets.

5. Application on fuzzy score function

In this section, we present a fuzzy score function based on methodical approach for decision-making problem with fuzzy information.

Definition 5.1. Let $S : M \to [0, 1]$. The Fuzzy score function (in short, FSF) is

$$S(M) = \frac{1}{k} \sum_{i=1}^{k} \mu_{M_i}$$

that represents the average of positiveness of the fuzzy component μ_M.

The following essential steps are proposed the precise way to deal with select the proper attributes and alternative in the decision-making situation using fuzzy sets.

Step 1: problem field selection: Consider the universe of discourse (set of objects) m, the set of alternatives n, the set of decision attributes p.

Step 2: Construct a fuzzy matrix of alternative verses objects and object verses decision attributes.

Calculation part:

Step 3: Frame the in-discernibility relation R on m.

Step 4: Construct the fuzzy nano topologies τ_j and ν_k.

Step 5: Find the score values by Definition 5.1 each of the entries of the $FNts$.

Conclusion Part:

Step 6: Organize the fuzzy score values of the alternatives $\tau_1 \leqslant \tau_2 \leqslant \cdots \leqslant \tau_n$ and the attributes $\nu_1 \leqslant \nu_2 \leqslant \cdots \leqslant \nu_p$. Choose the attribute ν_p for the alternative τ_1 and ν_{p-1} for the alternative τ_2 etc. If $n < p$, then ignore ν_k, where $k = 1, 2, \cdots, n - p$.

5.1. Numerical example

An application in decision making problem of candidates choosing their batches based on their skills using fuzzy score function.

Step 1: problem field selection:

Consider the following tables giving informations of five candidates and their skills. Name of the candidates are Candidate_1, Candidate_2, Candidate_3, Candidate_4 and Candidate_5 and their skills are Experience, Computer knowledge, Communication skill, Higher education and Young age. Based on their skills, they choose their batches to work. Name of the Batches are Batch_1, Batch_2, Batch_3, Batch_4 and Batch_5. The data in Tables 1 and 2 are explained by the membership functions of the candidates and batches, respectively.

Step 2:

Table 1: Fuzzy values for candidates.

Skills \ Candidates	Candidate_1	Candidate_2	Candidate_3	Candidate_4	Candidate_5
Experience	0.3	0.5	0.7	0.7	0.6
Computer knowledge	0.5	0.4	0.4	0.6	0.7
Communication skill	0.6	0.7	0.6	0.4	0.5
Higher education	0.7	0.6	0.7	0.8	0.6
Young age	0.6	0.5	0.3	0.7	0.7

Table 2: Fuzzy values for batches.

Batches \ Skills	Experience	Computer knowledge	Communication skill	Higher education	Young age
Batch_1	0.0	0.2	0.7	0.9	0.2
Batch_2	0.0	0.9	0.2	0.2	0.2
Batch_3	0.9	0.1	0.3	0.1	0.2
Batch_4	0.6	0.1	0.2	0.2	0.9
Batch_5	0.0	0.1	0.9	0.1	0.3

Step 3: Construct the in-discernibility relation for the correlation between the skills given as $U\backslash R =$ {{Experience}, {Computer knowledge}, {Communication skill}, {Higher education}, {Young age}}.

Step 4:

1. Form fuzzy nano topologies for (τ_j):
   (i) $\tau_1^* = \{0_F, 1_F, 0.3, 0.5, 0.6, 0.7\}$;
   (ii) $\tau_2^* = \{0_F, 1_F, 0.5, 0.4, 0.7, 0.6\}$;
   (iii) $\tau_3^* = \{0_F, 1_F, 0.7, 0.4, 0.6, 0.3\}$;
   (iv) $\tau_4^* = \{0_F, 1_F, 0.7, 0.6, 0.4, 0.8\}$;
   (v) $\tau_5^* = \{0_F, 1_F, 0.6, 0.7, 0.5\}$.
2. Form fuzzy nano topologies for (ν_k):
   (i) $\nu_1^* = \{0_F, 1_F, 0.2, 0.7, 0.9\}$;
   (ii) $\nu_2^* = \{0_F, 1_F, 0.9, 0.2\}$;
   (iii) $\nu_3^* = \{0_F, 1_F, 0.9, 0.1, 0.3, 0.2\}$;
   (iv) $\nu_4^* = \{0_F, 1_F, 0.6, 0.1, 0.2, 0.9\}$;

(v) $\nu_5^* = \{0_F, 1_F, 0.1, 0.9, 0.3\}$.

Step 5:

1. Fuzzy score values for candidates:
   (i) $NSF(\tau_1) = 0.5166$;
   (ii) $NSF(\tau_2) = 0.5333$;
   (iii) $NSF(\tau_3) = 0.5$;
   (iv) $NSF(\tau_4) = 0.5833$;
   (v) $NSF(\tau_5) = 0.56$.
2. Fuzzy score values for batches:
   (i) $NSF(\nu_1) = 0.56$;
   (ii) $NSF(\nu_2) = 0.525$;
   (iii) $NSF(\nu_3) = 0.4166$;
   (iv) $NSF(\nu_4) = 0.4666$;
   (v) $NSF(\nu_5) = 0.46$.

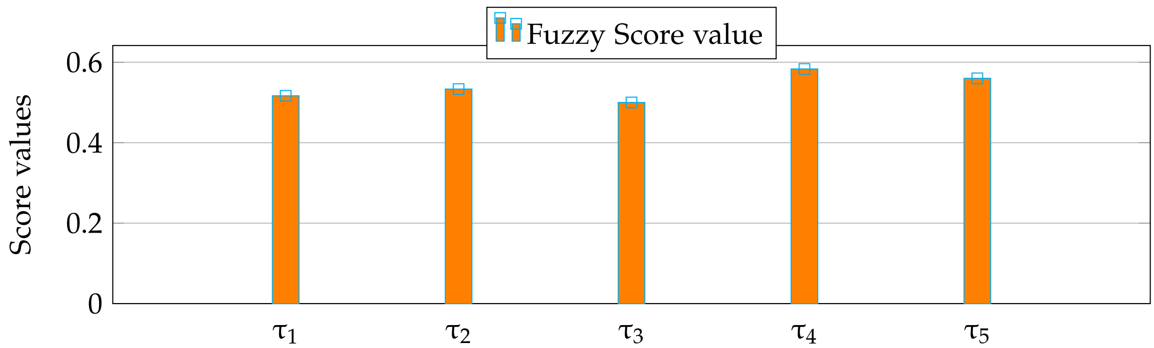

Figure 1: Fuzzy score values for candidates.

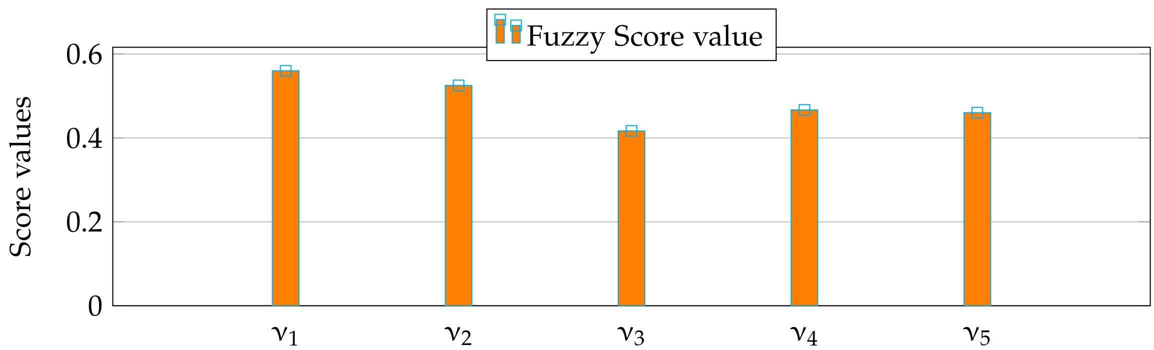

Figure 2: Fuzzy score values for batches.

Step 6: final decision: Arrange fuzzy score values for the alternatives τ_1, τ_2, τ_3, τ_4, τ_5 and the attributes ν_1, ν_2, ν_3, ν_4, ν_5 in ascending order. We get the following sequences $\tau_3 \leqslant \tau_1 \leqslant \tau_2 \leqslant \tau_5 \leqslant \tau_4$ and $\nu_3 \leqslant \nu_5 \leqslant \nu_4 \leqslant \nu_2 \leqslant \nu_1$. Thus the Candidate$_3$ goes to Batch$_1$, Candidate$_1$ goes to Batch$_2$, Candidate$_2$ goes to Batch$_4$, Candidate$_5$ goes to Batch$_5$ and Candidate$_4$ goes to Batch$_3$.

6. Conclusion

In this paper, we have studied FNMNor and StFNMNor spaces using FNMo and FNMc sets. They also discovered their relationships with one another and with pre-existing spaces. We also examined some fundamental features and characterizations of the previously described spaces. A new fuzzy set MADM technique has been introduced and applied to a situation involving a selection committee. The findings are crucial for improving the picture fuzzy set awareness that is available for decision-making applications. The MADM technique will be used in more practical applications in the future, as well as the practical interval valued fuzzy nano topological logic method for forecasting difficulties.

Acknowledgment

The authors would like to thank the editors and the anonymous reviewers for their valuable comments and suggestions which have helped immensely in improving the quality of the paper.

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