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New exact-analytical solutions for the mKdV equation

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Abstract

In this paper, the sine-cosine, the extended tanh method has been used to obtain solutions of the modified KdV(mKdV) equation. New periodic solutions are formally derived.

Keywords: The nonlinear evolutions equations, the sine-cosine method, the extended tanh method, the modified KdV equation.

1 Introduction

The investigation of exact traveling wave solutions to nonlinear evolutions equations plays an important role in the study of nonlinear physical phenomena. Applications of the results obtained for the mKdV equation in this paper is to description of ion-acoustic waves in an unmagnetized plasma.

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The aim of the present paper is to extend the sine-cosine and the extended tanh methods to finding new exact solutions for mKdV equation where is expressed as follows[7]

$$u_t + 6su^2u_x - u_{xxx} = 0, \quad s = \pm 1 \tag{1}$$

2 Analysis of method

For the sine-cosine and the extended tanh methods, we first unite the independent variable x and t into one wave variable $\xi=x-ct$, to carry out a PDE in two independent variables

$$P(u, u_x, u_t, u_{xx}, \dots) = 0 \tag{2}$$

into an ODE

$$Q(u, u', u'', u''', \dots). \tag{3}$$

Eq.(3) is then integrated as long as all terms contain derivative. Usually the integration constants are considered to be zeros in view of the localized solutions.

2.1. The sine-cosine method

For this method the solutions of the reduced ODE equation can be expressed in the form

$$u(x,t) = \begin{cases} \lambda \cos^\beta(\mu\xi) & |\mu\xi| \leq \frac{\pi}{2} \\ 0 & \text{else} \end{cases} \tag{4}$$

Where λ, μ, β are parameters that will be determined. [2,3,4,5]

Substituting the cosine assumption and their derivatives into the reduce ODE gives a trigonometric equation of $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ terms. The parameters are then determined by first balancing the exponents of each pair of cosine determine k . We next collect all coefficients of the same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ where these coefficients have to vanish. This gives a system of algebraic equations among the unknowns c, λ, μ, ξ that will be determined.

2.2. The extended tanh method

The extended tanh method introduces a new independent variable $Y = \tanh(\mu\xi)$, that leads to the changes of derivatives:

$$\frac{d}{d\xi} = \mu(1-Y^2) \frac{d}{dY}, \quad \frac{d^2}{d\xi^2} = \mu^2(1-Y^2)(-2Y \frac{d}{dY} + (1-Y^2) \frac{d^2}{dY^2}). \tag{5}$$

The solutions for the extended tanh method can be proposed as a finite power series in Y in the form

$$u(\mu\xi) = S(Y) = \sum_{k=0}^M a_k Y^k + \sum_{k=1}^M b_k Y^{-k} \tag{6}$$

Where the parameter M is a positive, in most cases, that will be determined. To determine the parameter M , we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. We then collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $a_k (k = 0, \dots, M), b_k (k = 1, \dots, M), \mu, \xi$, and c . Having determined these parameters we obtain an analytic solution $u(x,t)$ in a closed form. [1,6]

3 Application of methods

3.1. Using the sine-cosine method

Substituting $u(x,t)=u(\xi)$, where the wave variable is $\xi=x-ct$, carries out the mKdV equation(1) to the ODE

$$-cu' + 6su^2u' - u''' = 0. \tag{7}$$

Once integrating (7), setting the constant of integrating to zero, we obtain

$$-cu + 2su^3 - u'' = 0. \tag{8}$$

Substituting cosine anstaz (4) into (8) gives

$$-c\lambda \cos^\beta(\mu\xi) + 2s\lambda^3 \cos^{3\beta}(\mu\xi) + \lambda\mu^2 \beta^2 \cos^\beta(\mu\xi) + \lambda\mu^2 \beta(\beta-1)\cos^{\beta-2}(\mu\xi) = 0. \tag{9}$$

Equating the exponents of the second and the last cosine functions in above equation, collecting coefficients of each pair of cosine functions of like exponents, and setting it equal to zero, we obtain the following system of algebraic equations:

$$\begin{cases} \beta - 1 \neq 0, \\ \beta - 2 = 3\beta, \\ c = \mu^2 \beta^2, \\ 2s\lambda^3 = \lambda\mu^2 \beta(\beta - 1). \end{cases} \tag{10}$$

so that this gives

$$\begin{cases} \beta = -1, \\ c = \mu^2, \\ \lambda = \sqrt{s}\mu. \end{cases} \tag{11}$$

and μ is any real number.

Substituting (11) into (4), we find the following solution for modified KdV equation:

$$u(x,t) = \begin{cases} \sqrt{s}\mu \sec(x - \mu^2 t) & |\mu\xi| \leq \frac{\pi}{2} \\ 0 & \text{else.} \end{cases}$$

3.1. Using the extended tanh method

As shown before, the mKdV equation(1) can be transformed to the ODE

$$-cu + 2su^3 - u'' = 0. \tag{12}$$

Balancing u'' with u^3 gives $M+2=3M$, so that $M=1$, therefore the extended tanh method assumes that finite expansion

$$u(\mu\xi) = S(Y) = a_0 + a_1Y + b_1Y^{-1} \tag{13}$$

where $Y = \tanh(\mu\xi)$. Substituting (13) into (12) and using (5), collecting the coefficients of $Y^j (-3 \leq j \leq 3)$ and equating this coefficients to zero and solving the system of algebraic for a_0, a_1, b_1, μ and c we find the following set of

$$\begin{cases} a_0 = 0, \\ a_1 = 0 \\ b_1 = \sqrt{s}\mu, \\ c = 2\mu^2. \end{cases} \tag{14}$$

and μ is any real number. Substituting (14), into (6) gives

$$u(x,t) = \sqrt{s}\mu \tanh^{-1}\{\mu(x - 2\mu^2 t)\}.$$

Conclusions

The sine-cosine, the extended tanh methods were used to investigate nonlinear mKdV equation. The work emphasized our belief the two methods are powerful technique to handle nonlinear

digestive equation, hence these methods can be used in a wider context. The validity of these methods has been tested by applying to modified KdV equation. Finally it is a promising and powerful method for other nonlinear equations in mathematical physics.

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