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The soft topology of soft ω^*-open sets and soft almost Lindelofness

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Abstract

In this paper, We use the soft closure operator to introduce soft ω^*-open sets as a new class of soft sets. We prove that this class of soft sets forms a soft topology that lies strictly between the soft topology of soft θ-open sets and the soft topology of soft ω-open sets. Also, we show that the soft topology of soft ω^*-open sets contain the soft co-countable topology and is independent of the topology of soft open sets. Furthermore, several results regarding soft almost Lindelofness are given. In addition to these, we investigate the correspondences between the novel notions in soft topology and their general topological analogs.

Keywords: Soft ω-open sets, soft θ-open sets, soft regularity, soft hyperconnectedness, soft almost Lindelofness.

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1. Introduction and preliminaries

In engineering, medical research, finance, environment, and other professions, the majority of real-world issues are rife with uncertainty. Molodtsov [27] proposed soft set theory as a mathematical tool for handling uncertainty in 1999. The drawbacks of earlier theories, such as fuzzy set theory [32], rough set theory [29], and others, are not present in this theory. In particular, the nature of parameter sets connected to soft sets offers a consistent foundation for modelling ambiguous data. As a result, soft set theory has quickly developed in a short amount of time and has seen a variety of real-world applications.

The fundamental set-theoretic concepts and techniques are the focus of the branch of topology known as "general topology" in mathematics. In topology and other related fields of mathematics, a set of axioms known as the Kuratowski closure axioms [24] can be used to create a topological structure on a set. They share the same characteristics as the more popular open set concept. The closure system of axioms has practical applications and is important for domain theory (see [26] for more details).

Shabir and Naz [31] invented another branch of topology called "soft topology," which is a combination of soft set theory and topology, and was influenced by the basic tenets of classical topological space. It concentrates on creating the mechanism for all soft sets. Soft topology is considered one of the branches of topology that has gained interest among researchers in recent years [8–12, 14–18, 20, 28]. Of course, there is still an opportunity for a lot of research papers.

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We hope that by establishing a sound new class of soft sets in soft topological spaces, we will open the door to several articles on the topic in the future. Soft ω-open sets, for example, were introduced as a generalization of soft open sets in soft topological spaces in [7], and other research papers on the topic, such as [1–5], have also been published.

In this paper, we use the soft closure operator to introduce soft ω^*-open sets as a new class of soft sets. We prove that this class of soft sets forms a soft topology that lies strictly between the soft topology of soft θ-open sets and the soft topology of soft ω-open sets. Also, we show that the soft topology of soft ω^*-open sets contain the soft co-countable topology and is independent of the topology of soft open sets. Furthermore, several results regarding soft almost Lindelofness are given. In addition to these, we investigate the correspondences between the novel notions in soft topology and their general topological analogs.

The body of the paper is structured as follows. In Section 2, we use the soft closure operator to introduce soft ω^*-open sets as a new class of soft sets. We show that this class of soft sets has a soft topology that is strictly between the soft topologies of soft θ-open sets and soft ω-open sets. We also show that the soft topology of soft ω^*-open sets contain the soft countable topology and is independent of the soft open set topology. In Section 3, we introduce several results regarding soft almost Lindelof STSs.

We follow the concepts and terminology used in [6, 7] in this paper. In this paper, topological space and soft topological space will be referred to as ST and STS, respectively. Let (H, β, D) be an STS and (H, μ) be a TS. Let $K \in SS(H, D)$ and $A \subseteq H$. Throughout this paper, β^c will denote the collection of all soft closed sets of (H, β, D), and μ^c will denote the collection of all closed sets of (H, μ), with $Cl_\beta(K)$, $Int_\beta(K)$, $Bd_\beta(K)$, $Cl_\mu(A)$, and $Int_\mu(A)$ denoting the soft closure of K in (H, β, D), the soft interior of K in (H, β, D), the soft boundary of K in (H, β, D), the closure of A in (H, μ), and the interior of A in (H, μ), respectively. In this paper, for any non-empty set H and any set of parameters D, $\mathcal{S}_{coc}$ will denote the soft topology $\{K \in SS(H, D) : 1_D - K \in\in CSS(H, D)\} \cup \{0_D\}$ on H relative to D.

The following definitions and results will be used in the sequel.

Now we recall some preliminaries that will be used in the sequel.

Definition 1.1 ([21])**.** Let (H, μ) be a TS and $A \subseteq H$. Then

(a) A is said to be an ω^*-open set in (H, μ) if for every $a \in A$, there exist $B \in \mu$ such that $a \in B$ and $B - A$ is countable;

(b) the collection of all ω^*-open subsets of (H, μ) will be denoted by μ_{ω^*};

(c) A is said to be an ω^*-closed subset of (H, μ) if $H - A \in \mu_{\omega^*}$.

Theorem 1.2 ([22])**.** *Let (H, β, D) be an STS and let $K \in SS(H, D)$. Then*

(a) *a soft point $h_d \in SP(H, D)$ is in the soft θ-closure of K ($h_d \widetilde{\in} Cl_\theta(K)$) if $M \widetilde{\cap} Cl_\beta(M) \neq 0_D$ for any $M \in \beta$ with $h_d \widetilde{\in} M$;*

(b) *K is soft θ-closed in (H, β, D) if $Cl_\theta(K) = K$;*

(c) *K is soft θ-open in (H, β, D) if $1_D - K$ is soft θ-closed in (H, β, D);*

(d) *the family of all soft θ-open in (H, β, D) will be denoted by β_θ.*

Theorem 1.3 ([22])**.** *Let (H, β, D) be an STS. Then*

(a) *(H, β_θ, D) is an STS;*

(b) *$\beta_\theta \subseteq \beta$ and $\beta_\theta \neq \beta$ in general.*

Definition 1.4. An STS (H, β, D) is said to be

(a) [23] soft regular if for any $h_d \in SP(H, D)$ and any $K \in \beta$ such that $h_d \widetilde{\in} K$ there exists $M \in \beta$ such that $h_d \widetilde{\in} M$ and $Cl_\beta(M) \widetilde{\subseteq} K$;

(b) [30] soft locally indiscrete if $\beta \subseteq \beta^c$;

(c) [25] soft hyperconnected if $K\widetilde{\cap}M \neq 0_D$ for every $K, M \in \beta - \{0_D\}$;
(d) [19] soft Lindelof if for every $\mathcal{A} \subseteq \beta$ such that $\widetilde{\cup}_{A\in\mathcal{A}} A = 1_D$, there exists a countable subfamily $\mathcal{A}_1 \subseteq \mathcal{A}$ such that $\widetilde{\cup}_{A\in\mathcal{A}_1} A = 1_D$;
(e) [13] soft almost Lindelof if for every $\mathcal{A} \subseteq \beta$ such that $\widetilde{\cup}_{A\in\mathcal{A}} A = 1_D$, there exists a countable subfamily $\mathcal{A}_1 \subseteq \mathcal{A}$ such that $\widetilde{\cup}_{A\in\mathcal{A}_1} Cl_\beta(A) = 1_D$.

2. Soft ω^*-open sets

In this section, we use the soft closure operator to introduce soft ω^*-open sets as a new class of soft sets. We show that this class of soft sets has a soft topology that is strictly between the soft topologies of soft θ-open sets and soft ω-open sets. We also show that the soft topology of soft ω^*-open sets contains the soft countable topology and is independent of the soft open set topology.

Definition 2.1. Let (H, β, D) be an STS and let $K \in SS(H, D)$. Then

(a) K is called a soft ω^*-open set in (H, β, D) if for all $d_h \widetilde{\in} K$, there exists $M \in \beta$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - K \in CSS(H, D)$, the collection of all soft ω^*-open sets in (H, β, D) will be denoted by β_{ω^*};
(b) K is called a soft ω^*-closed set in (H, β, D) if $1_D - K \in \beta_{\omega^*}$.

Theorem 2.2. *Let (H, β, D) be an STS and let $K \in SS(H, D)$. Then $K \in \beta_{\omega^*}$ if and only if for every $d_h \widetilde{\in} K$, there exist $M \in \beta$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - N \widetilde{\subseteq} K$.*

Proof.

Necessity. Suppose that $K \in \beta_{\omega^*}$ and let $d_h \widetilde{\in} K$. Then there exists $M \in \beta$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - K \in CSS(H, D)$. Put $N = Cl_\beta(M) - K$. Then $N \in CSS(H, D)$ and $Cl_\beta(M) - N = Cl_\beta(M) - (Cl_\beta(M) - K) \widetilde{\subseteq} K$.

Sufficiency. Suppose that for every $d_h \widetilde{\in} K$, there exist $M \in \beta$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - N \widetilde{\subseteq} K$. Let $d_h \widetilde{\in} K$. Then there exist $M \in \beta$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - N \widetilde{\subseteq} K$. Thus, $Cl_\beta(M) - K \widetilde{\subseteq} N$ and hence, $Cl_\beta(M) - K \in CSS(H, D)$. Therefore, $K \in \beta_{\omega^*}$. □

Theorem 2.3. *For any STS (H, β, D), (H, β_{ω^*}, D) is an STS.*

Proof.

(a) $0_D \in \beta_{\omega^*}$ follows directly from the definition. To see that $1_D \in \beta_{\omega^*}$, let $d_h \widetilde{\in} 1_D$. Then we have $d_h \widetilde{\in} 1_D \in \beta$ and $Cl_\beta(1_D) - 1_D = 1_D - 1_D = 0_D \in CSS(H, D)$. Thus, $1_D \in \beta_{\omega^*}$.

(b) Let $T, S \in \beta_{\omega^*}$ and let $d_h \widetilde{\in} T\widetilde{\cap}S$. Then $d_h \widetilde{\in} T$ and $d_h \widetilde{\in} S$, and so there exist $M, N \in \beta$ such that $d_h \widetilde{\in} M\widetilde{\cap}N \in \beta$ and $Cl_\beta(M) - T, Cl_\beta(N) - S \in CSS(H, D)$. Since $Cl_\beta(M\widetilde{\cap}N) - (T\widetilde{\cap}S) \widetilde{\subseteq} (Cl_\beta(M) \widetilde{\cap} Cl_\beta(N)) - (T\widetilde{\cap}S) \widetilde{\subseteq} (Cl_\beta(M) - T) \widetilde{\cup} (Cl_\beta(N) - S)$, then $Cl_\beta(M\widetilde{\cap}N) - (T\widetilde{\cap}S) \in CSS(H, D)$. Therefore, $T\widetilde{\cap}S \in \beta_{\omega^*}$.

(c) Let $\{K_\alpha : \alpha \in \Delta\} \subseteq \beta_{\omega^*}$ and let $d_h \widetilde{\in} \widetilde{\cup}_{\alpha\in\Delta} K_\alpha$. Then there exists $\alpha_0 \in \Delta$ such that $d_h \widetilde{\in} K_{\alpha_0} \in \beta_{\omega^*}$. Thus, there exists $M \in \beta$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - K_{\alpha_0} \in CSS(H, D)$. Since $Cl_\beta(M) - (\widetilde{\cup}_{\alpha\in\Delta} K_\alpha) \widetilde{\subseteq} Cl_\beta(M) - K_{\alpha_0}$, then $Cl_\beta(M) - (\widetilde{\cup}_{\alpha\in\Delta} K_\alpha) \in CSS(H, D)$. Therefore, $\widetilde{\cup}_{\alpha\in\Delta} K_\alpha \in \beta_{\omega^*}$. □

Theorem 2.4. *For any STS (H, β, D), $\beta_{\omega^*} \subseteq \beta_\omega$.*

Proof. Let $K \in \beta_{\omega^*}$ and let $d_h \widetilde{\in} K$. Then there exists $M \in \beta$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - K \in CSS(H, D)$. Since $M - K \widetilde{\subseteq} Cl_\beta(M) - K$, then $M - K \in CSS(H, D)$. Therefore, $K \in \beta_\omega$. □

The following example shows that the inclusion in Theorem 2.4 cannot be replaced by equality in general.

Example 2.5. Let $H = \mathbb{R}$, $D = \{a, b\}$, and $\beta = \{T \in SS(H, D) : a_0 \widetilde{\in} T\} \cup \{0_D\}$. Let $K = \{(a, \{0, 1\}), (b, \emptyset)\}$. Then $a_0 \widetilde{\in} K$ and so $K \in \beta$. Thus, by Theorem 2 of [7], $K \in \beta_\omega$. To see that $K \notin \beta_{\omega^*}$, suppose to the contrary that $K \in \beta_{\omega^*}$. Then there exists $M \in \beta$ such that $a_0 \widetilde{\in} M$ and $Cl_\beta(M) - K \in CSS(H, D)$. Since $a_0 \widetilde{\in} Cl_\beta(M) \in \beta^c$, then $Cl_\beta(M) = 1_D$ and so $1_D - K \in CSS(H, D)$. But $1_D - K \notin CSS(H, D)$, a contradiction.

Example 2.5 shows also that $\beta \not\subseteq \beta_{\omega^*}$ in general.

Theorem 2.6. *For any soft regular STS* (H, β, D), $\beta_{\omega^*} = \beta_\omega$.

Proof. By Theorem 2.4, we have $\beta_{\omega^*} \subseteq \beta_\omega$. To show that $\beta_\omega \subseteq \beta_{\omega^*}$, let $K \in \beta_\omega$ and let $d_h \tilde{\in} K$. Then there exists $M \in \beta$ such that $d_h \tilde{\in} M$ and $M - K \in CSS(H, D)$. Since (H, β, D) is soft regular, then there exists $T \in \beta$ such that $d_h \tilde{\in} T \tilde{\subseteq} Cl_\beta(T) \tilde{\subseteq} M$. Since $Cl_\beta(T) - K \tilde{\subseteq} M - K$, then $Cl_\beta(T) - K \in CSS(H, D)$. Therefore, $K \in \beta_{\omega^*}$. □

Corollary 2.7. *For any soft locally indiscrete STS* (H, β, D), $\beta_{\omega^*} = \beta_\omega$.

Corollary 2.8. *For any soft regular STS* (H, β, D), $\beta \subseteq \beta_{\omega^*}$.

Proof. Follows from Theorem 2.6 and Theorem 2 of [7]. □

Corollary 2.9. *For any soft locally indiscrete STS* (H, β, D), $\beta \subseteq \beta_{\omega^*}$.

The converse of each of Theorem 2.6 and Corollary 2.8 need not be true in general.

Example 2.10. Let $H = \{0, 1\}$, $D = \{a\}$, and $\beta = \{0_D, 1_D, a_0\}$. Then $\beta \subseteq \beta_{\omega^*} = SS(H, D)$.

Theorem 2.11. *For any STS* (H, β, D), $(\beta_\theta)_\omega \subseteq \beta_{\omega^*}$.

Proof. Let $K \in (\beta_\theta)_\omega$ and let $d_h \tilde{\in} K$. Then there exists $M \in \beta_\theta$ such that $d_h \tilde{\in} M$ and $M - K \in CSS(H, D)$. Since $d_h \tilde{\in} M \in \beta_\theta$, then there exists $T \in \beta$ such that $d_h \tilde{\in} T \tilde{\subseteq} Cl_\beta(T) \tilde{\subseteq} M$. Since $Cl_\beta(T) - K \tilde{\subseteq} M - K$, then $Cl_\beta(T) - K \in CSS(H, D)$. Therefore, $K \in \beta_\omega$. □

Corollary 2.12. *For any STS* (H, β, D), $\beta_\theta \subseteq \beta_{\omega^*}$.

Proof. By Theorem 2 of [7], $\beta_\theta \subseteq (\beta_\theta)_\omega$. So, by Theorem 2.11, $\beta_\theta \subseteq \beta_{\omega^*}$. □

The following shows that the inclusion in Theorem 2.11 cannot be replaced by equality in general.

Example 2.13. Let $H = \mathbb{R}$, $D = \{a, b\}$, and $\beta = \left\{T \in SS(H, D) : a_0 \tilde{\notin} T\right\} \cup \{1_D\}$. Let $K = a_1$. To see that $K \in \beta_{\omega^*} - (\beta_\theta)_\omega$. Then $a_0 \tilde{\in} K$ and so $K \in \beta$. Thus, by Theorem 2 of [7], $K \in \beta_\omega$. To see that $K \in \beta_{\omega^*}$, suppose to the contrary, let $b_x \tilde{\in} K$. Then $b_x = a_1$. Take $M = K$. Then $b_x \tilde{\in} M \in \beta$ and $Cl_\beta(M) - M = a_{\{0,1\}} \in CSS(H, D)$. Therefore, $K \in \beta_{\omega^*}$. If $K \in (\beta_\theta)_\omega$, then there exists $T \in \beta_\theta$ such that $a_1 \tilde{\in} T$ and $T - K \in CSS(H, D)$. Since $a_1 \tilde{\in} T \in \beta_\theta$, then there exists $S \in \beta$ such that $a_1 \tilde{\in} S \tilde{\subseteq} Cl_\beta(S) = S \tilde{\cup} a_0 \tilde{\subseteq} T$. So, $a_0 \tilde{\in} T$ and thus $T = 1_D$. This implies that $1_D - K \in CSS(H, D)$, which is impossible.

Definition 2.14. Let (H, β, D) be an STS and let $K \in SS(H, D)$. A soft point $d_h \in SP(H, D)$ is said to be a soft $*$-condensation point of K if for every $M \in \beta$ with $d_h \tilde{\in} M$, $K \tilde{\cap} Cl_\beta(M) \notin CSS(H, D)$. The soft union of all soft $*$-condensation points of K will be denoted by $Cond^*(K)$.

Theorem 2.15. *Let* (H, β, D) *be an STS and let* $K \in SS(H, D)$. *Then* $K \in (\beta_{\omega^*})^c$ *if and only if* $Cond^*(K) \tilde{\subseteq} K$.

Proof.

Necessity. Suppose that $K \in (\beta_{\omega^*})^c$. Suppose to the contrary that there exists $d_h \tilde{\in} Cond^*(K) - K$. Since $d_h \tilde{\in} 1_D - K \in \beta_{\omega^*}$. Then there exists $M \in \beta$ such that $d_h \tilde{\in} M$ and $Cl_\beta(M) - (1_D - K) = K \tilde{\cap} Cl_\beta(M) \in CSS(H, D)$. On the other hand, since $d_h \tilde{\in} Cond^*(K)$, then $K \tilde{\cap} Cl_\beta(M) \notin CSS(H, D)$, a contradiction.

Sufficiency. Suppose that $Cond^*(K) \tilde{\subseteq} K$. Let $d_h \tilde{\in} 1_D - K$. Then $d_h \tilde{\notin} Cond^*(K)$ and so there exists $M \in \beta$ with $d_h \tilde{\in} M$, $K \tilde{\cap} Cl_\beta(M) = Cl_\beta(M) - (1_D - K) \in CSS(H, D)$. Therefore, $1_D - K \in \beta_{\omega^*}$. Hence, $K \in (\beta_{\omega^*})^c$. □

Corollary 2.16. *For any STS* (H, β, D), $CSS(H, D) \subseteq (\beta_{\omega^*})^c$.

Proof. Let $K \in CSS(H, D)$. Then $Cond^*(K) = 0_D$ and so $Cond^*(K)\widetilde{\subseteq}K$. Hence, by Theorem 2.15, $K \in (\beta_{\omega^*})^c$. □

Corollary 2.17. *For any STS* (H, β, D), $\mathcal{S}_{coc} \subseteq \beta_{\omega^*}$.

Proof. Let $K \in \delta_{coc} - \{0_D\}$. Then by Corollary 2.16, $1_D - K \in CSS(H, D) \subseteq (\beta_{\omega^*})^c$. Hence, $K \in \beta_{\omega^*}$. □

Corollary 2.18. *Let* (H, β, D) *be an STS such that* H *is countable. Then* $\beta_{\omega^*} = SS(H, D)$.

Proof. Let $K \in SS(H, D) - \{0_D\}$. Since H is countable, then $1_D - K \in CSS(H, D)$. Thus, by Corollary 2.16, $1_D - K \in (\beta_{\omega^*})^c$. Hence, $K \in \beta_{\omega^*}$. □

According to Corollary 2.18, we can find many examples to show that the inclusion in Corollary 2.17 cannot be replaced by equality in general. However, we have the following result.

Theorem 2.19. *If* (H, β, D) *is a soft hyperconnected STS, then* $\beta_{\omega^*} = \mathcal{S}_{coc}$.

Proof. To see that $\beta_{\omega^*} \subseteq \mathcal{S}_{coc}$, let $K \in \beta_{\omega^*} - \{0_D\}$. Pick $d_h\widetilde{\in}K$. Then there exists $M \in \beta$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - K \in CSS(H, D)$. Since (H, β, D) is soft hyperconnected, then $Cl_\beta(M) = 1_D$ and so $1_D - K \in CSS(H, D)$. Hence, $K \in \mathcal{S}_{coc}$. Therefore, $\beta_{\omega^*} \subseteq \mathcal{S}_{coc}$. On the other hand, by Corollary 2.17, $\mathcal{S}_{coc} \subseteq \beta_{\omega^*}$. □

Theorem 2.20. *Let* (H, β, D) *be an STS, then* $\beta_{\omega^*} = \beta_\omega$ *if and only if* $\beta \subseteq \beta_{\omega^*}$.

Proof.

Necessity. Suppose that $\beta_{\omega^*} = \beta_\omega$. Since, by Theorem 2 of [7], $\beta \subseteq \beta_\omega$, then $\beta \subseteq \beta_{\omega^*}$.

Sufficiency. Suppose that $\beta \subseteq \beta_{\omega^*}$. Then by Corollary 2.17, $\beta \cup \mathcal{S}_{coc} \subseteq \beta_{\omega^*}$. Therefore, by Corollary 2 of [7], $\beta_\omega \subseteq \beta_{\omega^*}$. On the other hand, by Theorem 2.4, we have $\beta_{\omega^*} \subseteq \beta_\omega$. Therefore, $\beta_{\omega^*} = \beta_\omega$. □

Theorem 2.21. *Let* (H, β, D) *be an STS. If* $K \in \beta$ *such that* $Bd_\beta(K) \in CSS(H, D)$, *then* $K \in \beta_{\omega^*}$.

Proof. Suppose that $K \in \beta$ such that $Bd_\beta(K) \in CSS(H, D)$. Let $d_h\widetilde{\in}K$. Then we have $d_h\widetilde{\in}K \in \beta$ and $Cl_\beta(K) - K = Cl_\beta(K) - Int_\beta(K) = Bd_\beta(K) \in CSS(H, D)$. Hence, $K \in \beta_{\omega^*}$. □

Corollary 2.22. *Let* (H, β, D) *be an STS which has a soft base* $\mathcal{A}$ *such that* $Bd_\beta(A) \in CSS(H, D)$ *for each* $A \in \mathcal{A}$. *Then* $\beta_{\omega^*} = \beta_\omega$.

Proof. By Theorem 2.21, $\mathcal{A} \subseteq \beta_{\omega^*}$ and so $\beta \subseteq \beta_{\omega^*}$. Thus, by Theorem 2.20, $\beta_{\omega^*} = \beta_\omega$. □

Theorem 2.23. *Let* (H, β, D) *be an STS. Then for all* $d \in D$, $(\beta_{\omega^*})_d \subseteq (\beta_d)_{\omega^*}$.

Proof. Let $d \in D$. Let $V \in (\beta_{\omega^*})_d$ and let $h \in V$. Choose $K \in \beta_{\omega^*}$ such that $K(d) = V$. Since $d_h\widetilde{\in}K \in \beta_{\omega^*}$, then by Theorem 2.2 there exist $M \in \beta$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in}M$ and $Cl_\beta(M) - N \widetilde{\subseteq}K$. Now, we have $h \in M(d) \in \beta_d$, $N(d)$ is a countable set. On the other hand, by Proposition 4 of [31], $Cl_{\beta_d}(M(d)) - N(d) \subseteq \left(Cl_\beta(M)\right)(d) - N(d) \subseteq K(d) = V$. Hence, $V \in (\beta_d)_{\omega^*}$. □

Corollary 2.24. *Let* (H, β, D) *be an STS. If* $K \in \beta_{\omega^*}$, *then* $K(d) \in (\beta_d)_{\omega^*}$ *for all* $d \in D$.

Proof. Let $K \in \beta_{\omega^*}$ and let $d \in D$. Then $K(d) \in (\beta_{\omega^*})_d$ and by Theorem 2.23, $K(d) \in (\beta_d)_{\omega^*}$. □

Theorem 2.25. *Let* $\{(H, \delta_d) : d \in D\}$ *be an indexed family of STSs. Then* $(\oplus_{d\in D}\delta_d)_{\omega^*} = \oplus_{d\in D}(\delta_d)_{\omega^*}$.

Proof. By Theorem 3.11 of [6] and Theorem 2.23, $((\oplus_{d\in D}\delta_d)_{\omega^*})_d \subseteq ((\oplus_{d\in D}\delta_d)_d)_{\omega^*} = (\delta_d)_{\omega^*}$ for every $d \in D$. Hence, $(\oplus_{d\in D}\delta_d)_{\omega^*} \subseteq \oplus_{d\in D}(\delta_d)_{\omega^*}$. To see that $\oplus_{d\in D}(\delta_d)_{\omega^*} \subseteq (\oplus_{d\in D}\delta_d)_{\omega^*}$, by Theorem 3.5 of [6], it is sufficient to see that $d_V \in (\oplus_{d\in D}\delta_d)_{\omega^*}$ for all $d \in D$ and $V \in (\delta_d)_{\omega^*}$. Let $d \in D$ and $V \in (\delta_d)_{\omega^*}$. Let $d_h\widetilde{\in}d_V$. Then $h \in V \in (\delta_d)_{\omega^*}$. So, there exists $U \in \delta_d$ such that $h \in U$ and $Cl_{\delta_d}(U) - V$ is countable.

We have $d_h \tilde{\in} d_U \in \oplus_{d\in D}\delta_d$. Also, for every $e \in D$, $\left(Cl_{\oplus_{d\in D}\delta_d}(d_U) - d_V\right)(e) = \left(Cl_{\oplus_{d\in D}\delta_d}(d_U)\right)(e) - (d_V)(e) = \left(d_{Cl_{\delta_d}(U)}\right)(e) - V = \begin{cases} Cl_{\delta_d}(U) - V, & \text{if } e = d, \\ \emptyset, & \text{if } e \neq d. \end{cases}$ Thus, $Cl_{\oplus_{d\in D}\delta_d}(d_U) - d_V \in CSS(H, D)$. It follows that $d_V \in (\oplus_{d\in D}\delta_d)_{\omega^*}$. □

Corollary 2.26. *For any TS* (H, δ) *and any set of parameters* D, $(\tau(\delta))_{\omega^*} = \tau(\delta_{\omega^*})$.

Proof. Let $\delta = \delta_d$ for all $d \in D$. Then $\tau(\delta) = \oplus_{d\in D}\delta_d$. So, by Theorem 2.25,

$$(\tau(\delta))_{\omega^*} = (\oplus_{d\in D}\delta_d)_{\omega^*} = \oplus_{d\in D}(\delta_d)_{\omega^*} = \oplus_{d\in D}\delta_{\omega^*} = \tau(\delta_{\omega^*}).$$

□

Theorem 2.27. *For any STS* (H, β, D), $(\beta_{\omega^*})_{\omega^*} \subseteq \beta_{\omega^*}$.

Proof. Let $K \in (\beta_{\omega^*})_{\omega^*}$ and let $d_h \tilde{\in} K$. By Theorem 2.2, there exist $M \in \beta_{\omega^*}$ and $N \in CSS(H, D)$ such that $d_h \tilde{\in} M$ and $Cl_{\beta_{\omega^*}}(M) - N \tilde{\subseteq} K$. Again by Theorem 2.2, there exist $T \in \beta$ and $S \in CSS(H, D)$ such that $d_h \tilde{\in} T$ and $Cl_\beta(T) - S \tilde{\subseteq} M$. Now $N \tilde{\cup} S \in CSS(H, D)$ and

$$Cl_\beta(T) - (N \tilde{\cup} S) \tilde{\subseteq} M - N \tilde{\subseteq} Cl_{\beta_{\omega^*}}(M) - N \tilde{\subseteq} K.$$

Hence, by Theorem 2.2, $K \in \beta_{\omega^*}$. □

Problem 2.28. Let (H, β, D) be an STS. Is it true that $(\beta_{\omega^*})_{\omega^*} = \beta_{\omega^*}$?

Lemma 2.29. *If* (H, β, D) *and* (H, γ, D) *are STSs such that* $\beta \subseteq \gamma$, *then* $\beta_{\omega^*} \subseteq \gamma_{\omega^*}$.

Proof. Let $K \in \beta_{\omega^*} - \{0_D\}$ and let $d_h \tilde{\in} K$. Then by Theorem 2.2, there exists $M \in \beta$ (hence, $M \in \gamma$) and $N \in CSS(H, D)$ such that $d_h \tilde{\in} M$ and $Cl_\beta(M) - N \tilde{\subseteq} K$. Since $\beta \subseteq \gamma$, then $Cl_\gamma(M) \tilde{\subseteq} Cl_\beta(M)$ and hence, $Cl_\gamma(M) - N \tilde{\subseteq} Cl_\beta(M) - N \tilde{\subseteq} K$. Therefore, $K \in \gamma_{\omega^*}$. □

The following result is a partial answer for Problem 2.28:

Theorem 2.30. *For any soft regular STS* (H, β, D), $(\beta_{\omega^*})_{\omega^*} = \beta_{\omega^*}$.

Proof. By Corollary 2.8, $\beta \subseteq \beta_{\omega^*}$. So by Lemma 2.29, $\beta_{\omega^*} \subseteq (\beta_{\omega^*})_{\omega^*}$. Also, by Theorem 2.27, $(\beta_{\omega^*})_{\omega^*} \subseteq \beta_{\omega^*}$. Hence, $(\beta_{\omega^*})_{\omega^*} = \beta_{\omega^*}$. □

Theorem 2.31. *Let* (H, β, D) *be an STS and* Y *be a non empty subset of* H. *Then* $(\beta_{\omega^*})_Y \subseteq (\beta_Y)_{\omega^*}$.

Proof. Let $T \in (\beta_{\omega^*})_Y$ and $d_y \tilde{\in} T$. Choose $K \in \beta_{\omega^*}$ such that $T = K \tilde{\cap} C_Y$. Since $d_y \tilde{\in} K \in \beta_{\omega^*}$, then there exists $M \in \beta$ such that $d_y \tilde{\in} M$ and $Cl_\beta(M) - K \in CSS(H, D)$. Let $S = M \tilde{\cap} C_Y$. Then $d_y \tilde{\in} S \in \beta_Y$ and $Cl_{\beta_Y}(S) \tilde{\subseteq} Cl_\beta(M)$. Since $Cl_{\beta_Y}(S) - T = Cl_{\beta_Y}(S) - (K \tilde{\cap} C_Y) = Cl_{\beta_Y}(S) - K \tilde{\subseteq} Cl_\beta(M) - K \tilde{\subseteq} Cl_\beta(M) - K$. Therefore, $Cl_{\beta_Y}(S) - T \in CSS(H, D)$. □

The following example will show that the inclusion in Theorem 2.31 cannot be replaced by equality even if $C_Y \in \beta^c$:

Example 2.32. Let $H = [0, 1]$, $D = \{a\}$, and $\beta = \{T \in SS(H, D) : 0 \in T(a)\} \cup \{0_D\}$ and let $Y = (0, 1]$. Then $C_Y \in \beta^c$, $\beta_Y = SS(Y, D)$, and $(\beta_Y)_{\omega^*} = SS(Y, D)$. Since (H, β, D) is a soft hyperconnected, then by Theorem 2.19, $\beta_{\omega^*} = \mathcal{S}_{coc}$ and so $(\beta_{\omega^*})_Y = \mathcal{S}_{coc} \neq (\beta_Y)_{\omega^*}$.

Theorem 2.33. *Let* (H, β, D) *be an STS and* Y *be a non empty subset of* H *such that* $C_Y \in \beta \cap \beta^c$. *Then* $(\beta_Y)_{\omega^*} = (\beta_{\omega^*})_Y$.

Proof. To show that $(\beta_Y)_{\omega^*} \subseteq (\beta_{\omega^*})_Y$, let $K \in (\beta_Y)_{\omega^*} - \{0_D\}$. Pick $d_y \widetilde{\in} K$. Then there exists $M \in \beta_Y$ such that $d_y \widetilde{\in} M$ and $Cl_{\beta_Y}(M) - K \in CSS(Y, D)$. Since $C_Y \in \beta$, then $M \in \beta$. Since $C_Y \in \beta^c$, then $Cl_{\beta_Y}(M) = Cl_\beta(M)$. Thus, $Cl_\beta(M) - K \in CSS(H, D)$ and hence $K \in \beta_{\omega^*}$. Since $K = K \widetilde{\cap} C_Y$, then $K \in (\beta_{\omega^*})_Y$. This shows that $(\beta_Y)_{\omega^*} \subseteq (\beta_{\omega^*})_Y$. On the other hand, by Theorem 2.31, $(\beta_{\omega^*})_Y \subseteq (\beta_Y)_{\omega^*}$. □

Theorem 2.34. *Let* (H, β, D) *be an STS. If* $K \in (\beta_{\omega^*})^c$, *then* $K \widetilde{\subseteq} Int_\beta(T) \widetilde{\cup} N$ *for some* $T \in \beta^c$ *and* $N \in CSS(H, D)$.

Proof. If $K = 1_D$, then we choose $T = K$ and $N = 0_D$ to get $K \widetilde{\subseteq} Int_\beta(T) \widetilde{\cup} N$. If $K \neq 1_D$, choose $d_h \widetilde{\in} 1_D - K \in \beta_{\omega^*}$. Then by Theorem 2.2, there exist $M \in \beta$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - N \widetilde{\subseteq} 1_D - K$. Put $T = 1_D - M$. Then we have $T \in \beta^c$, $N \in CSS(H, D)$, and

$$K \widetilde{\subseteq} 1_D - \left(Cl_\beta(M) - N\right) = \left(1_D - Cl_\beta(M)\right) \widetilde{\cup} N = Int_\beta(1_D - M) \widetilde{\cup} N = Int_\beta(T) \widetilde{\cup} N.$$

□

Theorem 2.35. *Let* (H, β, D) *be an STS and* $K \in SS(H, D) - \{1_D\}$. *Then* $K \in (\beta_{\omega^*})^c$ *if and only if for each* $d_h \widetilde{\in} 1_D - K$, *there exist* $T \in \beta^c$ *and* $N \in CSS(H, D)$ *such that* $d_h \widetilde{\in} 1_D - T$ *and* $K \widetilde{\subseteq} Int_\beta(T) \widetilde{\cup} N$.

Proof.

Necessity. Let $K \in (\beta_{\omega^*})^c$ and let $d_h \widetilde{\in} 1_D - K$. Then by Theorem 2.2, there exist $M \in \beta$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in} M$ and $Cl_\beta(M) - N \widetilde{\subseteq} 1_D - K$. Put $T = 1_D - M$. Then we have $T \in \beta^c$, $N \in CSS(H, D)$, and

$$K \widetilde{\subseteq} 1_D - \left(Cl_\beta(M) - N\right) = \left(1_D - Cl_\beta(M)\right) \widetilde{\cup} N = Int_\beta(1_D - M) \widetilde{\cup} N = Int_\beta(T) \widetilde{\cup} N.$$

Sufficiency. Suppose that for each $d_h \widetilde{\in} 1_D - K$, there exist $T \in \beta^c$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in} 1_D - T$ and $K \widetilde{\subseteq} Int_\beta(T) \widetilde{\cup} N$. We will show that $1_D - K \in \beta_{\omega^*}$. Let $d_h \widetilde{\in} 1_D - K$. Then there exist $T \in \beta^c$ and $N \in CSS(H, D)$ such that $d_h \widetilde{\in} 1_D - T$ and $K \widetilde{\subseteq} Int_\beta(T) \widetilde{\cup} N$. Let $M = 1_D - T$. Then $d_h \widetilde{\in} M \in \beta$ and $Cl_\beta(M) - N = 1_D - (Int_\beta(T) \widetilde{\cup} N) \widetilde{\subseteq} 1_D - K$. Hence, $1_D - K \in \beta_{\omega^*}$. □

3. Soft almost Lindelofness

In this section, we introduce several results regarding soft almost Lindelof STSs.

Lemma 3.1. *Let* (H, β, D) *be an STS and let* $\mathcal{N}$ *be a soft base of* (H, β, D). *Then* (H, β, D) *is soft almost Lindelof if and only if for every* $\mathcal{A} \subseteq \mathcal{N}$ *with* $\widetilde{\cup}_{A \in \mathcal{A}} A = 1_D$ *there exists a countable subfamily* $\mathcal{A}_1 \subseteq \mathcal{A}$ *such that* $\widetilde{\cup}_{A \in \mathcal{A}_1} A = 1_D$.

Proof.

Necessity. It is obvious.

Sufficiency. Suppose that for every $\mathcal{A} \subseteq \mathcal{N}$ with $\widetilde{\cup}_{A \in \mathcal{A}} A = 1_D$ there exists a countable subfamily $\mathcal{A}_1 \subseteq \mathcal{A}$ such that $\widetilde{\cup}_{A \in \mathcal{A}_1} A = 1_D$. Let $\mathcal{T} \subseteq \beta - \{0_D\}$ such that $\widetilde{\cup}_{T \in \mathcal{T}} T = 1_D$. For every $K \in \mathcal{T}$, there is $\mathcal{A}_K \subseteq \mathcal{N}$ such that $\widetilde{\cup}_{A \in \mathcal{A}_K} A = K$. Let $\mathcal{S} = \{A : A \in \mathcal{A}_K, K \in T\}$. Then $\widetilde{\cup}_{A \in \mathcal{S}} A = 1_D$ and by assumption, there exists a countable subfamily $\mathcal{S}_1 \subseteq \mathcal{S}$ such that $\widetilde{\cup}_{A \in \mathcal{S}_1} A = 1_D$. For each $A \in \mathcal{S}_1$, choose $T(A) \in \mathcal{T}$ such that $A \widetilde{\subseteq} T(A)$. Then $\{T(A) : A \in \mathcal{S}_1\}$ is a countable subfamily of $\mathcal{T}$ with $\widetilde{\cup}_{A \in \mathcal{S}_1} T(A) = 1_D$. Hence, (H, β, D) is soft almost Lindelof. □

Theorem 3.2. *Let* $\{(H, \mu_d) : d \in D\}$ *be an indexed family of STSs. Then* $(H, \oplus_{d \in D} \mu_d, D)$ *is soft almost Lindelof if and only if* D *is countable and* (H, μ_d) *is almost Lindelof for all* $d \in D$.

Proof.

Necessity. Suppose that $(H, \oplus_{d \in D} \mu_d, D)$ is soft almost Lindelof. Since $\{d_H : d \in D\} \subseteq \oplus_{d \in D} \mu_d$ with $\widetilde{\cup}_{d \in D} d_H = 1_D$, then there exists a countable subset $D_1 \subseteq D$ such that

$$\widetilde{\cup}_{d \in D_1} Cl_{\oplus_{d \in D} \mu_d}(d_H) = \widetilde{\cup}_{d \in D_1} d_{Cl_{\mu_d}(H)} = \widetilde{\cup}_{d \in D_1} d_H = 1_D,$$

which implies that $D = D_1$. Hence, D is countable. Let $a \in D$. To show that (H, μ_a) is almost Lindelof, let $\mathcal{U} \subseteq \mu_a$ with $\cup_{U\in\mathcal{U}} U = H$. Let $\mathcal{A} = \{a_U : U \in \mathcal{U}\} \cup \{d_H : d \in D - \{a\}\}$. Then $\mathcal{A} \subseteq \oplus_{d\in D}\mu_d$ with $\widetilde{\cup}_{A\in\mathcal{A}} A = 1_D$. Thus, there exists a countable subfamily $\mathcal{A}_1 \subseteq \mathcal{A}$ such that $\widetilde{\cup}_{A\in\mathcal{A}_1} Cl_{\oplus_{d\in D}\mu_d}(A) = 1_D$. Now for each $d \in D - \{a\}$, $Cl_{\oplus_{d\in D}\mu_d}(d_H) = d_{Cl_{\mu_d}(H)} = d_H$. And, for each $U \in \mathcal{U}$, $Cl_{\oplus_{d\in D}\mu_d}(a_U) = a_{Cl_{\mu_a}(U)}$. Therefore, there exists a countable subfamily $\mathcal{U}_1 \subseteq \mathcal{U}$ such that $\cup_{U\in\mathcal{U}_1} U = H$. It follows that (H, μ_a) is almost Lindelof.

Sufficiency. Suppose that D is countable and (H, μ_d) is almost Lindelof for all $d \in D$. Let $\mathcal{B} = \{d_U : d \in D$ and $U \in \mu_d\}$. By Theorem 3.5 of [6], $\mathcal{B}$ is a soft base of $\oplus_{d\in D}\mu_d$. We apply Lemma 3.1. Let $\mathcal{A} \subseteq \mathcal{B}$ such that $\widetilde{\cup}_{A\in\mathcal{A}} A = 1_D$. For each $d \in D$, let $\sigma_d = \{U \subseteq H : d_U \in \mathcal{A}\}$. Then for each $d \in D$, $\sigma_d \subseteq \mu_d$ with $\cup_{U\in\sigma_d} U = H$ and so, there exists a countable subfamily $\rho_d \subseteq \sigma_d$ such that $\cup_{U\in\rho_d} Cl_{\mu_d}(U) = H$. Let $\mathcal{A}_1 = \{d_U : d \in D$ and $U \in \rho_d\}$. Since D is countable, then $\{d_U : d \in D$ and $U \in \rho_d\}$ is countable. Therefore, we have $\mathcal{A}_1$ is a countable subfamily of $\mathcal{A}$ with $\widetilde{\cup}_{A\in\mathcal{A}_1} A = 1_D$. It follows that $(H, \oplus_{d\in D}\mu_d, D)$ is soft almost Lindelof. □

Theorem 3.3. *Let $\{(H, \mu_d) : d \in D\}$ be an indexed family of STSs. Then $(H, (\oplus_{d\in D}\mu_d)_{\omega^*}, D)$ is soft Lindelof if and only if D is countable and $(H, (\mu_d)_{\omega^*})$ is Lindelof for all $d \in D$.*

Proof. Follows from Theorem 2.25 and Theorem 38 of [7]. □

Theorem 3.4. *If (H, β, D) is soft almost Lindelof, then (H, β_{ω^*}, D) is soft Lindelof.*

Proof. Let $\mathcal{K} \subseteq \beta_{\omega^*}$ such that $1_D = \widetilde{\cup}\mathcal{K}$. For each $d_m \widetilde{\in} 1_D$, choose $K_{d_m} \in \mathcal{K}$ such that $d_m \widetilde{\in} K_{d_m}$. Thus, for each $d_m \widetilde{\in} 1_D$, there exists $M_{d_m} \in \beta$ such that $d_m \widetilde{\in} M_{d_m}$ and $Cl_\beta(M_{d_m}) - K_{d_m} = N_{d_m} \in CSS(H, D)$. Since (H, β, D) is soft almost Lindelof and $\widetilde{\cup}_{d_m \widetilde{\in} 1_D} M_{d_m} = 1_D$, then there exists a countable subset $\mathcal{T} \subseteq SP(H, D)$ such that $\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} Cl_\beta(M_{d_m}) = \left(\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} \left(Cl_\beta(M_{d_m}) - K_{d_m}\right)\right) \widetilde{\cup} \left(\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} K_{d_m}\right) = \left(\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} N_{d_m}\right) \widetilde{\cup} \left(\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} K_{d_m}\right)$. Since $\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} N_{d_m} \in CSS(H, D)$, then there exists a countable subset $\mathcal{K}_1 \subseteq \mathcal{K}$ such that $\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} N_{d_m} \widetilde{\subseteq} \widetilde{\cup}_{K\in\mathcal{K}_1} K$. Thus, $1_D = \left(\widetilde{\cup}_{K\in\mathcal{K}_1} K\right) \widetilde{\cup} \left(\widetilde{\cup}_{d_m \widetilde{\in} \mathcal{T}} K_{d_m}\right)$. Therefore, (H, β_{ω^*}, D) is soft Lindelof. □

Corollary 3.5. *If (H, β, D) is soft Lindelof, then (H, β_{ω^*}, D) is soft Lindelof.*

Proof. This follows from Theorem 3.4 and the fact that soft Lindelof STSs are soft almost Lindelof. □

Problem 3.6. Let (H, β, D) be an STS such that (H, β_{ω^*}, D) is soft Lindelof. Is it true that (H, β, D) is soft Lindelof?

4. Conclusion

Following the work of Shabir and Naz [31], several different soft set types have been explored. Soft semi-open sets, soft pre-open sets, soft regular open sets, soft theta open sets, soft omega open sets, and so forth are a few examples.

In this paper, we developed a new type of soft sets called soft ω^*-open sets using the soft closure operator. We establish that the soft topology formed by this class of soft sets strictly falls between that of soft ω-open sets and that of soft θ-open sets. Moreover, we demonstrate that the soft co-countable topology is included in the soft topology of soft ω^*-open sets and is independent of the topology of soft open sets. Additionally, we presented several results concerning soft almost Lindelof STSs. Finally, it studied how the unique concept of soft topology corresponds to its general topological analog.

In future studies, the following topics could be considered:

1) define new classes of soft functions using soft ω^*-open sets;
2) investigate the behavior of soft ω^*-open sets in the context of product STSs;
3) define the soft ω^*-closure and the soft ω^*-interior operators.

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