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Regularly open sets in intuitionistic fuzzy topological spaces

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Abstract

The aim of this paper is to introduce and study the concepts of generalization of intuitionistic fuzzy continuous functions via intuitionistic r-fuzzy regular open sets.

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1. Introduction

After the introduction of fuzzy sets by Zadeh [13], there have been a number of generalizations of this fundamental concept. The theory of fuzzy sets has several applications in real-life situations, and many scholars have researched fuzzy set theory. After the introduction of the concept of fuzzy sets, several research studies were conducted on the generalizations of fuzzy sets. The integration between fuzzy sets and some uncertainty approaches such as soft sets and rough sets has been discussed in [2, 4, 5]. The idea of intuitionistic fuzzy sets suggested by Atanassov [3] is one of the extensions of fuzzy sets with better applicability. Applications of intuitionistic fuzzy sets appear in various fields, including medical diagnosis, optimization problems, and multi-criteria decision making [8–10]. Using the notion of intuitionistic fuzzy sets, Coker [6] introduced the notion of intuitionistic smooth fuzzy topological spaces. Samanta and Mondal [11, 12] introduced the definitions of the intuitionistic smooth fuzzy topological space in Sŏstak sense. The aim of this paper is to introduce and study the concepts of generalization of intuitionistic fuzzy continuous functions via intuitionistic r-fuzzy regular open sets.

2. Preliminaries

Throughout this paper, let X be a non-empty set, I the unit interval $[0,1]$, and $I_0 = (0,1]$. The family of all intuitionistic fuzzy sets on X is denoted by I^X. By $\bar{0}$ and $\bar{1}$, we denote the smallest and the greatest intuitionistic fuzzy sets on X. For an intuitionistic fuzzy set $\lambda \in I^X$, $\bar{1} - \lambda$ denotes its complement.

Definition 2.1 ([3]). Let X be a nonempty set and I the closed interval $[0,1]$. An intuitionistic fuzzy set A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$, where the mappings $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each $x \in X$ to the set A, respectively, and $0 \leqslant \mu_A(x) + \gamma_A(x) \leqslant 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an intuitionistic fuzzy set of the following form $A = \{\langle x, \mu_A(x), \bar{1} - \mu_A(x) \rangle : x \in X\}$.

Definition 2.2 ([3]). Let A and B be intuitionistic fuzzy sets of the form $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$. Then

1. $A \leqslant B$ if and only if $\mu_A(x) \leqslant \mu_B(x)$ and $\gamma_A(x) \geqslant \gamma_B(x)$;
2. $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X\}$;
3. $A \wedge B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle : x \in X\}$;
4. $A \vee B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle : x \in X\}$.

We will use the notation $A = \langle x, \mu_A, \gamma_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$. A constant fuzzy set α taking value $\alpha \in [0,1]$ will be denoted by $\underline{\alpha}$. The intuitionistic fuzzy sets $\bar{0}$ and $\bar{1}$ are defined by $\bar{0} = \{\langle x, \underline{0}, \underline{1} \rangle : x \in X\}$ and $\bar{1} = \{\langle x, \underline{1}, \underline{0} \rangle : x \in X\}$. Let f be a mapping from an ordinary set X into an ordinary set Y. If $B = \{\langle y, \mu_B(y), \gamma_B(y) \rangle : y \in Y\}$ is an intuitionistic fuzzy set in Y, then the inverse image of B under f is intuitionistic fuzzy set defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle : x \in X\}$. The image of intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ under f is an intuitionistic fuzzy set defined by $f(A) = \{\langle y, f(\mu_A)(y), f(\gamma_A)(y) \rangle : y \in Y\}$, where $f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x), & f^{-1}(y) \neq 0, \\ 0, & \text{otherwise,} \end{cases}$

and $f(\gamma_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \gamma_A(x), & f^{-1}(y) \neq 0, \\ 1, & \text{otherwise,} \end{cases}$ for each $y \in Y$.

Definition 2.3 ([11, 12]). An intuitionistic gradation of openness on X is an ordered pair $(\tau, \tau^\star)$ of functions from I^X to I such that

1. $\tau(\lambda) + \tau^\star(\lambda) \leqslant 1$ for all $\lambda \in I^X$;
2. $\tau(\bar{0}) = \tau(\bar{1}) = 1$, $\tau^\star(\bar{0}) = \tau^\star(\bar{1}) = 0$;
3. $\tau(\lambda_1 \wedge \lambda_2) \geqslant \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^\star(\lambda_1 \wedge \lambda_2) \leqslant \tau^\star(\lambda_1) \vee \tau^\star(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$;
4. $\tau(\bigvee\limits_{i \in \Gamma} \lambda_i) \geqslant \bigwedge\limits_{i \in \Gamma} \tau(\lambda_i)$ and $\tau^\star(\bigvee\limits_{i \in \Gamma} \lambda_i) \leqslant \bigvee\limits_{i \in \Gamma} \tau^\star(\lambda_i)$ for each $\lambda_i \in I^X, i \in \Gamma$.

The triplet $(X, \tau, \tau^\star)$ is called an intuitionistic smooth fuzzy topological space.

Definition 2.4. An intuitionistic fuzzy set λ in an intuitionistic smooth fuzzy topological space $(X, \tau, \tau^\star)$ is called an intuitionistic r-fuzzy open if $\tau(\lambda) \geqslant r$ and $\tau^\star(\lambda) \leqslant 1 - r$ for each $r \in I_0$, λ is called an intuitionistic r-fuzzy closed if and only if $\bar{1} - \lambda$ is an intuitionistic r-fuzzy open set. We set $\tau_r = \{\lambda \in I^X : \tau(\lambda) \geqslant r, \tau^\star(\lambda) \leqslant 1 - r\}$.

Theorem 2.5 ([11, 12]). *Let $(X, \tau, \tau^\star)$ be an intuitionistic smooth fuzzy topological space. Then for each $r \in I_0$, $\lambda \in I^X$ we define operators* Cl, Int $: I^X \times I_0 \to I^X$ *as follows*

$$\text{Cl}(\lambda, r) = \bigwedge\{\mu \in I^X \mid \lambda \leqslant \mu, \tau(\bar{1} - \mu) \geqslant r, \tau^\star(\bar{1} - \mu) \leqslant 1 - r\},$$

$$\text{Int}(\lambda, r) = \bigvee\{\mu \in I^X \mid \mu \leqslant \lambda, \tau(\mu) \geqslant r, \tau^\star(\mu) \leqslant \bar{1} - r\}.$$

Definition 2.6 ([7])**.** Let X be a nonempty set and $c \in X$ a fixed element in X. If $a \in (0, 1]$ and $b \in [0, 1)$ are two fixed real numbers such that $a + b \leqslant 1$, then the intuitionistic fuzzy set $c(a, b) = \langle x, c_a, 1 - c_{1-b} \rangle$ is called an intuitionistic fuzzy point in X, where a denotes the degree of membership of $c(a, b)$, b the degree of non-membership of $c(a, b)$, and $c \in X$ the support of $c(a, b)$.

Definition 2.7. Let (X, τ) be an intuitionistic fuzzy topological space on X and $c(a, b)$ an intuitionistic fuzzy point in X. An intuitionistic fuzzy set A is called q-neighbourhood of $c(a, b)$, denoted by $\mathcal{N}_q(c(a, b))$, if there exists an intuitionistic fuzzy open set U in X such that $c(a, b) \, q \, U$ and $U \leqslant A$.

Definition 2.8. An intuitionistic fuzzy set λ of an intuitionistic fuzzy topological space $(X, \tau, \tau^\star)$ is said to be intuitionistic r-fuzzy semiopen [1] if $\lambda \leqslant \mathrm{Cl}(\mathrm{Int}(\lambda, r), r)$.

3. Intuitionistic smooth fuzzy open/closed mappings

Definition 3.1. An intuitionistic fuzzy mapping $f : X \to Y$ is called an intuitionistic r-fuzzy weakly continuous if for each intuitionistic r-fuzzy open set μ of Y, $f^{-1}(\mu) \leqslant \mathrm{Int}(f^{-1}(\mathrm{Cl}(\mu, r)), r)$.

Theorem 3.2. *If $f : X \to Y$ is an intuitionistic r-fuzzy weakly continuous, intuitionistic r-fuzzy open mapping, then f is intuitionistic r-fuzzy almost continuous.*

Proof. Since f is intuitionistic r-fuzzy weakly continuous, for every intuitionistic r-fuzzy open set μ in Y, $f^{-1}(\mu) \leqslant \mathrm{Int}(f^{-1}(\mathrm{Cl}(\mu, r)), r)$. Since f is intuitionistic r-fuzzy open, there is an intuitionistic r-fuzzy open set ν in X such that $f(\nu) \leqslant \mu$ or $\nu \leqslant f^{-1}(\mu) \leqslant \mathrm{Int}(f^{-1}(\mathrm{Cl}(\mu, r)), r)$. Again, since μ is an intuitionistic r-fuzzy open set, $f(\nu) \leqslant \mathrm{Int}(\mathrm{Cl}(\mu, r), r)$, or $\nu \leqslant f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mu, r)), r)$, or $\nu \leqslant \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mu, r), r)), r)$, that is, $\mathrm{Int}(f^{-1}(\mathrm{Cl}(\mu, r)), r) \leqslant \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mu, r), r)), r)$. Hence $\nu \leqslant f^{-1}(\mu) \leqslant \mathrm{Int}(f^{-1}(\mathrm{Cl}(\mu, r)), r) \leqslant \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mu, r), r)), r)$. Hence f is intuitionistic r-fuzzy almost continuous. $\square$

Theorem 3.3. *If $f : X \to Y$ is an intuitionistic r-fuzzy open and intuitionistic r-fuzzy continuous mapping and $g : Y \to Z$ is an intuitionistic fuzzy mapping, then $g \circ f$ is intuitionistic r-fuzzy almost continuous if and only if g is intuitionistic r-fuzzy almost continuous.*

Proof. Straightforward. $\square$

Definition 3.4. An intuitionistic fuzzy mapping $f : X \to Y$ is intuitionistic r-fuzzy almost open (closed) if for every intuitionistic r-fuzzy regularly open (closed) set μ in X, $f(\mu)$ is intuitionistic r-fuzzy open (closed) in Y.

Obviously, an intuitionistic r-fuzzy open mapping is intuitionistic r-fuzzy almost open, but the converse is not true, which is shown by the following example:

Example 3.5. An intuitionistic fuzzy mapping $f : X \to Y$, which is intuitionistic r-fuzzy almost open, need not to be intuitionistic r-fuzzy open. Let $X = [0, 1]$. The intuitionistic fuzzy subsets $\lambda_1, \lambda_2, \lambda_3 \in I^X$ are defined as follows:

$$\lambda_1 = \{\langle x, \mu_{\lambda_1}(x), \gamma_{\lambda_1}(x) \rangle : x \in X\}, \quad \lambda_2 = \{\langle x, \mu_{\lambda_2}(x), \gamma_{\lambda_2}(x) \rangle : x \in X\}, \quad \lambda_3 = \{\langle x, \mu_{\lambda_3}(x), \gamma_{\lambda_3}(x) \rangle : x \in X\},$$

where

$$\mu_{\lambda_1}(x) = \begin{cases} x, & \text{if } 0 \leqslant x \leqslant \frac{1}{2}, \\ 1 - x, & \text{if } \frac{1}{2} \leqslant x \leqslant 1, \end{cases} \qquad \gamma_{\lambda_1}(x) = \begin{cases} 1 - x, & \text{if } 0 \leqslant x \leqslant \frac{1}{2}, \\ x, & \text{if } \frac{1}{2} \leqslant x \leqslant 1, \end{cases}$$

$$\mu_{\lambda_2}(x) = \begin{cases} 2x, & \text{if } 0 \leqslant x \leqslant \frac{1}{2}, \\ 0, & \text{if } \frac{1}{2} < x \leqslant 1, \end{cases} \qquad \gamma_{\lambda_2}(x) = \begin{cases} 1 - 2x, & \text{if } 0 \leqslant x \leqslant \frac{1}{2}, \\ 1, & \text{if } \frac{1}{2} < x \leqslant 1, \end{cases}$$

$$\mu_{\lambda_3}(x) = \begin{cases} 1, & \text{if } 0 \leqslant x \leqslant \frac{1}{2}, \\ 0, & \text{if } \frac{1}{2} < x \leqslant 1, \end{cases} \qquad \gamma_{\lambda_3}(x) = \begin{cases} 0, & \text{if } 0 \leqslant x \leqslant \frac{1}{2}, \\ 1, & \text{if } \frac{1}{2} < x \leqslant 1. \end{cases}$$

We define $\tau, \tau^\star : I^X \to I$ as

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda \in \{\lambda_2, \lambda_3\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \tau^\star(\lambda) = \begin{cases} 0, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda \in \{\lambda_2, \lambda_3\}, \\ 1, & \text{otherwise}. \end{cases}$$

We define $\sigma, \sigma^\star : I^X \to I$ as

$$\sigma(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1, \\ 0, & \text{otherwise}, \end{cases} \qquad \sigma^\star(\lambda) = \begin{cases} 0, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1, \\ 1, & \text{otherwise}. \end{cases}$$

Consider the mapping $f : (X, \tau, \tau^\star) \to (X, \sigma, \sigma^\star)$ defined as $f(x) = 2x$ for $x \in [0, 1]$. Then $f(\bar{0}) = \bar{0}$, $f(\bar{1}) = \bar{1}$, $f(b)$ and $f(c)$ are not intuitionistic r-fuzzy open. Hence f is not intuitionistic r-fuzzy open, but it is intuitionistic r-fuzzy almost open, since the images of intuitionistic r-fuzzy regular open sets $\bar{0}$ and $\bar{1}$ are intuitionistic r-fuzzy open.

Theorem 3.6. *If* $f : X \to Y$ *is an intuitionistic r-fuzzy almost closed mapping of X onto Y, then for every intuitionistic r-fuzzy regularly open set σ in X and for all intuitionistic r-fuzzy singletons π in Y such that $f^{-1}(\pi) \leqslant \sigma$, we have $\pi \leqslant \text{Int}(f(\sigma), r)$.*

Proof. Since σ is intuitionistic r-fuzzy regularly open, $\bar{1} - \sigma$ is intuitionistic r-fuzzy regularly closed. Now, $f(\bar{1} - \sigma)$ is intuitionistic r-fuzzy closed in Y. Since $f^{-1}(\pi) \leqslant \sigma$, $\pi \leqslant f(\sigma)$, which implies $\bar{1} - \pi \geqslant f(\bar{1} - \sigma)$ and therefore there exists an intuitionistic r-fuzzy open set μ such that $\pi \leqslant \mu$ and $\mu < \bar{1} - f(\bar{1} - \sigma)$ or $\pi \leqslant \mu \leqslant f(\sigma)$. Hence $\pi \leqslant \text{Int}(f(\sigma), r)$. $\square$

Theorem 3.7. *If* $f : X \to Y$ *is an intuitionistic fuzzy bijective mapping, then following statements are equivalent:*

1. *f is intuitionistic r-fuzzy almost open;*
2. *f is intuitionistic r-fuzzy almost closed;*
3. *f^{-1} is intuitionistic r-fuzzy almost continuous.*

Proof.

$(1) \Rightarrow (2)$: Let σ be an intuitionistic r-fuzzy regularly closed set in X. Then $\bar{1} - \sigma$ is intuitionistic r-fuzzy regularly open, and $f(\bar{1} - \sigma)$ is intuitionistic r-fuzzy open in Y, or $\bar{1} - f(\sigma)$ is intuitionistic r-fuzzy open in Y. Hence $f(\sigma)$ is intuitionistic r-fuzzy closed in Y.

$(2) \Rightarrow (3)$: Let σ be an intuitionistic r-fuzzy regularly closed set in X. Then $f(\sigma)$ is intuitionistic r-fuzzy closed in Y. Since $f(\sigma) = (f^{-1})^{-1}(\sigma)$ and $f^{-1} : Y \to X$, we find that the inverse image of intuitionistic r-fuzzy regularly closed set σ in X is intuitionistic r-fuzzy closed in Y. Hence f^{-1} is intuitionistic r-fuzzy almost continuous.

$(3) \Rightarrow (1)$: Let σ be an intuitionistic r-fuzzy regularly open set in X, then since f^{-1} is intuitionistic r-fuzzy almost continuous, $(f^{-1})^{-1}(\sigma) = f(\sigma)$ is intuitionistic r-fuzzy open in Y and hence f is intuitionistic r-fuzzy almost open. $\square$

Theorem 3.8. *Suppose that* $f : X \to Y$ *and* $g : Y \to Z$ *are two intuitionistic fuzzy surjective maps. Then if* f *is intuitionistic r-fuzzy almost continuous and if* $g \circ f$ *is intuitionistic r-fuzzy open (resp. fuzzy closed), then* g *is also intuitionistic r-fuzzy almost open (resp. intuitionistic r-fuzzy almost closed).*

Proof. Let f be intuitionistic r-fuzzy almost continuous and let $g \circ f$ be intuitionistic r-fuzzy open (resp. fuzzy closed). Let σ be any intuitionistic r-fuzzy regularly open (resp. intuitionistic r-fuzzy regularly closed) set in Y. Then $f^{-1}(\sigma)$ is intuitionistic r-fuzzy open (resp. closed) in X. Now $g \circ f$ is intuitionistic r-fuzzy open (resp. closed) so that $(g \circ f)(f^{-1}(\sigma))$ is intuitionistic r-fuzzy open (resp. closed) in Z. But $(g \circ f)(f^{-1}(\sigma)) = g(\sigma)$. Thus $g(\sigma)$ is intuitionistic r-fuzzy open (resp. closed) in Z. Hence the map g is intuitionistic r-fuzzy almost open (resp. almost closed). $\square$

Lemma 3.9. *If* $f : X \to Y$ *is an intuitionistic r-fuzzy almost continuous, intuitionistic r-fuzzy almost open map, then the inverse image of every intuitionistic r-fuzzy regularly open (closed) set is intuitionistic r-fuzzy regularly open (resp. closed).*

Proof. Let μ be any intuitionistic r-fuzzy regularly open set in Y. Then $f^{-1}(\mu)$ is intuitionistic r-fuzzy open in X, and therefore $f^{-1}(\mu) \leqslant \operatorname{Int}(\operatorname{Cl}(f^{-1}(\mu), r), r)$. Since f is intuitionistic r-fuzzy almost continuous, and $\operatorname{Cl}(\mu, r)$ is intuitionistic r-fuzzy regularly closed set, then $f^{-1}(\operatorname{Cl}(\mu, r))$ is intuitionistic r-fuzzy closed and $\operatorname{Int}(\operatorname{Cl}(f^{-1}(\mu), r), r) \leqslant \operatorname{Cl}(f^{-1}(\mu), r) \leqslant f^{-1}(\operatorname{Cl}(\mu, r))$. Also f is intuitionistic r-fuzzy almost open, so that $\operatorname{Int}(\operatorname{Cl}(f^{-1}(\mu), r), r)$ being intuitionistic r-fuzzy regularly open in X, $f(\operatorname{Int}(\operatorname{Cl}(f^{-1}(\mu), r), r))$ is intuitionistic r-fuzzy open. Hence $f(\operatorname{Int}(\operatorname{Cl}(f^{-1}(\mu), r), r)) \leqslant \operatorname{Int}(\operatorname{Cl}(f(f^{-1}(\mu)), r), r) = \operatorname{Int}(\operatorname{Cl}(\mu, r), r) = \mu$. Thus $\operatorname{Int}(\operatorname{Cl}(f^{-1}(\mu), r), r) \leqslant f^{-1}(\mu)$. Hence $\operatorname{Int}(\operatorname{Cl}(f^{-1}(\mu), r), r) = f^{-1}(\mu)$. For the case of an intuitionistic r-fuzzy regularly closed set, we obtain the result by considering that $\bar{1} - \mu$ is an intuitionistic r-fuzzy regularly closed set in X, and $f^{-1}(\bar{1} - \mu) = \bar{1} - f^{-1}(\mu)$. $\square$

Definition 3.10. An intuitionistic r-fuzzy topological space $(X, \tau, \tau^\star)$ is said to be intuitionistic r-fuzzy almost regular if for every pair consisting of an intuitionistic fuzzy singleton π and an intuitionistic fuzzy r-regularly closed fuzzy set σ such that $\pi \leqslant \bar{1} - \sigma$, there exist two intuitionistic r-fuzzy open sets μ and γ such that $\pi \leqslant \mu$, $\sigma \leqslant \gamma$ and $\mu \leqslant \bar{1} - \gamma$.

Theorem 3.11. *For an intuitionistic fuzzy topological space the following are equivalent:*

1. $(X, \tau, \tau^\star)$ *is intuitionistic r-fuzzy almost regular;*
2. *for each intuitionistic r-fuzzy singleton π and each intuitionistic r-fuzzy regularly open set γ containing π, there exists an intuitionistic r-fuzzy regularly open set μ such that $\pi \leqslant \mu \leqslant \operatorname{Cl}(\mu, r) \leqslant \gamma$;*
3. *for each intuitionistic fuzzy singleton π and each intuitionistic r-fuzzy neighbourhood θ of π, there exists an intuitionistic r-fuzzy regularly open neighbourhood γ of π such that $\operatorname{Cl}(\gamma, r) \leqslant \operatorname{Int}(\operatorname{Cl}(\theta, r), r)$;*
4. *for each intuitionistic fuzzy singleton π and each intuitionistic r-fuzzy neighbourhood θ of π, there exists an intuitionistic r-fuzzy open neighbourhood γ of π such that $\operatorname{Cl}(\gamma, r) \leqslant \operatorname{Int}(\operatorname{Cl}(\theta, r), r)$;*
5. *for every intuitionistic r-fuzzy regularly closed set σ and each intuitionistic fuzzy singleton π such that $\pi \leqslant \bar{1} - \sigma$, there exist intuitionistic r-fuzzy open sets μ and γ such that $\pi \leqslant \mu$, $\sigma \leqslant \gamma$ and $\operatorname{Cl}(\mu, r) \leqslant \bar{1} - \operatorname{Cl}(\gamma, r)$.*

Proof.

$(1) \Rightarrow (2)$: If γ is an intuitionistic r-fuzzy regularly open set such that $\pi \leqslant \gamma$, then $\pi \leqslant \bar{1} - (\bar{1} - \gamma)$, $\bar{1} - \gamma$ is a intuitionistic r-fuzzy regularly closed set. Then there exist intuitionistic r-fuzzy open sets θ and ν such that $\pi \leqslant \theta$, $\bar{1} - \gamma \leqslant \nu$ and $\theta \leqslant \bar{1} - \nu$, and then $\operatorname{Cl}(\theta, r) \leqslant \bar{1} - \nu \leqslant \bar{1} - (\bar{1} - \gamma) = \gamma$. Thus $\pi \leqslant \theta \leqslant \operatorname{Cl}(\theta, r) \leqslant \gamma$. Again, $\theta \leqslant \operatorname{Int}(\operatorname{Cl}(\theta, r), r) \leqslant \operatorname{Cl}(\theta, r) \leqslant \gamma$. Therefore, if we put $\operatorname{Int}(\operatorname{Cl}(\theta, r), r) = \nu$, then $\theta \leqslant \nu \leqslant \operatorname{Cl}(\mu, r) \leqslant \gamma$. Hence $\pi \leqslant \nu \leqslant \operatorname{Cl}(\mu, r) \leqslant \gamma$, where ν is an intuitionistic r-fuzzy regularly open set.

$(2) \Rightarrow (3)$ and $(3) \Rightarrow (4)$: Obvious.

$(4) \Rightarrow (5)$: Let σ be an intuitionistic r-fuzzy regularly closed set and π be an intuitionistic fuzzy singleton such that $\pi \leqslant \bar{1} - \sigma$, that is, $\bar{1} - \sigma$ is a neighbourhood of π. Then there exists an intuitionistic r-fuzzy open set γ such that $\pi \leqslant \gamma \leqslant \operatorname{Cl}(\gamma, r) \leqslant \bar{1} - \sigma$. Again γ is a neighbourhood of π, there exists an intuitionistic r-fuzzy open set ν such that $\pi \leqslant \nu \leqslant \operatorname{Cl}(\mu, r) \leqslant \gamma$. Then ν and γ are intuitionistic r-fuzzy open sets such that $\pi \leqslant \nu$, $\sigma \leqslant \gamma$ and $\operatorname{Cl}(\mu, r) \leqslant \bar{1} - \operatorname{Cl}(\gamma, r)$.

$(5) \Rightarrow (1)$: Let σ be an intuitionistic r-fuzzy regularly closed set and π be an intuitionistic r-fuzzy singleton such that $\pi \leqslant \bar{1} - \sigma$. Then there exist intuitionistic r-fuzzy open sets ν and γ such $\pi \leqslant \nu$, $\sigma \leqslant \gamma$ and $\operatorname{Cl}(\mu, r) \leqslant \bar{1} - \operatorname{Cl}(\gamma, r)$ or $\nu \leqslant \operatorname{Cl}(\mu, r) \leqslant \bar{1} - \operatorname{Cl}(\gamma, r) \leqslant \bar{1} - \gamma$, that is, $\nu \leqslant \bar{1} - \gamma$. Hence $(X, \tau, \tau^\star)$ is intuitionistic r-fuzzy almost regular. $\square$

Lemma 3.12. *Let σ and θ be two intuitionistic fuzzy subsets of $(X, \tau, \tau^\star)$. If σ and θ can be separated by disjoint intuitionistic r-fuzzy open sets, then they can be separated by disjoint intuitionistic r-fuzzy regularly open sets.*

Proof. Let ν and γ be two intuitionistic r-fuzzy open sets such that $\sigma \leqslant \nu$, $\theta \leqslant \gamma$ and $\nu \leqslant \bar{1} - \gamma$. Then $\mathrm{Cl}(\mu, r) \leqslant \bar{1} - \gamma$, or $\mathrm{Int}(\mathrm{Cl}(\nu, r), r) \leqslant \bar{1} - \gamma$, or $\bar{1} - \mathrm{Int}(\mathrm{Cl}(\nu, r), r) \geqslant \gamma$ implies that $\mathrm{Cl}(\gamma, r) \leqslant \bar{1} - \mathrm{Int}(\mathrm{Cl}(\nu, r), r)$, or $\mathrm{Int}(\mathrm{Cl}(\gamma, r), r) \leqslant \bar{1} - \mathrm{Int}(\mathrm{Cl}(\nu, r), r)$, or $\mathrm{Int}(\mathrm{Cl}(\nu, r), r) \leqslant \bar{1} - \mathrm{Int}(\mathrm{Cl}(\gamma, r), r)$. Hence σ and θ can be separated by disjoint intuitionistic r-fuzzy regularly open sets. $\qquad\square$

Theorem 3.13. *If $f : (X, \tau, \tau^\star) \to (Y, \sigma, \sigma^\star)$ is an intuitionistic r-fuzzy almost continuous, intuitionistic r-fuzzy open and injective map, then $(Y, \sigma, \sigma^\star)$ is intuitionistic r-fuzzy almost regular if $(X, \tau, \tau^\star)$ is intuitionistic r-fuzzy almost regular.*

Proof. Let π be an intuitionistic r-fuzzy singleton and ρ be an intuitionistic r-fuzzy regularly open set in Y, and $\pi \leqslant \rho$. Then $f^{-1}(\pi)$ and $f^{-1}(\rho)$ are respectively, an intuitionistic fuzzy singleton and an intuitionistic r-fuzzy regularly open set in X. Since X is intuitionistic r-fuzzy almost regular, there exists an intuitionistic r-fuzzy open set μ in X such that $f^{-1}(\pi) \leqslant \mu \leqslant \mathrm{Cl}(\mu, r) \leqslant f^{-1}(\rho)$, or $\pi \leqslant f(\mu) \leqslant f(\mathrm{Cl}(\mu, r)) \leqslant \rho$, or $\pi \leqslant f(\mu) \leqslant \mathrm{Cl}(f(\mu), r) \leqslant \rho$, where $f(\mu)$ is an intuitionistic r-fuzzy open set in Y. Hence the space $(Y, \sigma, \sigma^\star)$ is intuitionistic r-fuzzy almost regular. $\qquad\square$

Definition 3.14. An intuitionistic fuzzy topological space $(X, \tau, \tau^\star)$ is said to be intuitionistic r-fuzzy almost normal if for every pair of intuitionistic fuzzy sets σ and θ, $\sigma \leqslant \bar{1} - \theta$ in X, where σ is intuitionistic r-fuzzy closed and θ is intuitionistic r-fuzzy regularly closed, there exist intuitionistic r-fuzzy open sets μ and γ such that $\sigma \leqslant \mu$, $\theta \leqslant \gamma$ and $\mu \leqslant \bar{1} - \gamma$.

Remark 3.15. Every intuitionistic r-fuzzy almost normal space is intuitionistic r-fuzzy almost regular. That the converse does not hold, is shown by the following example.

Example 3.16. Let $X = \{a, b\}$ and the intuitionistic fuzzy sets defined by

$$\lambda_1 = \langle x, (\frac{a}{0.1}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \rangle, \qquad \lambda_2 = \langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.9}, \frac{b}{0.8}) \rangle, \qquad \lambda_3 = \langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.7}, \frac{b}{0.9}) \rangle,$$

$$\lambda_4 = \langle x, (\frac{a}{0.7}, \frac{b}{0.2}), (\frac{a}{0.3}, \frac{b}{0.8}) \rangle, \qquad \lambda_5 = \langle x, (\frac{a}{0.3}, \frac{b}{0.2}), (\frac{a}{0.7}, \frac{b}{0.8}) \rangle, \qquad \lambda_6 = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}), (\frac{a}{0.6}, \frac{b}{0.6}) \rangle,$$

$$\lambda_7 = \langle x, (\frac{a}{0.3}, \frac{b}{0.9}), (\frac{a}{0.7}, \frac{b}{0.1}) \rangle, \qquad \lambda_8 = \langle x, (\frac{a}{0.9}, \frac{b}{0.7}), (\frac{a}{0.1}, \frac{b}{0.3}) \rangle, \qquad \lambda_9 = \langle x, (\frac{a}{0.9}, \frac{b}{0.2}), (\frac{a}{0.1}, \frac{b}{0.8}) \rangle.$$

We define $\tau, \tau^\star : I^X \to I$ as

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda \in \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \tau^\star(\lambda) = \begin{cases} 0, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda \in \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9\}, \\ 1, & \text{otherwise}. \end{cases}$$

Then except λ_4 all the intuitionistic r-fuzzy open sets are intuitionistic r-fuzzy regularly open. Then $(X, \tau, \tau^\star)$ is intuitionistic r-fuzzy almost regular but not intuitionistic r-fuzzy almost normal.

Theorem 3.17. *For an intuitionistic fuzzy space $(X, \tau, \tau^\star)$ the following are equivalent.*

1. *$(X, \tau, \tau^\star)$ is intuitionistic r-fuzzy almost normal.*
2. *For every intuitionistic r-fuzzy closed set σ and every intuitionistic regularly open set θ containing σ, there exists an intuitionistic r-fuzzy open set μ such that $\sigma \leqslant \mu \leqslant \mathrm{Cl}(\mu, r) \leqslant \theta$.*
3. *For every intuitionistic r-fuzzy regularly closed set σ and intuitionistic r-fuzzy open set θ containing σ, there exists an intuitionistic r-fuzzy open set μ such that $\sigma \leqslant \mu \leqslant \mathrm{Cl}(\mu, r) \leqslant \theta$.*
4. *For every pair of intuitionistic r-fuzzy sets consisting of an intuitionistic r-fuzzy closed set σ and an intuitionistic r-fuzzy regularly closed set θ, there exist intuitionistic r-fuzzy open sets μ and γ such that $\sigma \leqslant \mu$, $\theta \leqslant \gamma$ and $\mu \leqslant \bar{1} - \gamma$.*

Proof.

(1) $\Rightarrow$ (2): Let $\sigma \leqslant \theta$, that is, $\sigma \leqslant \bar{1} - (\bar{1} - \theta)$, where $\bar{1} - \theta$ is an intuitionistic r-fuzzy regularly closed set. Since $(X, \tau, \tau^\star)$ is intuitionistic r-fuzzy almost normal, there exist intuitionistic r-fuzzy open sets μ and γ such that $\sigma \leqslant \mu$, $\bar{1} - \theta \leqslant \gamma$ and $\mu \leqslant \bar{1} - \gamma$. Then $\sigma \leqslant \gamma \leqslant \bar{1} - \gamma \leqslant \theta$ but $\bar{1} - \gamma$ is an intuitionistic r-fuzzy closed set, so that $\sigma \leqslant \mu \leqslant \mathrm{Cl}(\mu, r) \leqslant \theta$.

(2) $\Rightarrow$ (3): Obvious.

(3) $\Rightarrow$ (4): If σ and θ are two intuitionistic r-fuzzy regularly closed sets such that $\sigma \leqslant \overline{1} - \theta$, then $\overline{1} - \theta$ is an intuitionistic r-fuzzy regularly open set containing σ, and by hypothesis there exists an intuitionistic r-fuzzy open set μ such that $\sigma \leqslant \mu \leqslant Cl(\mu, r) \leqslant \overline{1} - \theta$. Again since μ is an intuitionistic r-fuzzy regularly open set containing an intuitionistic r-fuzzy regularly closed set σ, there exists an intuitionistic r-fuzzy regularly open set ω such that $\sigma \leqslant \omega \leqslant Cl(\omega, r) \leqslant \mu$. Now let $\overline{1} - Cl(\mu, r) = \gamma$, then $\sigma \leqslant \omega$, $\theta \leqslant \gamma$ and $Cl(\omega, r) \leqslant \overline{1} - Cl(\gamma, r)$.

(4) $\Rightarrow$ (1): Let σ and θ be two intuitionistic r-fuzzy sets in X, where σ is intuitionistic r-fuzzy closed and θ is intuitionistic r-fuzzy regularly closed, and $\sigma \leqslant \overline{1} - \theta$. By hypothesis, there exist two intuitionistic r-fuzzy open sets μ and γ such that $\sigma \leqslant \mu$, $\theta \leqslant \gamma$ and $Cl(\mu, r) \leqslant \overline{1} - Cl(\gamma, r)$. We have $\mu \leqslant Cl(\mu, r) \leqslant \overline{1} - Cl(\gamma, r) \leqslant \overline{1} - \gamma$, that is, $\mu \leqslant \overline{1} - \gamma$. Hence $(X, \tau, \tau^\star)$ is intuitionistic r-fuzzy almost normal. $\square$

Theorem 3.18. *Every intuitionistic r-fuzzy semi-normal, intuitionistic r-fuzzy almost normal space is intuitionistic r-fuzzy normal.*

Proof. Let μ be any intuitionistic r-fuzzy open set containing an intuitionistic r-fuzzy closed set ρ. Then by intuitionistic r-fuzzy semi-normality, there exists an intuitionistic r-fuzzy semi-open set ω such that $\rho \leqslant \omega \leqslant Int(Cl(\omega, r), r) \leqslant \mu$. Since $Int(Cl(\omega, r), r)$ is an intuitionistic r-fuzzy regularly open set containing the intuitionistic r-fuzzy closed set ρ, therefore by intuitionistic almost normality there exists an intuitionistic r-fuzzy open set γ such that $\rho \leqslant \gamma \leqslant Cl(\gamma, r) \leqslant Int(Cl(\mu, r), r) \leqslant \mu$. Thus the space is intuitionistic r-fuzzy normal. $\square$

Corollary 3.19. *An intuitionistic r-fuzzy almost normal space is intuitionistic r-fuzzy normal if and only if it is intuitionistic r-fuzzy semi-normal.*

Theorem 3.20. *Let $f : X \to Y$ be an intuitionistic r-fuzzy open and intuitionistic r-fuzzy continuous mapping from an intuitionistic r-fuzzy almost normal space X to an intuitionistic fuzzy space Y. Then Y is also intuitionistic r-fuzzy almost normal.*

Proof. Let σ and θ be intuitionistic fuzzy subsets of Y, where σ is intuitionistic r-fuzzy closed, θ is intuitionistic r-fuzzy regularly open and $\sigma \leqslant \theta$. Then $f^{-1}(\sigma)$ and $f^{-1}(\theta)$ are respectively intuitionistic r-fuzzy closed and intuitionistic r-fuzzy regularly open sets in Y, and $f^{-1}(\sigma) \leqslant f^{-1}(\theta)$. Since X is intuitionistic r-fuzzy almost normal, there exists an intuitionistic r-fuzzy open set μ such that $f^{-1}(\sigma) \leqslant \mu \leqslant Cl(\mu, r) \leqslant f^{-1}(\theta)$, or $\sigma \leqslant f(\mu) \leqslant Cl(\mu, r) \leqslant f(Cl(\mu, r)) \leqslant \theta$. Thus we have $\sigma \leqslant f(\mu) \leqslant Cl(f(\mu), r) \leqslant \theta$, where $f(\mu)$ is an intuitionistic r-fuzzy open set in Y. Hence $(Y, \sigma, \sigma^\star)$ is intuitionistic r-fuzzy almost normal. $\square$

Definition 3.21. An intuitionistic fuzzy space $(X, \tau, \tau^\star)$ is said to be intuitionistic r-fuzzy mildly normal if for every pair of intuitionistic r-fuzzy regularly closed sets ρ and σ such that $\rho \leqslant \overline{1} - \sigma$, there exist intuitionistic r-fuzzy open sets μ and γ such that $\rho \leqslant \overline{1} - \mu$, $\sigma \leqslant \overline{1} - \gamma$ and $\mu \leqslant \overline{1} - \gamma$.

Every intuitionistic r-fuzzy almost normal space is intuitionistic r-fuzzy mildly normal but the converse does not hold. Consider the following counter example.

Example 3.22. Let $X = [0, 1]$. The intuitionistic r-fuzzy subsets $\lambda_1, \lambda_2, \lambda_3 \in I^X$ are defined as follows:

$$\lambda_1 = \{\langle x, \mu_{\lambda_1}(x), \gamma_{\lambda_1}(x)\rangle : x \in X\}, \quad \lambda_2 = \{\langle x, \mu_{\lambda_2}(x), \gamma_{\lambda_2}(x)\rangle : x \in X\}, \quad \lambda_3 = \{\langle x, \mu_{\lambda_3}(x), \gamma_{\lambda_3}(x)\rangle : x \in X\},$$

where

$$\mu_{\lambda_1}(x) = \begin{cases} \frac{1}{8}, & \text{if } x = 0, \\ \frac{1}{2}, & \text{if } 0 < x \leqslant 1, \end{cases} \qquad \gamma_{\lambda_1}(x) = \begin{cases} \frac{7}{8}, & \text{if } x = 0, \\ \frac{1}{2}, & \text{if } 0 < x \leqslant 1, \end{cases}$$

$$\mu_{\lambda_2}(x) = \begin{cases} \frac{7}{8}, & \text{if } x = 0, \\ \frac{1}{2}, & \text{if } 0 < x \leqslant 1, \end{cases} \qquad \gamma_{\lambda_2}(x) = \begin{cases} \frac{1}{8}, & \text{if } x = 0, \\ \frac{1}{2}, & \text{if } 0 < x \leqslant 1, \end{cases}$$

$$\mu_{\lambda_3}(x) = \begin{cases} \frac{3}{4}, & \text{if } x = 0, \\ \frac{1}{2}, & \text{if } 0 < x \leqslant 1, \end{cases} \qquad \gamma_{\lambda_3}(x) = \begin{cases} \frac{1}{4}, & \text{if } x = 0, \\ \frac{1}{2}, & \text{if } 0 < x \leqslant 1. \end{cases}$$

We define $\tau, \tau^\star : I^X \to I$ as

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda \in \{\lambda_1, \lambda_2\}, \\ 0, & \text{otherwise}, \end{cases} \qquad \tau^\star(\lambda) = \begin{cases} 0, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda \in \{\lambda_1, \lambda_2\}, \\ 1, & \text{otherwise}. \end{cases}$$

Consider λ_1 and λ_2 are intuitionistic r-fuzzy regularly open as well as intuitionistic r-fuzzy regularly closed, but λ_3 is not intuitionistic r-fuzzy regularly open. Clearly the space $(X, \tau, \tau^\star)$ is intuitionistic r-fuzzy mildly normal but not intuitionistic r-fuzzy almost mildly normal.

Theorem 3.23. *For an intuitionistic fuzzy space $(X, \tau, \tau^\star)$ the following are equivalent.*

1. *X is intuitionistic r-fuzzy mildly normal.*
2. *For every intuitionistic r-fuzzy regularly closed set σ and every intuitionistic r-fuzzy regularly open set μ containing σ, there exists an intuitionistic r-fuzzy open set γ such that $\sigma \leqslant \gamma \leqslant \text{Cl}(\gamma, r) \leqslant \mu$.*
3. *For every intuitionistic r-fuzzy regularly open set μ containing an intuitionistic r-fuzzy regularly closed set σ, there exists an intuitionistic r-fuzzy regularly open set γ such that $\sigma \leqslant \gamma \leqslant \text{Cl}(\gamma, r) \leqslant \mu$.*
4. *For every pair of intuitionistic r-fuzzy regularly closed sets σ and θ, there exist intuitionistic r-fuzzy open sets μ and γ such that $\sigma \leqslant \mu$, $\theta \leqslant \gamma$ and $\text{Cl}(\mu, r) \leqslant \bar{1} - \text{Cl}(\gamma, r)$.*

Proof.

$(1) \Rightarrow (2)$: Since $\sigma \leqslant \mu$ or $\sigma \leqslant \bar{1} - (\bar{1} - \mu)$, where $\bar{1} - \mu$ is an intuitionistic r-fuzzy regularly closed set, and X is intuitionistic r-fuzzy mildly normal, there exist intuitionistic r-fuzzy open sets γ and ω such that $\sigma \leqslant \gamma$, $\bar{1} - \mu \leqslant \omega$ and $\gamma \leqslant \bar{1} - \omega$ or $\text{Cl}(\gamma, r) \leqslant \bar{1} - \omega$. We have $\sigma \leqslant \gamma \leqslant \text{Cl}(\gamma, r) \leqslant \bar{1} - \omega \leqslant \theta$.

$(2) \Rightarrow (3)$: Since $\sigma \leqslant \mu$, where μ is an intuitionistic r-fuzzy regularly open and σ is an intuitionistic r-fuzzy regularly closed set, there exists a fuzzy open set γ with $\sigma \leqslant \gamma \leqslant \text{Cl}(\gamma, r) \leqslant \mu$. Since γ is intuitionistic r-fuzzy open, we have $\sigma \leqslant \gamma \leqslant \text{Int}(\text{Cl}(\gamma, r), r) \leqslant \theta$. Let $\text{Int}(\text{Cl}(\gamma, r), r) = \omega$. Then $\sigma \leqslant \omega \leqslant \text{Cl}(\omega, r) \leqslant \theta$, where ω is an intuitionistic r-fuzzy regularly open set.

$(3) \Rightarrow (4)$ and $(4) \Rightarrow (1)$: Are simple. $\qquad\square$

Theorem 3.24. *Every intuitionistic r-fuzzy almost continuous, intuitionistic r-fuzzy almost closed and intuitionistic r-fuzzy open image of an intuitionistic r-fuzzy mildly normal space is intuitionistic r-fuzzy mildly normal.*

Proof. Let $f : X \to Y$ be an intuitionistic r-fuzzy almost continuous, intuitionistic r-fuzzy open and intuitionistic r-fuzzy almost closed mapping and let X be intuitionistic r-fuzzy mildly normal. Let σ and θ be intuitionistic r-fuzzy regularly closed sets and $\sigma \leqslant \bar{1} - \theta$. Then $f^{-1}(\sigma)$ and $f^{-1}(\theta)$ are intuitionistic r-fuzzy regularly closed sets in X. Since X is intuitionistic r-fuzzy mildly normal, there exist intuitionistic r-fuzzy open sets μ and γ such that $f^{-1}(\sigma) \leqslant \mu \leqslant f^{-1}(\theta) \leqslant \gamma$ and $\mu \leqslant \bar{1} - \gamma$, where $f(\mu)$ and $f(\gamma)$ are intuitionistic r-fuzzy open sets in Y. Hence the space Y is also intuitionistic r-fuzzy mildly normal. $\qquad\square$

Theorem 3.25. *An intuitionistic r-fuzzy almost continuous, intuitionistic r-fuzzy closed image of an intuitionistic r-fuzzy normal space is intuitionistic r-fuzzy mildly normal.*

Proof. Let $f : X \to Y$ be an intuitionistic r-fuzzy almost continuous and intuitionistic r-fuzzy closed mapping and let X be intuitionistic r-fuzzy normal space. Let σ and θ be two intuitionistic r-fuzzy regularly closed subsets of Y such that $\sigma \leqslant \bar{1} - \theta$. Then $f^{-1}(\sigma)$ and $f^{-1}(\theta)$ are intuitionistic r-fuzzy closed sets in X. Since X is intuitionistic r-fuzzy normal, there exist intuitionistic r-fuzzy open sets μ and γ in X, such that $f^{-1}(\sigma) \leqslant \mu$, $f^{-1}(\theta) \leqslant \gamma$ and $\mu \leqslant \bar{1} - \gamma$. Now let $\omega = \{p : f^{-1}(p) \leqslant \mu$, where π is an intuitionistic r-fuzzy singleton$\}$ and $\gamma = \{q : f^{-1}(q) \leqslant \gamma$, where q is an intuitionistic r-fuzzy singleton$\}$. Then $\sigma \leqslant \omega$, $\theta \leqslant \gamma$ and $m \leqslant \bar{1} - \theta$. Hence the space Y is intuitionistic r-fuzzy mildly normal. $\qquad\square$

Acknowledgments

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