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Soft bi-continuity and related soft functions

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Abstract

In this article, we start with some properties of several types of soft continuous and soft open functions. We primarily focus on studying soft continuous (soft open) and soft irresolute (soft anti-irresolute) functions. We show that soft continuous and soft irresolute functions are independent and correspondingly soft open and soft anti-irresolute functions. On the other hand, soft bi-continuity implies soft bi-irresoluteness but not the other way round. Moreover, we find conditions under which soft bi-irresoluteness and soft bi-continuity are similar.

Keywords: Soft continuous, soft semicontinuous, soft somewhat continuous, soft open, soft somewhat open, soft semiopen.

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1. Introduction

Molodtsov [31], a Russian mathematician, proposed the theory of soft sets in 1999. Soft sets were introduced as a method for modeling mathematical problems involving uncertainty. Since there are no restricted criteria to classify the objects in the current theory, researchers are free to choose the type of parameters they need, simplifying the decision-making process and making the method more efficient in the absence of partial data. Other theories, such as fuzzy set theory, ambiguous set theory, and rough set theory, can be considered mathematical approaches for dealing with uncertainties but they have their own difficulties. According to Molodtsov, soft set theory can be implemented in a variety of disciplines. Maji et al. [30] explored the soft set theory's (detailed) theoretical structure. They developed several operators for soft set theory in particular. Other mathematical models, such as soft group theory and soft ring theory, have been studied in soft settings since Maji's contribution. In 2011, the concept of soft topology was defined by Shabir and Naz [33] and Çağman et al. [20] independently. After that, different methods

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of generating soft topologies were given by different researchers (see, [1, 8, 11, 17]). Recently, it has been introduced and investigated various kinds of soft separation axioms in [5, 7, 14].

In 2013, Nazmul and Samanta [32] defined soft continuity and soft openness of functions. Undoubtedly, these concepts have a role not less than continuity and openness in classical topology. Then various generalized types of soft continuity and soft openness of functions appeared in the literature. Namely: soft semicontinuous [29], soft β-continuous [34], soft somewhat continuous [13], soft somewhere dense continuous [6, 12], soft semi-open [29], soft β-open [34], soft somewhat open functions [13], soft somewhere dense open [6, 15], and so on. In this paper, we continue working on these classes of soft functions and provide more results and characterizations. Particularly, we study the connection between soft bi-continuous and soft bi-irresolute functions.

2. Preliminaries

This section introduces some key terms and definitions that will be utilized in the sequel. We will refer to X as an initial universe, E as a set of parameters, and $P(X)$ as the power set of X from now on.

Definition 2.1 ([31])**.** A pair $\langle F, E\rangle = \{(e, F(e)) : e \in E\}$ is said to be a soft set over X, where $F : E \to \mathcal{P}(X)$ is a (crisp) map. The class of all soft sets on X is expressed by $\mathcal{P}(X, E)$. If $L \subseteq E$, then it will be expressed by $\mathcal{P}(X, L)$.

Definition 2.2 ([10, 32])**.** A soft set $\langle F, E\rangle$ over X is called

(i) a soft element if $F(e) = \{x\}$ for all $e \in E$, where $x \in X$. It is denoted by $(\{x\}, E)$;
(ii) a soft point if $e \in E$ and $x \in X$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \neq e$, $e' \in E$. It is denoted by x_e. An expression $x_e \sqsubseteq \langle F, E\rangle$ means that $x \in F(e)$.

Definition 2.3 ([9])**.** The complement of $\langle F, E\rangle$ is a soft set $\langle X, E\rangle \setminus \langle F, E\rangle$ (or simply $\langle F, E\rangle^c$), where $F^c : E \to \mathcal{P}(X)$ is given by $F^c(e) = X \setminus F(e)$ for all $e \in E$.

Definition 2.4 ([30])**.** A soft subset $\langle F, E\rangle$ over X is called

(i) null if $F(e) = \emptyset$ for any $e \in E$;
(ii) absolute if $F(e) = X$ for any $e \in E$.

The null and absolute soft sets are respectively expressed by $\widetilde{\Phi}$ and $\widetilde{X}$. Clearly, $\widetilde{X}^c = \widetilde{\Phi}$ and $\widetilde{\Phi}^c = \widetilde{X}$.

Definition 2.5 ([30])**.** Let $E_1, E_2 \subseteq E$. It is said that $\langle G, E_1\rangle$ is a soft subset of $\langle H, E_2\rangle$ (written by $\langle G, E_1\rangle \sqsubseteq \langle H, E_2\rangle$) if $E_1 \subseteq E_2$ and $F(e) \subseteq G(e)$ for any $e \in E_1$. We say $\langle G, E_1\rangle = \langle H, E_2\rangle$ if $\langle G, E_1\rangle \sqsubseteq \langle H, E_2\rangle$ and $\langle H, E_2\rangle \sqsubseteq \langle G, E_1\rangle$.

Definition 2.6 ([9])**.** Let $\{\langle F_\alpha, E\rangle : \alpha \in \Lambda\}$ be a collection of soft sets over X, where Λ is any index set.

(i) The intersection of $\langle F_\alpha, E\rangle$, for $\alpha \in \Lambda$, is a soft set $\langle G, E\rangle$ such that $G(e) = \bigcap_{\alpha\in\Lambda} F_\alpha(e)$ for each $e \in E$ and is denoted by $\langle G, E\rangle = \bigsqcap_{\alpha\in\Lambda} \langle F_\alpha, E\rangle$.
(ii) The union of $\langle F_\alpha, E\rangle$, for $\alpha \in \Lambda$, is a soft set $\langle G, E\rangle$ such that $G(e) = \bigcup_{\alpha\in\Lambda} F_\alpha(e)$ for each $e \in E$ and is denoted by $\langle G, E\rangle = \bigsqcup_{\alpha\in\Lambda} \langle F_\alpha, E\rangle$.

Definition 2.7 ([33])**.** A subfamily $\mathcal{T}$ of $\mathcal{P}(X, E)$ is called a soft topology on X if

(c1) $\widetilde{\Phi}$ and $\widetilde{X}$ belong to $\mathcal{T}$;
(c2) finite intersection of sets from $\mathcal{T}$ belongs to $\mathcal{T}$; and
(c3) any union of sets from $\mathcal{T}$ belongs to $\mathcal{T}$.

Terminologically, we call $(X, \mathcal{T}, E)$ a soft topological space on X. The elements of $\mathcal{T}$ are called soft open sets, and their complements are called soft closed sets.

Henceforward, $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ mean soft topological spaces.

Definition 2.8 ([20]). A subfamily $\mathcal{B} \subseteq \mathcal{T}$ is called a soft base for the soft topology $\mathcal{T}$ if each element of $\mathcal{T}$ is a union of elements of $\mathcal{B}$.

Definition 2.9 ([33]). Let $\langle Y, E\rangle$ be a non-null soft subset of $(X, \mathcal{T}, E)$. Then $\mathcal{T}_Y := \{\langle G, E\rangle \sqcap \langle Y, E\rangle : \langle G, E\rangle \sqsubseteq \mathcal{T}\}$ is called a soft relative topology on Y and $(Y, \mathcal{T}_Y, E)$ is a soft subspace of $(X, \mathcal{T}, E)$.

Definition 2.10 ([2]). Let $\mathcal{F}$ be a family of soft subsets over X. The intersection of all soft topologies on X containing $\mathcal{F}$ is called a soft topology generated by $\mathcal{F}$.

Definition 2.11 ([33]). Let $\langle F, E\rangle$ be a soft subset of $(X, \mathcal{T}, E)$. The soft interior of $\langle F, E\rangle$ is the largest soft open set contained in $\langle F, E\rangle$ and is denoted by $\mathrm{Int}_X(\langle F, E\rangle)$ (or shortly $\mathrm{Int}(\langle F, E\rangle)$). The soft closure of $\langle F, E\rangle$ is the smallest soft closed set which contains $\langle F, E\rangle$ and is denoted by $\mathrm{Cl}_X(\langle F, E\rangle)$ (or simply $\mathrm{Cl}(\langle F, E\rangle)$).

Lemma 2.12 ([25]). *For a soft subset $\langle G, E\rangle$ of $(X, \mathcal{T}, E)$,*

$$\mathrm{Int}(\langle G, E\rangle^c) = (\mathrm{Cl}(\langle G, E\rangle))^c \textit{ and } \mathrm{Cl}(\langle G, E\rangle^c) = (\mathrm{Int}(\langle G, E\rangle))^c.$$

Definition 2.13. A soft subset $\langle G, E\rangle$ of $(X, \mathcal{T}, E)$ is called soft dense [22] (resp. soft nowhere dense [22], soft semiopen [21], soft β-open [34], soft somewhat open [13], soft somewhere dense open [3, 12]) if $\mathrm{Cl}(\langle G, E\rangle) = \widetilde{X}$ (resp. $\mathrm{Int}(\mathrm{Cl}(\langle G, E\rangle)) = \widetilde{\Phi}$, $\langle G, E\rangle \sqsubseteq \mathrm{Cl}(\mathrm{Int}(\langle G, E\rangle))$, $\langle G, E\rangle \sqsubseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(\langle G, E\rangle)))$, $\mathrm{Int}(\langle G, E\rangle) \neq \widetilde{\Phi}$ or $\langle G, E\rangle = \widetilde{\Phi}$, $\mathrm{Int}(\mathrm{Cl}(\langle G, E\rangle)) \neq \widetilde{\Phi}$ or $\langle G, E\rangle = \widetilde{\Phi}$).

Definition 2.14 ([19]). A soft topological space $(X, \mathcal{T}, E)$ is called

(i) soft T_0 if for each $x_e, y_{e'}$ over X with $x_e \neq y_{e'}$, there exist soft open sets $\langle G, E\rangle, \langle H, E\rangle$ such that $x_e \in \langle G, E\rangle$, $y_{e'} \notin \langle G, E\rangle$ or $y_{e'} \in \langle H, E\rangle$, $x_e \notin \langle H, E\rangle$;
(ii) soft T_1 if for each $x_e, y_{e'}$ over X with $x_e \neq y_{e'}$, there exist soft open sets $\langle G, E\rangle, \langle H, E\rangle$ such that $x_e \in \langle G, E\rangle$, $y_{e'} \notin \langle G, E\rangle$ and $y_{e'} \in \langle H, E\rangle$, $x_e \notin \langle H, E\rangle$;
(iii) soft T_2 (soft Hausdorff) if for each $x_e, y_{e'}$ over X with $x_e \neq y_{e'}$, there exist soft open sets $\langle G, E\rangle, \langle H, E\rangle$ containing $x_e, y_{e'}$, respectively, such that $\langle G, E\rangle \sqcap \langle H, E\rangle = \widetilde{\Phi}$;
(iv) soft regular if for each soft closed set $\langle F, E\rangle$ and each soft point x_e with $x_e \notin \langle F, E\rangle$, there exist soft open sets $\langle G, E\rangle, \langle H, E\rangle$ such that $x_e \in \langle G, E\rangle$, $\langle F, E\rangle \sqsubseteq \langle H, E\rangle$ and $\langle G, E\rangle \sqcap \langle H, E\rangle = \widetilde{\Phi}$.

Definition 2.15 ([24]). Let $\langle A, E\rangle$ be a subset of $(X, \mathcal{T}, E)$. A point x_e in $\langle A, E\rangle$ is said to be soft isolated if there exists a soft open set $\langle G, E\rangle$ over X containing x_e such that $\langle A, E\rangle \sqcap \langle G, E\rangle = \{x_e\}$.

Definition 2.16. A subset $\langle A, E\rangle$ of $(X, \mathcal{T}, E)$ is called soft perfect if it soft closed and contains no isolated soft points. A space $(X, \mathcal{T}, E)$ is soft perfect if no soft point over X is isolated or equivalently no one point soft set is soft open.

Definition 2.17. A space $(X, \mathcal{T}, E)$ is called

(i) soft compact [16] if every cover of $\widetilde{X}$ by soft open sets has a finite subcover;
(ii) soft locally compact [18] if each soft point has a soft compact neighborhood.

3. Some soft functions and their relations

Definition 3.1. A soft function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is called

(i) soft continuous [32] (resp. soft semicontinuous [29], soft somewhat continuous [13], soft somewhere dense continuous [6], soft β-continuous [34]) if $f^{-1}(\langle B, E\rangle)$ is a soft open (resp. soft semiopen, soft somewhat open, soft somewhere dense open, β-open) set in $(X, \mathcal{T}, E)$ for each soft open set $\langle B, E\rangle$ in $(Y, \mathcal{S}, E')$;

(ii) soft open [32] (resp. soft semiopen [29], soft somewhat open [13], soft somewhere dense open [6]) if $f(\langle A, E\rangle)$ is a soft open (resp. soft semiopen, soft somewhat open, soft soft somewhere dense open) set in $(Y, \mathcal{S}, E')$ for each soft open set $\langle A, E\rangle$ in $(X, \mathcal{T}, E)$;

(iii) soft irresolute [29] if $f^{-1}(\langle B, E\rangle)$ is a soft semiopen set in $(X, \mathcal{T}, E)$ for each soft semiopen set $\langle B, E\rangle$ in $(Y, \mathcal{S}, E')$;

(iv) soft irresolute open [27] if $f(\langle A, E\rangle)$ is a soft semiopen set in $(Y, \mathcal{S}, E')$ for each soft semiopen set $\langle A, E\rangle$ in $(X, \mathcal{T}, E)$; to avoid confusion with the term soft open, we call this function soft anti-irresolute;

(v) soft closed [32] if $f(\langle A, E\rangle)$ is a soft closed set in $(Y, \mathcal{S}, E')$ for each soft closed set $\langle A, E\rangle$ in $(X, \mathcal{T}, E)$.

We call a function f soft bi-continuous (resp. soft bi-irresolute) if it is both soft open and soft continuous (resp. soft irresolute and soft anti-irresolute).

For the definition of soft functions between collections of all soft sets, we refer the reader to [28]. Henceforward, by the word "function" we mean "soft function".

The following diagram gives the connection between the types of soft functions defined above.

		soft continuous (soft open)		
		$\downarrow$		
soft irresolute (soft anti-irresolute)	$\longrightarrow$	soft semicontinuous (soft semiopen)	$\longrightarrow$	soft SW-continuous (soft SW-open)
		$\downarrow$		$\downarrow$
		soft β-continuous (soft β-open)	$\longrightarrow$	soft SD-continuous (soft SD-open)

Diagram I: Relationship between types of generalized soft functions

In the above diagram, we mean by SD, "somewhere dense" and SW, "somewhat". The reverses of the arrows are generally false. Counterexamples are available in [12, 13, 15]. It is only left to show that soft continuous and soft irresolute functions are independent and correspondingly soft open and soft anti-irresolute.

Example 3.2. Let $X = \mathbb{R}$ be the set of real numbers and $E = \{e_1, e_2\}$ be a set of parameters. Let $\mathcal{T}$ be the soft topology on X generated by

$$\{(e_1, B(e_1)), (e_2, B(e_2)) : B(e_1) = (a, b), B(e_2) = (c, d); a, b, c, d \in \mathbb{R}; a < b, c < d\}$$

and let $\mathcal{S}$ be another soft topology on X generated by

$$\{(e_1, B(e_1)), (e_2, B(e_2)) : B(e_1) = (a, b], B(e_2) = (c, d); a, b, c, d \in \mathbb{R}; a < b, c < d\}.$$

The identity function $f : (X, \mathcal{T}, E) \to (X, \mathcal{S}, E)$ is soft irresolute but not soft continuous.

Example 3.3. Let $X = \{a, b, c\}$ and $E = \{e\}$. Define the topology $\mathcal{T}$ on X by

$$\mathcal{T} = \{\widetilde{\Phi}, \langle\{a\}, E\rangle, \langle\{a, b\}, E\rangle, \widetilde{X}\}$$

and the function $f : (X, \mathcal{T}, E) \to (X, \mathcal{T}, E)$ by $f(a_e) = f(b_e) = b_e$ and $f(c_e) = c_e$. Then f is soft continuous but not soft irresolute.

The other examples can be concluded from the above ones.

Lemma 3.4 ([13]). *Let $\langle G,E\rangle,\langle H,E\rangle$ be subsets of $(X,\mathcal{T},E)$. If $\langle G,E\rangle$ is soft open and $\langle H,E\rangle$ is soft semiopen, then $\langle G,E\rangle\sqcap\langle H,E\rangle$ is soft semiopen over X.*

Lemma 3.5 ([13]). *Let $\langle G,E\rangle$ be a non-null subset of $(X,\mathcal{T},E)$. If $\langle G,E\rangle$ is soft semiopen, then $\mathrm{Int}(\langle G,E\rangle)\neq\widetilde{\Phi}$.*

Theorem 3.6. *If a function $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ is soft open and soft somewhat continuous injective, then f is soft irresolute.*

Proof. Let $\langle H,E'\rangle$ be a soft semiopen set in $(Y,\mathcal{T},E')$ and let $x_e\sqsubseteq f^{-1}(\langle H,E'\rangle)$. It needs to show that $x_e\sqsubseteq \mathrm{Cl}(\mathrm{Int}(f^{-1}(\langle H,E'\rangle)))$. Let $\langle U,E\rangle$ be a soft open set containing x_e. Since f is soft open, then $f(\langle U,E\rangle)$ is a soft open set containing $f(x_e)$. Therefore $\langle H,E'\rangle\sqcap f(\langle U,E\rangle)\neq\widetilde{\Phi}$. By Lemma 3.4, $\langle H,E'\rangle\sqcap f(\langle U,E\rangle)$ is soft semiopen and so Lemma 3.5 guarantees that there is a non-null soft open set $\langle V,E'\rangle$ over Y such that $\langle V,E'\rangle\sqsubseteq\langle H,E'\rangle\sqcap f(\langle U,E\rangle)$. Now, we have

$$f^{-1}(\langle V,E'\rangle\sqsubseteq f^{-1}(\langle H,E'\rangle\bigsqcap f(\langle U,E\rangle))\sqsubseteq f^{-1}(\langle H,E'\rangle)\bigsqcap\langle U,E\rangle.$$

Since f is soft somewhat continuous, then there is soft open $\langle R,E\rangle$ over X such that

$$\widetilde{\Phi}\neq\langle R,E\rangle\sqsubseteq f^{-1}(\langle V,E'\rangle)\sqsubseteq f^{-1}(\langle H,E'\rangle)\bigsqcap\langle U,E\rangle,$$

which implies that

$$\mathrm{Int}(f^{-1}(\langle H,E'\rangle))\bigsqcap\langle U,E\rangle\neq\widetilde{\Phi}$$

and so $x_e\sqsubseteq\mathrm{Cl}(\mathrm{Int}(f^{-1}(\langle H,E'\rangle)))$. Hence f is is soft irresolute. □

We shall note that the injectivity of f in the above theorem cannot be neglected.

Example 3.7. Let $X=\mathbb{R}$ be the set of real numbers, $Y=\{0,1\}$ and $E=\{e_1,e_2\}$ be a set of parameters. Let $\mathcal{T}$ be the soft topology on X generated by $\{(e_1,B(e_1)),(e_2,B(e_2)):B(e_1)=(a,b),B(e_2)=(c,d);a,b,c,d\in\mathbb{R};a<b,c<d\}$ and let $\mathcal{S}$ be the soft discrete topology on Y. Define the soft function $f:(X,\mathcal{T},E)\to(X,\mathcal{S},E)$ by

$$f(x_e)=\begin{cases}0_{e_1}, & \text{if } x_e\sqsubseteq\{(e_1,[0,\infty)),(e_2,\{-1\})\},\\ 1_{e_2}, & \text{otherwise.}\end{cases}$$

Then f is soft open and soft somewhat continuous but not soft irresolute as it is not injective.

Lemma 3.8. *A function $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ is soft open if and only if $f^{-1}(\mathrm{Cl}(\langle V,E'\rangle))\sqsubseteq\mathrm{Cl}(f^{-1}(\langle V,E'\rangle))$, for each set $\langle V,E'\rangle$ over Y.*

Proof. From the definition. □

Lemma 3.9 ([21, Theorem 3.3]). *Let $\langle A,E\rangle,\langle B,E\rangle$ be subsets of $(X,\mathcal{T},E)$. If $\langle A,E\rangle$ is soft semiopen and $\langle A,E\rangle\sqsubseteq\langle B,E\rangle\sqsubseteq\mathrm{Cl}(\langle A,E\rangle)$, then $\langle B,E\rangle$ is also soft semiopen.*

Theorem 3.10. *If a function $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ is soft open and soft semicontinuous, then f is soft irresolute.*

Proof. Let $\langle H,E'\rangle$ be a soft semiopen set over Y. Then there exists a soft open $\langle V,E'\rangle$ such that

$$\langle V,E'\rangle\sqsubseteq\langle H,E'\rangle\sqsubseteq\mathrm{Cl}(\langle V,E'\rangle).$$

Therefore,

$$f^{-1}(\langle V,E'\rangle)\sqsubseteq f^{-1}(\langle H,E'\rangle)\sqsubseteq f^{-1}(\mathrm{Cl}(\langle V,E'\rangle)).$$

Since f is soft open, by Lemma 3.8, $f^{-1}(\mathrm{Cl}(\langle V,E'\rangle))\sqsubseteq\mathrm{Cl}(f^{-1}(\langle V,E'\rangle))$. This means that

$$f^{-1}(\langle V,E'\rangle)\sqsubseteq f^{-1}(\langle H,E'\rangle)\sqsubseteq\mathrm{Cl}(f^{-1}(\langle V,E'\rangle)).$$

By soft semicontinuity, $f^{-1}(\langle V,E'\rangle)$ is soft semiopen and so, by Lemma 3.9, $f^{-1}(\langle H,E'\rangle)$ is soft semiopen. This finishes the proof. □

In the above result, one may notice that the soft semicontinuity replaces injective soft somewhat continuity (in Theorem 3.6), but this is not the case. The following function is injective soft somewhat continuous but not semicontinuous, (c.f. Corollary 3.13).

Example 3.11 ([13]). Let $X = \mathbb{R}$ be the set of real numbers and $E = \{e\}$ be a set of parameters. Let $\mathcal{T}$ be the soft topology on $\mathbb{R}$ generated by $\{(e, B(e)) : B(e) = (a, b); a, b \in \mathbb{R}; a < b\}$. Define the soft function $f : (X, \mathcal{T}, E) \to (X, \mathcal{T}, E)$ by

$$f(x_e) = \begin{cases} x_e, & \text{if } x_e \not\sqsubseteq \langle\{0,1\}, E\rangle, \\ 0_e, & \text{if } x_e = 1_e, \\ 1_e, & \text{if } x_e = 0_e. \end{cases}$$

From Theorems 3.6 and 3.10 and Diagram I, we obtain following.

Corollary 3.12. *If a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft bi-continuous, then f is soft irresolute.*

Corollary 3.13. *If a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft open and soft somewhat continuous injective, then f is soft semicontinuous.*

Lemma 3.14. *Let $\langle A, E\rangle, \langle B, E\rangle$ be subsets of $(X, \mathcal{T}, E)$. If $\langle A, E\rangle$ is soft open and $\langle B, E\rangle$ is soft β-open, then $\langle A, E\rangle \sqcap \langle B, E\rangle$ is soft β-open in $\langle A, E\rangle$.*

Proof. Given the soft sets $\langle A, E\rangle, \langle B, E\rangle$, then

$$\begin{aligned} \langle A, E\rangle \sqcap \langle B, E\rangle &\sqsubseteq \langle A, E\rangle \sqcap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(\langle B, E\rangle))) \\ &\sqsubseteq \mathrm{Cl}(\langle A, E\rangle \sqcap \mathrm{Int}(\mathrm{Cl}(\langle B, E\rangle))) \\ &= \mathrm{Cl}(\langle A, E\rangle \sqcap \mathrm{Int}(\mathrm{Cl}(\langle B, E\rangle))) \sqcap \langle A, E\rangle \\ &= \mathrm{Cl}_A(\langle A, E\rangle \sqcap \mathrm{Int}(\mathrm{Cl}(\langle B, E\rangle))). \end{aligned}$$

Since $\langle A, E\rangle \sqcap \mathrm{Int}(\mathrm{Cl}(\langle B, E\rangle))$ is soft open in $\langle A, E\rangle$, by [33, Theorem 2],

$$\begin{aligned} \mathrm{Int}_A(\langle A, E\rangle \sqcap \mathrm{Int}(\mathrm{Cl}(\langle B, E\rangle))) &= \langle A, E\rangle \sqcap \mathrm{Int}(\mathrm{Cl}(\langle B, E\rangle)), \\ \langle A, E\rangle \sqcap \langle B, E\rangle &\sqsubseteq \mathrm{Cl}_A(\mathrm{Int}_A(\langle A, E\rangle \sqcap \mathrm{Cl}(\langle B, E\rangle) \sqcap \langle A, E\rangle)) \\ &\sqsubseteq \mathrm{Cl}_A(\mathrm{Int}_A(\mathrm{Cl}_A(\langle A, E\rangle \sqcap \langle B, E\rangle))). \end{aligned}$$

The lemma is proved. □

Theorem 3.15. *A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft β-continuous if and only if f is soft somewhere dense continuous on each soft open set over X.*

Proof. Suppose that f is a soft β-continuous function and $\langle U, E\rangle$ is any soft open set over X. Let $\langle H, E'\rangle$ be soft open over Y. By Lemma 3.14, $f^{-1}(\langle H, E'\rangle) \sqcap \langle U, E\rangle$ is a soft β-open set in $\langle U, E\rangle$. If it is null, then by definition, $f^{-1}(\langle H, E'\rangle) \sqcap \langle U, E\rangle$ is soft somewhere dense. Otherwise, by [3, Proposition 2.8], $f^{-1}(\langle H, E'\rangle) \sqcap \langle U, E\rangle$ is soft somewhere dense in $\langle U, E\rangle$. Thus f is soft somewhere dense continuous on each soft open set over X.

Conversely, assume that f is a soft somewhere dense continuous function on each soft open set over X. Given a soft point x_e in X and let $\langle H, E'\rangle$ be soft open over Y containing $f(x_e)$. If $\langle G, E\rangle$ is a soft open set containing x_e, then $f^{-1}(\langle H, E'\rangle) \sqcap \langle G, E\rangle \neq \widetilde{\Phi}$. By assumption, $\widetilde{\Phi} \neq \mathrm{Int}(\mathrm{Cl}(f^{-1}(\langle H, E'\rangle \sqcap \langle G, E\rangle))) = \mathrm{Int}(\mathrm{Cl}(f^{-1}(\langle H, E'\rangle))) \sqcap \langle G, E\rangle$. This implies that $x_e \sqsubseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(\langle H, E'\rangle))))$. Hence f is soft β-continuous. □

Theorem 3.16 ([13]). *A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft semicontinuous if and only if f is soft somewhat continuous on each soft open set over X.*

Theorem 3.17. *If a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft continuous and soft somewhat open, then f is soft anti-irresolute.*

Proof. Let $\langle G, E\rangle$ be a soft semiopen set over X and let $y_{e'} \sqsubseteq f(\langle G, E\rangle)$. We have to obtain that $y_{e'} \sqsubseteq Cl(Int(f(\langle G, E\rangle)))$. Let $\langle V, E'\rangle$ be a soft open set over Y including $y_{e'}$. Since $f(x_e) = y_{e'}$, then $x_e \sqsubseteq \langle G, E\rangle$. Since f is soft continuous, then there exists a soft open set $\langle U, E\rangle$ containing x_e such that $f(\langle U, E\rangle) \sqsubseteq \langle V, E'\rangle$. Therefore $\langle U, E\rangle \sqcap \langle G, E\rangle \neq \widetilde{\Phi}$. By Lemmas 3.4 and 3.5, there is a soft open set $\langle C, E\rangle$ over X such that

$$\widetilde{\Phi} \neq \langle C, E\rangle \sqsubseteq \langle U, E\rangle \bigsqcap \langle G, E\rangle.$$

By somewhat openness of f, there is a soft open set $\langle S, E'\rangle$ over Y such that

$$\widetilde{\Phi} \neq \langle S, E'\rangle \sqsubseteq f(\langle C, E\rangle) \sqsubseteq f(\langle U, E\rangle \bigsqcap \langle G, E\rangle) \sqsubseteq \langle V, E'\rangle \bigsqcap f(\langle G, E\rangle).$$

This yields that

$$\langle V, E'\rangle \bigsqcap Int(f(\langle G, E\rangle)) \neq \widetilde{\Phi}.$$

Since $\langle V, E'\rangle$ was taken arbitrarily, so $y_{e'} \sqsubseteq Cl(Int(f(\langle G, E\rangle)))$. This completes the proof. □

From the above result and Diagram I, we obtain following.

Corollary 3.18. *If a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft bi-continuous, then f is soft anti-irresolute.*

Corollary 3.19. *If a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft open and soft somewhat continuous, then f is soft semiopen.*

From Corollaries 3.12 and 3.18, we obtain following.

Theorem 3.20. *If a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft bi-continuous, then f is soft bi-irresolute.*

In general, the converse of the above result is false.

Example 3.21. Let $X = \{a, b, c\}$ and $E = \{e\}$. Consider the soft topology $\mathcal{T}$ on X given in Example 3.3. Let $\mathcal{S} = \{\widetilde{\Phi}, \langle\{a\}, E\rangle, \langle\{a, b\}, E\rangle, \langle\{a, b\}, E\rangle, \widetilde{X}\}$ be another soft topology on X. The identity function $f : (X, \mathcal{T}, E) \to (X, \mathcal{S}, E)$ is soft bi-irresolute but not soft bi-continuous.

Lemma 3.22 ([26, Theorem 2.2.23]). *A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft continuous at x_e if and only if for every soft net $\{x^d_e : d \in D\}$ over X converges to x_e, where D is a directed set, the net $\{f(x^d_e) : d \in D\}$ converges to $f(x_e)$.*

Theorem 3.23. *Let $(X, \mathcal{T}, E)$ be a soft space and let $(Y, \mathcal{S}, E')$ be a soft perfect Hausdorff. If $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is a soft anti-irresolute and soft semicontinuous function, then f is soft continuous.*

Proof. If f is possibly not soft continuous at some point x_e over X, by Lemma 3.22, there exists a soft net $\{x^d_e : d \in D\}$ that converges to x_e and $\{f(x^d_e) : d \in D\}$ does not converge to $f(x_e)$. Therefore, there exists a soft open set $\langle V, E'\rangle$ over $f(X)$ containing $f(x_e)$ such that $\langle V, E'\rangle \sqcap \{f(x^d_e)\} = \widetilde{\Phi}$ for every $d \in D$. By soft Hausdorfness of $(Y, \mathcal{S}, E')$, for each $d \in D$, there exists a soft open set $\langle W_d, E'\rangle$ over $f(X)$ including $f(x^d_e)$ such that $\langle V, E'\rangle \sqcap \langle W_d, E'\rangle = \widetilde{\Phi}$. This implies that $\langle V, E'\rangle \sqcap \left(\bigsqcup_{d\in D} \langle W_d, E'\rangle\right) = \widetilde{\Phi}$. Set $\langle Z, E'\rangle = \bigsqcup_{d\in D} \langle W_d, E'\rangle$. Then $\langle Z, E'\rangle$ is a soft open set over Y. By soft semicontinuity, $f^{-1}(\langle Z, E'\rangle)$ is soft semiopen over X. Since $f^{-1}(\langle Z, E'\rangle)$ includes the soft net $\{x^d_e : d \in D\}$, then x_e shall be a soft limit point of $f^{-1}(\langle Z, E'\rangle)$. Thus $f^{-1}(\langle Z, E'\rangle) \bigsqcup \{x_e\}$ is also soft semiopen. Since f is soft anti-irresolute, then $f(f^{-1}(\langle Z, E'\rangle) \bigsqcup \{x_e\}) = \langle Z, E'\rangle \bigsqcup \{f(x_e)\}$ is a soft semiopen set. But $\langle V, E'\rangle$ and $\langle Z, E'\rangle$ have no soft points in common, so $\langle Z, E'\rangle \bigsqcup \{f(x_e)\} \sqcap \langle V, E'\rangle = \{f(x_e)\}$ and thus, by Lemma 3.4, $\{f(x_e)\}$ is soft semiopen. By Lemma 3.5, $\{f(x_e)\}$ is soft open, which contradicts the soft perfectness of $(Y, \mathcal{S}, E')$. Hence f shall be continuous. □

From Diagram I, we obtain following.

Corollary 3.24. *Let* $(X, \mathcal{T}, E)$ *be a soft space and let* $(Y, \mathcal{S}, E')$ *be a soft perfect Hausdorff. If* $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ *is a soft bi-irresolute function, then* f *is continuous.*

Merging Theorems 3.17 and 3.23 together, one can have following.

Theorem 3.25. *Let* $(X, \mathcal{T}, E)$ *be a soft space and let* $(Y, \mathcal{S}, E')$ *be a soft perfect Hausdorff. Then a function* $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ *is soft anti-irresolute and soft semicontinuous if and only if* f *is soft continuous and soft somewhat open.*

Theorem 3.26. *A function* $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ *is soft semiopen if and only if* f *is soft somewhat open on each soft open set over* Y*.*

Proof. Suppose that f is a soft semiopen function and $\langle V, E'\rangle$ is any soft open set over Y. Let $\langle G, E\rangle$ be soft open over X. By [13, Lemma 3.21], $f(\langle G, E\rangle) \sqcap \langle V, E'\rangle$ is soft semiopen in $\langle V, E'\rangle$. If it is null, then by definition, $f(\langle G, E\rangle) \sqcap \langle V, E'\rangle$ is soft somewhat open. Otherwise, by Lemma 3.5, $f(\langle G, E\rangle) \sqcap \langle V, E'\rangle$ is soft somewhat open in $\langle V, E'\rangle$. Thus f is soft somewhat open on each soft open set over Y.

Conversely, assume that f is a soft somewhat open function on each soft open set over Y. Given $y_{e'} \in \widetilde{Y}$ and let $\langle G, E\rangle$ be soft open over X such that $y_{e'} \sqsubseteq f(\langle G, E\rangle)$. If $\langle H, E'\rangle$ is a soft open set containing $y_{e'}$, then $f(\langle G, E\rangle) \sqcap \langle H, E'\rangle \neq \widetilde{\Phi}$. By assumption,

$$\widetilde{\Phi} \neq \mathrm{Int}(f(\langle G, E\rangle) \bigsqcap \langle H, E'\rangle) = \mathrm{Int}(f(\langle G, E\rangle)) \bigsqcap \langle H, E'\rangle.$$

This implies that

$$y_{e'} \sqsubseteq \mathrm{Cl}(\mathrm{Int}(f(\langle G, E\rangle))).$$

Hence f is soft semiopen. □

Lemma 3.27 ([26, Theorem 3.1.6])**.** *Let* f *be a function from a soft compact space* $(X, \mathcal{T}, E)$ *onto a soft space* $(Y, \mathcal{S}, E')$*. If* f *is soft continuous, then* $(Y, \mathcal{S}, E')$ *is soft compact.*

Definition 3.28 ([4])**.** A soft set $\langle A, E\rangle$ from $(X, \mathcal{T}, E)$ is called stable if there exists a subset Y of X such that $A(e) = Y$ for each $e \in E$.

Definition 3.29. We call a soft space $(X, \mathcal{T}, E)$ stable if each soft open is stable.

Lemma 3.30. *If* $(X, \mathcal{T}, E)$ *is a stable soft Hausdorff space, then each soft compact is soft closed.*

Proof. Follows from Lemma 7 and Theorem 8 in [4]. □

Theorem 3.31. *Let* f *be a soft continuous function from a soft locally compact space* $(X, \mathcal{T}, E)$ *into a stable soft Hausdorff space* $(Y, \mathcal{S}, E')$*. Then* f *is soft somewhere dense open if and only if* f *is soft somewhat open.*

Proof. By Default, each soft somewhat open function is soft somewhere dense open.

Conversely, let $\langle G, E\rangle$ be a soft open set over X. By soft locally compactness of X, there is a non-null soft open set $\langle U, E\rangle$ such that $\mathrm{Cl}(\langle U, E\rangle) \sqsubseteq \langle G, E\rangle$ and $\mathrm{Cl}(\langle U, E\rangle)$ is soft compact. Since f is soft continuous, by Lemmas 3.27 and 3.30, $f(\mathrm{Cl}(\langle U, E\rangle))$ is soft closed. The soft somewhere dense openness of f guarantees that $\widetilde{\Phi} \neq \mathrm{Int}(f(\mathrm{Cl}(\langle U, E\rangle))) \sqsubseteq \mathrm{Int}(f(\langle G, E\rangle))$. Thus f is soft somewhat open. □

Theorem 3.32. *Let* f *be a function from a soft regular space* $(X, \mathcal{T}, E)$ *into a soft perfect Hausdorff space* $(Y, \mathcal{S}, E')$*. If* f *is soft bi-irresolute and soft closed, then* f *is soft open.*

Proof. Suppose otherwise that f is not soft open. Then there is a soft open set $\langle U, E\rangle$ such that $f(\langle U, E\rangle)$ is not soft open. By soft Hausdorffness of $(Y, \mathcal{S}, E')$, for each $y^1_{e'}, y^2_{e'} \Subset (Y, \mathcal{S}, E')$ with $y^1_{e'} \neq y^2_{e'}$, there exist soft open sets $\langle V_1, E'\rangle, \langle V_2, E'\rangle$ containing $y^1_{e'}, y^2_{e'}$, respectively, such that $\langle V_1, E'\rangle \sqcap \langle V_2, E'\rangle = \widetilde{\Phi}$. Since $y_{e'} \Subset \langle V_y, E'\rangle$ (for some soft open set $\langle V_y, E'\rangle$) for each $y_{e'}$ over Y, so $\widetilde{Y} = \bigsqcup_{y_{e'} \in \widetilde{Y}} \langle V_y, E'\rangle$.

Now, since $\langle U, E\rangle \sqsubseteq \widetilde{X}$, so

$$\langle U, E\rangle \bigsqcap f^{-1}(\widetilde{Y}) = \langle U, E\rangle \bigsqcap f^{-1}(\bigsqcup_{y_{e'} \in \widetilde{Y}} \langle V_y, E'\rangle) = \langle U, E\rangle \bigsqcap \bigsqcup_{y_{e'} \in \widetilde{Y}} f^{-1}(\langle V_y, E'\rangle).$$

Therefore, $f(\langle U, E\rangle) = \bigsqcup_{y_{e'} \in \widetilde{Y}} f(\langle U, E\rangle \sqcap f^{-1}(\langle V_y, E'\rangle))$. Since $f(\langle U, E\rangle)$ is not soft open, so there exists $y^*_{e'} \in \widetilde{Y}$ for which $f(\langle U, E\rangle \sqcap f^{-1}(\langle V_{y^*}, E'\rangle))$ is not soft open. By Corollary 3.24, f is soft continuous, so the set $\langle G, E\rangle = \langle U, E\rangle \sqcap f^{-1}(\langle V_{y^*}, E'\rangle)$ is soft open. Since $f(\langle G, E\rangle)$ is not soft open, there is $x_e \Subset \langle G, E\rangle$ such that $f(x_e) \not\Subset \mathrm{Int}(f(\langle G, E\rangle))$.

$$\begin{aligned}&\text{Since } (X, \mathcal{T}, E) \text{ is soft regular, there exists a soft open set } \langle O, E\rangle \text{ over X such that}\\ &x_e \Subset \mathrm{Cl}(\langle O, E\rangle) \sqsubseteq \langle G, E\rangle. \text{ Since f is soft closed, then } f(\mathrm{Cl}(\langle G, E\rangle)) \text{ is soft closed.}\end{aligned} \tag{3.1}$$

Since $f(x_e) \not\Subset \mathrm{Int}(f(\langle G, E\rangle))$, then $f(x_e) \not\Subset \mathrm{Int}(f(\mathrm{Cl}(\langle O, E\rangle)))$ and so $f(x_e) \Subset \mathrm{Cl}(\widetilde{Y} \backslash f(\mathrm{Cl}(\langle O, E\rangle)))$. Set $\langle W, E'\rangle = \widetilde{Y} \backslash f(\mathrm{Cl}(\langle O, E\rangle))$. Then $\langle W, E'\rangle \bigsqcup \{f(x_e)\}$ is soft semiopen over Y. By soft irresoluteness,

$$f^{-1}(\langle W, E'\rangle \bigsqcup \{f(x_e)\})$$

is soft semiopen over X. By Lemma 3.4,

$$\langle Z, E\rangle = f^{-1}(\langle W, E'\rangle \bigsqcup \{f(x_e)\}) \bigsqcap \mathrm{Int}(\mathrm{Cl}(\langle O, E\rangle))$$

is soft semiopen. Since f is soft anti-irresolute, then

$$\begin{aligned} f(\langle Z, E\rangle) &= f(f^{-1}(\langle W, E'\rangle \bigsqcup \{f(x_e)\}) \bigsqcap \mathrm{Int}(\mathrm{Cl}(\langle O, E\rangle))) \\ &= \langle W, E'\rangle \bigsqcup \{f(x_e)\} \bigsqcap f(\mathrm{Int}(\mathrm{Cl}(\langle O, E\rangle))) \\ &= \widetilde{Y} \backslash f(\mathrm{Cl}(\langle O, E\rangle \bigsqcup \{f(x_e)\}) \bigsqcap f(\mathrm{Int}(\mathrm{Cl}(\langle O, E\rangle))) = \{f(x_e)\} \end{aligned}$$

is a soft semiopen set over Y. By Lemma 3.5, $\{f(x_e)\}$ is soft open, which is a contradiction to the assumption that $(Y, \mathcal{S}, E')$ is soft perfect. Hence f is soft open. □

The following example shows that the soft closedness of f in the above theorem is essential.

Example 3.33. Consider the soft topological space $(X, \mathcal{T}, E)$ given in Example 3.11. Let $f : (X, \mathcal{T}, E) \to (X, \mathcal{T}, E)$ be the function defined by $f(x_e) = \sin(x_e)$. We claim that f is both soft irresolute and soft anti-irresolute but neither soft open nor soft closed. We now show that f is not open, take the soft open set $\{(e, (0, \pi))\}$, so $f(\{(e, (0, \pi))\}) = \{(e, (0, 1])\}$, which is not a soft open set. The set $\langle C, E\rangle = \bigsqcup_{n \in \mathbb{N}} \{(e, [2n\pi + \frac{1}{n+1}, (2n+1)\pi - \frac{1}{n+1}])\}$ is soft closed, but $f(\langle C, E\rangle) = \{(e, (0, 1])\}$ is not soft closed. Since all sets of the forms $\{(e, (a, b))\}, \{(e, (a, b])\}, \{(e, [a, b))\}$, and $\{(e, [a, b])\}$ are soft semiopen, then their images/preimages under f is either one of them or their unions and the class of soft semiopen sets is closed under arbitrary unions. This concludes that f is soft bi-irresolute.

From Corollary 3.24 and Theorem 3.32, we get following.

Theorem 3.34. *Let f a function from a soft regular space $(X, \mathcal{T}, E)$ into a soft perfect Hausdorff space $(Y, \mathcal{S}, E')$ be soft closed. Then f is soft bi-irresolute if and only if f is soft bi-continuous.*

Theorem 3.35. *Let $(X, \mathcal{T}, E)$ be a soft locally compact Hausdorff space and let $(Y, \mathcal{S}, E')$ be a stable soft perfect Hausdorff space. Then $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is a soft bi-irresolute function if and only if f is soft bi-continuous.*

Proof. The proof is mainly the same as Theorem 3.32 except the conclusion made in (3.1). We now prove (3.1) by using the given assumption. By soft locally compactness, for each $x_e \Subset \langle G, E\rangle$, there exists a soft compact neighborhood $\langle K, E\rangle$ over X such that

$$x_e \Subset \langle K, E\rangle \sqsubseteq \langle G, E\rangle.$$

Since f is soft continuous, by Lemma 3.27, $f(\langle K, E\rangle)$ is soft compact over Y. Since $(Y, \mathcal{S}, E')$ is soft Hausdorff, by Lemma 3.30, $f(\langle K, E\rangle)$ is soft closed. For the rest of the proof, $\langle K, E\rangle$ will play the role of $Cl(\langle G, E\rangle)$. Thus, the result is proved. □

Remark 3.36. At this place, we shall remark that some results in this paper may not be true (at least Theorem 3.35, which is the main result) if we use the concept of soft element. For more discussion on this direction, we refer the reader to [4, 23].

4. Conclusion and future work

A soft open function transforms soft open sets to soft open sets, while a soft continuous function does the converse. Different classes of soft open and soft continuous functions have been defined with respect to different classes of soft open sets. We have studied the properties of several classes of soft continuous and soft open functions. We focused on finding the relationships between soft continuous (soft open) and soft irresolute (soft anti-irresolute) functions. We demonstrated that soft continuous functions are independent of soft irresolute functions and soft open functions are independent of soft anti-irresolute functions. On the other hand, each soft bi-continuous function is soft bi-irresolute. The converse is not possible in general. We found some conditions under which soft bi-irresoluteness and soft bi-continuity become equivalent. At this point, we shall highlight that our results are based on the concept of soft point. The Remark 3.36 confirms that all the results obtained in this work may not be true when we replace a soft point by a soft element.

As a piece of future work, we can examine our findings with respect to soft elements or some other theories of soft points. A soft irresolute function sends soft semiopen sets to soft semiopen sets. One can replace soft semiopen sets with some other type of soft open sets and study the direction discussed in this paper.

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