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Soft functions via soft semi ω-open sets

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Abstract

In this paper, we introduce the concepts of soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute functions by using soft semi ω-open sets. We characterize them and discuss their main properties with the help of examples. We investigate under what conditions the restriction of soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute functions are respectively soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute. Also, we investigate under what conditions the composition of two soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute functions are respectively soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute. In addition to these, we examine the connection between the new classes of soft functions and their corresponding general topological concepts.

Keywords: Soft semi continuous functions, soft ω-continuous functions, semi ω-continuous, ω-irresolute, semi ω-irresolute functions, soft induced topological spaces.

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1. Introduction and preliminaries

The bulk of real-world problems in engineering, health, economics, the environment, and other fields include uncertainty. Molodtsov [23] proposed soft set theory as a mathematical approach for minimizing uncertainty in 1999. This is free of the shortcomings of previous theories such as fuzzy set theory [29], rough set theory [27], and so on. The structure of parameter sets, especially those linked to soft sets, offers a consistent framework for modeling uncertain data. This results in the fast expansion of soft set theory in a short period of time, as well as a large spectrum of soft set real-world applications.

The part of the topology that deals with the fundamental definitions of set theory and topological structures are known as general topology, and it is one of the major fields of mathematics. The majority of other fields of topology, such as algebraic, geometrical, and differential topology, are built upon it. Shabir and Naz [28] created a new branch of topology known as "soft topology," which is a combination of soft set theory and topology inspired by the usual postulates of ordinary topological space. It focuses on the overall development of the soft-set system. The study in [28] was particularly important in the development of the field of soft topology ([7–12, 14–16, 24, 25]).

Generalizations of soft open sets play an effective role in soft topology through their use to improve on some known results or to open the door to redefine and investigate some of the soft topological concepts

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such as soft compactness, soft correlation, soft class axioms, soft assignments, etc. Hdeib [19] defined and investigated ω-open sets as a generalization of open sets. Al Ghour and Hamed [6] have extended ω-open sets to include STSs. Then via soft ω-open sets several research papers have appeared. In particular, soft semi ω-open sets were introduced in [2].

In this paper, we introduce the concepts of soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute functions by using soft semi ω-open sets. We characterize them and discuss their main properties with the help of examples. We investigate under what conditions the restriction of soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute functions are respectively soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute. Also, we investigate under what conditions the composition of two soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute functions are respectively soft semi ω-continuous, soft ω-irresolute, and soft semi ω-irresolute. In addition to these, we examine the connection between the new classes of soft functions and their corresponding general topological concepts. In the next work, we hope to find an application for our new soft topological notions in a decision-making problem.

In this paper, we follow the notions and terminologies presented in [2, 5, 6]. Soft topological space and topological space will be referred to as STS and TS, respectively, throughout this paper.

Let (Z, γ, D) be a STS, (Z, μ) be a TS, $H \in SS(Z, D)$, and $U \subseteq Z$. Throughout this paper, $Cl_\gamma(H)$, $Int_\gamma(H)$, $Cl_\mu(U)$, and $Int_\mu(U)$ will denote the soft closure of H in (Z, γ, D), the soft interior of H in (Z, γ, D), the closure of U in (Z, μ), and the interior of U in (Z, μ), respectively, γ^c and μ^c will denote the family of all soft closed sets in (Z, γ, D) and the family of all closed sets in (Z, μ), respectively.

The following definitions and results will be used in the sequel.

Definition 1.1 ([19])**.** Let (Z, μ) be a TS, $D \subseteq Z$, and $z \in Z$. Then z is a condensation point of D if for each $U \in \mu$ with $z \in U$, the set $U \cap D$ is uncountable. D is called an ω-closed set in (Z, μ) if it contains all its condensation points. D is called an ω-open set in (Z, μ) if $Z - D$ is an ω-closed set in (Z, μ). The family of all ω-open sets in (Z, μ) will be denoted by μ_ω.

Definition 1.2. A function $p : (Z, \mu) \longrightarrow (W, \lambda)$ between the TSs (Z, μ) and (W, λ) is said to be

(a) ω-continuous if $p^{-1}(U) \in \mu_\omega$ for every $U \in \lambda$, [20];
(b) semi continuous if $p^{-1}(U) \in SO\,(Z, \mu)$ for every $U \in \lambda$, [21];
(c) irresolute if $p^{-1}(U) \in SO\,(Z, \mu)$ for every $U \in SO\,(W, \lambda)$, [17];
(d) semi ω-continuous if $p^{-1}(U) \in S\omega O\,(Z, \mu)$ for every $U \in \lambda$, [13];
(e) ω-irresolute if $p^{-1}(U) \in \mu_\omega$ for every $U \in \lambda_\omega$, [13];
(f) semi ω-irresolute if $p^{-1}(U) \in S\omega O\,(Z, \mu)$ for every $U \in S\omega O\,(W, \lambda)$, [13].

Definition 1.3. A soft function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is said to be

(a) soft ω-continuous if $f_{pu}^{-1}(H) \in \gamma_\omega$ for every $H \in \delta$, [4];
(b) soft semi continuous if $f_{pu}^{-1}(H) \in SO\,(Z, \gamma, D)$ for every $H \in \delta$, [22];
(c) soft irresolute if $f_{pu}^{-1}(H) \in SO\,(Z, \gamma, D)$ for every $SO\,(W, \delta, E)$, [22];
(d) soft precontinuous $f_{pu}^{-1}(H) \in PO\,(Z, \gamma, D)$ for every $H \in \delta$, [1].

Definition 1.4. A soft set $H \in SS(Z, D)$ defined by

(a) $H(d) = \begin{cases} X, & \text{if } d = a, \\ \emptyset, & \text{if } d \neq a, \end{cases}$ is denoted by a_X, [5];
(b) $H(d) = X$ for all $d \in D$ is denoted by C_X, [5];
(c) $H(d) = \begin{cases} \{z\}, & \text{if } d = a, \\ \emptyset, & \text{if } d \neq a, \end{cases}$ is denoted by a_z and is called a soft point, the set of all soft points in $SS(Z, D)$ is denoted by $SP\,(Z, D)$, [18].

In Definition 1.4 (b), $C_\emptyset$ is the null soft set.

Definition 1.5 ([18])**.** Let $H \in SS(Z, D)$ and $a_z \in SP(Z, D)$. Then a_z is said to belong to H (notation: $a_z \widetilde{\in} H$) if $z \in H(a)$.

Theorem 1.6 ([26])**.** *For any TS* (Z, μ), *the family* $\{H \in SS(Z, D) : H(d) \in \mu$ *for all* $d \in D\}$ *is a soft topology on* Z *relative to* D. *This soft topology will be denoted by* $\tau(\mu)$.

2. Soft semi ω-continuous functions

In this section, we introduce the class of soft semi ω-continuous functions, which lies strictly between the classes of soft ω-continuous functions and soft semi continuous functions. We study the relationship between soft semi ω-continuous functions between STSs and semi ω-continuous functions between TSs and provide many characterizations of soft semi ω-continuity.

Definition 2.1. A soft function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is said to be soft semi ω-continuous if $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$ for every $H \in \delta$.

Theorem 2.2. *For a soft function* $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$, *the following are equivalent.*

(a) f_{pu} *is soft semi ω-continuous.*
(b) *For every* $d_z \in SP(Z, D)$ *and for every* $H \in \delta$ *such that* $f_{pu}(d_z) \widetilde{\in} H$, *there exists* $K \in S\omega O(Z, \gamma, D)$ *such that* $d_z \widetilde{\in} K$ *and* $f_{pu}(K) \widetilde{\subseteq} H$.
(c) $f_{pu}^{-1}(T) \in S\omega C(Z, \gamma, D)$ *for every* $T \in \delta^c$.
(d) $f_{pu}(S\omega\text{-}Cl_\gamma(N)) \widetilde{\subseteq} Cl_\delta(f_{pu}(N))$ *for every* $N \in SS(Z, D)$.
(e) $f_{pu}^{-1}(Int_\delta(M)) \widetilde{\subseteq} S\omega\text{-}Int_\gamma\left(f_{pu}^{-1}(M)\right)$ *for every* $M \in SS(W, E)$.

Proof.

(a) $\Longrightarrow$ (b): Let $d_z \in SP(Z, D)$ and $H \in \delta$ such that $f_{pu}(d_z) \widetilde{\in} H$. Then $d_z \widetilde{\in} f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Put $K = f_{pu}^{-1}(H)$. Then $d_z \widetilde{\in} K \in S\omega O(Z, \gamma, D)$ and $f_{pu}(K) = f_{pu}\left(f_{pu}^{-1}(H)\right) \widetilde{\subseteq} H$.

(b) $\Longrightarrow$ (c): Let $T \in \delta^c$. We will show that $1_D - f_{pu}^{-1}(T) \in S\omega O(Z, \gamma, D)$. For every $d_z \widetilde{\in} 1_D - f_{pu}^{-1}(T) = f_{pu}^{-1}(1_E - T)$, we have $f_{pu}(d_z) \widetilde{\in} 1_E - T \in \delta$, and by (b), there exists $K_{d_z} \in S\omega O(Z, \gamma, D)$ such that $d_z \widetilde{\in} K_{d_z}$ and $f_{pu}(K_{d_z}) \widetilde{\subseteq} 1_E - T$; hence, $d_z \widetilde{\in} K_{d_z} \widetilde{\subseteq} f_{pu}^{-1}(f_{pu}(K)) \widetilde{\subseteq} f_{pu}^{-1}(1_E - T) = 1_D - f_{pu}^{-1}(T)$. Therefore, $1_D - f_{pu}^{-1}(T) = \widetilde{\bigcup}_{d_z \widetilde{\in} 1_D - f_{pu}^{-1}(T)} K_{d_z}$. Thus, by Theorem 11 of [2], $1_D - f_{pu}^{-1}(T) \in S\omega O(Z, \gamma, D)$.

(c) $\Longrightarrow$ (d): Let $N \in SS(Z, D)$, then $Cl_\delta(f_{pu}(N)) \in \delta^c$. So, by Theorem (c), $f_{pu}^{-1}(Cl_\delta(f_{pu}(N))) \in S\omega C(Z, \gamma, D)$. Since $N \widetilde{\subseteq} f_{pu}^{-1}(Cl_\delta(f_{pu}(N))) \in S\omega C(Z, \gamma, D)$, then $S\omega\text{-}Cl_\gamma(N) \widetilde{\subseteq} f_{pu}^{-1}(Cl_\delta(f_{pu}(N)))$ and so $f_{pu}(S\omega\text{-}Cl_\gamma(N)) \widetilde{\subseteq} Cl_\delta(f_{pu}(N))$.

(d) $\Longrightarrow$ (a): Let $H \in \delta$. Then $1_E - H \in \delta^c$. So by (d), $f_{pu}(S\omega\text{-}Cl_\gamma\left(f_{pu}^{-1}(1_E - H)\right)) \widetilde{\subseteq} Cl_\delta\left(f_{pu}(f_{pu}^{-1}(1_E - H))\right) \widetilde{\subseteq} Cl_\delta(1_E - H) = 1_E - H$. And thus, $S\omega\text{-}Cl_\gamma\left(f_{pu}^{-1}(1_E - H)\right) \widetilde{\subseteq} f_{pu}^{-1}\left(f_{pu}(S\omega\text{-}Cl_\gamma\left(f_{pu}^{-1}(1_E - H)\right))\right) \widetilde{\subseteq} f_{pu}^{-1}(1_E - H)$. Therefore, $S\omega\text{-}Cl_\gamma\left(f_{pu}^{-1}(1_E - H)\right) = f_{pu}^{-1}(1_E - H)$, and hence $f_{pu}^{-1}(1_E - H) = 1_D - f_{pu}^{-1}(H) \in S\omega C(Z, \gamma, D)$. This shows that $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. It follows that f_{pu} is soft semi ω-continuous.

(a) $\Longrightarrow$ (e): Let $M \in SS(W, E)$, then $Int_\delta(M) \in \delta$ and by (a), $f_{pu}^{-1}(Int_\delta(M)) \in S\omega O(Z, \gamma, D)$. Since $f_{pu}^{-1}(Int_\delta(M)) \widetilde{\subseteq} f_{pu}^{-1}(M)$, then $f_{pu}^{-1}(Int_\delta(M)) \widetilde{\subseteq} S\omega\text{-}Int_\gamma\left(f_{pu}^{-1}(M)\right)$.

(e) $\Longrightarrow$ (a): Let $H \in \delta$. Then by (e), $f_{pu}^{-1}(H) = f_{pu}^{-1}(Int_\delta(H)) \widetilde{\subseteq} S\omega\text{-}Int_\gamma\left(f_{pu}^{-1}(H)\right)$. And so $S\omega\text{-}Int_\gamma\left(f_{pu}^{-1}(H)\right) = f_{pu}^{-1}(H)$. Thus, $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Hence, f_{pu} is soft semi ω-continuous. □

Theorem 2.3. *If* $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ *is a soft function such that* (Z, γ, D) *is soft locally countable, then* f_{pu} *is soft semi ω-continuous.*

Proof. Let $H \in \delta$. Since (Z, γ, D) is soft locally countable, then by Corollary 5 of [6], $S\omega O(Z, \gamma, D) = SS(Z, D)$. Hence, $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Therefore, f_{pu} is soft semi ω-continuous. □

Corollary 2.4. *If $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is a soft function such that Z is a countable set, then f_{pu} is soft semi ω-continuous.*

Theorem 2.5. *Every soft ω-continuous function is soft semi ω-continuous.*

Proof. Let $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ be soft ω-continuous and let $H \in \delta$. Since f_{pu} is soft ω-continuous, then $f_{pu}^{-1}(H) \in \gamma_\omega$. By Theorem 5 of [2], $\gamma_\omega \subseteq S\omega O(Z, \gamma, D)$. So, $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Therefore, f_{pu} is soft semi ω-continuous. □

The following example shows that the converse of Theorem 2.5 need not to be true in general.

Example 2.6. Let $Z = \mathbb{R}$, $W = \{a, b\}$, $D = \mathbb{Z}$, $\gamma = \{C_V : V \subseteq \mathbb{R} \text{ and } 1 \notin V\} \cup \{C_V : V \subseteq \mathbb{R}, 1 \in V \text{ and } \mathbb{R} - V$ is finite$\}$, and $\delta = SS(W, D)$. Define $p : Z \longrightarrow W$ and $u : D \longrightarrow D$ as $p(z) = \begin{cases} a, & \text{if } z \in \mathbb{N}, \\ b, & \text{if } z \in \mathbb{R} - \mathbb{N}, \end{cases}$ and $u(d) = d$ for all $d \in D$. Consider the function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$. Since $C_{\{a\}} \notin \delta$ but $f_{pu}^{-1}(C_{\{a\}}) = C_{\mathbb{N}} \notin \gamma_\omega$, then f_{pu} is not soft ω-continuous. Since $C_{\mathbb{N}-\{1\}} \in \gamma_\omega$ and $C_{\mathbb{N}-\{1\}} \widetilde{\subseteq} C_{\mathbb{N}} \widetilde{\subseteq} Cl_\gamma(C_{\mathbb{N}-\{1\}})$, then $C_{\mathbb{N}} \in S\omega O(Z, \gamma, D)$ and so $f_{pu}^{-1}(C_{\{a\}}) = C_{\mathbb{N}} \in S\omega O(Z, \gamma, D)$. Also, since $f_{pu}^{-1}(C_{\{b\}}) = C_{\mathbb{R}-\mathbb{N}} \in \gamma_\omega$, then by Theorem 5 of [2], $f_{pu}^{-1}(C_{\{b\}}) \in S\omega O(Z, \gamma, D)$. Therefore, f_{pu} is soft semi ω-continuous.

Theorem 2.7. *Every soft semi continuous function is soft semi ω-continuous.*

Proof. Let $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ be soft semi continuous and let $H \in \delta$. Since f_{pu} is soft semi continuous, then $f_{pu}^{-1}(H) \in SO(Z, \gamma, D)$. Since by Theorem 6 of [2], $SO(Z, \gamma, D) \subseteq S\omega O(Z, \gamma, D)$, then, $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Therefore, f_{pu} is soft semi ω-continuous. □

Corollary 2.8. *Every soft ω_s-continuous function is soft semi ω-continuous.*

Proof. Follows form Theorem 2.7 and Theorem 6 of [2]. □

The following example shows that the converse of Theorem 2.7 need not to be true in general.

Example 2.9. Let $Z = \{1, 2, 3\}$, $W = \{a, b\}$, $D = \mathbb{Z}$, $\gamma = \{C_V : V \in \{\emptyset, Z, \{1\}, \{2, 3\}\}\}$, and $\delta = SS(W, D)$. Define $p : Z \longrightarrow W$ and $u : D \longrightarrow D$ by $p(1) = p(2) = a$, $p(3) = b$, and $u(d) = d$ for all $d \in D$. Consider the function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$. Since Z is a countable set, then by Corollary 2.4, f_{pu} soft semi ω-continuous. On the other hand, since $C_{\{a\}} \in \delta$ but $f_{pu}^{-1}\left(C_{\{a\}}\right) = C_{\{1,2\}} \notin SO(Z, \gamma, D)$, then f_{pu} not soft semi continuous.

Theorem 2.10. *For any two STSs (Z, γ, D) and (W, δ, E), and any soft function $f_{pu} : SS(Z, D) \longrightarrow SS(W, E)$, the following are equivalent.*

(a) $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ *is soft semi ω-continuous.*
(b) $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ *is soft semi continuous.*
(c) $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ *is soft ω_s-continuous.*

Proof. Follows from the definitions and Theorem 9 of [2]. □

Theorem 2.11. *Let (Z, γ, D) and (W, δ, E) be two STSs and let $f_{pu} : SS(Z, D) \longrightarrow SS(W, E)$ be a soft function. If $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous, then $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous.*

Proof. Suppose that $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous. Let $H \in \delta$. Since $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous, then $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma_\omega, D)$. Since by Theorem 10 of [2], $S\omega O(Z, \gamma_\omega, D) \subseteq S\omega O(Z, \gamma, D)$, then, $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Therefore, $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous. □

The following example shows that the converse of Theorem 2.11 need not to be true in general.

Example 2.12. Let (Z, γ, D), (W, δ, D), $p : Z \longrightarrow W$, and $u : D \longrightarrow D$ be as in Example 2.6. We proved in Example 2.9 that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$ is semi ω-continuous. On the other hand, since $C_{\{a\}} \notin \delta$ but $f_{pu}^{-1}(C_{\{a\}}) = C_{\mathbb{N}} \notin S\omega O(Z, \gamma_\omega, D)$, then $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, D)$ is not semi ω-continuous.

The following two examples will show that soft semi ω-continuity and soft precontinuity are independent notions.

Example 2.13. Let $Z = \{1, 2\}$, $W = \{a, b\}$, $D = \mathbb{Z}$, $\gamma = \{C_V : V \in \{\emptyset, Z, \{1\}\}\}$, and $\delta = SS(W, D)$. Define $p : Z \longrightarrow W$ and $u : D \longrightarrow D$ by $p(1) = a$, $p(2) = b$, and $u(d) = d$ for all $d \in D$. Consider the function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$. By Corollary 2.4, f_{pu} soft semi ω-continuous. On the other hand, since $C_{\{b\}} \in \delta$ but $f_{pu}^{-1}\left(C_{\{b\}}\right) = C_{\{b\}} \notin PO\left(Z, \gamma, D\right)$, then f_{pu} not soft precontinuous.

Example 2.14. Let $Z = \mathbb{R}$, $W = \{a, b\}$, $D = \mathbb{Z}$, μ be the usual topology on $\mathbb{R}$, $\gamma = \{C_V : V \in \mu\}$, and $\delta = \left\{0_D, 1_D, C_{\{a\}}\right\}$. Define $p : Z \longrightarrow W$ and $u : D \longrightarrow D$ as $p(z) = \begin{cases} a, & \text{if } z \in \mathbb{Q}, \\ b, & \text{if } z \in \mathbb{R} - \mathbb{Q}, \end{cases}$ and $u(d) = d$ for all $d \in D$. Consider the soft function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$. Since $f_{pu}^{-1}\left(C_{\{a\}}\right) = C_{\mathbb{Q}} \in PO\left(Z, \gamma, D\right) - S\omega O\left(Z, \gamma, D\right)$, then f_{pu} is soft precontinuous but not soft semi ω-continuous.

Lemma 2.15. *Let (Z, μ) be a TS and let D be a set of parameters. Let $H \in SS(Z, D)$. Then $H \in SO(Z, \tau\left(\mu\right), D)$ if and only if $H\left(d\right) \in SO(Z, \mu)$ for all $d \in D$.*

Proof. For each $d \in D$, put $\mu_d = \mu$. Then $\tau\left(\mu\right) = \underset{d \in D}{\oplus} \mu_d$. Then by Lemma 4.9 of [3], we get the result. □

Theorem 2.16. *Let $p : (Z, \mu) \longrightarrow (W, \lambda)$ be a function between two TSs and let $u : D \longrightarrow E$ be a function between two sets of parameters. Then $p : (Z, \mu) \longrightarrow (W, \lambda)$ is semi continuous if and only if $f_{pu} : (Z, \tau\left(\mu\right), D) \longrightarrow (W, \tau\left(\lambda\right), E)$ is soft semi continuous.*

Proof.

Necessity. Let $p : (Z, \mu) \longrightarrow (W, \lambda)$ be semi continuous. Let $H \in \tau\left(\lambda\right)$. Then for each $d \in D$, $H(u\left(d\right)) \in \lambda$, and thus $p^{-1}(H(u\left(d\right))) \in SO(Z, \mu)$. Therefore, $\left(f_{pu}^{-1}(H)\right)\left(d\right) = p^{-1}(H(u\left(d\right))) \in SO(Z, \mu)$ for each $d \in D$. Hence, by Lemma 2.15, $f_{pu}^{-1}(H) \in SO(Z, \tau\left(\mu\right), D)$. It follows that $f_{pu} : (Z, \tau\left(\mu\right), D) \longrightarrow (W, \tau\left(\lambda\right), E)$ is soft semi continuous.

Sufficiency. Let $f_{pu} : (Z, \tau\left(\mu\right), D) \longrightarrow (W, \tau\left(\lambda\right), E)$ be soft semi continuous. Let $V \in \lambda$. Then $C_V \in \tau\left(\lambda\right)$, and so $f_{pu}^{-1}(C_V) \in SO(Z, \tau\left(\mu\right), D)$. Choose $d \in D$. Then, by Lemma 2.15, $\left(f_{pu}^{-1}(C_V)\right)\left(d\right) = p^{-1}(V) \in SO(Z, \mu)$. It follows that $p : (Z, \mu) \longrightarrow (W, \lambda)$ is semi continuous. □

Lemma 2.17. *Let (Z, μ) be a TS and let D be a set of parameters. Let $H \in SS(Z, D)$. Then $H \in S\omega O(Z, \tau\left(\mu\right), D)$ if and only if $H\left(d\right) \in S\omega O(Z, \mu)$ for all $d \in D$.*

Proof. For each $d \in D$, put $\mu_d = \mu$. Then $\tau\left(\mu\right) = \underset{d \in D}{\oplus} \mu_d$. Then by Lemma 4.7 of [3], we get the result. □

Theorem 2.18. *Let $p : (Z, \mu) \longrightarrow (W, \lambda)$ be a function between two TSs and let $u : D \longrightarrow E$ be a function between two sets of parameters. Then $p : (Z, \mu) \longrightarrow (W, \lambda)$ is semi ω-continuous if and only if $f_{pu} : (Z, \tau\left(\mu\right), D) \longrightarrow (W, \tau\left(\lambda\right), E)$ is soft semi ω-continuous.*

Proof.

Necessity. Let $p : (Z, \mu) \longrightarrow (W, \lambda)$ be semi ω-continuous. Let $H \in \tau\left(\lambda\right)$. Then for each $d \in D$, $H(u\left(d\right)) \in \lambda$, and thus $p^{-1}(H(u\left(d\right))) \in S\omega O(Z, \mu)$. Therefore, $\left(f_{pu}^{-1}(H)\right)\left(d\right) = p^{-1}(H(u\left(d\right))) \in S\omega O(Z, \mu)$ for each $d \in D$. Hence, by Lemma 2.17, $f_{pu}^{-1}(H) \in S\omega O(Z, \tau\left(\mu\right), D)$. It follows that $f_{pu} : (Z, \tau\left(\mu\right), D) \longrightarrow (W, \tau\left(\lambda\right), E)$ is soft semi ω-continuous.

Sufficiency. Let $f_{pu} : (Z, \tau\left(\mu\right), D) \longrightarrow (W, \tau\left(\lambda\right), E)$ be soft semi ω-continuous. Let $V \in \lambda$. Then $C_V \in \tau\left(\lambda\right)$, and so $f_{pu}^{-1}(C_V) \in S\omega O(Z, \tau\left(\mu\right), D)$. Choose $d \in D$. Then, by Lemma 2.17, $\left(f_{pu}^{-1}(C_V)\right)\left(d\right) = p^{-1}(V) \in S\omega O(Z, \mu)$. It follows that $p : (Z, \mu) \longrightarrow (W, \lambda)$ is semi ω-continuous. □

Theorem 2.19. *If $f_{p_1u_1}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ is soft semi ω-continuous and $f_{p_2u_2}:(W,\delta,E)\longrightarrow(S,\beta,A)$ is soft continuous, then $f_{(p_2\circ p_1)(u_2\circ u_1)}:(Z,\gamma,D)\longrightarrow(S,\beta,A)$ is soft semi ω-continuous.*

Proof. Let $H\in\beta$. Since $f_{p_2u_2}:(W,\delta,E)\longrightarrow(S,\beta,A)$ is soft continuous, then $f^{-1}_{p_2u_2}(H)\in\delta$. Since $f_{p_1u_1}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ is soft semi ω-continuous, $f^{-1}_{p_1u_1}\left(f^{-1}_{p_2u_2}(H)\right)=f^{-1}_{(p_2\circ p_1)(u_2\circ u_1)}(K)\in S\omega O(Z,\gamma,D)$. It follows that $f_{(p_2\circ p_1)(u_2\circ u_1)}:(Z,\gamma,D)\longrightarrow(S,\beta,A)$ is soft semi ω-continuous. □

Corollary 2.20. *Let (Z,γ,D), (W,δ,E), and (S,β,A) be three STSs and let $p:Z\longrightarrow W\times S$ and $u:D\longrightarrow E\times A$ be functions such that $f_{pu}:(Z,\gamma,D)\longrightarrow(W\times S,\delta\times\beta,E\times A)$ is soft semi ω-continuous, then $f_{\pi_w\pi_e}\circ f_{pu}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ and $f_{\pi_s\pi_a}\circ f_{pu}:(Z,\gamma,D)\longrightarrow(S,\beta,A)$ are soft semi ω-continuous functions.*

Proof. Consider the soft functions $f_{\pi_w\pi_e}:(W\times S,\delta\times\beta,E\times A)\longrightarrow(W,\delta,E)$ and $f_{\pi_s\pi_a}:(W\times S,\delta\times\beta,E\times A)\longrightarrow(S,\beta,A)$. To see that $f_{\pi_w\pi_e}$ is soft continuous, let $H\in\delta$. Then $f^{-1}_{\pi_w\pi_e}(H)=H\times 1_A\in\delta\times\beta$. Similarly, we can see that $f_{\pi_s\pi_a}$ is soft continuous. Therefore, by Theorem 2.19, we get the result. □

Theorem 2.21. *Let $f_{pu}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ be a soft function and let $p_1:Z\longrightarrow Z\times W$ and $u_1:D\longrightarrow D\times E$ be functions defined by $p_1(z)=(z,p(z))$ and $u_1(d)=(d,u(d))$. Then $f_{p_1u_1}:(Z,\gamma,D)\longrightarrow(Z\times W,\gamma\times\delta,D\times E)$ is soft semi ω-continuous if and only if $f_{pu}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ is soft semi ω-continuous.*

Proof.

Necessity. Suppose that $f_{p_1u_1}:(Z,\gamma,D)\longrightarrow(Z\times W,\gamma\times\delta,D\times E)$ is soft semi ω-continuous. Since $f_{pu}=f_{\pi_w\pi_e}\circ f_{p_1u_1}$, then by Corollary 2.20, $f_{pu}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ is soft semi ω-continuous.

Sufficieny. Suppose that $f_{pu}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ is soft semi ω-continuous. Let $d_z\in SP(Z,D)$ and let $G\in\gamma\times\delta$ such that $f_{p_1u_1}(d_z)=(u_1(d))_{p_1(z)}=(d,u(d))_{(z,p(z))}\widetilde{\in}G$. Choose $H\in\gamma$ and $K\in\delta$ such that $(d,u(d))_{(z,p(z))}\widetilde{\in}H\times K\widetilde{\subseteq}G$. Since $f_{pu}:(Z,\gamma,D)\longrightarrow(W,\delta,E)$ is soft semi ω-continuous and $d_z\widetilde{\in}H\in\gamma$, then there exists $M\in S\omega O(Z,\gamma,D)$ such that $d_z\widetilde{\in}M$ and $f_{pu}(M)\widetilde{\subseteq}H$. Put $N=H\widetilde{\cap}M$. Then by Theorem 12 of [2], $N\in S\omega O(Z,\gamma,D)$. Moreover, $d_z\widetilde{\in}N$ and $f_{pu}(N)\widetilde{\subseteq}H$. Therefore, we have $f_{p_1u_1}(N)\widetilde{\subseteq}H\times K\widetilde{\subseteq}G$. It follows that $f_{p_1u_1}:(Z,\gamma,D)\longrightarrow(Z\times W,\gamma\times\delta,D\times E)$ is soft semi ω-continuous. □

The following example will show that the composition of two soft semi ω-continuous functions need not to be a soft semi ω-continuous function.

Example 2.22. Let $Z=\mathbb{R}$, $W=\{a,b\}$, μ be the usual topology on $\mathbb{R}$, $\lambda=\{\emptyset,W,\{b\}\}$, and $D=[0,1]$. Define $p_1:Z\longrightarrow W$, $p_2:W\longrightarrow W$, and $u_1,u_2:D\longrightarrow D$ as follows

$$p_1(z)=\begin{cases} b, & \text{if } z\in\mathbb{R}-\mathbb{Q},\\ a, & \text{if } z\in\mathbb{Q},\end{cases}\quad,\quad p_2(a)=b,\ p_2(b)=a,\quad\text{and}\quad u_1(d)=u_2(d)=d\text{ for all } d\in D.$$

Then $p_1:(Z,\mu)\longrightarrow(W,\lambda)$ and $p_2:(W,\lambda)\longrightarrow(W,\lambda)$ are semi ω-continuous. On the other hand, since $\{b\}\in\lambda$ while $(p_2\circ p_1)^{-1}(\{b\})=\mathbb{Q}\notin S\omega O(Z,\mu)$, then $p_2\circ p_1:(Z,\mu)\longrightarrow(W,\lambda)$ is not semi ω-continuous. Therefore, by Theorem 2.18, $f_{p_1u_1}:(Z,\tau(\mu),D)\longrightarrow(W,\tau(\lambda),D)$ and $f_{p_2u_2}:(W,\tau(\lambda),D)\longrightarrow(W,\tau(\lambda),D)$ are soft semi ω-continuous functions while $f_{p_2u_2}\circ f_{p_1u_1}=f_{(p_2\circ p_1)(u_2\circ u_1)}:(W,\tau(\lambda),D)\longrightarrow(W,\tau(\lambda),D)$ is not soft semi ω-continuous.

In Theorem 2.19, the condition that "$f_{p_2u_2}$ is soft continuous" cannot be replaced by "$f_{p_2u_2}$ is soft ω-continuous":

Example 2.23. Let $Z=\mathbb{R}$, $W=\{0,1,2\}$, $S=\{a,b\}$, μ be the usual topology on $\mathbb{R}$, $\lambda=\{\emptyset,W,\{0\},\{0,1\}\}$, $\nu=\{\emptyset,S,\{a\}\}$, and $D=[0,1]$. Define $p_1:Z\longrightarrow W$, $p_2:W\longrightarrow S$, and $u_1,u_2:D\longrightarrow D$ as follows

$$p_1(z)=\begin{cases} 1, & \text{if } z\in\mathbb{R}-\mathbb{Q},\\ 2, & \text{if } z\in\mathbb{Q},\end{cases}\qquad p_2(0)=p_2(2)=a,\ p_2(1)=b,\quad\text{and}\quad u_1(d)=u_2(d)=d\text{ for all } d\in D.$$

Then $p_1 : (Z, \mu) \longrightarrow (W, \lambda)$ is ω-continuous and $p_2 : (W, \lambda) \longrightarrow (S, \nu)$ is semi ω-continuous. On the other hand, since $\{a\} \in \nu$ while $(p_2 \circ p_1)^{-1}(\{a\}) = \mathbb{Q} \notin S\omega O(Z, \mu)$, then $p_2 \circ p_1 : (Z, \mu) \longrightarrow (W, \lambda)$ is not semi ω-continuous. Therefore, by Corollary 2.6 of [4], $f_{p_2 u_2} : (W, \tau(\lambda), D) \longrightarrow (S, \tau(\nu), D)$ is soft ω-continuous and by Theorem 2.19, $f_{p_1 u_1} : (Z, \tau(\mu), D) \longrightarrow (W, \tau(\lambda), D)$ is soft semi ω-continuous while $f_{p_2 u_2} \circ f_{p_1 u_1} = f_{(p_2 \circ p_1)(u_2 \circ u_1)} : (W, \tau(\lambda), D) \longrightarrow (S, \tau(\nu), D)$ is not soft semi ω-continuous.

Theorem 2.24. *If $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous and $X \subseteq Z$ such that $C_X \in \gamma - \{0_D\}$, then the soft restriction $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous.*

Proof. Let $H \in \delta$. Since $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous, then $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Thus, by Theorem 12 of [2], $f_{(p_{|X})u}^{-1}(H) = f_{pu}^{-1}(H) \widetilde{\cap} C_X \in S\omega O(Z, \gamma, D)$. Hence, by Theorem 14 of [2], $f_{(p_{|X})u}^{-1}(H) \in S\omega O(X, \gamma_X, D)$. It follows that $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous. □

Corollary 2.25. *$f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ be a soft function. Let $\{C_{Z_\alpha} : j \in J\} \subseteq \gamma$ such that $1_D = \widetilde{\cup}\{C_{Z_j} : j \in J\}$. If for each $j \in J$, $f_{\left(p_{|Z_j}\right)u} : \left(Z_j, \gamma_{Z_j}, D\right) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous, then $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous.*

Proof. Let $d_z \in SP(Z, D)$. We show that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous at d_z. Since $1_D = \widetilde{\cup}\{C_{Z_j} : j \in J\}$, then there exists $j_\circ \in J$ such that $d_z \widetilde{\in} C_{Z_{j_\circ}}$. Therefore, by Theorem 2.24, it follows that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous. □

Note that Theorem 2.24 is not true if we take $C_X \in \gamma_\omega - \{0_D\}$ as it is shown in the next example.

Example 2.26. Consider $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$ be as in Example 2.6. Take $X = (\mathbb{R} - \mathbb{N}) \cup \{1\}$. Then $C_X \in \gamma_\omega - \{0_D\}$ and $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is not soft semi ω-continuous since $C_{\{a\}} \in \delta$ but $f_{(p_{|X})u}^{-1}\left(C_{\{a\}}\right) = C_{\{1\}} \notin S\omega O(X, \gamma_X, D)$.

3. Soft ω-irresolute functions

In this section, we introduce and investigate soft ω-irresoluteness which is a strong form of soft ω-continuity.

Definition 3.1. A soft function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is said to be soft ω-irresolute if $f_{pu}^{-1}(H) \in \gamma_\omega$ for every $H \in \delta_\omega$.

Theorem 3.2. *For a soft function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$, the following conditions are equivalent.*

(a) *f_{pu} is soft ω-irresolute.*
(b) *For a soft base $\mathcal{S}$ of (W, δ, E), $f_{pu}^{-1}(S - F) \in \gamma_\omega$ for all $S \in \mathcal{S}$ and $F \in CSS(W, E)$.*
(c) *For each $M \in (\delta_\omega)^c$, $f_{pu}^{-1}(M) \in (\gamma_\omega)^c$.*
(d) *For each $N \in SS(Z, D)$, $f_{pu}(Cl_{\gamma_\omega}(N)) \widetilde{\subseteq} Cl_{\delta_\omega}(f_{pu}(N))$.*
(e) *For each $T \in SS(W, E)$, $Cl_{\gamma_\omega}\left(f_{pu}^{-1}(T)\right) \widetilde{\subseteq} f_{pu}^{-1}(Cl_{\delta_\omega}(T))$.*

Proof.

(a) $\Longrightarrow$ (b): Let $S \in \mathcal{S}$ and $F \in CSS(W, E)$. Then $S - F \in \delta_\omega$ and by (a), $f_{pu}^{-1}(S - F) \in \gamma_\omega$.

(b) $\Longrightarrow$ (c): Let $M \in (\delta_\omega)^c$. We will show that $1_D - f_{pu}^{-1}(M) \in \gamma_\omega$. Let $d_z \widetilde{\in} 1_D - f_{pu}^{-1}(M) = f_{pu}^{-1}(1_E - M)$. Then $f_{pu}(d_z) \widetilde{\in} 1_E - M \in \delta_\omega$. Choose $K \in \delta$ and $F \in CSS(W, E)$ such that $f_{pu}(d_z) \widetilde{\in} K - F \widetilde{\subseteq} 1_E - M$. Since $\mathcal{S}$ is a soft base of (W, δ, E) and $f_{pu}(d_z) \widetilde{\in} K \in \delta$, then there exists $S \in \mathcal{S}$ such that $f_{pu}(d_z) \widetilde{\in} S \widetilde{\subseteq} K$. Thus, we have $f_{pu}(d_z) \widetilde{\in} S - F \widetilde{\subseteq} K - F \widetilde{\subseteq} 1_E - M$, and hence $d_z \widetilde{\in} f_{pu}^{-1}(S - F) \widetilde{\subseteq} f_{pu}^{-1}(1_E - M) = 1_D - f_{pu}^{-1}(M)$. By (b), $f_{pu}^{-1}(S - F) \in \gamma_\omega$. It follows that $1_D - f_{pu}^{-1}(M) \in \gamma_\omega$.

(c) $\Longrightarrow$ (d): Let $N \in SS(Z,D)$. Then $Cl_{\delta_\omega}(f_{pu}(N)) \in (\delta_\omega)^c$. Thus by (c), $f_{pu}^{-1}(Cl_{\delta_\omega}(f_{pu}(N))) \in (\gamma_\omega)^c$. Since $N \widetilde{\subseteq} f_{pu}^{-1}(f_{pu}(N)) \widetilde{\subseteq} f_{pu}^{-1}(Cl_{\delta_\omega}(f_{pu}(N)))$, then $Cl_{\gamma_\omega}(N) \widetilde{\subseteq} f_{pu}^{-1}(Cl_{\delta_\omega}(f_{pu}(N)))$ and thus, $f_{pu}(Cl_{\gamma_\omega}(N)) \widetilde{\subseteq} f_{pu}(f_{pu}^{-1}(Cl_{\delta_\omega}(f_{pu}(N)))) \widetilde{\subseteq} Cl_{\delta_\omega}(f_{pu}(N))$.

(d) $\Longrightarrow$ (e): Let $T \in SS(W,E)$. Then $f_{pu}^{-1}(T) \in SS(Z,D)$. Thus by (d),

$$f_{pu}(Cl_{\gamma_\omega}(f_{pu}^{-1}(T))) \widetilde{\subseteq} Cl_{\delta_\omega}(f_{pu}(f_{pu}^{-1}(T))) \widetilde{\subseteq} Cl_{\delta_\omega}(T).$$

Therefore, $Cl_{\gamma_\omega}(f_{pu}^{-1}(T)) \widetilde{\subseteq} f_{pu}^{-1}(f_{pu}(Cl_{\gamma_\omega}(f_{pu}^{-1}(T)))) \widetilde{\subseteq} f_{pu}^{-1}(Cl_{\delta_\omega}(T))$.

(e) $\Longrightarrow$ (a): Let $H \in \delta_\omega$. Then $1_E - H \in (\delta_\omega)^c$. By (e),

$$Cl_{\gamma_\omega}(1_D - f_{pu}^{-1}(H)) = Cl_{\gamma_\omega}(f_{pu}^{-1}(1_E - H)) \widetilde{\subseteq} f_{pu}^{-1}(Cl_{\delta_\omega}(1_E - H)) = f_{pu}^{-1}(1_E - H) = 1_D - f_{pu}^{-1}(H).$$

Therefore, $1_D - f_{pu}^{-1}(H) \in (\gamma_\omega)^c$ and thus, $f_{pu}^{-1}(H) \in \gamma_\omega$. It follows that f_{pu} is soft ω-irresolute. □

Theorem 3.3. *If $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ is soft ω-irresolute, then $p : (Z,\gamma_d) \longrightarrow (W,\delta_{u(d)})$ is ω-irresolute for all $d \in D$.*

Proof. Let $d \in D$. To see that $p : (Z,\gamma_d) \longrightarrow (W,\delta_{u(d)})$ is ω-irresolute, let $V \in (\delta_{u(d)})_\omega$ and $z \in p^{-1}(V)$. By Theorem 7 of [6], $(\delta_{u(d)})_\omega = (\delta_\omega)_{u(d)}$ and so $V \in (\delta_\omega)_{u(d)}$. Choose $H \in \delta_\omega$ such that $H(u(d)) = V$. Since $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ is soft ω-irresolute, then $f_{pu}^{-1}(H) \in \gamma_\omega$. Since by Theorem 7 of [6], $(\gamma_\omega)_d = (\gamma_d)_\omega$, then $z \in (f_{pu}^{-1}(H))(d) \in (\gamma_d)_\omega$. To show that $(f_{pu}^{-1}(H))(d) \subseteq p^{-1}(V)$, let $x \in (f_{pu}^{-1}(H))(d)$, then $u(d)_{p(x)} \widetilde{\in} H$ and so $p(x) \in H(u(d)) = V$. Hence, $x \in p^{-1}(V)$. It follows that $p^{-1}(V) \in (\gamma_d)_\omega$. This shows that is $p : (Z,\gamma_d) \longrightarrow (W,\delta_{u(d)})$ is ω-irresolute. □

Theorem 3.4. *Let $p : (Z,\mu) \longrightarrow (W,\lambda)$ be a function between two TSs and let $u : D \longrightarrow E$ be a function between two sets of parameters. Then $p : (Z,\mu) \longrightarrow (W,\lambda)$ is ω-irresolute if and only if $f_{pu} : (Z,\tau(\mu),D) \longrightarrow (W,\tau(\lambda),E)$ is soft ω-irresolute.*

Proof.

Necessity. Let $p : (Z,\mu) \longrightarrow (W,\lambda)$ be ω-irresolute. Let $H \in (\tau(\lambda))_\omega$. Since by Corollary 4 of [6], $(\tau(\lambda))_\omega = \tau(\lambda_\omega)$, then $H \in \tau(\lambda_\omega)$. So for each $d \in D$, $H(u(d)) \in \lambda_\omega$. Thus for every $d \in D$, we have $(f_{pu}^{-1}(H))(d) = p^{-1}(H(u(d))) \in \mu_\omega$. Hence, $f_{pu}^{-1}(H) \in \tau(\mu_\omega)$ and again by Corollary 4 of [6], $f_{pu}^{-1}(H) \in (\tau(\mu))_\omega$. It follows that $f_{pu} : (Z,\tau(\mu),D) \longrightarrow (W,\tau(\lambda),E)$ is soft ω-irresolute.

Sufficiency. Let $f_{pu} : (Z,\tau(\mu),D) \longrightarrow (W,\tau(\lambda),E)$ be soft ω-irresolute. Let $V \in \lambda_\omega$. Then $C_V \in \tau(\lambda_\omega) = (\tau(\lambda))_\omega$, and so $f_{pu}^{-1}(C_V) \in (\tau(\mu))_\omega = \tau(\mu_\omega)$. Choose $d \in D$. Then $(f_{pu}^{-1}(C_V))(d) = p^{-1}(V) \in \mu_\omega$. It follows that $p : (Z,\mu) \longrightarrow (W,\lambda)$ is ω-irresolute. □

Theorem 3.5. *Soft ω-irresolute functions are soft ω-continuous.*

Proof. Let $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ be soft ω-irresolute and let $H \in \delta$. Then by Theorem 2 of [6], $H \in \delta_\omega$. Since f_{pu} is soft ω-irresolute, then $f_{pu}^{-1}(H) \in \gamma_\omega$. Therefore, f_{pu} is soft ω-continuous. □

The soft function $f_{p_1u_1} : (Z,\tau(\mu),D) \longrightarrow (W,\tau(\lambda),D)$ in Example 2.22 is soft ω-continuous but not soft ω-irresolute. Thus, the converse of Theorem 3.5 is not true in general.

Theorem 3.6. *If $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ is a soft function such that (Z,γ,D) is soft locally countable, then f_{pu} is soft ω-irresolute.*

Proof. Let $H \in \delta_\omega$. Since (Z,γ,D) is soft locally countable, then by Corollary 5 of [6], $\delta_\omega = SS(Z,D)$. Hence, $f_{pu}^{-1}(H) \in \delta_\omega$. Therefore, f_{pu} is soft ω-irresolute. □

Corollary 3.7. *If $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ is a soft function such that Z is a countable set, then f_{pu} is soft ω-irresolute.*

Corollary 3.8. *If $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is a soft function such that (Z, γ, D) is soft locally countable, then f_{pu} is soft ω-continuous.*

Proof. Follows from Theorems 3.5 and 3.6. □

Corollary 3.9. *If $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is a soft function such that Z is a countable set, then f_{pu} is soft ω-continuous.*

By the following two examples we will show that soft continuity and soft ω-irresoluteness are independent.

Example 3.10. Let $Z = \mathbb{Q}$ and $D = \{a, b\}$. Let $\mu = \{\emptyset, Z, \mathbb{N}\}$ and λ be the cofinite topology on Z. Let $p : Z \longrightarrow Z$ and $u : D \longrightarrow D$ be the identities functions. Then by Corollary 3.7, $f_{pu} : (Z, \tau(\mu), D) \longrightarrow (Z, \tau(\lambda), D)$ is soft ω-irresolute. On the other hand, since $C_{\mathbb{Q}-\{1\}} \in \tau(\lambda)$ but $f_{pu}^{-1}\left(C_{\mathbb{Q}-\{1\}}\right) = C_{\mathbb{Q}-\{1\}} \notin \tau(\mu)$, then $f_{pu} : (Z, \tau(\mu), D) \longrightarrow (Z, \tau(\lambda), D)$ is not soft continuous.

Example 3.11. Let $Z = \mathbb{R}$ and $D = [0, 1]$. Let μ be the usual topology on Z and $\lambda = \{\emptyset, Z, \{\pi\}\}$. Define $p : Z \longrightarrow Z$ and $u : E \longrightarrow E$ as follows: $p(z) = \begin{cases} \pi, & \text{if } z \in \mathbb{R} - \mathbb{N}, \\ z, & \text{if } z \in \mathbb{N}, \end{cases}$ and $u(d) = d$ for all $d \in D$. Consider the function $p : (Z, \mu) \longrightarrow (Z, \lambda)$. Since $p^{-1}(\{\pi\}) = \mathbb{R} - \mathbb{N} \in \mu$, then $p : (Z, \mu) \longrightarrow (Z, \lambda)$ is continuous. On the other hand, since $\mathbb{R} - \{\pi\} \in \lambda_\omega$ while $p^{-1}(\mathbb{R} - \{\pi\}) = \mathbb{N} \notin \mu$, then $p : (Z, \mu) \longrightarrow (Z, \lambda)$ is not ω-irresolute. Thus, by Theorem 5.31 of [5] and Theorem 3.4, $f_{pu} : (Z, \tau(\mu), D) \longrightarrow (Z, \tau(\lambda), D)$ is soft continuous but not soft ω-irresolute.

Theorem 3.12. *If $f_{p_1u_1} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ and $f_{p_2u_2} : (W, \delta, E) \longrightarrow (S, \beta, A)$ are soft ω-irresolute functions, then $f_{(p_2 \circ p_1)(u_2 \circ u_1)} : (Z, \gamma, D) \longrightarrow (S, \beta, A)$ is soft ω-irresolute.*

Proof. Let $H \in \beta_\omega$. Since $f_{p_2u_2} : (W, \delta, E) \longrightarrow (S, \beta, A)$ is ω-irresolute, then $f_{p_2u_2}^{-1}(H) \in \delta_\omega$. Since $f_{p_1u_1} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft ω-irresolute, $f_{p_1u_1}^{-1}\left(f_{p_2u_2}^{-1}(H)\right) = f_{(p_2 \circ p_1)(u_2 \circ u_1)}^{-1}(K) \in \gamma_\omega$. It follows that $f_{(p_2 \circ p_1)(u_2 \circ u_1)} : (Z, \gamma, D) \longrightarrow (S, \beta, A)$ is soft ω-irresolute. □

Theorem 3.13. *If $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft ω-irresolute and X is a non-empty subset of Z, then the soft restriction $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is soft ω-irresolute.*

Proof. Let $H \in \delta_\omega$. Since $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft ω-irresolute, then $f_{pu}^{-1}(H) \in \gamma_\omega$. Thus, $f_{(p_{|X})u}^{-1}(H) = f_{pu}^{-1}(H) \tilde{\cap} C_X \in (\gamma_\omega)_X$. Hence, by Theorem 15 of [6], $f_{(p_{|X})u}^{-1}(H) \in (\gamma_X)_\omega$. It follows that $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is soft ω-irresolute. □

Theorem 3.14. *Let $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ be a soft function and let $1_D = C_X \tilde{\cup} C_Y$, where $C_X, C_Y \in (\gamma_\omega)^c - \{0_D\}$. If $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ and $f_{(p_{|Y})u} : (Y, \gamma_Y, D) \longrightarrow (W, \delta, E)$ are soft ω-irresolute functions, then $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft ω-irresolute.*

Proof. We apply Theorem 3.2 (c). Let $K \in (\delta_\omega)^c$. Then

$$\begin{aligned} f_{pu}^{-1}(K) = f_{pu}^{-1}(K) \tilde{\cap} 1_D &= f_{pu}^{-1}(K) \tilde{\cap} \left(C_X \tilde{\cup} C_Y\right) \\ &= \left(f_{pu}^{-1}(K) \tilde{\cap} C_X\right) \tilde{\cup} \left(f_{pu}^{-1}(K) \tilde{\cap} C_Y\right) = \left(f_{(p_{|X})u}^{-1}(K)\right) \tilde{\cup} \left(f_{(p_{|Y})u}^{-1}(K)\right). \end{aligned}$$

Since $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ and $f_{(p_{|Y})u} : (Y, \gamma_Y, D) \longrightarrow (W, \delta, E)$ are soft ω-irresolute functions, then $f_{(p_{|X})u}^{-1}(K) \in ((\gamma_X)_\omega)^c$ and $f_{(p_{|Y})u}^{-1}(K) \in ((\gamma_Y)_\omega)^c$. So, by Theorem 15 of [6], $f_{(p_{|X})u}^{-1}(K) \in ((\gamma_\omega)_X)^c$ and $f_{(p_{|Y})u}^{-1}(K) \in ((\gamma_\omega)_Y)^c$. Since $C_X, C_Y \in (\gamma_\omega)^c$, then $f_{(p_{|X})u}^{-1}(K), f_{(p_{|Y})u}^{-1}(K) \in (\gamma_\omega)^c$. Hence, $f_{pu}^{-1}(K) \in (\gamma_\omega)^c$. It follows that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft ω-irresolute. □

Lemma 3.15. *If* $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ *is soft open, then* $f_{pu}^{-1}(Cl_\delta(M)) \widetilde{\subseteq} Cl_\gamma\left(f_{pu}^{-1}(M)\right)$ *for each* $M \in SS(W, E)$.

Proof. Suppose that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft open and let $M \in SS(W, E)$. Suppose to the contrary that there exists such that $d_z \widetilde{\in} f_{pu}^{-1}(Cl_\delta(M)) - Cl_\gamma\left(f_{pu}^{-1}(M)\right)$. Then $f_{pu}(d_z) \widetilde{\in} Cl_\delta(M)$ and there exists $G \in \gamma$ such that $d_z \widetilde{\in} G$ and $G \widetilde{\cap} f_{pu}^{-1}(M) = 0_D$. Since f_{pu} is soft open, then we have $f_{pu}(d_z) \widetilde{\in} f_{pu}(G) \in \delta$. Since $f_{pu}(d_z) \widetilde{\in} Cl_\delta(M)$, then $f_{pu}(G) \widetilde{\cap} M \neq 0_E$. Thus, there exists $a_x \widetilde{\in} G$ such that $f_{pu}(a_x) \widetilde{\in} M$. Hence, $a_x \widetilde{\in} G \widetilde{\cap} f_{pu}^{-1}(M)$. But $G \widetilde{\cap} f_{pu}^{-1}(M) = 0_D$, a contradiction. □

Theorem 3.16. *Let* $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ *be soft open and soft* ω*-irresolute. Then for each* $M \in S\omega O(W, \delta, E)$, $f_{pu}^{-1}(M) \in S\omega O(Z, \gamma, D)$.

Proof. Let $M \in S\omega O(W, \delta, E)$. Then there exists $K \in \delta_\omega$ such that $K \widetilde{\subseteq} M \widetilde{\subseteq} Cl_\delta(K)$ and so, $f_{pu}^{-1}(K) \widetilde{\subseteq} f_{pu}^{-1}(M) \widetilde{\subseteq} f_{pu}^{-1}(Cl_\delta(K))$. Since f_{pu} is soft ω-irresolute, then $f_{pu}^{-1}(K) \in \gamma_\omega$. Since f_{pu} is soft open, then by Lemma 3.15, $f_{pu}^{-1}(Cl_\delta(K)) \widetilde{\subseteq} Cl_\gamma\left(f_{pu}^{-1}(K)\right)$. Therefore, $f_{pu}^{-1}(M) \in S\omega O(Z, \gamma, D)$. □

Theorem 3.17. *Let* $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ *be soft continuous such that* $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta_\omega, E)$ *is soft open. Then for each* $N \in S\omega O(Z, \gamma, D)$, $f_{pu}(N) \in S\omega O(W, \delta, E)$.

Proof. Let $N \in S\omega O(Z, \gamma, D)$. Then there exists $S \in \delta_\omega$ such that $S \widetilde{\subseteq} N \widetilde{\subseteq} Cl_\gamma(S)$ and so, $f_{pu}(S) \widetilde{\subseteq} f_{pu}(N) \widetilde{\subseteq} f_{pu}(Cl_\gamma(N))$. Since $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta_\omega, E)$ is soft open, then $f_{pu}(S) \in \delta_\omega$. Since $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft continuous, then $f_{pu}(Cl_\gamma(N)) \widetilde{\subseteq} Cl_\delta(f_{pu}(N))$. Therefore, $f_{pu}(N) \in S\omega O(W, \delta, E)$. □

Theorem 3.18. *Let* $f_{p_1u_1} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ *be soft continuous, soft open, and surjective such that* $f_{p_1u_1} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta_\omega, E)$ *is soft continuous and soft open. Then* $f_{p_2u_2} : (W, \delta, E) \longrightarrow (S, \beta, A)$ *is soft semi* ω*-continuous if and only if* $f_{(p_2 \circ p_1)(u_2 \circ u_1)} : (Z, \gamma, D) \longrightarrow (S, \beta, A)$ *is soft semi* ω*-continuous.*

Proof.

Necessity. Suppose that $f_{p_2u_2} : (W, \delta, E) \longrightarrow (S, \beta, A)$ is soft semi ω-continuous. Let $K \in \beta$. Then $f_{p_2u_2}^{-1}(K) \in S\omega O(W, \delta, E)$. Thus, by Theorem 3.16, $f_{p_1u_1}^{-1}\left(f_{p_2u_2}^{-1}(K)\right) = f_{(p_2 \circ p_1)(u_2 \circ u_1)}^{-1}(K) \in S\omega O(Z, \gamma, D)$.

Sufficiency. Suppose that $f_{(p_2 \circ p_1)(u_2 \circ u_1)} : (Z, \gamma, D) \longrightarrow (S, \beta, A)$ is soft semi ω-continuous. Let $K \in \beta$. Then $f_{(p_2 \circ p_1)(u_2 \circ u_1)}^{-1}(K) = f_{p_1u_1}^{-1}\left(f_{p_2u_2}^{-1}(K)\right) \in S\omega O(Z, \gamma, D)$. Thus, by Theorem 3.17, $f_{p_1u_1}\left(f_{p_1u_1}^{-1}\left(f_{p_2u_2}^{-1}(K)\right)\right) \in S\omega O(W, \delta, E)$. Since $f_{p_1u_1} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is surjective, then $f_{p_1u_1}\left(f_{p_1u_1}^{-1}\left(f_{p_2u_2}^{-1}(K)\right)\right) = f_{p_2u_2}^{-1}(K)$. □

4. Soft semi ω-irresolutefunctions

In this section, we introduce the class of soft semi ω-irresolute functions, which contained strictly in class of soft semi ω-continuous functions. We show that soft ω-irresoluteness and soft semi ω-irresoluteness are independent notions. Moreover, we study the relationship between soft semi ω-irresolute functions beween STSs and semi ω-irresolute functions between TS and provide many characterizations of soft semi ω-continuity.

Definition 4.1. A soft function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is said to be soft semi ω-irresolute if $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$ for every $H \in S\omega O(W, \delta, E)$.

Theorem 4.2. *For a soft function* $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$, *the following are equivalent.*

(a) f_{pu} *is soft semi* ω*-irresolute.*
(b) *For every* $d_z \in SP(Z, D)$ *and for every* $H \in S\omega O(W, \delta, E)$ *such that* $f_{pu}(d_z) \widetilde{\in} H$, *there exists* $K \in S\omega O(Z, \gamma, D)$ *such that* $d_z \widetilde{\in} K$ *and* $f_{pu}(K) \widetilde{\subseteq} H$.

(c) $f_{pu}^{-1}(T) \in S\omega C(Z,\gamma,D)$ *for every* $T \in S\omega C(W,\delta,E)$.

(d) $f_{pu}(S\omega\text{-}Cl_{\gamma}(N)) \widetilde{\subseteq} S\omega\text{-}Cl_{\delta}(f_{pu}(N))$ *for every* $N \in SS(Z,D)$.

(e) $f_{pu}^{-1}(S\omega\text{-}Int_{\delta}(M)) \widetilde{\subseteq} S\omega\text{-}Int_{\gamma}\left(f_{pu}^{-1}(M)\right)$ *for every* $M \in SS(W,E)$.

Proof.

(a) $\Longrightarrow$ (b): Let $d_z \in SP(Z,D)$ and $H \in S\omega O(W,\delta,E)$ such that $f_{pu}(d_z) \widetilde{\in} H$. Then $d_z \widetilde{\in} f_{pu}^{-1}(H) \in S\omega O(Z,\gamma,D)$. Put $K = f_{pu}^{-1}(H)$. Then $d_z \widetilde{\in} K \in S\omega O(Z,\gamma,D)$ and $f_{pu}(K) \widetilde{\subseteq} H$.

(b) $\Longrightarrow$ (c): Let $T \in S\omega C(W,\delta,E)$. We will show that $1_D - f_{pu}^{-1}(T) \in S\omega O(Z,\gamma,D)$. For every $d_z \widetilde{\in} 1_D - f_{pu}^{-1}(T) = f_{pu}^{-1}(1_E - T)$, we have $f_{pu}(d_z) \widetilde{\in} 1_E - T \in S\omega O(Z,\gamma,D)$, and by (b), there exists $K_{d_z} \in S\omega O(Z,\gamma,D)$ such that $d_z \widetilde{\in} K_{d_z}$ and $f_{pu}(K_{d_z}) \widetilde{\subseteq} 1_E - T$; hence, $d_z \widetilde{\in} K_{d_z} \widetilde{\subseteq} f_{pu}^{-1}(f_{pu}(K)) \widetilde{\subseteq} f_{pu}^{-1}(1_E - T) = 1_D - f_{pu}^{-1}(T)$. Therefore, $1_D - f_{pu}^{-1}(T) = \widetilde{\bigcup}_{d_z \widetilde{\in} 1_D - f_{pu}^{-1}(T)} K_{d_z}$. Thus, by Theorem 11 of [2], $1_D - f_{pu}^{-1}(T) \in S\omega O(Z,\gamma,D)$.

(c) $\Longrightarrow$ (d): Let $N \in SS(Z,D)$, then $S\omega\text{-}Cl_{\delta}(f_{pu}(N)) \in S\omega C(W,\delta,E)$. So, by (c), $f_{pu}^{-1}(S\omega\text{-}Cl_{\delta}(f_{pu}(N))) \in S\omega C(Z,\gamma,D)$. Since $N \widetilde{\subseteq} f_{pu}^{-1}(S\omega\text{-}Cl_{\delta}(f_{pu}(N))) \in S\omega C(Z,\gamma,D)$, then $S\omega\text{-}Cl_{\gamma}(N) \widetilde{\subseteq} f_{pu}^{-1}(S\omega\text{-}Cl_{\delta}(f_{pu}(N)))$, and so $f_{pu}(S\omega\text{-}Cl_{\gamma}(N)) \widetilde{\subseteq} f_{pu}(f_{pu}^{-1}(S\omega\text{-}Cl_{\delta}(f_{pu}(N)))) \widetilde{\subseteq} S\omega\text{-}Cl_{\delta}(f_{pu}(N))$.

(d) $\Longrightarrow$ (a): Let $H \in S\omega O(W,\delta,E)$. Then $1_E - H \in S\omega O(W,\delta,E)$. So by (d), $f_{pu}(S\omega\text{-}Cl_{\gamma}\left(f_{pu}^{-1}(1_E - H)\right)) \widetilde{\subseteq} S\omega\text{-}Cl_{\delta}(f_{pu}(f_{pu}^{-1}(1_E - H))) \widetilde{\subseteq} S\omega\text{-}Cl_{\delta}(1_E - H) = 1_E - H$. And thus, $S\omega\text{-}Cl_{\gamma}\left(f_{pu}^{-1}(1_E - H)\right) \widetilde{\subseteq} f_{pu}^{-1}\left(f_{pu}(S\omega\text{-}Cl_{\gamma}\left(f_{pu}^{-1}(1_E - H)\right))\right) \widetilde{\subseteq} f_{pu}^{-1}(1_E - H)$. Therefore, $S\omega\text{-}Cl_{\gamma}\left(f_{pu}^{-1}(1_E - H)\right) = f_{pu}^{-1}(1_E - H)$, and hence $f_{pu}^{-1}(1_E - H) = 1_D - f_{pu}^{-1}(H) \in S\omega C(Z,\gamma,D)$. This shows that $f_{pu}^{-1}(H) \in S\omega O(Z,\gamma,D)$. It follows that f_{pu} is soft semi ω-irresolute.

(a) $\Longrightarrow$ (e): Let $M \in SS(W,E)$, then $S\omega\text{-}Int_{\delta}(M) \in S\omega O(W,\delta,E)$ and so $f_{pu}^{-1}(S\omega\text{-}Int_{\delta}(M)) \in S\omega O(Z,\gamma,D)$. Since $f_{pu}^{-1}(S\omega\text{-}Int_{\delta}(M)) \widetilde{\subseteq} f_{pu}^{-1}(M)$, then $f_{pu}^{-1}(S\omega\text{-}Int_{\delta}(M)) \widetilde{\subseteq} S\omega\text{-}Int_{\gamma}\left(f_{pu}^{-1}(M)\right)$.

(e) $\Longrightarrow$ (a): Let $H \in S\omega O(W,\delta,E)$. Then $f_{pu}^{-1}(H) = f_{pu}^{-1}(f_{pu}^{-1}(S\omega\text{-}Int_{\delta}(H)) \widetilde{\subseteq} S\omega\text{-}Int_{\gamma}\left(f_{pu}^{-1}(M)\right)) \widetilde{\subseteq} S\omega\text{-}Int_{\gamma}\left(f_{pu}^{-1}(H)\right)$. And so $S\omega\text{-}Int_{\gamma}\left(f_{pu}^{-1}(H)\right) = f_{pu}^{-1}(H)$. Thus, $f_{pu}^{-1}(H) \in S\omega O(Z,\gamma,D)$. Hence, $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ is soft semi ω-irresolute. □

Theorem 4.3. *If* $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ *is a soft function such that* (Z,γ,D) *is soft locally countable, then* f_{pu} *is soft semi* ω*-irresolute.*

Proof. Let $H \in S\omega O(W,\delta,E)$. Since (Z,γ,D) is soft locally countable, then by Corollary 5 of [6], $S\omega O(Z,\gamma,D) = SS(Z,D)$. Hence, $f_{pu}^{-1}(H) \in S\omega O(Z,\gamma,D)$. Therefore, f_{pu} is soft semi ω-irresolute. □

Corollary 4.4. *If* $f_{pu} : (Z,\gamma,D) \longrightarrow (W,\delta,E)$ *is a soft function such that* Z *is a countable set, then* f_{pu} *is soft semi* ω*-irresolute.*

Theorem 4.5. *Let* $p : (Z,\mu) \longrightarrow (W,\lambda)$ *be a function between two TSs and let* $u : D \longrightarrow E$ *be a function between two sets of parameters. Then* $p : (Z,\mu) \longrightarrow (W,\lambda)$ *is semi* ω*-irresolute if and only if* $f_{pu} : (Z,\tau(\mu),D) \longrightarrow (W,\tau(\lambda),E)$ *is soft semi* ω*-irresolute.*

Proof.

Necessity. Let $p : (Z,\mu) \longrightarrow (W,\lambda)$ be semi ω-irresolute. Let $H \in S\omega O(W,\tau(\lambda),E)$. Then by Lemma 2.17, for each $d \in D$, $H(u(d)) \in S\omega O(W,\lambda)$, and thus $p^{-1}(H(u(d))) \in S\omega O(Z,\mu)$. Therefore, $\left(f_{pu}^{-1}(H)\right)(d) = p^{-1}(H(u(d))) \in S\omega O(Z,\mu)$ for each $d \in D$. Hence, by Lemma 2.17, $f_{pu}^{-1}(H) \in S\omega O(Z,\tau(\mu),D)$. It follows that $f_{pu} : (Z,\tau(\mu),D) \longrightarrow (W,\tau(\lambda),E)$ is soft semi ω-irresolute.

Sufficiency. Let $f_{pu} : (Z,\tau(\mu),D) \longrightarrow (W,\tau(\lambda),E)$ be soft semi ω-irresolute. Let $V \in S\omega O(W,\lambda)$. Then by Lemma 2.17, $C_V \in S\omega O(W,\tau(\lambda),E)$. Thus, $f_{pu}^{-1}(C_V) \in S\omega O(Z,\tau(\mu),D)$. Choose $d \in D$. Then, by Lemma 2.17, $\left(f_{pu}^{-1}(C_V)\right)(d) = p^{-1}(V) \in S\omega O(Z,\mu)$. It follows that $p : (Z,\mu) \longrightarrow (W,\lambda)$ is semi ω-irresolute. □

Theorem 4.6. *For any two STSs* (Z, γ, D) *and* (W, δ, E)*, and any soft function* $f_{pu} : SS(Z, D) \longrightarrow SS(W, E)$*, the following are equivalent.*

(a) $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta_\omega, E)$ *is soft semi* ω*-irresolute.*
(b) $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta_\omega, E)$ *is soft irresolute.*

Proof.
(a) $\Longrightarrow$ (b): Let $H \in SO(W, \delta_\omega, E)$. Then by Theorem 9 of [2], $H \in S\omega O(W, \delta_\omega, E)$. So by (a), $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma_\omega, D)$. Thus, again by Theorem 9 of [2] we have $f_{pu}^{-1}(H) \in SO(Z, \gamma_\omega, D)$.

(b) $\Longrightarrow$ (a): Let $H \in S\omega O(W, \delta_\omega, E)$. Then by Theorem 9 of [2], $H \in SO(W, \delta_\omega, E)$. So by (b), $f_{pu}^{-1}(H) \in SO(Z, \gamma_\omega, D)$. Thus, again by Theorem 9 of [2] we have $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma_\omega, D)$. □

Theorem 4.7. *Let* (Z, γ, D) *and* (W, δ, E) *be two STSs and let* $f_{pu} : SS(Z, D) \longrightarrow SS(W, E)$ *be a soft function. If* $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ *is soft semi* ω*-irresolute, then* $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ *is soft semi* ω*-irresolute.*

Proof. Suppose that $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute. Let $H \in S\omega O(W, \delta, E)$. Since $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute, then $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma_\omega, D)$. Since by Theorem 10 of [2], $S\omega O(Z, \gamma_\omega, D) \subseteq S\omega O(Z, \gamma, D)$, then, $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Therefore, $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-continuous. □

The following example shows that the converse of Theorem 4.7 need not to be true in general.

Example 4.8. Let (Z, γ, D), (W, δ, D), $p : Z \longrightarrow W$, and $u : D \longrightarrow D$ be as in Example 2.6. It is not difficult to chech that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$ is semi ω-irresolute. On the other hand, since $C_{\{a\}} \notin S\omega O(W, \delta, E)$ but $f_{pu}^{-1}(C_{\{a\}}) = C_{\mathbb{N}} \notin S\omega O(Z, \gamma_\omega, D)$, then $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, D)$ is not soft semi ω-irresolute.

Theorem 4.9. *Let* (Z, γ, D) *and* (W, δ, E) *be two STSs and let* $f_{pu} : SS(Z, D) \longrightarrow SS(W, E)$ *be a soft function. If* $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ *is soft semi* ω*-irresolute, then* $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta_\omega, E)$ *is soft irresolute.*

Proof. Suppose that $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute. Let $H \in SO(W, \delta_\omega, E)$. Since by Theorems 9 and 10 of [2], $S\omega O(W, \delta_\omega, E) = SO(W, \delta_\omega, E) \subseteq SO(W, \delta, E) \subseteq S\omega O(W, \delta, E)$, then $H \in S\omega O(W, \delta, E)$. Since $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute, then $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma_\omega, D)$. So by Theorem 9 of [2] we have $f_{pu}^{-1}(H) \in SO(Z, \gamma_\omega, D)$. Therefore, $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (W, \delta_\omega, E)$ is soft irresolute. □

The converse of Theorem 4.9 is false as the following example shows.

Example 4.10. Let $Z = \mathbb{R}$, $D = \mathbb{Z}$, $\gamma = \{C_V : V \subseteq \mathbb{R} \text{ and } 1 \notin V\} \cup \{C_V : V \subseteq \mathbb{R}, 1 \in V \text{ and } \mathbb{R} - V \text{ is finite}\}$. Let $p : Z \longrightarrow Z$ and $u : D \longrightarrow D$ be the identities functions. Then $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (Z, \gamma_\omega, D)$ is soft irresolute. On the other hand, since $C_{\mathbb{Q}} \in S\omega O(Z, \gamma, D)$ but $f_{pu}^{-1}(C_{\mathbb{Q}}) = C_{\mathbb{Q}} \notin S\omega O(Z, \gamma_\omega, D) = SO(Z, \gamma_\omega, D)$, then $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (Z, \gamma, D)$ is not soft semi ω-irresolute.

Definition 4.11. A soft function $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is said to be strongly soft semi ω-irresolute if $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$ for every $H \in \delta_\omega$.

Theorem 4.12. *Every soft strongly semi* ω*-irresolute function is soft semi* ω*-continuous.*

Proof. Let $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ be soft strongly semi ω-irresolute and let $H \in \delta$. Then by Theorem 2 of [6], $H \in \delta_\omega$. Since f_{pu} is soft strongly semi ω-irresolute, then $f_{pu}^{-1}(H) \in S\omega O(Z, \gamma, D)$. Therefore, f_{pu} is soft semi ω-continuous. □

The soft function $f_{p_1 u_1} : (Z, \tau(\mu), D) \longrightarrow (W, \tau(\lambda), D)$ in Example 2.22 is soft semi ω-continuous but not soft semi ω-irresolute. Thus, the converse of Theorem 4.12 is not true in general.

The following two examples will show that soft ω-irresoluteness and soft semi ω-irresoluteness are independent notions.

Example 4.13. Let $f_{pu} : (Z, \gamma_\omega, D) \longrightarrow (Z, \gamma, D)$ be as in Example 4.10. Then f_{pu} is not soft semi ω-irresolute. To see that f_{pu} is soft ω-irresolute, let $H \in \gamma_\omega$, then by Theorem 5 of [6], $f^{-1}_{pu}(H) = H \in (\gamma_\omega)_\omega$. Thus, f_{pu} is soft ω-irresolute.

Example 4.14. Let $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$ be as in Example 2.6. Then f_{pu} is soft semi ω-irresolute but not soft ω-irresolute.

Theorem 4.15. *Every soft ω-irresolute soft open function is soft semi ω-irresolute.*

Proof. Let $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ be soft ω-irresolute and soft open function. Let $H \in S\omega O\,(W, \delta, E)$. Then there exists $K \in \delta_\omega$ such that $K \widetilde{\subseteq} H \widetilde{\subseteq} Cl_\delta(K)$ and so $f^{-1}_{pu}\,(K) \widetilde{\subseteq} f^{-1}_{pu}\,(H) \widetilde{\subseteq} f^{-1}_{pu}\,(Cl_\delta(K))$. Since f_{pu} is soft ω-irresolute, then $f^{-1}_{pu}(K) \in \gamma_\omega$. Since f_{pu} is soft open, then by Lemma 3.15, $f^{-1}_{pu}\,(Cl_\delta\,(K)) \widetilde{\subseteq} Cl_\gamma\left(f^{-1}_{pu}\,(K)\right)$. Thus, we have $f^{-1}_{pu}\,(K) \widetilde{\subseteq} f^{-1}_{pu}\,(H) \widetilde{\subseteq} Cl_\gamma\left(f^{-1}_{pu}\,(K)\right)$ with $f^{-1}_{pu}(K) \in \gamma_\omega$. Hence, $f^{-1}_{pu}\,(H) \in S\omega O\,(Z, \gamma, D)$. □

Theorem 4.16. *If $f_{p_1u_1} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute and $f_{p_2u_2} : (W, \delta, E) \longrightarrow (S, \beta, A)$ is soft semi ω-irresolute, then $f_{(p_2\circ p_1)(u_2\circ u_1)} : (Z, \gamma, D) \longrightarrow (S, \beta, A)$ is soft semi ω-irresolute.*

Proof. Let $H \in S\omega O\,(S, \beta, A)$. Since $f_{p_2u_2} : (W, \delta, E) \longrightarrow (S, \beta, A)$ is soft semi ω-irresolute, then $f^{-1}_{p_2u_2}(H) \in S\omega O\,(W, \delta, E)$. Since $f_{p_1u_1} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute, $f^{-1}_{p_1u_1}\left(f^{-1}_{p_2u_2}(H)\right) = f^{-1}_{(p_2\circ p_1)(u_2\circ u_1)}(K) \in S\omega O\,(Z, \gamma, D)$. It follows that $f_{(p_2\circ p_1)(u_2\circ u_1)} : (Z, \gamma, D) \longrightarrow (S, \beta, A)$ is soft semi ω-irresolute. □

Theorem 4.17. *If $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute and $X \subseteq Z$ such that $C_X \in \gamma - \{0_D\}$, then the soft restriction $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute.*

Proof. Let $H \in S\omega O\,(W, \delta, E)$. Since $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute, then $f^{-1}_{pu}(H) \in S\omega O\,(Z, \gamma, D)$. Thus, by Theorem 12 of [2], $f^{-1}_{(p_{|X})u}(H) = f^{-1}_{pu}(H) \widetilde{\cap} C_X \in S\omega O\,(Z, \gamma, D)$. Hence, by Theorem 14 of [2], $f^{-1}_{(p_{|X})u}(H) \in S\omega O\,(X, \gamma_X, D)$. It follows that $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute. □

Corollary 4.18. *Let $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ be a soft function. Let $\{C_{Z_\alpha} : j \in J\} \subseteq \gamma$ such that $1_D = \widetilde{\cup}\{C_{Z_j} : j \in J\}$. If for each $j \in J$, $f_{\left(p_{|Z_j}\right)u} : \left(Z_j, \gamma_{Z_j}, D\right) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute, then $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute.*

Proof. Let $d_z \in SP(Z, D)$. We show that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute at d_z. Since $1_D = \widetilde{\cup}\{C_{Z_j} : j \in J\}$, then there exists $j_\circ \in J$ such that $d_z \widetilde{\in} C_{Z_{j_\circ}}$. Therefore, by Theorem 4.17, it follows that $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, E)$ is soft semi ω-irresolute. □

Note that Theorem 4.17 is not true if we take $C_X \in \gamma_\omega - \{0_D\}$ as it is shown in the next example.

Example 4.19. Consider $f_{pu} : (Z, \gamma, D) \longrightarrow (W, \delta, D)$ be as in Example 2.6. Take $X = (\mathbb{R} - \mathbb{N}) \cup \{1\}$. Then $C_X \in \gamma_\omega - \{0_D\}$ and $f_{(p_{|X})u} : (X, \gamma_X, D) \longrightarrow (W, \delta, E)$ is not soft semi ω-irresolute since $C_{\{a\}} \in S\omega O\,(W, \delta, E)$ but $f^{-1}_{(p_{|X})u}\left(C_{\{a\}}\right) = C_{\{1\}} \notin S\omega O\,(X, \gamma_X, D)$.

5. Conclusion

The concept of "soft sets" was proposed as an effective tool to deal with uncertainty and vagueness. Topologists employ this concept to define and study STSs.

In this paper, soft semi ω-continuity, soft ω-irresoluteness, and soft semi ω-irresoluteness are introduced as extensions of semi ω-continuity, ω-irresoluteness, and semi ω-irresoluteness general topological concepts, respectively. Several characterizations, relationships, and examples of the new soft topological concepts are given. Also, several restrictions and composition theorems are obtained. In addition to these, the connections between the new classes of soft functions and their corresponding general topological concepts are justified.

The following topics could be considered in future studies:

1) to define soft ω-homeomorphisms;
2) to define soft separation axioms via semi ω-open sets.

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