J. Math. Computer Sci., 30 (2023), 190–203

Online: ISSN 2008-949X

Journal of Mathematics and Computer Science

Journal Homepage: www.isr-publications.com/jmcs

Fuzzy hypersoft contra maps, homeomorphisms, and application in Covid-19 diagnosis using Hamming distance

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Abstract

This paper aims to introduce and study fuzzy hypersoft contra open, fuzzy hypersoft contra semi open, fuzzy hypersoft contra closed, and fuzzy hypersoft contra semi closed maps in fuzzy hypersoft topological spaces. Basic properties of fuzzy hypersoft contra open, contra semi open, contra closed and contra semi closed maps are analyzed with examples. Also, the relation between fuzzy hypersoft contra open maps, contra semi open maps, contra closed maps and contra semi closed maps is discussed. It is extended to fuzzy hypersoft contra homeomorphism, contra semi homeomorphism, contra C-homeomorphism and its related characteristics are also investigated. The fuzzy hypersoft set measure Hamming distance can be applied in real-world decision-making problems containing more uncertain and inadequate data. By applying Hamming distance between the Covid-19 patients and the other patients, a better decision can be taken in the Covid-19 diagnosis. This paper proposes a method to diagnose Covid-19 using Hamming distance of fuzzy hypersoft sets. The association between the patients and the symptoms is formulated as fuzzy hypersoft sets in which the Hamming distance measure is applied to decide on Covid-19 diagnosis.

Keywords: Fuzzy hypersoft contra open maps, fuzzy hypersoft contra semi open maps, fuzzy hypersoft contra closed maps, fuzzy hypersoft contra semi closed maps, fuzzy hypersoft contra homeomorphism, fuzzy hypersoft contra semi homeomorphism, fuzzy hypersoft contra C-homeomorphism, Hamming distance.

2020 MSC: 03E72, 54A10, 54A40, 54C05, 54C10.

1. Introduction

The real-world decision-making problems in medical diagnosis, engineering, economics, management,

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doi: 10.22436/jmcs.030.03.01

Received: 2022-09-19 *Revised:* 2022-11-02 *Accepted:* 2022-11-10

computer science, artificial intelligence, social sciences, environmental science and sociology contain more uncertain and inadequate data. Traditional mathematical methods cannot deal with these kinds of problems due to imprecise data. To deal with the problems with uncertainty, Zadeh [14] introduced the fuzzy set in 1965 which contains the membership value in [0,1]. A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called the membership value of an element in that set. The topological structure on fuzzy set was undertaken by Chang [5] as fuzzy topological space. Molodstov [8] introduced a new mathematical tool, soft set theory in 1999 to deal with uncertainties in which a soft set is a collection of approximate descriptions of an object. A soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory has several applications in different fields such as decision-making, optimization, forecasting, data analysis etc. Shabir and Naz [11] presented soft topological spaces.

Smarandache [12] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing a function with a multi-argument function described in the Cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in decision-making issues involving a different kinds of attributes. Saeed et al. [9, 10] studied the fundamentals of hypersoft set theory by introducing aggregate operators, relations,functionsn, matrices and operations on hypersoft matrices. Abbas et al. [1] defined the basic operations on hypersoft sets and hypersoft point in the fuzzy, intuitionistic and neutrosophic environments. Ajay and Charisma [3] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology. Ajay et al. [4] defined fuzzy hypersoft semi-open sets and developed an application in multiattribute group decision-making. The concept of contra continuous function in general topology was introduced by Dontchev [6] in 1996. Vadivel et al. [13] introduced generalized fuzzy contra e-continuous functions in fuzzy topological spaces. Ahsan et al. [2] studied a theoretical and analytical approach for fundamental framework of composite mappings on fuzzy hypersoft classes. Kacprzyk [7] proposed various distance measures like Hamming, Euclidean, normalized Hamming and normalized Euclidean distances for fuzzy sets.

In this paper, we develop the concept of fuzzy hypersoft contra open mapping, contra semi open mapping, contra closed mapping and contra semi closed mapping in fuzzy hypersoft topological spaces and some of their basic properties are analyzed with examples. Added to that, we introduce and discuss some characterizations and properties of fuzzy hypersoft contra homeomorphisms, fuzzy hypersoft contra semi homeomorphisms and fuzzy hypersoft contra C-homeomorphisms. Also, an application in Covid-19 diagnosis is explained with the example using Hamming distance by formulating the association between the patients and symptoms as fuzzy hypersoft sets. The Hamming distance is used to measure the uncertainty in decision making problems.

2. Preliminaries

Definition 2.1 ([14])**.** Let $\mathfrak{M}$ be an initial universe. A function λ from $\mathfrak{M}$ into the unit interval I is called a fuzzy set in $\mathfrak{M}$. For every $\mathfrak{m} \in \mathfrak{M}$, $\lambda(\mathfrak{m}) \in I$ is called the grade of membership of $\mathfrak{m}$ in λ. Some authors say that λ is a fuzzy subset of $\mathfrak{M}$ instead of saying that λ is a fuzzy set in $\mathfrak{M}$. The class of all fuzzy sets from $\mathfrak{M}$ into the closed unit interval I will be denoted by $I^{\mathfrak{M}}$.

Definition 2.2 ([8])**.** Let $\mathfrak{M}$ be an initial universe, Q be a set of parameters and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. A pair $(\tilde{H}, Q)$ is called the a soft set over $\mathfrak{M}$ where $\tilde{H}$ is a mapping $\tilde{H} : Q \to \mathcal{P}(\mathfrak{M})$. In other words, the soft set is a parametrized family of subsets of the set $\mathfrak{M}$.

Definition 2.3 ([12])**.** Let $\mathfrak{M}$ be an initial universe, and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$, and $q_1, q_2, q_3, \dots, q_n$ for $n \geqslant 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $Q_1, Q_2, \dots, Q_n$ with $Q_i \cap Q_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, \dots, n\}$. Consider the cartesian product

$Q_1 \times Q_2 \times \cdots \times Q_n$. Then the pair $(\tilde{H}, Q_1 \times Q_2 \times \cdots \times Q_n)$, where $\tilde{H} : Q_1 \times Q_2 \times \cdots \times Q_n \to \mathcal{P}(\mathfrak{M})$ is called a hypersoft set over $\mathfrak{M}$.

Definition 2.4 ([1]). Let $\mathfrak{M}$ be an initial universal set and $Q_1, Q_2, \ldots, Q_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\mathfrak{M})$ be the set of all fuzzy sets of $\mathfrak{M}$. Let E_i be the nonempty subset of the pair Q_i for each $i = 1, 2, \ldots, n$. Consider the cartesian products $E_1 \times E_2 \times \cdots \times E_n$ and $Q_1 \times Q_2 \times \cdots \times Q_n$. A fuzzy hypersoft set (briefly, FHSs) over $\mathfrak{M}$ is defined as the pair $(\tilde{H}, E_1 \times E_2 \times \cdots \times E_n)$, where $\tilde{H} : E_1 \times E_2 \times \cdots \times E_n \to \mathcal{P}(\mathfrak{M})$ and $\tilde{H}(E_1 \times E_2 \times \cdots \times E_n) = \{(q, \langle m, \mu_{\tilde{H}(q)}(m)\rangle : m \in \mathfrak{M}) : q \in E_1 \times E_2 \times \cdots \times E_n \subseteq Q_1 \times Q_2 \times \cdots \times Q_n\}$, where $\mu_{\tilde{H}(q)}(m)$ is the membership value such that $\mu_{\tilde{H}(q)}(m) \in [0, 1]$.

Definition 2.5 ([1]). Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be two FHSs's over $\mathfrak{M}$. Then $(\tilde{H}, \wedge_1)$ is the fuzzy hypersoft subset of $(\tilde{G}, \wedge_2)$ if $\mu_{\tilde{H}(q)}(m) \leqslant \mu_{\tilde{G}(q)}(m)$. It is denoted by $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2)$.

Definition 2.6 ([1]). Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be FHSs's over $\mathfrak{M}$. $(\tilde{H}, \wedge_1)$ is equal to $(\tilde{G}, \wedge_1)$ if $\mu_{\tilde{H}(q)}(m) = \mu_{\tilde{G}(q)}(m)$.

Definition 2.7 ([1]). A FHSs $(\tilde{H}, \wedge)$ over the universe set $\mathfrak{M}$ is said to be

(i) null fuzzy hypersoft set if $\mu_{\tilde{H}(q)}(m) = 0$, $\forall q \in \wedge$ and $m \in \mathfrak{M}$, it is denoted by $\tilde{0}_{(\mathfrak{M},Q)}$;
(ii) absolute fuzzy hypersoft set if $\mu_{\tilde{H}(q)}(m) = 1$ $\forall q \in \wedge$ and $m \in \mathfrak{M}$, it is denoted by $\tilde{1}_{(\mathfrak{M},Q)}$.

Clearly, $\tilde{0}^c_{(\mathfrak{M},Q)} = \tilde{1}_{(\mathfrak{M},Q)}$ and $\tilde{1}^c_{(\mathfrak{M},Q)} = \tilde{0}_{(\mathfrak{M},Q)}$.

Definition 2.8 ([1]). Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge)$ be FHSs over $\mathfrak{M}$. $(\tilde{H}, \wedge)^C$ is the complement of $(\tilde{H}, \wedge)$ if $\mu^C_{\tilde{H}(q)}(m) = \tilde{1}^c_{(\mathfrak{M},Q)} - \mu_{\tilde{H}(q)}(m)$, where $\forall q \in \wedge$ and $\forall m \in \mathfrak{M}$. It is clear that $((\tilde{H}, \wedge)^C)^C = (\tilde{H}, \wedge)$.

Definition 2.9 ([1, 3]). Let $\mathfrak{M}$ be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be FHSs's over $\mathfrak{M}$.

(i) Extended union $(\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(m), & \text{if } q \in \wedge_1 - \wedge_2, \\ \mu_{\tilde{G}(q)}(m), & \text{if } q \in \wedge_2 - \wedge_1, \\ \max\{\mu_{\tilde{H}(q)}(m), \mu_{\tilde{G}(q)}(m)\}, & \text{if } q \in \wedge_1 \cap \wedge_2. \end{cases}$$

(ii) Extended intersection $(\tilde{H}, \wedge_1) \cap (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cap (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(m), & \text{if } q \in \wedge_1 - \wedge_2, \\ \mu_{\tilde{G}(q)}(m), & \text{if } q \in \wedge_2 - \wedge_1, \\ \min\{\mu_{\tilde{H}(q)}(m), \mu_{\tilde{G}(q)}(m)\}, & \text{if } q \in \wedge_1 \cap \wedge_2. \end{cases}$$

Definition 2.10 ([3]). Let $(\mathfrak{M}, Q)$ be the family of all FHSs's over the universe set $\mathfrak{M}$ and $\tau \subseteq \text{FHSs}(\mathfrak{M}, Q)$. Then τ is said to be a fuzzy hypersoft topology (briefly, FHSt) on $\mathfrak{M}$ if

(i) $\tilde{0}_{(\mathfrak{M},Q)}$ and $\tilde{1}_{(\mathfrak{M},Q)}$ belong to τ;
(ii) the union of any number of FHSs's in τ belongs to τ;
(iii) the intersection of finite number of FHSs's in τ belongs to τ.

Then $(\mathfrak{M}, Q, \tau)$ is called a fuzzy hypersoft toplogical space (briefly, FHSts) over $\mathfrak{M}$. Each member of τ is said to be fuzzy hypersoft open set (briefly, FHSos). A FHSs $(\tilde{H}, \wedge)$ is called a fuzzy hypersoft closed set (briefly, FHScs) if its complement $(\tilde{H}, \wedge)^C$ is FHSos.

Definition 2.11 ([3]). Let $(\mathfrak{M}, Q, \tau)$ be a FHSts over $\mathfrak{M}$ and $(\tilde{H}, \wedge)$ be a FHSs in $\mathfrak{M}$. Then,

(i) the fuzzy hypersoft interior (briefly, FHSint) of $(\tilde{H}, \wedge)$ is defined as $\text{FHSint}(\tilde{H}, \wedge) = \cup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$, where $(\tilde{G}, \wedge)$ is FHSos$\}$;

(ii) the fuzzy hypersoft closure (briefly, $FHScl$) of $(\tilde{H}, \wedge)$ is defined as $FHScl(\tilde{H}, \wedge) = \cap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$, where $(\tilde{G}, \wedge)$ is $FHScs\}$;
(iii) the fuzzy hypersoft semi interior (briefly, $FHS\mathcal{S}int$) of $(\tilde{H}, \wedge)$ is defined as $FHS\mathcal{S}int(\tilde{H}, \wedge) = \cup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$, where $(\tilde{G}, \wedge)$ is $FHS\mathcal{S}os\}$;
(iv) the fuzzy hypersoft semi closure (briefly, $FHS\mathcal{S}cl$) of $(\tilde{H}, \wedge)$ is defined as $FHS\mathcal{S}cl(\tilde{H}, \wedge) = \cap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$, where $(\tilde{G}, \wedge)$ is $FHS\mathcal{S}cs\}$.

Definition 2.12 ([4]). Let $(\mathfrak{M}, Q, \tau)$ be a FHSts over $\mathfrak{M}$ and $(\tilde{H}, \wedge)$ be a FHSs in $\mathfrak{M}$. Then, $(\tilde{H}, \wedge)$ is called the fuzzy hypersoft semiopen set (briefly, FHSSos) if $(\tilde{H}, \wedge) \subseteq FHScl(int(\tilde{H}, \wedge))$.

A FHSs $(\tilde{H}, \wedge)$ is called a fuzzy hypersoft semiclosed set (briefly, FHSScs) if its complement $(\tilde{H}, \wedge)^c$ is a FHSSos.

Definition 2.13 ([2]). Let $(\mathfrak{M}, Q)$ and $(\mathfrak{N}, R)$ be classes of FHSs's over $\mathfrak{M}$ and $\mathfrak{N}$ with attributes Q and R, respectively. Let $\eta : \mathfrak{M} \to \mathfrak{N}$ and $\zeta : Q \to R$ be mappings. Then a FHS mappings $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ is defined as follows, for a FHSs $(\tilde{H}, \wedge)_A$ in $(\mathfrak{M}, Q)$, $f(\tilde{H}, \wedge)_A$ is a FHSs in $(\mathfrak{N}, R)$ obtained as follows, for $\beta \in \zeta(Q) \subseteq R$ and $\mathfrak{n} \in \mathfrak{N}$, $\mathfrak{k}(\tilde{H}, \wedge)_A(\beta)(\mathfrak{n}) = \bigcup_{\alpha \in \zeta^{-1}(\beta) \bigcap A, s \in \eta^{-1}(\mathfrak{n})} (\alpha)\mu_s \mathfrak{k}(\tilde{H}, \wedge)_A$ is called a fuzzy hypersoft image of a FHSs $(\tilde{H}, \wedge)$. Hence $((\tilde{H}, \wedge)_A, \mathfrak{k}(\tilde{H}, \wedge)_A) \in \mathfrak{k}$, where $(\tilde{H}, \wedge)_A \subseteq (\mathfrak{M}, Q), \mathfrak{k}(\tilde{H}, \wedge)_A \subseteq (\mathfrak{N}, R)$.

Definition 2.14 ([2]). If $\mathfrak{k} : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ be a FHS mapping, then FHS class $(\mathfrak{M}, Q)$ is called the domain of $\mathfrak{k}$ and the FHS class $(\tilde{G}, \wedge) \in (\mathfrak{N}, R) : (\tilde{G}, \wedge) = \mathfrak{k}(\tilde{H}, \wedge))$ for some $(\tilde{H}, \wedge) \in (\mathfrak{M}, Q)$ is called the range of $\mathfrak{k}$. The FHS class $(\mathfrak{N}, R)$ is called co-domain of $\mathfrak{k}$.

Definition 2.15 ([2]). If $\mathfrak{k} : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ be a FHS mapping and $(\tilde{G}, \wedge)_B$, a FHSs in $(\mathfrak{N}, R)$, where $\eta : \mathfrak{M} \to \mathfrak{N}$, $\zeta : Q \to R$ and $B \subseteq R$, then $\mathfrak{k}^{-1}(\tilde{G}, \wedge)_B$ is a FHSs in $(\mathfrak{M}, Q)$ defined as follows, for $\alpha \in \zeta^{-1}(B) \subseteq Q$ and $\mathfrak{m} \in \mathfrak{M}$, $\mathfrak{k}^{-1}(\tilde{G}, \wedge)_B(\alpha)(\mathfrak{m}) = (\zeta(\alpha))\mu_p(\mathfrak{m})\mathfrak{k}^{-1}(\tilde{G}, \wedge)_B$ is called a FHS inverse image of $(\tilde{G}, \wedge)_B$.

Definition 2.16 ([2]). Let $\mathfrak{k} = (\eta, \zeta)$ be a FHS mapping of a FHS class $(\mathfrak{M}, Q)$ into a FHS class $(\mathfrak{N}, R)$. Then $\mathfrak{k}$ is said to be a

(i) one-one (or injection) FHS mapping if for both $\eta : \mathfrak{M} \to \mathfrak{N}$ and $\zeta : Q \to R$ are one-one;
(ii) onto (or surjection) FHS mapping if for both $\eta : \mathfrak{M} \to \mathfrak{N}$ and $\zeta : Q \to R$ are onto.

If $\mathfrak{k}$ is both one-one and onto, then $\mathfrak{k}$ is called a one-one onto (or bijective) correspondance of FHS mapping.

Definition 2.17 ([2]). If $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ and $\mathfrak{g} = (\mathfrak{m}, \mathfrak{n}) : (\mathfrak{N}, R) \to (\mathfrak{O}, S)$ are two FHS mappings, then their composite $\mathfrak{g} \circ \mathfrak{k}$ is a FHS mapping of $(\mathfrak{M}, Q)$ into $(\mathfrak{O}, S)$ such that for every $(\tilde{H}, \wedge)_A \in (\mathfrak{M}, Q)$, $(\mathfrak{g} \circ \mathfrak{k})(\tilde{H}, \wedge)_A = \mathfrak{g}(\mathfrak{k}(\tilde{H}, \wedge)_A)$. For $\beta \in \mathfrak{n}(R) \subseteq N$ and $p \in \mathfrak{O}$, it is defined as $\mathfrak{g}(\mathfrak{k}(\tilde{H}, \wedge)_A(\beta)(p) = \bigcup_{\alpha \in \mathfrak{n}^{-1}(\beta) \bigcap \mathfrak{k}(A), s \in \mathfrak{m}^{-1}(p)} (\alpha)\mu_s$.

Definition 2.18 ([2]). Let $\mathfrak{k} = (\eta, \zeta)$ be a FHS mapping, where $\eta : \mathfrak{M} \to \mathfrak{M}$ and $\zeta : Q \to Q$. Then $\mathfrak{k}$ is said to be a FHS identity mapping if for both η and ζ are identity mappings.

Definition 2.19 ([2]). A one-one onto FHS mapping $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ is called FHS invertable mapping. Its FHS inverse mapping is denoted by $\mathfrak{k}^{-1} = (\eta^{-1}, \zeta^{-1}) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$.

Definition 2.20 ([7]). Consider two fuzzy sets $\tilde{H}$ and $\tilde{G}$ over the fuzzy initial universe $\mathfrak{M}$. The hamming distance for these two sets are given by $d_H(\tilde{H}, \tilde{G}) = \sum_{i=1}^{n} |\mu_H^i - \mu_G^i|$.

3. Fuzzy hypersoft contra open mapping

In this section, fuzzy hypersoft contra open maps are introduced and their characteristics are studied.

Definition 3.1. Consider any two FHSts $(\mathfrak{M}, Q, \tau_1)$ and $(\mathfrak{N}, R, \tau_2)$. A map $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called as fuzzy hypersoft:

(i) continuous (in short, FHS continuous) if the inverse image of each FHSos in $(\mathfrak{N}, R, \tau_2)$ is a FHSos in $(\mathfrak{M}, Q, \tau_1)$;

(ii) semi-continuous (in short, FHS semi continuous) if the inverse image of each FHSos in $(\mathfrak{N}, R, \tau_2)$ is a FHSSos in $(\mathfrak{M}, Q, \tau_1)$.

Definition 3.2. A map $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called a fuzzy hypersoft irresolute map (in short, FHS irresolute) if $\mathfrak{k}^{-1}(\tilde{G}, \wedge)$ is a FHSSos in $(\mathfrak{M}, Q, \tau_1)$ for every FHSSos $(\tilde{G}, \wedge)$ of $(\mathfrak{N}, R, \tau_2)$.

Definition 3.3. A mapping $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called fuzzy hypersoft

(i) open (in short, FHS open) if the image of every FHSos of $(\mathfrak{M}, Q, \tau_1)$ is FHSos in $(\mathfrak{N}, R, \tau_2)$;

(ii) semi open (in short, FHS semi open) if the image of every FHSos of $(\mathfrak{M}, Q, \tau_1)$ is FHSSos in $(\mathfrak{N}, R, \tau_2)$.

Definition 3.4. A mapping $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called fuzzy hypersoft

(i) contra open (in short, FHS contra open) if the image of every FHSos of $(\mathfrak{M}, Q, \tau_1)$ is FHScs in $(\mathfrak{N}, R, \tau_2)$;

(ii) contra semi open (in short, FHS contra semi open) if the image of every FHSos of $(\mathfrak{M}, Q, \tau_1)$ is FHSScs in $(\mathfrak{N}, R, \tau_2)$.

Example 3.5. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the FHS initial universes and the attributes be $Q = Q_1 \times Q_2$ and $R = Q_1' \times Q_2'$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, \quad Q_2 = \{b_1, b_2\}, \quad Q_1' = \{c_1, c_2, c_3\}, \quad Q_2' = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, Q)$ and $(\mathfrak{N}, R)$ be the classes of FHS sets. Let the FHSs's $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$ and $(\tilde{H}_4, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \\ \langle (a_2, b_2), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle \end{matrix} \right\}, \quad (\tilde{H}_2, \wedge_2) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \end{matrix} \right\},$$

$$(\tilde{H}_3, \wedge_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \end{matrix} \right\}, \quad (\tilde{H}_4, \wedge_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \end{matrix} \right\},$$

$\tau_1 = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is FHSts.

Let the FHSs's $(\tilde{G}_1, \wedge_1)$, $(\tilde{G}_2, \wedge_2)$, $(\tilde{G}_3, \wedge_3)$ and $(\tilde{G}_4, \wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1, \wedge_1) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{n_1}{0.4}, \frac{n_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle \end{matrix} \right\}, \quad (\tilde{G}_2, \wedge_2) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \end{matrix} \right\},$$

$$(\tilde{G}_3, \wedge_3) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \end{matrix} \right\}, \quad (\tilde{G}_4, \wedge_3) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{n_1}{0.4}, \frac{n_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \end{matrix} \right\},$$

$\tau_2 = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_2), (\tilde{G}_3, \wedge_3), (\tilde{G}_4, \wedge_3)\}$ is FHSts.

Let $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ be a FHS mapping as follows:

$$\begin{gathered} \eta(m_1) = n_2, \quad \eta(m_2) = n_1, \\ \zeta(a_1, b_1) = (c_2, d_1), \ \zeta(a_2, b_1) = (c_1, d_2), \ \zeta(a_1, b_2) = (c_2, d_2), \\ \mathfrak{k}(\tilde{H}_1, \wedge_1) = (\tilde{G}_1, \wedge_1)^c, \ \mathfrak{k}(\tilde{H}_2, \wedge_2) = (\tilde{G}_2, \wedge_2)^c, \\ \mathfrak{k}(\tilde{H}_3, \wedge_3) = (\tilde{G}_3, \wedge_3)^c, \ \mathfrak{k}(\tilde{H}_4, \wedge_3) = (\tilde{G}_4, \wedge_3)^c, \end{gathered}$$

$\therefore \mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ is FHS contra open.

Example 3.6. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the FHS initial universes and the attributes be $Q = Q_1 \times Q_2$ and $R = Q_1' \times Q_2'$, respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\quad Q_2 = \{b_1, b_2\},\quad Q_1' = \{c_1, c_2, c_3\},\quad Q_2' = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, Q)$ and $(\mathfrak{N}, R)$ be the classes of FHS sets. Let the FHSs $(\tilde{H}_1, \wedge_1)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{m_1}{0.5}, \frac{m_2}{0.2}\}\rangle, \\ \langle (a_2, b_1), \{\frac{m_1}{0.5}, \frac{m_2}{0.5}\}\rangle \end{Bmatrix},$$

$\tau_1 = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1)\}$ is FHSts. Let the FHSs$(\tilde{G}_1, \wedge_1)$, $(\tilde{G}_2, \wedge_2)$, $(\tilde{G}_3, \wedge_3)$, $(\tilde{G}_4, \wedge_3)$, $(\tilde{G}_5, \wedge_1)$ and $(\tilde{G}_6, \wedge_2)$ be defined as

$$(\tilde{G}_1, \wedge_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{n_1}{0.2}, \frac{n_2}{0.4}\}\rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.3}, \frac{n_2}{0.5}\}\rangle \end{Bmatrix},\quad (\tilde{G}_2, \wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{n_1}{0.8}, \frac{n_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.5}\}\rangle \end{Bmatrix},\quad (\tilde{G}_3, \wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{n_1}{0.8}, \frac{n_2}{0.7}\}\rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.3}, \frac{n_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.5}\}\rangle \end{Bmatrix},$$

$$(\tilde{G}_4, \wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{n_1}{0.2}, \frac{n_2}{0.4}\}\rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.3}, \frac{n_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.5}\}\rangle \end{Bmatrix},\quad (\tilde{G}_5, \wedge_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{n_1}{0.8}, \frac{n_2}{0.5}\}\rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.5}\}\rangle \end{Bmatrix},\quad (\tilde{G}_6, \wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{n_1}{0.4}, \frac{n_2}{0.6}\}\rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.4}\}\rangle \end{Bmatrix},$$

$\tau_2 = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_2), (\tilde{G}_3, \wedge_3)$ and $(\tilde{G}_4, \wedge_3)\}$ is FHSts.

Let $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ be a FHS mapping as follows:

$$\eta(m_1) = n_2, \eta(m_2) = n_1, \zeta(a_1, b_1) = (c_2, d_1), \zeta(a_2, b_1) = (c_1, d_2),\ \zeta(a_1, b_2) = (c_2, d_2),\ \mathfrak{k}(\tilde{H}_1, \wedge_1) = (\tilde{G}_5, \wedge_1)^c,$$

$(\tilde{H}_1, \wedge_1)$ is FHSos in $\mathfrak{M}$ and $\mathfrak{k}(\tilde{H}_1, \wedge_1) = (\tilde{G}_5, \wedge_1)^c$ is FHSScs in $\mathfrak{N}$. $\therefore$ $\mathfrak{k}$ is FHS contra semi open.

Theorem 3.7. *Each* FHS *contra open mapping is a* FHS *contra semi open mapping. But the converse need not to be true.*

Proof. Let $(\tilde{H}, \wedge)$ be a FHSos in $\mathfrak{M}$. Since $\mathfrak{k}$ is FHS contra open mapping, $\mathfrak{k}(\tilde{H}, \wedge)$ is a FHScs in $\mathfrak{N}$. Since every FHScs is a FHSScs [4], $\mathfrak{k}(\tilde{H}, \wedge)$ is a FHSScs in $\mathfrak{N}$. Hence $\mathfrak{k}$ is a FHS contra semi open mapping. □

Example 3.8. In Example 3.6, $\mathfrak{k}$ is FHS contra semi open but not FHS contra open because $(\tilde{H}_1, \wedge_1)$ is FHSos in $\mathfrak{M}$ but $\mathfrak{k}(\tilde{H}_1, \wedge_1) = (\tilde{G}_5, \wedge_1)^c$ is not FHScs in $\mathfrak{N}$.

Theorem 3.9. *A mapping* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *is* FHS *contra open iff for every* FHSs $(\tilde{H}, \wedge)$ *of* $(\mathfrak{M}, Q, \tau_1)$, $\mathfrak{k}(\mathrm{FHSint}(\tilde{H}, \wedge)) \supseteq \mathrm{FHScl}(\mathfrak{k}(\tilde{H}, \wedge))$.

Proof.

Necessity: Let $\mathfrak{k}$ be a FHS contra open mapping and $(\tilde{H}, \wedge)$ be a FHSos in $(\mathfrak{M}, Q, \tau_1)$. Now, $\mathrm{FHSint}(\tilde{H}, \wedge) \subseteq (\tilde{H}, \wedge)$ implies $\mathfrak{k}(\mathrm{FHSint}(\tilde{H}, \wedge)) \subseteq \mathfrak{k}(\tilde{H}, \wedge)$. Since, $\mathfrak{k}$ is a FHS contra open mapping, $\mathfrak{k}(\mathrm{FHSint}(\tilde{H}, \wedge))$ is FHScs in $(\mathfrak{N}, R, \tau_2)$ such that $\mathfrak{k}(\mathrm{FHSint}(\tilde{H}, \wedge)) \supseteq \mathfrak{k}(\tilde{H}, \wedge)$. Therefore, $\mathfrak{k}(\mathrm{FHSint}(\tilde{H}, \wedge)) \supseteq \mathrm{FHScl}(\mathfrak{k}(\tilde{H}, \wedge))$.

Sufficiency: Assume $(\tilde{H}, \wedge)$ is a FHSos of $(\mathfrak{M}, Q, \tau_1)$. Then $\mathfrak{k}(\tilde{H}, \wedge) = \mathfrak{k}(\mathrm{FHSint}(\tilde{H}, \wedge)) \supseteq \mathrm{FHScl}(\mathfrak{k}(\tilde{H}, \wedge))$. But $\mathrm{FHScl}(\mathfrak{k}(\tilde{H}, \wedge)) \supseteq \mathfrak{k}(\tilde{H}, \wedge)$. So, $\mathfrak{k}(\tilde{H}, \wedge) = \mathrm{FHScl}(\mathfrak{k}(\tilde{H}, \wedge))$ which implies $\mathfrak{k}(\tilde{H}, \wedge)$ is a FHScs of $(\mathfrak{N}, R, \tau_2)$ and hence $\mathfrak{k}$ is a FHS contra open mapping. □

Theorem 3.10. *If* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *is a* FHS *contra open mapping, then* $\mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))$ *for every* FHSs $(\tilde{G}, \wedge)$ *of* $(\mathfrak{N}, R, \tau_2)$.

Proof. Let $(\tilde{G}, \wedge)$ be a FHSs of $(\mathfrak{N}, R, \tau_2)$. Then, $\mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))$ is a FHSos in $(\mathfrak{M}, Q, \tau_1)$. Since $\mathfrak{k}$ is FHS contra open map, $\mathfrak{k}(\mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)))$ is FHScs in $(\mathfrak{N}, R, \tau_2)$ and hence $\mathfrak{k}(\mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))) \subseteq \mathrm{FHScl}(\mathfrak{k}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))) \subseteq \mathrm{FHScl}(\tilde{G}, \wedge)$. Thus, $\mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))$. □

Theorem 3.11. *A mapping* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *is* FHS *contra semi open (resp.* FHS *contra open) iff for each* FHSs $(\tilde{G}, \wedge)$ *of* $(\mathfrak{N}, R, \tau_2)$ *and for each* FHSos$(\tilde{H}, \wedge)$ *of* $(\mathfrak{M}, Q, \tau_1)$ *containing* $\mathfrak{k}^{-1}(\tilde{G}, \wedge)$*, there is a* FHSSos *(resp.* FHSos*)* $(\tilde{A}, \wedge)$ *of* $(\mathfrak{N}, R, \tau_2)$ *such that* $(\tilde{G}, \wedge) \subseteq (\tilde{A}, \wedge)$ *and* $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \subseteq (\tilde{H}, \wedge)$.

Proof.

Necessity: Assume $\mathfrak{k}$ is a FHS contra semi open mapping. Let $(\tilde{G}, \wedge)$ be the FHSs of $(\mathfrak{N}, R, \tau_2)$ and $(\tilde{H}, \wedge)$ is a FHSos of $(\mathfrak{M}, Q, \tau_1)$ such that $\mathfrak{k}^{-1}(\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$. Then, $(\tilde{A}, \wedge) = (\mathfrak{k}^{-1}(\tilde{H}, \wedge)^c)^c$ is FHSSos (resp. FHSos) of $(\mathfrak{N}, R, \tau_2)$ such that $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \subseteq (\tilde{H}, \wedge)$.

Sufficiency: Assume $(\tilde{H}, \wedge)$ is a FHSos of $(\mathfrak{M}, Q, \tau_1)$. Then, $\mathfrak{k}^{-1}((\mathfrak{k}(\tilde{H}, \wedge))^c) \subseteq (\tilde{H}, \wedge)^c$ and $(\tilde{H}, \wedge)^c$ is FHScs in $(\mathfrak{M}, Q, \tau_1)$. By hypothesis, there is a FHSSos (resp. FHSos) $(\tilde{A}, \wedge)$ of $(\mathfrak{N}, R, \tau_2)$ such that $(\mathfrak{k}(\tilde{H}, \wedge))^c \subseteq (\tilde{A}, \wedge)$ and $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \subseteq (\tilde{H}, \wedge)^c$. Therefore, $(\tilde{H}, \wedge) \subseteq (\mathfrak{k}^{-1}(\tilde{A}, \wedge))^c$. Hence, $(\tilde{A}, \wedge)^c \subseteq \mathfrak{k}(\tilde{H}, \wedge) \subseteq \mathfrak{k}((\mathfrak{k}^{-1}(\tilde{A}, \wedge))^c) \subseteq (\tilde{A}, \wedge)^c$ which implies $\mathfrak{k}(\tilde{H}, \wedge) = (\tilde{A}, \wedge)^c$. Since, $(\tilde{A}, \wedge)^c$ is FHSScs (resp. FHScs) of $(\mathfrak{N}, R, \tau_2)$, $\mathfrak{k}(\tilde{H}, \wedge)$ is FHSScs (resp. FHScs)in $(\mathfrak{N}, R, \tau_2)$ and thus $\mathfrak{k}$ is FHS contra semi open (resp. FHS contra open) mapping. □

Theorem 3.12. *A mapping $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is FHS contra semi open (resp. FHSO) iff*

$$\mathfrak{k}^{-1}(\mathrm{FHSScl}(\tilde{G}, \wedge)) \supseteq \mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))(\textit{resp.}\mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge)) \supseteq \mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)))$$

for every FHSs$(\tilde{G}, \wedge)$ *of* $(\mathfrak{N}, R, \tau_2)$.

Proof.

Necessity: Assume $\mathfrak{k}$ is a FHS contra semi open (resp. FHS contra open) mapping. For any FHSs$(\tilde{G}, \wedge)$ of $(\mathfrak{N}, R, \tau_2)$, $\mathfrak{k}^{-1}(\tilde{G}, \wedge) \subseteq \mathrm{FHScl}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))$. Therefore, by Theorem 3.11, there exits a FHSSos (resp. FHSos) $(\tilde{A}, \wedge)$ in $(\mathfrak{N}, R, \tau_2)$ such that $(\tilde{G}, \wedge) \supseteq (\tilde{A}, \wedge)$ and $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \supseteq \mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))$. Therefore, we obtain that $\mathfrak{k}^{-1}(\mathrm{FHSScl}(\tilde{G}, \wedge)) \supseteq \mathfrak{k}^{-1}(\tilde{A}, \wedge) \supseteq \mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))$ (resp. $\mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge)) \supseteq \mathfrak{k}^{-1}(\tilde{A}, \wedge) \supseteq \mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge))$).

Sufficiency: Assume $(\tilde{G}, \wedge)$ is a FHSs of $(\mathfrak{N}, R, \tau_2)$ and $(\tilde{H}, \wedge)$ is a FHScs of $(\mathfrak{M}, Q, \tau_1)$ containing $\mathfrak{k}^{-1}(\tilde{G}, \wedge)$. Put $(\tilde{A}, \wedge) = \mathrm{FHScl}(\tilde{G}, \wedge)$. Then, $(\tilde{G}, \wedge) \subseteq (\tilde{A}, \wedge)$ and $(\tilde{A}, \wedge)$ is FHSScs (resp. FHScs) and $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \subseteq \mathrm{FHSint}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq (\tilde{H}, \wedge)$. Then, by Theorem 3.11, $\mathfrak{k}$ is FHS contra semi open (resp. FHS contra open) mapping. □

Theorem 3.13. *If $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ and $\mathfrak{g} : (\mathfrak{N}, R, \tau_2) \to (\mathfrak{O}, S, \tau_3)$ be two* FHS *mappings and $\mathfrak{g} \circ \mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{O}, S, \tau_3)$ is* FHS *contra open mapping, and if $\mathfrak{g} : (\mathfrak{N}, R, \tau_2) \to (\mathfrak{O}, S, \tau_3)$ is* FHS *contra irresolute, then $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is* FHS *semi open mapping.*

Proof. Let $(\tilde{H}, \wedge)$ be a FHSos in $(\mathfrak{M}, Q, \tau_1)$. Then, $\mathfrak{g} \circ \mathfrak{k}(\tilde{H}, \wedge)$ is FHScs of $(\mathfrak{O}, S, \tau_3)$ because $\mathfrak{g} \circ \mathfrak{k}$ is FHS contra open mapping. Since, every FHScs is a FHSScs, $\mathfrak{g} \circ \mathfrak{k}(\tilde{H}, \wedge)$ is a FHSScs of $(\mathfrak{O}, S, \tau_3)$. Since, $\mathfrak{g}$ is FHS contra irresolute, $\mathfrak{g}^{-1}(\mathfrak{g} \circ \mathfrak{k}(\tilde{H}, \wedge)) = \mathfrak{k}(\tilde{H}, \wedge)$ is FHSSos in $(\mathfrak{N}, R, \tau_2)$. Hence, $\mathfrak{k}$ is FHS semi open mapping. □

Theorem 3.14. *If $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is* FHS *open and $\mathfrak{g} : (\mathfrak{N}, R, \tau_2) \to (\mathfrak{O}, S, \tau_3)$ is* FHS *contra semi open (resp.* FHS *contra open) mappings, then $\mathfrak{g} \circ \mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{O}, S, \tau_3)$ is* FHS *contra semi open (resp.* FHS *contra open) map.*

Proof. Let $(\tilde{H}, \wedge)$ be a FHSos in $(\mathfrak{M}, Q, \tau_1)$, then $\mathfrak{k}(\tilde{H}, \wedge)$ is a FHSos of $(\mathfrak{N}, R, \tau_2)$ because $\mathfrak{k}$ is a FHS open mapping. Since $\mathfrak{g}$ is a FHS contra semi open (resp. FHS contra open), $\mathfrak{g}(\mathfrak{k}(\tilde{H}, \wedge)) = (\mathfrak{g} \circ \mathfrak{k})(\tilde{H}, \wedge)$ is a FHSScs (resp. FHScs) of $(\mathfrak{O}, S, \tau_3)$. Hence, $\mathfrak{g} \circ \mathfrak{k}$ is FHS contra semi open (resp. FHS contra open) mapping. □

4. Fuzzy hypersoft contra closed mapping

In this section, fuzzy hypersoft contra closed maps are introduced and its characteristics atre studied.

Definition 4.1. A mapping $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called fuzzy hypersoft

(i) closed (in short, FHS closed) if the image of every FHScs of $(\mathfrak{M}, Q, \tau_1)$ is FHScs in $(\mathfrak{N}, R, \tau_2)$;

(ii) semi closed (in short, FHS semi closed) if the image of every FHScs of $(\mathfrak{M}, Q, \tau_1)$ is FHS$\mathcal{S}$cs in $(\mathfrak{N}, R, \tau_2)$.

Definition 4.2. A mapping $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called fuzzy hypersoft

(i) contra closed (in short, FHS contra closed) if the image of every FHScs of $(\mathfrak{M}, Q, \tau_1)$ is FHSos in $(\mathfrak{N}, R, \tau_2)$;

(ii) contra semi closed (in short, FHS contra semi closed) if the image of every FHScs of $(\mathfrak{M}, Q, \tau_1)$ is FHS$\mathcal{S}$os in $(\mathfrak{N}, R, \tau_2)$.

Example 4.3. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the FHS initial universes and the attributes be $Q = Q_1 \times Q_2$ and $R = Q_1' \times Q_2'$, respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, \quad Q_2 = \{b_1, b_2\}, \quad Q_1' = \{c_1, c_2, c_3\}, \quad Q_2' = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, Q)$ and $(\mathfrak{N}, R)$ be the classes of FHS sets. Let the FHSs's $(\tilde{H_1}, \wedge_1)$, $(\tilde{H_2}, \wedge_2)$, $(\tilde{H_3}, \wedge_3)$, and $(\tilde{H_4}, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H_1}, \wedge_1) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle \end{array} \right\}, \qquad (\tilde{H_2}, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \end{array} \right\},$$

$$(\tilde{H_3}, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \end{array} \right\}, \qquad (\tilde{H_4}, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \end{array} \right\},$$

$\tau_1 = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H_1}, \wedge_1), (\tilde{H_2}, \wedge_2), (\tilde{H_3}, \wedge_3), (\tilde{H_4}, \wedge_3)\}$ is FHSts.

Let the FHSs's $(\tilde{G_1}, \wedge_1)$, $(\tilde{G_2}, \wedge_2)$, $(\tilde{G_3}, \wedge_3)$ and $(\tilde{G_4}, \wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G_1}, \wedge_1) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{n_1}{0.4}, \frac{n_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle \end{array} \right\}, \qquad (\tilde{G_2}, \wedge_2) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \end{array} \right\},$$

$$(\tilde{G_3}, \wedge_3) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \end{array} \right\}, \qquad (\tilde{G_4}, \wedge_3) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{n_1}{0.4}, \frac{n_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \end{array} \right\},$$

$\tau_2 = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G_1}, \wedge_1), (\tilde{G_2}, \wedge_2), (\tilde{G_3}, \wedge_3), (\tilde{G_4}, \wedge_3)\}$ is FHSts.

Let $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ be a FHS mapping as follows:

$$\begin{gathered} \eta(m_1) = n_2, \quad \eta(m_2) = n_1, \\ \zeta(a_1, b_1) = (c_2, d_1), \quad \zeta(a_2, b_1) = (c_1, d_2), \quad \zeta(a_1, b_2) = (c_2, d_2), \\ \mathfrak{k}(\tilde{H_1}, \wedge_1)^c = (\tilde{G_1}, \wedge_1), \quad \mathfrak{k}(\tilde{H_2}, \wedge_2)^c = (\tilde{G_2}, \wedge_2), \\ \mathfrak{k}(\tilde{H_3}, \wedge_3)^c = (\tilde{G_3}, \wedge_3), \quad \mathfrak{k}(\tilde{H_4}, \wedge_3)^c = (\tilde{G_4}, \wedge_3), \end{gathered}$$

$\therefore \mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ is FHS contra closed.

Example 4.4. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the FHS initial universes and the attributes be $Q = Q_1 \times Q_2$ and $R = Q_1' \times Q_2'$, respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, \quad Q_2 = \{b_1, b_2\}, \quad Q_1' = \{c_1, c_2, c_3\}, \quad Q_2' = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, Q)$ and $(\mathfrak{N}, R)$ be the classes of FHS sets. Let the FHSs $(\tilde{H_1}, \wedge_1)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H_1}, \wedge_1) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{m_1}{0.5}, \frac{m_2}{0.2}\} \rangle, \\ \langle (a_2, b_1), \{\frac{m_1}{0.5}, \frac{m_2}{0.5}\} \rangle \end{array} \right\},$$

$\tau_1 = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1)\}$ is FHSts. Let the FHSs's $(\tilde{G}_1, \wedge_1)$, $(\tilde{G}_2, \wedge_2)$, $(\tilde{G}_3, \wedge_3)$, $(\tilde{G}_4, \wedge_3)$, $(\tilde{G}_5, \wedge_1)$, and $(\tilde{G}_6, \wedge_2)$ be defined as

$$(\tilde{G}_1, \wedge_1) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.2}, \frac{\mathfrak{n}_2}{0.4}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{matrix} \right\}, \quad (\tilde{G}_2, \wedge_2) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.8}, \frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{matrix} \right\}, \quad (\tilde{G}_3, \wedge_3) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.8}, \frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{matrix} \right\},$$

$$(\tilde{G}_4, \wedge_3) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.2}, \frac{\mathfrak{n}_2}{0.4}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{matrix} \right\}, \quad (\tilde{G}_5, \wedge_1) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.8}, \frac{\mathfrak{n}_2}{0.5}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{matrix} \right\}, \quad (\tilde{G}_6, \wedge_2) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.7}, \frac{\mathfrak{n}_2}{0.6}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.4}\}\rangle \end{matrix} \right\},$$

$\tau_2 = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_2), (\tilde{G}_3, \wedge_3), (\tilde{G}_4, \wedge_3)\}$ is FHSts.

Let $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ be a FHS mapping as follows:

$$\eta(\mathfrak{m}_1) = \mathfrak{n}_2, \eta(\mathfrak{m}_2) = \mathfrak{n}_1, \zeta(a_1, b_1) = (c_2, d_1), \zeta(a_2, b_1) = (c_1, d_2), \ \zeta(a_1, b_2) = (c_2, d_2), \mathfrak{k}(\tilde{H}_1, \wedge_1)^c = (\tilde{G}_5, \wedge_1),$$

$(\tilde{H}_1, \wedge_1)^c$ is FHScs in $\mathfrak{M}$ and $\mathfrak{k}(\tilde{H}_1, \wedge_1)^c = (\tilde{G}_5, \wedge_1)$ is FHSSos in $\mathfrak{N}$. $\therefore$ $\mathfrak{k}$ is FHS contra semi closed.

Theorem 4.5. *Every* FHS *contra closed mapping is a* FHS *contra semi closed mapping. But the converse need not be true.*

Proof. Let $(\tilde{H}, \wedge)$ be a FHScs in $\mathfrak{M}$. Since $\mathfrak{k}$ is a FHS contra closed mapping, $\mathfrak{k}(\tilde{H}, \wedge)$ is a FHSos in $\mathfrak{N}$. Since every FHSos is a FHSSos, $\mathfrak{k}(\tilde{H}, \wedge)$ is a FHSSos in $\mathfrak{N}$. Hence, $\mathfrak{k}$ is a FHS contra semi closed mapping. □

Example 4.6. In Example 4.4, $\mathfrak{k}$ is FHS contra semi closed but not FHS contra closed because $(\tilde{H}_1, \wedge_1)^c$ is FHScs in $\mathfrak{M}$ but $\mathfrak{k}(\tilde{H}_1, \wedge_1)^c = (\tilde{G}_5, \wedge_1)$ is not FHSos in $\mathfrak{N}$.

Theorem 4.7. *A mapping* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *is* FHS *contra semi closed (resp.* FHS *contra closed) iff for each* FHSs $(\tilde{G}, \wedge)$ *of* $(\mathfrak{N}, R, \tau_2)$ *and for each* FHScs$(\tilde{H}, \wedge)$ *of* $(\mathfrak{M}, Q, \tau_1)$ *containing* $\mathfrak{k}^{-1}(\tilde{G}, \wedge)$*, there is a* FHSScs *(resp.* FHScs*)* $(\tilde{A}, \wedge)$ *of* $(\mathfrak{N}, R, \tau_2)$ *such that* $(\tilde{G}, \wedge) \subseteq (\tilde{A}, \wedge)$ *and* $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \subseteq (\tilde{H}, \wedge)$.

Proof.

Necessity: Assume $\mathfrak{k}$ is a FHS contra semi closed (resp. FHS contra closed) mapping. Let $(\tilde{G}, \wedge)$ be the FHSs of $(\mathfrak{N}, R, \tau_2)$ and $(\tilde{H}, \wedge)$ is a FHScs of $(\mathfrak{M}, Q, \tau_1)$ such that $\mathfrak{k}^{-1}(\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$. Then, $(\tilde{A}, \wedge) = \tilde{1}_{(\mathfrak{N},Q)} - \mathfrak{k}^{-1}((\tilde{H}, \wedge)^c)$ is FHSScs (resp. FHScs) of $(\mathfrak{N}, R, \tau_2)$ such that $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \subseteq (\tilde{H}, \wedge)$.

Sufficiency: Assume $(\tilde{H}, \wedge)$ is a FHScs of $(\mathfrak{M}, Q, \tau_1)$. Then, $(\mathfrak{k}(\tilde{H}, \wedge))^c$ is a FHSs of $(\mathfrak{N}, R, \tau_2)$ and $(\tilde{H}, \wedge)^c$ is FHSos in $(\mathfrak{M}, Q, \tau_1)$ such that $\mathfrak{k}^{-1}((\mathfrak{k}(\tilde{H}, \wedge))^c) \subseteq (\tilde{H}, \wedge)^c$. By hypothesis, there is a FHSScs (resp. FHScs) $(\tilde{A}, \wedge)$ of $(\mathfrak{N}, R, \tau_2)$ such that $(\mathfrak{k}(\tilde{H}, \wedge))^c \subseteq (\tilde{A}, \wedge)$ and $\mathfrak{k}^{-1}(\tilde{A}, \wedge) \subseteq (\tilde{H}, \wedge)^c$. Therefore, $(\tilde{H}, \wedge) \subseteq (\mathfrak{k}^{-1}(\tilde{A}, \wedge))^c$. Hence, $(\tilde{A}, \wedge)^c \subseteq \mathfrak{k}(\tilde{H}, \wedge) \subseteq \mathfrak{k}(\mathfrak{k}^{-1}(\tilde{A}, \wedge))^c \subseteq (\tilde{A}, \wedge)^c$ which implies $\mathfrak{k}(\tilde{H}, \wedge) = (\tilde{A}, \wedge)^c$. Since, $(\tilde{A}, \wedge)^c$ is FHSSos (resp. FHSos) of $(\mathfrak{N}, R, \tau_2)$, $\mathfrak{k}(\tilde{H}, \wedge)$ is FHSSos (resp. FHSos) in $(\mathfrak{N}, R, \tau_2)$ and thus $\mathfrak{k}$ is FHS contra semi closed (resp. FHS contra closed) mapping. □

Theorem 4.8. *If* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *is* FHS *closed and* $\mathfrak{g} : (\mathfrak{N}, R, \tau_2) \to (\mathfrak{O}, S, \tau_3)$ *is* FHS *contra semi closed (resp.* FHS *contra closed) mappings, then* $\mathfrak{g} \circ \mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{O}, S, \tau_3)$ *is* FHS *contra semi closed (resp.* FHS *contra closed) map.*

Proof. Let $(\tilde{H}, \wedge)$ be a FHScs in $(\mathfrak{M}, Q, \tau_1)$ then $\mathfrak{k}(\tilde{H}, \wedge)$ is a FHScs of $(\mathfrak{N}, R, \tau_2)$ because $\mathfrak{k}$ is a FHS closed mapping. Since $\mathfrak{g}$ is a FHS contra semi closed (resp. FHS contra closed), $\mathfrak{g}(\mathfrak{k}(\tilde{H}, \wedge)) = (\mathfrak{g} \circ \mathfrak{k})(\tilde{H}, \wedge)$ is a FHSSos (resp. FHSos) of $(\mathfrak{O}, S, \tau_3)$. Hence, $\mathfrak{g} \circ \mathfrak{k}$ is FHS contra semi closed (resp. FHS contra closed) mapping. □

Theorem 4.9. *If* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *is*

(i) FHS *contra closed map, then* $\mathrm{FHSint}(\mathfrak{k}(\tilde{H}, \wedge)) \supseteq \mathfrak{k}(\mathrm{FHSint}(\tilde{H}, \wedge))$;

(ii) FHS *contra semi closed map, then* $\mathsf{FHSSint}(\mathfrak{k}(\tilde{H},\wedge)) \supseteq \mathfrak{k}(\mathsf{FHSint}(\tilde{H},\wedge))$.

Proof. Obvious. □

Theorem 4.10. *Let* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *and* $\mathfrak{g} : (\mathfrak{N}, R, \tau_2) \to (\mathfrak{O}, S, \tau_3)$ *be* FHS *contra semi closed (resp.* FHS *contra closed) mappings. If every* FHSSos *of* $(\mathfrak{N}, R, \tau_2)$ *is* FHSos*, then* $\mathfrak{g} \circ \mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{O}, S, \tau_3)$ *is* FHS *semi closed (resp.* FHS *closed).*

Proof. Let $(\tilde{H},\wedge)$ be a FHScs in $(\mathfrak{M}, Q, \tau_1)$. Then, $\mathfrak{k}(\tilde{H},\wedge)$ is FHSSos (resp. FHSos) of $(\mathfrak{N}, R, \tau_2)$ because $\mathfrak{k}$ is FHS contra semi closed (resp. FHS contra closed) mapping. By hypothesis, $\mathfrak{k}(\tilde{H},\wedge)$ is FHSos of $(\mathfrak{N}, R, \tau_2)$. Now $\mathfrak{g}(\mathfrak{k}(\tilde{H},\wedge)) = (\mathfrak{g} \circ \mathfrak{k})(\tilde{H},\wedge)$ is FHSScs (resp. FHScs) in $(\mathfrak{O}, S, \tau_3)$ because $\mathfrak{g}$ is FHS contra semi closed (resp. FHS contra closed) mapping. Thus, $\mathfrak{g} \circ \mathfrak{k}$ is FHS semi closed (resp. FHS closed) mapping. □

Theorem 4.11. *Let* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *be a bijective mapping. Then the following statements are equivalent.*

(i) $\mathfrak{k}$ *is a* FHS *contra semi open (resp.* FHS *contra open) mapping.*
(ii) $\mathfrak{k}$ *is a* FHS *contra semi closed (resp.* FHS *contra closed) mapping.*
(iii) $\mathfrak{k}$ *is a* FHS *contra semi continuous (resp.* FHS *contra continuous) mapping.*

Proof.

(i) ⇒ (ii): Let us assume that $\mathfrak{k}$ is a FHS contra semi open (resp. FHS contra open) mapping. By definition, if $(\tilde{H},\wedge)$ is a FHSos in $(\mathfrak{M}, Q, \tau_1)$, then $\mathfrak{k}(\tilde{H},\wedge)$ is a FHSSos (resp. FHSos) in $(\mathfrak{N}, R, \tau_2)$. Here, $(\tilde{H},\wedge)$ is FHScs in $(\mathfrak{M}, Q, \tau_1)$. Then $\tilde{1}_{(\mathfrak{M},Q)} - (\tilde{H},\wedge)$ is a FHSos in $(\mathfrak{M}, Q, \tau_1)$. By assumption, $\mathfrak{k}(\tilde{1}_{(\mathfrak{M},Q)} - (\tilde{H},\wedge))$ is a FHSSos (resp.FHSos) in $(\mathfrak{N}, R, \tau_2)$. Hence, $\tilde{1}_{(\mathfrak{N},Q)} - \mathfrak{k}(\tilde{1}_{(\mathfrak{M},Q)} - (\tilde{H},\wedge))$ is a FHSSos (resp. FHSos) in $(\mathfrak{N}, R, \tau_2)$. Therefore, $\mathfrak{k}$ is a FHS semi closed (resp. FHS closed) mapping.

(ii) ⇒ (iii): Let $(\tilde{H},\wedge)$ be a FHScs in $(\mathfrak{M}, Q, \tau_1)$. By (ii), $\mathfrak{k}(\tilde{H},\wedge)$ is a FHSSos (resp. FHSos) in $(\mathfrak{N}, R, \tau_2)$. Hence, $\mathfrak{k}(\tilde{H},\wedge) = (\mathfrak{k}^{-1})^{-1}(\tilde{H},\wedge)$. So $\mathfrak{k}^{-1}$ is a FHSSos (resp. FHSos) in $(\mathfrak{N}, R, \tau_2)$. Hence, $\mathfrak{k}^{-1}$ is FHS semi continuous (resp. FHS continuous).

(iii) ⇒ (i): Let $(\tilde{H},\wedge)$ be a FHSos in $(\mathfrak{M}, Q, \tau_1)$. By (iii), $(\mathfrak{k}^{-1})^{-1}(\tilde{H},\wedge) = \mathfrak{k}(\tilde{H},\wedge)$ is a FHS semi open (resp. FHS open) mapping. □

5. Fuzzy hypersoft contra homeomorphism

Fuzzy hypersoft contra homeomorphism is introduced and studied in this section.

Definition 5.1. A bijection $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called a fuzzy hypersoft contra homeomorphism (in short, FHS contra homeomorphism) (resp. fuzzy hypersoft contra semi homeomorphism (in short, FHS contra semi homeomorphism)) if $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra continuous (resp. FHS contra semi continuous) mappings.

Example 5.2. In Example 4.3,

$$\mathfrak{k}(\tilde{H_1},\wedge_1) = (\tilde{G_1},\wedge_1)^c, \ \ \mathfrak{k}(\tilde{H_2},\wedge_2) = (\tilde{G_2},\wedge_2)^c, \ \ \mathfrak{k}(\tilde{H_3},\wedge_3) = (\tilde{G_3},\wedge_3)^c, \ \ \mathfrak{k}(\tilde{H_4},\wedge_3) = (\tilde{G_4},\wedge_3)^c$$

and

$$\mathfrak{k}^{-1}(\tilde{G_1},\wedge_1) = (\tilde{H_1},\wedge_1)^c, \ \ \mathfrak{k}^{-1}(\tilde{G_2},\wedge_2) = (\tilde{H_2},\wedge_2)^c, \ \ \mathfrak{k}^{-1}(\tilde{G_3},\wedge_3) = (\tilde{H_3},\wedge_3)^c, \ \ \mathfrak{k}^{-1}(\tilde{G_4},\wedge_3) = (\tilde{H_4},\wedge_3)^c.$$

Hence $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra continuous. ∴ $\mathfrak{k}$ is FHS contra homeomorphism.

Theorem 5.3. *Each* FHS *contra homeomorphism is a* FHS *contra semi homeomorphism. But not conversely.*

Proof. Let $\mathfrak{k}$ be FHS contra homeomorphism, then $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra continuous. But every FHS contra continuous function is FHS contra semi continuous. Hence, $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra semi continuous. Therefore, $\mathfrak{k}$ is a FHS contra semi homeomorphism. □

Example 5.4. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $Q = Q_1 \times Q_2$ and $R = Q_1' \times Q_2'$, respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, \quad Q_2 = \{b_1, b_2\}, \quad Q_1' = \{c_1, c_2, c_3\}, \quad Q_2' = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, Q)$ and $(\mathfrak{N}, R)$ be the classes of FHS sets. Let the FHSs's $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_3)$, and $(\tilde{H}_5, \wedge_1)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{array} \right\}, \quad (\tilde{H}_2, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{array} \right\}, \quad (\tilde{H}_3, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{array} \right\},$$

$$(\tilde{H}_4, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{array} \right\}, \quad (\tilde{H}_5, \wedge_1) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{array} \right\},$$

$\tau_1 = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is FHSts.

Let the FHSs's $(\tilde{G}_1, \wedge_1)$, $(\tilde{G}_2, \wedge_1)$, $(\tilde{G}_3, \wedge_2)$, $(\tilde{G}_4, \wedge_3)$, and $(\tilde{G}_5, \wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1, \wedge_1) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.7}, \frac{\mathfrak{n}_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.8}\} \rangle \end{array} \right\}, \quad (\tilde{G}_2, \wedge_1) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\} \rangle \end{array} \right\}, \quad (\tilde{G}_3, \wedge_2) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.7}, \frac{\mathfrak{n}_2}{0.8}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{array} \right\},$$

$$(\tilde{G}_4, \wedge_3) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.7}, \frac{\mathfrak{n}_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{array} \right\}, \quad (\tilde{G}_5, \wedge_3) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{array} \right\},$$

$\tau_2 = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1)\}$ is FHSts.

Let $\mathfrak{k} = (\eta, \zeta) : (\mathfrak{M}, Q) \to (\mathfrak{N}, R)$ be a FHS mapping as follows:

$$\begin{gathered} \eta(\mathfrak{m}_1) = \mathfrak{n}_2, \quad \eta(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \zeta(a_1, b_1) = (c_2, d_1), \quad \zeta(a_2, b_1) = (c_1, d_2), \quad \zeta(a_1, b_2) = (c_2, d_2), \\ \mathfrak{k}(\tilde{H}_1, \wedge_1) = (\tilde{G}_2, \wedge_1)^c, \quad \mathfrak{k}(\tilde{H}_2, \wedge_2) = (\tilde{G}_3, \wedge_2)^c, \quad \mathfrak{k}(\tilde{H}_3, \wedge_3) = (\tilde{G}_4, \wedge_3)^c, \\ \mathfrak{k}(\tilde{H}_4, \wedge_3) = (\tilde{G}_5, \wedge_3)^c, \quad \mathfrak{k}(\tilde{H}_5, \wedge_1) = (\tilde{G}_1, \wedge_1)^c, \quad \mathfrak{k}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_5, \wedge_1)^c. \end{gathered}$$

Here $\mathfrak{k}^{-1}$ is FHS contra semi continuous because $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, and $(\tilde{H}_4, \wedge_3)$ are FHSos's in $\mathfrak{M}$ and $\mathfrak{k}(\tilde{H}_1, \wedge_1)$, $\mathfrak{k}(\tilde{H}_2, \wedge_2)$, $\mathfrak{k}(\tilde{H}_3, \wedge_3)$, and $\mathfrak{k}(\tilde{H}_4, \wedge_3)$ are FHS$\mathcal{S}$cs's in $\mathfrak{N}$. Also, $\mathfrak{k}$ is FHS contra semi continuous because $(\tilde{G}_1, \wedge_1)$ is FHSos in $\mathfrak{N}$ and $\mathfrak{k}^{-1}(\tilde{G}_1, \wedge_1)$ is FHS$\mathcal{S}$cs in $\mathfrak{M}$. Hence $\mathfrak{k}$ is FHS contra semi homeomorphism. But $\mathfrak{k}$ is not FHS contra homeomorphism because $(\tilde{G}_1, \wedge_1)$ is FHSos in $\mathfrak{N}$ but $\mathfrak{k}^{-1}(\tilde{G}_1, \wedge_1)$ is not FHScs in $\mathfrak{M}$.

Theorem 5.5. *Let $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ be a bijective mapping. If $\mathfrak{k}$ is* FHS *contra semi continuous (resp.* FHS *contra continuous), then the following statements are equivalent:*

(i) $\mathfrak{k}$ *is a* FHS *contra semi closed (resp.* FHS *contra closed) mapping;*
(ii) $\mathfrak{k}$ *is a* FHS *contra semi open (resp.* FHS *contra open) mapping;*
(iii) $\mathfrak{k}^{-1}$ *is a* FHS *contra semi homeomorphism (resp.* FHS *contra semi homeomorphism).*

Proof.

(i)⇒(ii): Assume that $\mathfrak{k}$ is a bijective mapping and a FHS contra semi closed (resp. FHS contra closed) mapping. Here, $\mathfrak{k}^{-1}$ is a FHS contra semi continuous (resp. FHS contra continuous) mapping. We knowthat each FHSos in $(\mathfrak{M}, Q, \tau_1)$ is a FHS$\mathcal{S}$cs (resp. FHScs) in $(\mathfrak{N}, R, \tau_2)$. Hence, $\mathfrak{k}$ is a FHS contra semi open (resp. FHS contra open) mapping.

(ii)⇒ (ii): Let $\mathfrak{k}$ be a bijective and FHS contra semi open (resp. FHS contra open) mapping. Further, $\mathfrak{k}$ is a FHS contra semi continuous (resp. FHS contra continuous) mapping. Hence, $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra semi continuous (resp. FHS contra continuous). Therefore, $\mathfrak{k}$ is a FHS contra semi homeomorphism (resp. FHS contra homeomorphism).

(iii)⇒(i): Let $\mathfrak{k}$ be a FHS contra semi homeomorphism (resp. FHS contra homeomorphism). Then $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra semi continuous (resp. FHS contra continuous). Since each FHScs in $(\mathfrak{M}, Q, \tau_1)$ is a FHS$\mathcal{S}$os in $(\mathfrak{N}, R, \tau_2)$, $\mathfrak{k}$ is a FHS contra semi closed (resp. FHS contra closed) mapping. □

Theorem 5.6. *Let $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ and $\mathfrak{g} : (\mathfrak{N}, R, \tau_2) \to (\mathfrak{O}, S, \tau_3)$ be two* FHS *mappings. Then the following hold.*

(i) *If $\mathfrak{g} \circ \mathfrak{k}$ is* FHS *contra semi open (resp.* FHS *contra open) and $\mathfrak{k}$ is* FHS *continuous, then $\mathfrak{g}$ is* FHS *contra semi open (resp.* FHS *contra open).*

(ii) *If $\mathfrak{g} \circ \mathfrak{k}$ is* FHS *open and $\mathfrak{g}$ is* FHS *contra semi continuous (resp.* FHS *contra continuous), then $\mathfrak{k}$ is* FHS *contra semi open (resp.* FHS *contra open).*

Proof. Obvious. □

6. Fuzzy hypersoft contra C-homeomorphism

Fuzzy hypersoft contra C-homeomorphism is introduced and studied in this section.

Definition 6.1. A bijection $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is called a fuzzy hypersoft contra C-homeomorphism (in short, FHS contra C-homeomorphism) if $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra irresolute mappings.

Theorem 6.2. *Each* FHS *contra C-homeomorphism is a* FHS *contra semi homeomorphism. But not conversely.*

Proof. Let us assume that $(\tilde{G}, \wedge)$ is a FHSos in $(\mathfrak{N}, R, \tau_2)$. This shows that $(\tilde{G}, \wedge)$ is a FHS$\mathcal{S}$os in $(\mathfrak{N}, R, \tau_2)$. By assumption, $\mathfrak{k}^{-1}(\tilde{G}, \wedge)$ is a FHS$\mathcal{S}$cs in $(\mathfrak{M}, Q, \tau_1)$. Hence, $\mathfrak{k}$ is a FHS contra semi continuous mapping. Therefore, $\mathfrak{k}$ and $\mathfrak{k}^{-1}$ are FHS contra semi continuous mappings. Hence, $\mathfrak{k}$ is a FHS contra semi homeomorphism. □

Example 6.3. Consider the Example 5.4. Let the FHSs $(\tilde{H_6}, \wedge_2)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H_6}, \wedge_2) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{m_1}{0.3}, \frac{m_2}{0.4}\} \rangle, \\ \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \end{matrix} \right\}.$$

Let the FHSs $(\tilde{G_6}, \wedge_2)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G_6}, \wedge_2) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{n_1}{0.6}, \frac{n_2}{0.7}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.4}, \frac{n_2}{0.5}\} \rangle \end{matrix} \right\}, \quad \mathfrak{k}(\tilde{H_6}, \wedge_2) = (\tilde{G_6}, \wedge_2)^c, \quad \text{and} \quad \mathfrak{k}^{-1}(\tilde{G_6}, \wedge_2) = (\tilde{H_6}, \wedge_2)^c.$$

Here $\mathfrak{k}$ is FHS contra semi homeomorphism but not FHS contra C-homeomorphism because $(\tilde{G_6}, \wedge_2)$ is FHS$\mathcal{S}$os in $\mathfrak{N}$ but $\mathfrak{k}^{-1}(\tilde{G_6}, \wedge_2)$ is not FHS$\mathcal{S}$cs in $\mathfrak{M}$.

Theorem 6.4. *If $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ is a* FHS *contra C-homeomorphism, then* $\mathrm{FHS\mathcal{S}int}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))$ *for each* FHSs $(\tilde{G}, \wedge)$ *in* $(\mathfrak{N}, R, \tau_2)$.

Proof. Let $(\tilde{G}, \wedge)$ be a FHSs in $(\mathfrak{N}, R, \tau_2)$. Then, $\mathrm{FHScl}(\tilde{G}, \wedge)$ is a FHScs in $(\mathfrak{N}, R, \tau_2)$ and every FHScs is a FHS$\mathcal{S}$cs in $(\mathfrak{N}, R, \tau_2)$. Assume $\mathfrak{k}$ is FHS contra irresolute and $\mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))$ is a FHS$\mathcal{S}$os in $(\mathfrak{M}, Q, \tau_1)$. Then $\mathrm{FHS\mathcal{S}int}(\mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))) = \mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))$. Here,

$$\mathrm{FHS\mathcal{S}int}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathrm{FHS\mathcal{S}int}(\mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))) = \mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge)).$$

Therefore, $\mathrm{FHS\mathcal{S}int}(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathfrak{k}^{-1}(\mathrm{FHScl}(\tilde{G}, \wedge))$ for every FHSs $(\tilde{G}, \wedge)$ in $(\mathfrak{N}, R, \tau_2)$. □

Theorem 6.5. *Let* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *be a* FHS *contra C-homeomorphism. Then* $\mathsf{FHS}\mathcal{S}int(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathfrak{k}^{-1}(\mathsf{FHS}\mathcal{S}cl(\tilde{G}, \wedge))$ *for each* FHSs $(\tilde{G}, \wedge)$ *in* $(\mathfrak{N}, R, \tau_2)$.

Proof. Since $\mathfrak{k}$ is a FHS contra C-homeomorphism, $\mathfrak{k}$ is a FHS contra irresolute mapping. Let $(\tilde{G}, \wedge)$ be a FHSs in $(\mathfrak{N}, M, \sigma)$. Clearly, $\mathsf{FHS}\mathcal{S}cl(\tilde{G}, \wedge)$ is a FHS$\mathcal{S}$cs in $(\mathfrak{N}, R, \tau_2)$. Since, $\mathfrak{k}^{-1}(\tilde{G}, \wedge) \subseteq \mathfrak{k}^{-1}(\mathsf{FHS}\mathcal{S}cl(\tilde{G}, \wedge))$, $\mathsf{FHS}\mathcal{S}int(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathsf{FHS}\mathcal{S}int(\mathfrak{k}^{-1}(\mathsf{FHS}\mathcal{S}cl(\tilde{G}, \wedge))) \subseteq \mathfrak{k}^{-1}(\mathsf{FHS}\mathcal{S}cl(\tilde{G}, \wedge))$. Hence, $\mathsf{FHS}\mathcal{S}int(\mathfrak{k}^{-1}(\tilde{G}, \wedge)) \subseteq \mathfrak{k}^{-1}(\mathsf{FHS}\mathcal{S}cl(\tilde{G}, \wedge))$. □

Theorem 6.6. *If* $\mathfrak{k} : (\mathfrak{M}, Q, \tau_1) \to (\mathfrak{N}, R, \tau_2)$ *and* $\mathfrak{g} : (\mathfrak{N}, R, \tau_2) \to (\mathfrak{O}, S, \tau_3)$ *are* FHS *contra C-homeomorphisms, then* $\mathfrak{g} \circ \mathfrak{k}$ *is* FHS *C-homeomorphism.*

Proof. Let $\mathfrak{k}$ and $\mathfrak{g}$ be two FHS contra C-homeomorphisms. Assume $(\tilde{G}, \wedge)$ is a FHS$\mathcal{S}$cs in $(\mathfrak{O}, S, \tau_3)$. Then, $\mathfrak{g}^{-1}(\tilde{G}, \wedge)$ is a FHS$\mathcal{S}$os in $(\mathfrak{N}, R, \tau_2)$. Then, by hypothesis, $\mathfrak{k}^{-1}(\mathfrak{g}^{-1}(\tilde{G}, \wedge))$ is a FHS$\mathcal{S}$cs in $(\mathfrak{M}, Q, \tau_1)$. Hence, $(\mathfrak{g} \circ \mathfrak{k})^{-1}$ is a FHS irresolute mapping. Now, let $(\tilde{H}, \wedge)$ be a FHS$\mathcal{S}$cs in $(\mathfrak{M}, Q, \tau_1)$. Then, by presumption, $\mathfrak{k}(\mathfrak{g})$ is a FHS$\mathcal{S}$os in $(\mathfrak{N}, R, \tau_2)$. Then, by hypothesis, $\mathfrak{g}(\mathfrak{k}(\tilde{H}, \wedge))$ is a FHS$\mathcal{S}$cs in $(\mathfrak{O}, S, \tau_3)$. This implies that $\mathfrak{g} \circ \mathfrak{k}$ is a FHS irresolute mapping. Hence, $\mathfrak{g} \circ \mathfrak{k}$ is a FHS C-homeomorphism. □

7. Application in Covid-19 diagnosis using Hamming distance

In this section, Hamming distance is applied in an example to diagnose Covid-19.

Definition 7.1. Consider two FHSs's $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ over $\mathfrak{M}$. The Hamming distance for these two sets is given by $d_H((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) = \sum_{i=1}^{n} |\mu_H^i - \mu_G^i|$.

Example 7.2. Consider 2 patients visiting a hospital with the following symptoms: Fever, Dry cough, Headache, Body pain, Difficulty in breathing and Chest pain. The symptoms of Covid-19 patients can be categorized as

- Severe symptoms = Difficulty in breathing, Chest pain.
- Most common symptoms = Fever, Dry cough.
- Less common symptoms = Headache, Body pain.

Using the fuzzy hypersoft model problem, the examination can be done whether the patients have the possibility of suffering from Covid-19 or not. Let $\mathfrak{M}$ be the universal set $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ = {Covid-19, No Covid-19}. The attributes are given as:

$$\begin{aligned} Q_1 &= \{a_1 = \text{Difficulty in breathing}, a_2 = \text{Chest pain}\}, \\ Q_2 &= \{b_1 = \text{Fever}, b_2 = \text{Dry cough}\}, \\ Q_3 &= \{c_1 = \text{Headache}, c_2 = \text{Body pain}\}. \end{aligned}$$

We define the fuzzy hypersoft sets which give the degree of association between the Covid-19 patients and the Covid-19 symptoms and between the 2 patients visited and their symptoms. The FHSs $(\tilde{H}, \wedge)$ describes the evaluation of the Covid-19 patients and their symptoms as per the hospital records.

$$(\tilde{H}, \wedge) = \left\{ \begin{array}{l} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{1.0}, \frac{\mathfrak{m}_2}{0.2}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.2}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.2}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.1}\} \rangle \end{array} \right\}.$$

The FHSs 's $(\tilde{G}, \wedge), (\tilde{P}, \wedge)$ describe the evaluation of the 2 patients visited and their symptoms, respectively.

$$(\tilde{G}, \wedge) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{m_1}{0.0}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{m_1}{0.1}, \frac{m_2}{0.8}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle \end{Bmatrix}, \quad (\tilde{P}, \wedge) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{m_1}{0.7}, \frac{m_2}{0.2}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.4}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{m_1}{0.6}, \frac{m_2}{0.4}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.2}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{m_1}{0.7}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{m_1}{0.7}, \frac{m_2}{0.2}\} \rangle \end{Bmatrix}.$$

Using hamming distance, we get $d_H((\tilde{H}, \wedge), (\tilde{G}, \wedge)) = 12.2$ and $d_H((\tilde{H}, \wedge), (\tilde{P}, \wedge)) = 2.3$. As the distance between the Covid-19 patient and the 2nd patient is lesser than 1st patient, there is larger possibility for the 2nd patient suffering from Covid-19.

8. Conclusion

In this paper, FHS contra open, FHS contra semi open, FHS contra closed and FHS contra semi closed maps are introduced and their properties are analyzed with the examples. Then FHS contra open and FHS contra semi open maps are compared with FHS contra closed and FHS contra closed maps. In addition, these maps are extended to FHS contra homeomorphism, FHS contra semi homeomorphism and FHS contra C-homeomorphism. Further, an application in diagnosing Covid-19 using Hamming distance is discussed with an example. The shorter the distance between the Covid-19 patients and the other patients, the larger possibility of the other patients suffering from Covid-19. These findings will be helpful in the medical field in an attempt to make better decisions.

Acknowledgments

The authors are very grateful to the reviewers for their careful reading of the manuscript.

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