J. Math. Computer Sci., 30 (2023), 390–406

Online: ISSN 2008-949X

Journal of Mathematics and Computer Science

Journal Homepage: www.isr-publications.com/jmcs

Analysis of a malaria transmission mathematical model considering immigration

C. Taftaf[a,∗], H. Benazza[a], Y. Louartassi[b], Z. Hamidi[c]

[a]Lab-Mia, Faculty of Sciences, Mohammed V University in Rabat, Rabat, Morocco.

[b]Superior School of Technology Salé, Mohammed V University in Rabat, LASTIMI, Salé, Morocco.

[c]Laboratory M2PA, Department of mathematics & informatics, ENS. University Sidi Mohamed Ben Abdellah. Fez, Morocco.

Abstract

The aims of this paper are to study the local and global stability of the equilibrium points using a mathematical model for malaria disease. The model is based on five differential equations. The analysis of the stability was examined using the Lyapunov method. We prove that the disease free equilibrium point is locally and globally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$. On the other hand, the endemic equilibrium point is locally and globally asymptotically stable when $R_0 > 1$.

Keywords: Malaria, epidemic model, Lyapunov, endemic equilibrium, disease-free equilibrium.

2020 MSC: 92B05, 92B10, 93A10.

1. Introduction

Malaria is a contagious disease brought on by the plasmodium family that is transmitted from an infected Anopheles female mosquito to humans through a bite. A recent estimation by the World Health Organization indicates 229 million cases of infection over the world. Besides, the number of deaths as a consequence of the disease reached 409000 in 2019, the majority of infected people were pregnant women and young children, most of whom live in Africa. Given this state, an understanding of the factors underlying malaria spread may be requisite for the development of adequate prevention strategies.

Modeling the spread of malaria has been attempted by several researchers (see [1–4, 9, 12, 14]) since the beginning of the twentieth century. In [14], the first deterministic differential equation has been proposed to represent the crucial characteristics of malaria spread. The main infected zones that are in contact with the disease, and that are known to be contagious are also taken into account into the model. In addition, the author has showed that malaria can be eradicated if these parameters stay below a certain average. In [12], the author extended Ross's model, by integrating the dynamic effects emerging during the parasite's incubation stage. Furthermore, the author demonstrated that the correlation between the

∗Corresponding author

Email address: taftaf.chaimaa@gmail.com (C. Taftaf)

doi: 10.22436/jmcs.030.04.08

Received: 2022-07-04 *Revised:* 2022-10-19 *Accepted:* 2023-01-30

number of musquitoes and the transmission rate is weak. In [3], other features were added to Macdonald's model such as the intensity of the infection and the length of the immunity period. In [4], the authors presented a mathematical simulation of the dynamics of malaria transmission, including the structural age of the vector population and the number of bites by female mosquitoes. Here, the authors proposed that there are two levels of vulnerability within the human population: the most vulnerable are non-immune, and the least vulnerable are semi-immune. In this paper the authors provides the sufficient conditions ensuring the stability of the disease free equilibrium.

The quest to eradicate or at least stabilize malaria has taken too many years of continuous struggle and myriads of scientists endeavored to curb the spread of malaria. The malady has now reached some areas that were previously disease-free. This is thought to be largely due to increased mobility and other consequences of globalization. There are areas which are yet disease free, but their probability of contracting the virus has increased over the years given all these factors [6, 18].

Therefore, in this work, we propose a similar mathematical model on the basis of ordinary differential equations (ODE) where people and mosquitoes contact and spread the infection to one another. To better understand malaria transmission and the contribution of migration, we assume that the parasite enters the mosquito when the latter bites an infected human. We also assume that the mosquito remains infectious for life. Accordingly, the total number of bites depends both on the human and the mosquito population sizes. Presumably, recovered individuals are expected to acquire some immunity to the disease and should therefore not return to the category of susceptible individuals. The total population of humans is: $N_h(t) = S_h(t) + I_h(t) + R_h(t)$ and they are classified into three categories: Susceptible, S_h; Infectious, I_h; and Recovered, R_h.

Indeed, people enter the susceptible class either through birth (all newly born individual necessarily enter the susceptible population first at the natural birth rate b_h) or through immigration at a stable rate $\wedge_h$. Susceptible individuals become infected either by infected humans at a constant rate β_1 or by infected mosquitoes at a constant rate β_2. Contaminated individuals recover at the rate γ_h, and all populations represented by the compartments in the model on the next page are affected by a per capita density-dependent natural death and emigration rates $f_h(N_h) = \mu_{1h} + \mu_{2h}N_h$. The number of infected people is also reduced as a function of disease related deaths at the rate δ_h. On the other hand, there are two groups of mosquitoes: those who are susceptible (S_ν) and those who are infected (I_ν). In addition, the overall mosquito population is $N_\nu(t) = S_\nu(t) + I_\nu(t)$. So, when a susceptible mosquito bites an infectious human, the mosquito contracts the virus and enters the I_ν class. Individuals join the susceptible population at the rate $\wedge_\nu$, and those who are susceptible contract the infection at the rate $\beta_3 I_\nu$ and as in the case of humans, newly born mosquitoes (at a rate b_ν) first enter the susceptible population before contracting the virus. (Male mosquitoes are not included in our model since only female mosquitoes bite people and animals in order to obtain lipids and proteins, which are vital for their ability to reproduce). The number of susceptible female mosquitoes is reduced by the affine function: $f_\nu(N_\nu) = \mu_{1\nu} + \mu_{2\nu}N_\nu$ (mosquito mortality and emigration rates that depend on population per capita density). Infected individuals are also reduced by disease related death at the rate δ_ν.

The article is structured as follows. Section 2 will focus on the model solution's existence, positivity and boundedness. In Section 3 we will discuss whether the disease-free equilibrium exists and its local and global stability. In the last part of the article is shown how the endemic equilibrium is both locally and globally asymptotically stable.

2. The Malaria mathematical modelling

Figure 1 explains how malaria transmission proceeds in our mathematical model.

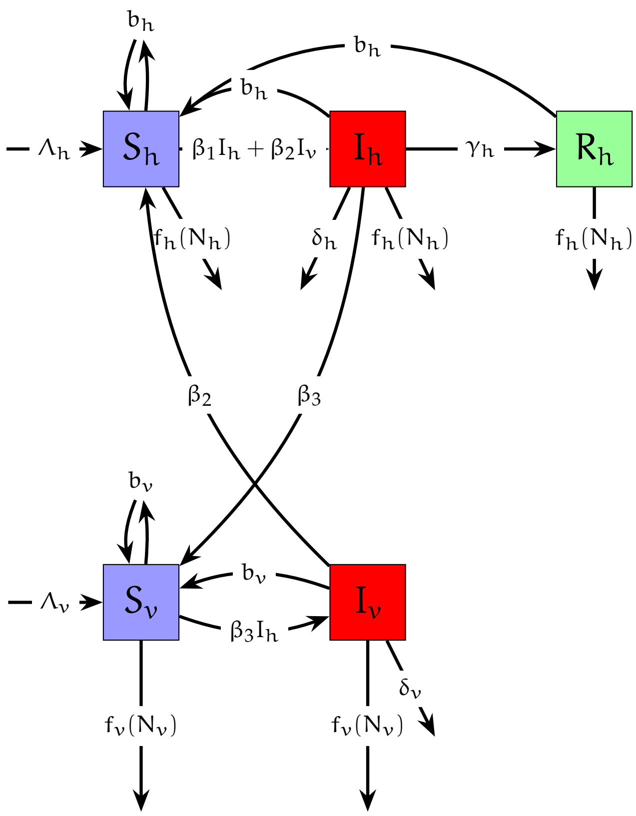

Figure 1: Malaria model flow chart.

From Figure 1 we have the following equations:

$$
\begin{cases}
\frac{dS_h}{dt} = g_h(N_h) - \beta_1 S_h I_h - \beta_2 S_h I_v - f_h(N_h) S_h, \\
\frac{dI_h}{dt} = \beta_1 S_h I_h + \beta_2 S_h I_v - I_h(\gamma_h + \delta_h + f_h(N_h)), \\
\frac{dR_h}{dt} = I_h \gamma_h - R_h f_h(N_h), \\
\frac{dS_v}{dt} = g_v(N_v) - \beta_3 S_v I_h - f_v(N_v) S_v, \\
\frac{dI_v}{dt} = \beta_3 I_h S_v - (\delta_v + f_v(N_v)) I_v,
\end{cases} \tag{2.1}
$$

where $f_h(N_h) = \mu_{1h} + \mu_{2h} N_h$ and $f_v(N_v) = \mu_{1v} + \mu_{2v} N_v$ represent the per capita density-dependent natural death and emigration rates for humans and mosquitoes, respectively (see [8]). The birth and emigration rates g_h and g_v are given by:

$$g_h(N_h) = \Lambda_h + b_h N_h, \qquad g_v(N_v) = \Lambda_v + b_v N_v,$$

with initial conditions

$$S_h(0) \geqslant 0, \quad I_h(0) \geqslant 0, \quad R_h(0) \geqslant 0, \quad S_v(0) \geqslant 0, \quad \text{and} \quad I_v(0) \geqslant 0. \tag{2.2}$$

Table 1 provides definitions for parameters and variables.

Table 1: Parameter and variable descriptions form model (2.1).

variables	Descriptions
S_h	How many humans are susceptible
I_h	How many humans had the disease
R_h	How many humans have been recovered
S_v	How many mosquitoes are susceptible
I_v	How many mosquitoes had the disease
N_h	Total human population
N_v	Total mosquito population
parameters	Descriptions
$\wedge_h$	Human's immigration rate
$\wedge_v$	Mosquito's immigration rate
b_h	Human's birth rate
b_v	Mosquito's birth rate
μ_{1h}	Density-independent part of the mortality (and emigration) rate for humans
μ_{2h}	Density-dependent part of the mortality (and emigration) rate for humans
μ_{1v}	Density-independent part of the mortality (and emigration) rate for mosquitoes
μ_{2v}	Density-dependent part of the mortality (and emigration) rate for mosquitoes
γ_h	Recovery rate for humans from the infectious state to the recovered state
δ_h	Disease-induced death rate for humans
δ_v	Disease-induced death rate for mosquitoes
β_1	Transmission rate of infection from an infectious man to a susceptible man
β_2	Transmission rate of infection from an infectious mosquito to a susceptible man
β_3	Transmission rate of infection from an infectious man to a susceptible mosquito

3. The model's mathematical analysis

3.1. Solution boundedness and positivity

It is necessary to show that the state variables are not negative at all times. By using [13], we will prove the positivity and boundness of the solution.

The total number of humans $N_h(t)$ is defined by $N_h(t) = S_h(t) + I_h(t) + R_h(t)$ and verifies the following equation

$$\frac{dN_h}{dt} = g_h(N_h) - f_h(N_h)N_h - \delta_h I_h = \Lambda_h - (\mu_{1h} - b_h)N_h - \mu_{2h}N_h^2 - \delta_h I_h. \tag{3.1}$$

The vectors population size $N_v(t)$ can be defined by $N_v(t) = S_v(t) + I_v(t)$ and verify the the following equation

$$\frac{dN_v}{dt} = g_v(N_v) - f_v(N_v)N_v - \delta_v I_v = \Lambda_v - (\mu_{1v} - b_v)N_v - \mu_{2v}N_v^2 - \delta_v I_v. \tag{3.2}$$

According to (3.1) and (3.2) we have

$$\frac{dN_h}{dt} \leqslant \Lambda_h - (\mu_{1h} - b_h)N_h \quad \text{and} \quad \frac{dN_v}{dt} \leqslant \Lambda_v - (\mu_{1v} - b_v)N_v.$$

So,

$$\begin{cases} \frac{dN_h}{dt} \leqslant 0, & \text{if } \frac{\Lambda_h}{\mu_{1h}-b_h} \leqslant N_h \text{ and } \mu_{1h} \geqslant b_h, \\ \frac{dN_v}{dt} \leqslant 0, & \text{if } \frac{\Lambda_v}{\mu_{1v}-b_v} \leqslant N_v \text{ and } \mu_{1v} \geqslant b_v. \end{cases}$$

Let $V_1 = \limsup_{t\to+\infty} N_h$, and $V_2 = \limsup_{t\to+\infty} N_v$. Then,

$$V_1 \leqslant \frac{\Lambda_h}{\mu_{1h} - b_h} \text{ and } V_2 \leqslant \frac{\Lambda_v}{\mu_{1v} - b_v}.$$

Consequently, the system's feasible region of (2.1) is

$$\Omega = \{(S_h, I_h, R_h, S_v, I_v) \in \mathbb{R}^5_+, \quad V_1 \leqslant \frac{\Lambda_h}{\mu_{1h} - b_h}, \quad \text{and} \quad V_2 \leqslant \frac{\Lambda_v}{\mu_{1v} - b_v}\}.$$

Lemma 3.1. *Let $(S_h, I_h, R_h, S_v, I_v)$ represents the system (2.1)'s solution with initial conditions (2.2). The closed set $\Omega = \{(S_h, I_h, R_h, S_v, I_v) \in \mathbb{R}^5_+,\ V_1 \leqslant \frac{\Lambda_h}{\mu_{1h}-b_h}$, and $V_2 \leqslant \frac{\Lambda_v}{\mu_{1v}-b_v}\}$ is attractive and positively invariant.*

Proof. Considering the next Lyapunov function

$$V(t) = (V_1(t), V_2(t)) = (S_h + I_h + R_h, S_v + I_v),$$

its derivative is

$$\frac{dV}{dt} = (\Lambda_h - (\mu_{1h} - b_h)V_1 - \mu_{2h}V_1^2 - \delta_h I_h, \Lambda_v - (\mu_{1v} - b_v)V_2 - \mu_{2v}V_2^2 - \delta_v I_v).$$

It is easy to prove that

$$\begin{cases} \frac{dV_1}{dt} \leqslant \Lambda_h - (\mu_{1h} - b_h)V_1 \leqslant 0, & \text{for } V_1 \geqslant \frac{\Lambda_h}{\mu_{1h}-b_h}, \\ \frac{dV_2}{dt} \leqslant \Lambda_v - (\mu_{1v} - b_v)V_2 \leqslant 0, & \text{for } V_2 \geqslant \frac{\Lambda_v}{\mu_{1v}-b_v}. \end{cases} \tag{3.3}$$

So, from (3.3), $\frac{dV}{dt} \leqslant 0$ proves that the set Ω is positively invariant, and by using the differential inequality in [5, 11] we get,

$$0 \leqslant (V_1, V_2) \leqslant (V_1(0)e^{-(\mu_{1h}-b_h)t} + \frac{\Lambda_h(1 - e^{-(\mu_{1h}-b_h)t})}{\mu_{1h} - b_h}, V_2(0)e^{-(\mu_{1v}-b_v)t} + \frac{\Lambda_v(1 - e^{-(\mu_{1v}-b_v)t})}{\mu_{1v} - b_v}).$$

Thus as $t \to \infty$, $0 \leqslant (V_1, V_2) \leqslant (\frac{\Lambda_h}{\mu_{1h}-b_h}, \frac{\Lambda_v}{\mu_{1v}-b_v})$, we conclude that Ω is an attracting set. □

The model (2.1) is correctly stated mathematically and epidemiologically within the domain. Therefore, studying the dynamics of this fundamental model in Ω is adequate.

3.2. *Disease-free equilibrium*

3.2.1. *Existence:*

The disease-free equilibrium of the system (2.1) is presented by: $E_1 = (S_h^0, 0, 0, S_v^0, 0)$, where

$$S_h^0 = \frac{-(\mu_{1h} - b_h) + \sqrt{(\mu_{1h} - b_h)^2 + 4\mu_{2h}\Lambda_h}}{2\mu_{2h}} \text{ and } S_v^0 = \frac{-(\mu_{1v} - b_v) + \sqrt{(\mu_{1v} - b_v)^2 + 4\mu_{2v}\Lambda_v}}{2\mu_{2v}}.$$

3.2.2. *Local stability*

The quantity R_0 can be used to characterize the dynamics of the disease:

$$R_0 = \frac{\beta_2\beta_3 S_h^0 S_v^0}{(B_h - \beta_1 S_h^0)(\mu_{2v}S_v^0 + \delta_v + \mu_{1v})},$$

with $B_h = \gamma_h + \delta_h + \mu_{1h} + \mu_{2h}S_h^0$.

Theorem 3.2. *If $R_0 < 1$, then the disease-free equilibrium point E_1 of the model (2.1) is locally asymptotically stable, otherwise unstable.*

Proof. The Jacobian matrix of disease-free equilibrium E_1 is:

$$J = \begin{pmatrix} -\sqrt{\Delta_h} & A_h - \beta_1 S_h^0 & A_h & 0 & -\beta_2 S_h^0 \\ 0 & -B_h + \beta_1 S_h^0 & 0 & 0 & \beta_2 S_h^0 \\ 0 & \gamma_h & -C_h & 0 & 0 \\ 0 & -\beta_3 S_v^0 & 0 & -\sqrt{\Delta_v} & A_v \\ 0 & \beta_3 S_v^0 & 0 & 0 & -\mu_{2v} S_v^0 - \delta_v - \mu_{1v} \end{pmatrix}$$

with $A_h = b_h - \mu_{2h} S_h^0$, $A_v = b_v - \mu_{2v} S_v^0$, $B_h = \gamma_h + \delta_h + \mu_{1h} + \mu_{2h} S_h^0$, $C_h = \mu_{1h} + \mu_{2h} S_h^0$, $\Delta_h = (\mu_{1h} - b_h)^2 + 4\mu_{2h}\Lambda_h$ and $\Delta_v = (\mu_{1v} - b_v)^2 + 4\mu_{2v}\Lambda_v$. Moreover, the characteristic equation of this matrix is given by:

$$\begin{aligned} &(\lambda + \sqrt{\Delta_h})(\lambda + C_h)(\lambda + \sqrt{\Delta_v}) \Big[\lambda^2 + \left(\mu_{1v} + \delta_v + \mu_{2v} S_v^0 + B_h - \beta_1 S_h^0\right)\lambda \\ &\quad + \left(B_h - \beta_1 S_h^0\right)\left(\mu_{1v} + \delta_v + \mu_{2v} S_v^0\right)\left(1 - \frac{\beta_2\beta_3 S_h^0 S_v^0}{\left(B_h - \beta_1 S_h^0\right)\left(\mu_{1v} + \delta_v + \mu_{2v} S_v^0\right)}\right)\Big] = 0, \end{aligned} \tag{3.4}$$

where $R_0 = \frac{\beta_2\beta_3 S_h^0 S_v^0}{\left(B_h - \beta_1 S_h^0\right)\left(\mu_{1v} + \delta_v + \mu_{2v} S_v^0\right)}$. The disease-free equilibrium E_1 is therefore locally asymptotically stable since all of the eigenvalues of the characteristic equation (3.4) have negative real parts if $R_0 < 1$. □

3.2.3. *Global stability*

We can write our main system the following:

$$\begin{cases} \frac{dS_h}{dt} = \wedge_h - (\mu_{1h} - b_h)S_h + b_h I_h + b_h R_h - (\beta_1 + \mu_{2h})S_h I_h - \beta_2 S_h I_v - \mu_{2h} S_h^2 - \mu_{2h} S_h R_h, \\ \frac{dI_h}{dt} = -(B_h - \mu_{2h} S_h^0) I_h + (\beta_1 - \mu_{2h}) S_h I_h + \beta_2 S_h I_v - \mu_{2h} I_h^2 - \mu_{2h} R_h I_h, \\ \frac{dR_h}{dt} = \gamma_h I_h - \mu_{1h} R_h - \mu_{2h} S_h R_h - \mu_{2h} I_h R_h - \mu_{2h} R_h^2, \\ \frac{dS_v}{dt} = \wedge_v - (\mu_{1v} - b_v) S_v + b_v I_v - \beta_3 S_v I_h - \mu_{2v} S_v^2 - \mu_{2v} I_v S_v, \\ \frac{dI_v}{dt} = \beta_3 S_v I_h - (\delta_v + \mu_{1v}) I_v - \mu_{2v} I_v S_v - \mu_{2v} I_v^2. \end{cases} \tag{3.5}$$

According to the above, we have the existence of a single disease-free equilibrium that verifies:

$$\begin{cases} \wedge_h = (\mu_{1h} - b_h) S_h^0 + \mu_{2h} S_h^{0^2}, \\ \wedge_v = (\mu_{1v} - b_v) S_v^0 + \mu_{2v} S_v^{0^2}. \end{cases} \tag{3.6}$$

Proposition 3.3. *If $R_0 < 1$, then the disease-free equilibrium point E_1 of the model (2.1) is globally asymptotically stable, otherwise unstable.*

Proof. We consider the following candidate Lyapunov function

$$V_{DFE} = (\mu_{1v} + \delta_v + \mu_{2v} S_v^0)\left(S_h^0\left(\frac{S_h}{S_h^0} - \ln\left(\frac{S_h}{S_h^0}\right) - 1\right) + I_h + R_h\right) + \beta_2 S_h^0\left(I_v + S_v^0\left(\frac{S_v}{S_v^0} - \ln\left(\frac{S_v}{S_v^0}\right) - 1\right)\right).$$

We set

$$Q = \beta_2 S_h^0 \quad \text{and} \quad P = \mu_{1v} + \delta_v + \mu_{2v} S_v^0.$$

Indeed, $V_{DFE} > 0$ for all $(S_h, I_h, R_h, S_v, I_v) \in \Omega \backslash E_1$ and for $(S_h, I_h, R_h, S_v, I_v) = E_1$, with $E_1 = (S_h^0, 0, 0, S_v^0, 0)$ we have, $V_{DFE}(E_1) = P\left(I_h^0 + R_h^0\right) + Q I_v^0 = 0$, which verifies Definition 4 in [10].

The derivative of Lyapunov's candidate function V_{DFE} along the path of the system is given by

$$\dot{V}_{DFE} = P\left(\frac{S_h - S_h^0}{S_h}\right)\dot{S}_h + Q\left(\frac{S_v - S_v^0}{S_v}\right)\dot{S}_v + P\dot{I}_h + Q\dot{I}_v + P\dot{R}_h.$$

Through the model (3.5), we get

$$
\begin{aligned}
\dot{V}_{DFE} &= P\left(1-\frac{S_h^0}{S_h}\right)(\wedge_h-(\mu_{1h}-b_h)S_h+b_hI_h+b_hR_h-(\beta_1+\mu_{2h})S_hI_h-\beta_2S_hI_v-\mu_{2h}S_h^2\\
&\quad-\mu_{2h}S_hR_h)+Q\left(1-\frac{S_v^0}{S_v}\right)(\wedge_v-(\mu_{1v}-b_v)S_v+b_vI_v-\beta_3S_vI_h-\mu_{2v}S_v^2-\mu_{2v}I_vS_v)\\
&\quad+P\left(-(B_h-\mu_{2h}S_h^0)I_h+(\beta_1-\mu_{2h})S_hI_h+\beta_2S_hI_v-\mu_{2h}I_h^2-\mu_{2h}R_hI_h\right)\\
&\quad+Q\left(\beta_3S_vI_h-(\delta_v+\mu_{1v})I_v-\mu_{2v}I_vS_v-\mu_{2v}I_v^2\right)+P\dot{R}_h\\
&= P\left(1-\frac{S_h^0}{S_h}\right)(\wedge_h-(\mu_{1h}-b_h)S_h+b_hI_h+b_hR_h-\mu_{2h}S_hI_h-\mu_{2h}S_h^2-\mu_{2h}S_hR_h)\\
&\quad-P\beta_2S_hI_v+P\beta_2S_h^0I_v+Q\left(1-\frac{S_v^0}{S_v}\right)(\wedge_v+b_vS_v+b_vI_v)\\
&\quad-Q\beta_3S_vI_h+Q\beta_3S_v^0I_h+Q\left(S_v-S_v^0\right)(-\mu_{1v}-\mu_{2v}S_v)-2Q\mu_{2v}S_vI_v+2\mu_{2v}QS_v^0I_v\\
&\quad+P\left(\mu_{2h}S_h^0I_h-\mu_{2h}S_hI_h-\mu_{2h}I_h^2-\mu_{2h}R_hI_h\right)\\
&\quad-P\left(B_h-\beta_1S_h^0\right)I_h+P\beta_2S_hI_v-Q\left(\delta_v+\mu_{1v}+\mu_{2v}S_v^0\right)I_v-Q\mu_{2v}I_v^2+Q\beta_3S_vI_h+P\dot{R}_h\\
&= Q\beta_3S_v^0I_h-P\left(B_h-\beta_1S_h^0\right)I_h+P\beta_2S_h^0I_v-Q\left((\delta_v+\mu_{1v}+\mu_{2v}S_v^0)\right)I_v\\
&\quad+P\left(1-\frac{S_h^0}{S_h}\right)(\wedge_h-(\mu_{1h}-b_h)S_h+b_hI_h+b_hR_h-\mu_{2h}S_hI_h-\mu_{2h}S_h^2-\mu_{2h}S_hR_h)\\
&\quad+Q\left(1-\frac{S_v^0}{S_v}\right)(\wedge_v+b_vS_v+b_vI_v)-Q\left(S_v-S_v^0\right)(\mu_{1v}+\mu_{2v}S_v)-2Q\mu_{2v}I_v\left(S_v-S_v^0\right)\\
&\quad+P\left(\mu_{2h}S_h^0I_h-\mu_{2h}S_hI_h-\mu_{2h}I_h^2-\mu_{2h}R_hI_h\right)-Q\mu_{2v}I_v^2+P\dot{R}_h\\
&= Q\beta_3S_v^0\left(-\frac{P\left(B_h-\beta_1S_h^0\right)}{Q\beta_3S_v^0}+1\right)I_h-\left(Q(\delta_v+\mu_{1v}+\mu_{2v}S_v^0)-P\beta_2S_h^0\right)I_v\\
&\quad+P\left(1-\frac{S_h^0}{S_h}\right)(\wedge_h-(\mu_{1h}-b_h)S_h+b_hI_h+b_hR_h-\mu_{2h}S_hI_h-\mu_{2h}S_h^2-\mu_{2h}S_hR_h)\\
&\quad+Q\left(1-\frac{S_v^0}{S_v}\right)(\wedge_v+b_vS_v+b_vI_v)-Q\left(S_v-S_v^0\right)(\mu_{1v}+\mu_{2v}S_v)-2Q\mu_{2v}I_v\left(S_v-S_v^0\right)\\
&\quad+P\left(\mu_{2h}S_h^0I_h-\mu_{2h}S_hI_h-\mu_{2h}I_h^2-\mu_{2h}R_hI_h\right)-Q\mu_{2v}I_v^2+P\dot{R}_h,
\end{aligned}
$$

by replacing the Q and P with their expressions in the first line we have,

$$
\begin{aligned}
\dot{V}_{DFE} &= \beta_2S_h^0\beta_3S_v^0\left(1-\frac{1}{R_0}\right)I_h\\
&\quad+P\left(1-\frac{S_h^0}{S_h}\right)(\wedge_h-(\mu_{1h}-b_h)S_h+b_hI_h+b_hR_h-\mu_{2h}S_hI_h-\mu_{2h}S_h^2-\mu_{2h}S_hR_h)\\
&\quad+Q\left(1-\frac{S_v^0}{S_v}\right)(\wedge_v+b_vS_v+b_vI_v)-Q\left(S_v-S_v^0\right)(\mu_{1v}+\mu_{2v}S_v)-2Q\mu_{2v}I_v\left(S_v-S_v^0\right)\\
&\quad+P\left(\mu_{2h}S_h^0I_h-\mu_{2h}S_hI_h-\mu_{2h}I_h^2-\mu_{2h}R_hI_h\right)-Q\mu_{2v}I_v^2+P\dot{R}_h,
\end{aligned}
$$

using the formulas (3.6) in $\dot{V}_{DFE}$ we get

$$
\begin{aligned}
\dot{V}_{DFE} &= \beta_2S_h^0\beta_3S_v^0\left(1-\frac{1}{R_0}\right)I_h\\
&\quad-\frac{P}{S_h}\left(S_h-S_h^0\right)^2(\mu_{1h}-b_h)-\frac{P}{S_h}\mu_{2h}\left(S_h-S_h^0\right)^2\left(S_h+S_h^0\right)-P\mu_{2h}I_h^2-P\mu_{2h}R_hI_h
\end{aligned}
$$

$$
\begin{aligned}
&- Q\mu_{2v} I_v^2 + P\left(S_h - S_h^0\right)\left(b_h \frac{I_h}{S_h} + b_h \frac{R_h}{S_h} - 2\mu_{2h} I_h - \mu_{2h} R_h\right)\\
&- \frac{Q}{S_v}\left(S_v - S_v^0\right)^2 (\mu_{1v} - b_v) - \frac{Q}{S_v}\mu_{2v}\left(S_v - S_v^0\right)^2\left(S_v + S_v^0\right)\\
&+ Q\left(S_v - S_v^0\right)\left(\frac{b_v}{S_v} - 2\mu_{2v}\right) I_v + P\dot{R}_h,
\end{aligned}
$$

by sitting

$$
\begin{cases} b_h = \mu_{1h} - \sqrt{\mu_{1h}^2 - 4\mu_{2h}\wedge_h}, \\ b_v = \mu_{1v} - \sqrt{\mu_{1v}^2 - 4\mu_{2v}\wedge_v}. \end{cases} \tag{3.7}
$$

So,

$$
\begin{cases} \wedge_h = (2\mu_{1h} - b_h)\dfrac{b_h}{4\mu_{2h}}, \\ \wedge_v = (2\mu_{1v} - b_v)\dfrac{b_v}{4\mu_{2v}}. \end{cases}
$$

If we use these relations in

$$
\begin{cases} S_h^0 = \dfrac{-(\mu_{1h} - b_h) + \sqrt{(\mu_{1h} - b_h)^2 + 4\mu_{2h}\Lambda_h}}{2\mu_{2h}}, \\ S_v^0 = \dfrac{-(\mu_{1v} - b_v) + \sqrt{(\mu_{1v} - b_v)^2 + 4\mu_{2v}\Lambda_v}}{2\mu_{2v}}, \end{cases}
$$

we get

$$
\begin{cases} b_h = 2S_h^0\mu_{2h}, \\ b_v = 2S_v^0\mu_{2v}. \end{cases} \tag{3.8}
$$

By exploiting the expressions of (3.8) in $P\left(S_h - S_h^0\right)\left(b_h \frac{I_h}{S_h} + b_h \frac{R_h}{S_h} - 2\mu_{2h} I_h - \mu_{2h} R_h\right)$ and $Q\left(S_v - S_v^0\right)\left(\frac{b_v}{S_v} - 2\mu_{2v}\right) I_v$, we have

$$
\begin{aligned}
&P\left(S_h - S_h^0\right)\left(b_h \frac{I_h}{S_h} + b_h \frac{R_h}{S_h} - 2\mu_{2h} I_h - \mu_{2h} R_h\right)\\
&= P\left(S_h - S_h^0\right)\left(\frac{b_h}{S_h} - 2\mu_{2h}\right) I_h + P\left(S_h - S_h^0\right)\left(\frac{b_h}{S_h} - \mu_{2h}\right) R_h,\\
&= P\left(S_h - S_h^0\right)\left(\frac{b_h}{S_h} - 2\mu_{2h}\right) I_h + P\left(S_h - S_h^0\right)\left(\frac{b_h}{S_h} - 2\mu_{2h}\right) R_h + P\mu_{2h}\left(S_h - S_h^0\right) R_h,\\
&= -2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 I_h - 2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 R_h + P\mu_{2h}\left(S_h - S_h^0\right) R_h,\\
&= -2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 I_h - 2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 R_h - P\mu_{2h} S_h^0 R_h + P\mu_{2h} S_h R_h,
\end{aligned}
$$

and

$$
Q\left(S_v - S_v^0\right)\left(\frac{b_v}{S_v} - 2\mu_{2v}\right) I_v = -2\frac{Q}{S_v}\mu_{2v}\left(S_v - S_v^0\right)^2 I_v.
$$

On the other hand, we use the relation of $\dot{R}_h$ in the system (3.5). So, by using these relations in the expression of $\dot{V}_{DFE}$, we get

$$
\begin{aligned}
\dot{V}_{DFE} &= \beta_2 S_h^0 \beta_3 S_v^0 \left(1 - \frac{1}{R_0}\right) I_h\\
&- \frac{P}{S_h}\left(S_h - S_h^0\right)^2 (\mu_{1h} - b_h) - \frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2\left(S_h + S_h^0\right) - P\mu_{2h} I_h^2 - P\mu_{2h} R_h I_h
\end{aligned}
$$

$$
\begin{aligned}
&- Q\mu_{2v}I_v^2 - 2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 I_h - 2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 R_h - P\mu_{2h}S_h^0 R_h + P\mu_{2h}S_h R_h \\
&- \frac{Q}{S_v}\left(S_v - S_v^0\right)^2(\mu_{1v} - b_v) - \frac{Q}{S_v}\mu_{2v}\left(S_v - S_v^0\right)^2\left(S_v + S_v^0\right) - 2\frac{Q}{S_v}\mu_{2v}\left(S_v - S_v^0\right)^2 I_v \\
&+ p\left(\gamma_h I_h - \mu_{1h}R_h - \mu_{2h}S_h R_h - \mu_{2h}I_h R_h - \mu_{2h}R_h^2\right) \\
= &\left(\beta_2 S_h^0 \beta_3 S_v^0\left(1 - \frac{1}{R_0}\right) + P\gamma_h\right)I_h \\
&- \frac{P}{S_h}\left(S_h - S_h^0\right)^2(\mu_{1h} - b_h) - \frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2\left(S_h + S_h^0\right) - P\mu_{2h}I_h^2 - 2P\mu_{2h}R_h I_h - Q\mu_{2v}I_v^2 \\
&- p\mu_{1h}R_h - p\mu_{2h}R_h^2 - 2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 I_h - 2\frac{P}{S_h}\mu_{2h}\left(S_h - S_h^0\right)^2 R_h - P\mu_{2h}S_h^0 R_h \\
&- \frac{Q}{S_v}\left(S_v - S_v^0\right)^2(\mu_{1v} - b_v) - \frac{Q}{S_v}\mu_{2v}\left(S_v - S_v^0\right)^2\left(S_v + S_v^0\right) - 2\frac{Q}{S_v}\mu_{2v}\left(S_v - S_v^0\right)^2 I_v.
\end{aligned}
$$

with $R_0 < 1$ and $P\gamma_h < \beta_2 S_h^0 \beta_3 S_v^0\left(\frac{1}{R_0} - 1\right)$, so $\dot{V}_{DFE}$ is negative. Then the disease-free equilibrium point E_1 of the model (2.1) is globally asymptotically stable if $R_0 < 1$. □

3.2.4. *Numerical simulation*

In the following section, we present a numerical simulation of the model. We aim to estimate and add other values from [8, 18] to the previous parameters of the system (3.5) . These numerical values are in the Table 2.

Table 2: Parameter values

parameters	Values	Reference
$\wedge_h$	0.02	Assumed
$\wedge_v$	25	[18]
μ_{1h}	4.212×10^{-4}	Assumed
μ_{2h}	10^{-7}	[8]
μ_{1v}	0.7	Assumed
μ_{2v}	2.279×10^{-4}	[8]
γ_h	3.704×10^{-3}	[8]
δ_h	0.01	[18]
δ_v	10^{-3}	Assumed
β_1	10^{-4}	Assumed
β_2	10^{-2}	[18]
β_3	2.2974×10^{-4}	Assumed
b_h	9.6062×10^{-6}	$\mu_{1h} - \sqrt{\mu_{1h}^2 - 4\mu_{2h}\wedge_h}$ (see (3.7))
b_v	0.0165	$\mu_{1v} - \sqrt{\mu_{1v}^2 - 4\mu_{2v}\wedge_v}$ (see (3.7))

With the following initial values $S_h(0) = 120$, $I_h(0) = 20$, $R_h(0) = 18$, $S_v(0) = 110$, and $I_v(0) = 70$ and using the data from the Table 2, the susceptible human population will initially decrease over time and then increase, according to the numerical simulation shown in Figure 2. Populations of infected and recovered humans will increase with time before converging to zero. This means that the disease-free equilibrium $E_1 = (S_h^0, 0, 0, S_v^0, 0) = (48.0311, 0, 0, 36.1395, 0)$ is stable for a reproduction number that is lower than 1 ($R_0 = 0.6029 < 1$). According to Figure 3, the population of infected mosquitoes and the susceptible mosquito population are both decreasing over time, indicating that the population would not suffer from malaria epidemic.

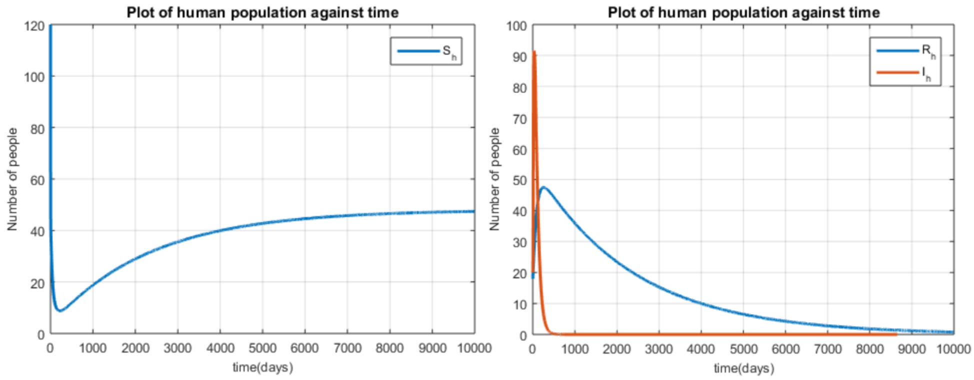

Figure 2: The simulation of the model (2.1) depending on time, shows convergence of solutions to the disease-free equilibrium for human population. The parameters values in Table 2 give $R_0 = 0.6029 < 1$.

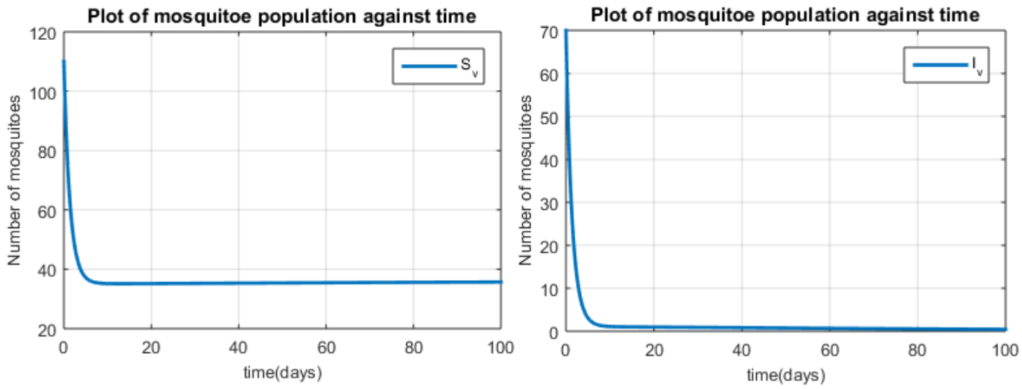

Figure 3: The simulation of the model (2.1) depending on time, shows convergence of solutions to the disease-free equilibrium for mosquito population.

3.3. Endemic equilibrium point

3.3.1. Existence

In the endemic equilibrium, the system (3.5), verifies

$$\begin{cases} \wedge_h = (\mu_{1h} - b_h)S_h^* - b_h I_h^* - b_h R_h^* + (\beta_1 + \mu_{2h})S_h^* I_h^* + \beta_2 S_h^* I_v^* + \mu_{2h} S_h^{*2} + \mu_{2h} S_h^* R_h^*, \\ (\gamma_h + \delta_h + \mu_{1h}) I_h^* = (\beta_1 - \mu_{2h}) S_h^* I_h^* + \beta_2 S_h^* I_v^* - \mu_{2h} I_h^{*2} - \mu_{2h} R_h^* I_h^*, \\ \mu_{1h} R_h^* = \gamma_h I_h^* - \mu_{2h} S_h^* R_h^* - \mu_{2h} I_h^* R_h^* - \mu_{2h} R_h^{*2}, \\ \wedge_v = (\mu_{1v} - b_v) S_v^* - b_v I_v^* + \beta_3 S_v^* I_h^* + \mu_{2v} S_v^{*2} + \mu_{2v} I_v^* S_v^*, \\ (\delta_v + \mu_{1v}) I_v^* = \beta_3 S_v^* I_h^* - \mu_{2v} I_v^* S_v^* - \mu_{2v} I_v^{*2}. \end{cases} \tag{3.9}$$

From the system (3.9) we have the existence of a single endemic equilibrium that verifies:

$$\begin{cases} \wedge_h = (\mu_{1h} - b_h)S_h^* + (\gamma_h + \delta_h + \mu_{1h} - b_h) I_h^* - b_h R_h^* + \mu_{2h}(2 S_h^* I_h^* + S_h^* R_h^* + R_h^* I_h^*) \\ \qquad + \mu_{2h}({S_h^*}^2 + {I_h^*}^2), \\ \wedge_v = (\mu_{1v} - b_v) S_v^* + (\delta_v + \mu_{1v} - b_v) I_v^* + 2\mu_{2v} I_v^* S_v^* + \mu_{2v}(S_v^{*2} + I_v^{*2}). \end{cases} \tag{3.10}$$

The endemic equilibrium of the system (3.9) is given by $E_2 = (S_h^*, I_h^*, R_h^*, S_v^*, I_v^*)$, where

$$R_h^* = \frac{-(\mu_{1h} - b_h)(\beta_1 - 2\mu_{2h}) + \sqrt{((\mu_{1h} - b_h)(\beta_1 - 2\mu_{2h}))^2 + 4R_0 \beta_1 \gamma_h (\mu_{1h} - b_h)(\beta_1 - \mu_{2h})}}{2R_0 \beta_1 \mu_{2h}},$$

$$S_h^* = \frac{b_h \gamma_h (\beta_1 - \mu_{2h}) + b_h \mu_{2h} R_h^* (2\mu_{2h} - \beta_1)}{\mu_{2h} \gamma_h (\beta_1 - \mu_{2h}) + 2\mu_{2h}^3 R_h^*},$$

$$I_h^* = \frac{(\mu_{1h} + \mu_{2h} S_h^* + \mu_{2h} R_h^*) R_h^*}{\gamma_h - \mu_{2h} R_h^*},$$
$$I_v^* = \frac{(\gamma_h + \delta_h + \mu_{1h}) I_h^* - (\beta_1 - \mu_{2h}) S_h^* I_h^* + \mu_{2h} {I_h^*}^2 + \mu_{2h} R_h^* I_h^*}{\beta_2 S_h^*},$$
$$S_v^* = \frac{b_v R_0 I_v^*}{\mu_{1v} - b_v + \mu_{2v} R_0 I_v^*}.$$

3.3.2. Global stability

Proposition 3.4. *If* $R_0 > 1$, *then the endemic equilibrium point* E_2 *of the model* (2.1) *is globally asymptotically stable, otherwise unstable.*

Proof. We consider the following candidate Lyaponov function

$$V_{EE} = \left(1 - \frac{\mu_{2h}(\gamma_h - 2\mu_{2h} R_h^*)}{\beta_1(\gamma_h - \mu_{2h} R_h^*)}\right) S_h^* \left(\frac{S_h}{S_h^*} - \log(\frac{S_h}{S_h^*}) - 1\right) + I_h^* \left(\frac{I_h}{I_h^*} - \log(\frac{I_h}{I_h^*}) - 1\right)$$
$$+ \frac{\mu_{2h} P {R_h^*}^2}{\gamma_h - \mu_{2h} R_h^*} \left(\frac{R_h}{R_h^*} - \log(\frac{R_h}{R_h^*}) - 1\right) + S_v^* \left(\frac{S_v}{S_v^*} - \log(\frac{S_v}{S_v^0}) - 1\right) + \frac{\mu_{1v} - b_v}{\mu_{2v} R_0} \left(\frac{I_v}{I_v^*} - \log(\frac{I_v}{I_v^*}) - 1\right).$$

We set $Q = 1 - \dfrac{\mu_{2h}(\gamma_h - 2\mu_{2h} R_h^*)}{\beta_1(\gamma_h - \mu_{2h} R_h^*)}$, $W = \dfrac{\mu_{2h} R_h^*}{\gamma_h - \mu_{2h} R_h^*}$, $H = \dfrac{(\mu_{1v} - b_v)}{\mu_{2v} R_0 I_v^*}$.

$V_{EE} > 0$ for all $(S_h, I_h, R_h, S_v, I_v) \in \Omega \backslash E_2$ and for $(S_h, I_h, R_h, S_v, I_v) = E_2$, we have

$$\left(\frac{S_h^*}{S_h^*} - \log(\frac{S_h^*}{S_h^*}) - 1\right) = \cdots = \left(\frac{I_v^*}{I_v^*} - \log(\frac{I_v^*}{I_v^*}) - 1\right) = 0.$$

So $V_{DFE}(E_2) = 0$, which verifies the Definition 4 in [10].

Derivative of Lyapunov's candidate function V_{DFE} along the path of the system is

$$\dot{V}_{EE} = Q\left(1 - \frac{S_h^*}{S_h}\right)\dot{S}_h + \left(1 - \frac{I_h^*}{I_h}\right)\dot{I}_h + W\left(1 - \frac{R_h^*}{R_h}\right)\dot{R}_h + \left(1 - \frac{S_v^*}{S_v}\right)\dot{S}_v + H\left(1 - \frac{I_v^*}{I_v}\right)\dot{I}_v.$$

Using (3.5), we get

$$\dot{V}_{EE} = Q\left(1 - \frac{S_h^*}{S_h}\right)(\wedge_h - (\mu_{1h} - b_h) S_h + b_h I_h + b_h R_h - (\beta_1 + \mu_{2h}) S_h I_h - \beta_2 S_h I_v - \mu_{2h} S_h^2$$
$$- \mu_{2h} S_h R_h) + (1 - \frac{I_h^*}{I_h})\left(-(\gamma_h + \delta_h + \mu_{1h}) I_h + (\beta_1 - \mu_{2h}) S_h I_h + \beta_2 S_h I_v - \mu_{2h} I_h^2 - \mu_{2h} R_h I_h\right)$$
$$+ W(1 - \frac{R_h^*}{R_h})(\gamma_h I_h - \mu_{1h} R_h - \mu_{2h} S_h R_h - \mu_{2h} I_h R_h - \mu_{2h} R_h^2)$$
$$+ (1 - \frac{S_v^*}{S_v})(\wedge_v - (\mu_{1v} - b_v) S_v + b_v I_v - \beta_3 S_v I_h - \mu_{2v} S_v^2 - \mu_{2v} I_v S_v)$$
$$+ H(1 - \frac{I_v^*}{I_v})(\beta_3 S_v I_h - (\delta_v + \mu_{1v}) I_v - \mu_{2v} I_v S_v - \mu_{2v} I_v^2).$$

According to the system (3.9), we obtain:

$$\dot{V}_{EE} = Q(1 - \frac{S_h^*}{S_h})((\mu_{1h} - b_h)(S_h^* - S_h) + b_h(I_h - I_h^*) + b_h(R_h - R_h^*) + (\beta_1 + \mu_{2h})(S_h^* I_h^* - S_h I_h)$$
$$+ \beta_2(S_h^* I_v^* - S_h I_v) + \mu_{2h}({S_h^*}^2 - S_h^2) + \mu_{2h}(S_h^* R_h^* - S_h R_h))$$
$$+ (1 - \frac{I_h^*}{I_h})((\beta_1 - \mu_{2h})(S_h - S_h^*) I_h + \beta_2(S_h I_v - \frac{S_h^* I_v^*}{I_h^*} I_h) + \mu_{2h}(I_h^* - I_h) I_h + \mu_{2h}(R_h^* - R_h) I_h)$$

$$
\begin{aligned}
&+W(1-\frac{R_h^*}{R_h})(\gamma_h(I_h-\frac{I_h^*}{R_h^*}R_h)+\mu_{2h}(S_h^*-S_h)R_h+\mu_{2h}(I_h^*-I_h)R_h+\mu_{2h}(R_h^*-R_h)R_h)\\
&+(1-\frac{S_v^*}{S_v})((\mu_{1v}-b_v)(S_v^*-S_v)+b_v(I_v-I_v^*)+\beta_3(S_v^*I_h^*-S_vI_h)+\mu_{2v}(S_v^{*2}-S_v^2)\\
&+\mu_{2v}(I_v^*S_v^*-I_vS_v))+H(1-\frac{I_v^*}{I_v})(\beta_3(S_vI_h-S_v^*I_h^*)+\mu_{2v}(S_v^*-S_v)I_v+\mu_{2v}(I_v^*-I_v)I_v)\\
=&-Q(\mu_{1h}-b_h)\frac{(S_h-S_h^*)^2}{S_h}-Q\mu_{2h}(S_h+S_h^*)\frac{(S_h-S_h^*)^2}{S_h}\\
&-(\mu_{1v}-b_v)\frac{(S_v-S_v^*)^2}{S_v}-\mu_{2v}\frac{(S_v+S_v^*)}{S_v}(S_v-S_v^*)^2-H\mu_{2v}(I_v-I_v^*)^2\\
&-\mu_{2h}(I_h-I_h^*)^2-W\mu_{2h}(R_h-R_h^*)^2\\
&+(1-\frac{I_h^*}{I_h})\mu_{2h}(R_h^*-R_h)I_h+W(1-\frac{R_h^*}{R_h})\gamma_h(I_h-\frac{I_h^*}{R_h^*}R_h)+W(1-\frac{R_h^*}{R_h})\mu_{2h}(I_h^*-I_h)R_h\\
&+Q(1-\frac{S_h^*}{S_h})(b_h(I_h-I_h^*)+b_h(R_h-R_h^*)+(\beta_1+\mu_{2h})(S_h^*I_h^*-S_hI_h)\\
&+\beta_2(S_h^*I_v^*-S_hI_v)+\mu_{2h}(S_h^*R_h^*-S_hR_h))\\
&+(1-\frac{I_h^*}{I_h})((\beta_1-\mu_{2h})(S_h-S_h^*)I_h+\beta_2(S_hI_v-\frac{S_h^*I_v^*}{I_h^*}I_h))\\
&+W\mu_{2h}(1-\frac{R_h^*}{R_h})(S_h^*-S_h)R_h+(1-\frac{S_v^*}{S_v})(b_v(I_v-I_v^*)+\beta_3(S_v^*I_h^*-S_vI_h)+\mu_{2v}(I_v^*S_v^*-I_vS_v))\\
&+H(1-\frac{I_v^*}{I_v})(\beta_3(S_vI_h-S_v^*I_h^*)+\mu_{2v}(S_v^*-S_v)I_v).
\end{aligned}
$$

We have

$$
\begin{aligned}
(1-\frac{I_h^*}{I_h})&\mu_{2h}(R_h^*-R_h)I_h+W(1-\frac{R_h^*}{R_h})\gamma_h(I_h-\frac{I_h^*}{R_h^*}R_h)+W(1-\frac{R_h^*}{R_h})\mu_{2h}(I_h^*-I_h)R_h\\
&=-\mu_{2h}(1+W)(I_h-I_h^*)(R_h-R_h^*)+W\gamma_h(R_h-R_h^*)(\frac{I_h}{R_h}-\frac{I_h}{R_h^*}-\frac{I_h^*}{R_h^*}+\frac{I_h}{R_h^*})\\
&=-(\mu_{2h}(1+W)-W\frac{\gamma_h}{R_h^*})(I_h-I_h^*)(R_h-R_h^*)-W\gamma_h(R_h-R_h^*)^2\frac{I_h}{R_hR_h^*}.
\end{aligned}
$$

As $W=\dfrac{\mu_{2h}R_h^*}{\gamma_h-\mu_{2h}R_h^*}$, we get, $\mu_{2h}(1+W)-W\dfrac{\gamma_h}{R_h^*}=0$, so

$$
\begin{aligned}
\dot{V}_{EE}=&-Q(\mu_{1h}-b_h)\frac{(S_h-S_h^*)^2}{S_h}-Q\mu_{2h}(S_h+S_h^*)\frac{(S_h-S_h^*)^2}{S_h}\\
&-(\mu_{1v}-b_v)\frac{(S_v-S_v^*)^2}{S_v}-\mu_{2v}\frac{(S_v+S_v^*)}{S_v}(S_v-S_v^*)^2-H\mu_{2v}(I_v-I_v^*)^2\\
&-\mu_{2h}(I_h-I_h^*)^2-\mu_{2h}W(R_h-R_h^*)^2-W\gamma_h(R_h-R_h^*)^2\frac{I_h}{R_hR_h^*}\\
&-Q(1-\frac{S_h^*}{S_h})(b_h(I_h-I_h^*)+b_h(R_h-R_h^*)+(\beta_1+\mu_{2h})(S_h^*I_h^*-S_hI_h)+\beta_2(S_h^*I_v^*-S_hI_v)\\
&+\mu_{2h}(S_h^*R_h^*-S_hR_h))+(1-\frac{I_h^*}{I_h})\left((\beta_1-\mu_{2h})(S_h-S_h^*)I_h+\beta_2(S_hI_v-\frac{S_h^*I_v^*}{I_h^*}I_h)\right)\\
&+W\mu_{2h}(1-\frac{R_h^*}{R_h})(S_h^*-S_h)R_h+(1-\frac{S_v^*}{S_v})\,(b_v(I_v-I_v^*)+\beta_3(S_v^*I_h^*-S_vI_h)+\mu_{2v}(I_v^*S_v^*-I_vS_v))\\
&+H(1-\frac{I_v^*}{I_v})\,(\beta_3(S_vI_h-S_v^*I_h^*)+\mu_{2v}(S_v^*-S_v)I_v)
\end{aligned}
$$

$$
\begin{aligned}
&= -Q(\mu_{1h}-b_h)\frac{(S_h-S_h^*)^2}{S_h} - Q\mu_{2h}(S_h+S_h^*)\frac{(S_h-S_h^*)^2}{S_h}\\
&\quad -(\mu_{1v}-b_v)\frac{(S_v-S_v^*)^2}{S_v} - \mu_{2v}\frac{(S_v+S_v^*)}{S_v}(S_v-S_v^*)^2 - H\mu_{2v}(I_v-I_v^*)^2 - W\gamma_h(R_h-R_h^*)^2\frac{I_h}{R_hR_h^*}\\
&\quad -\mu_{2h}(I_h-I_h^*)^2 - \mu_{2h}W(R_h-R_h^*)^2\frac{Qb_h}{S_h}(S_h-S_h^*)(I_h-I_h^*) + \frac{Qb_h}{S_h}(S_h-S_h^*)(R_h-R_h^*)\\
&\quad -\frac{Q(\beta_1+\mu_{2h})}{S_h}(S_h-S_h^*)(S_hI_h-S_h^*I_h^*) - \frac{Q\beta_2}{S_h}(S_h-S_h^*)(S_hI_v-S_h^*I_v^*)\\
&\quad -\frac{Q\mu_{2h}}{S_h}(S_h-S_h^*)(S_hR_h-S_h^*R_h^*) + (\beta_1-\mu_{2h})(I_h-I_h^*)(S_h-S_h^*)\\
&\quad +\frac{\beta_2}{I_h}(I_h-I_h^*)(S_hI_v-\frac{S_h^*I_v^*}{I_h^*}I_h) - W\mu_{2h}(R_h-R_h^*)(S_h-S_h^*)\\
&\quad +\frac{b_v}{S_v}(S_v-S_v^*)(I_v-I_v^*) - \frac{M\beta_3}{S_v}(S_v-S_v^*)(S_vI_h-S_v^*I_h^*) - \frac{\mu_{2v}}{S_v}(S_v-S_v^*)(I_vS_v-I_v^*S_v^*)\\
&\quad +\frac{H\beta_3}{I_v}(I_v-I_v^*)(S_vI_h-S_v^*I_h^*) - H\mu_{2v}(I_v-I_v^*)(S_v-S_v^*),
\end{aligned}
$$

since,

$$
\begin{aligned}
&\frac{Qb_h}{S_h}(S_h-S_h^*)(I_h-I_h^*) - \frac{Q(\beta_1+\mu_{2h})}{S_h}(S_h-S_h^*)(S_hI_h-S_h^*I_h^*) + (\beta_1-\mu_{2h})(I_h-I_h^*)(S_h-S_h^*)\\
&= \frac{(S_h-S_h^*)}{S_h}\left(Qb_h(I_h-I_h^*) - Q(\beta_1+\mu_{2h})(S_hI_h-S_h^*I_h^*) + (\beta_1-\mu_{2h})(I_h-I_h^*)S_h\right)\\
&= \frac{(S_h-S_h^*)}{S_h}(Qb_hI_h - Qb_hI_h^* - Q(\beta_1+\mu_{2h})S_hI_h + Q(\beta_1+\mu_{2h})S_h^*I_h^* + (\beta_1-\mu_{2h})I_hS_h\\
&\quad -(\beta_1-\mu_{2h})I_h^*S_h)\\
&= \frac{(S_h-S_h^*)}{S_h}\left((Qb_h - (Q(\beta_1+\mu_{2h}) - (\beta_1-\mu_{2h}))S_h)I_h + (Q(\beta_1+\mu_{2h})S_h^* - Qb_h - (\beta_1-\mu_{2h})S_h)I_h^*\right)\\
&= \frac{(S_h-S_h^*)}{S_h}((\frac{Qb_h}{Q(\beta_1+\mu_{2h})-(\beta_1-\mu_{2h})} - S_h)(Q(\beta_1+\mu_{2h})-(\beta_1-\mu_{2h}))I_h\\
&\quad +(\frac{Q(\beta_1+\mu_{2h})S_h^*-Qb_h}{(\beta_1-\mu_{2h})} - S_h)(\beta_1-\mu_{2h})I_h^*)\\
&= \frac{(S_h-S_h^*)}{S_h}((\frac{Qb_h}{(Q-1)\beta_1+(Q+1)\mu_{2h}} - S_h)((Q-1)\beta_1+(Q+1)\mu_{2h})I_h\\
&\quad +(\frac{Q(\beta_1+\mu_{2h})S_h^*-Qb_h}{(\beta_1-\mu_{2h})} - S_h)(\beta_1-\mu_{2h})I_h^*).
\end{aligned}
$$

With $S_h^* = \dfrac{Qb_h}{(Q-1)\beta_1+(Q+1)\mu_{2h}}$, we get $\dfrac{Q(\beta_1+\mu_{2h})S_h^*-Qb_h}{(\beta_1-\mu_{2h})} = \dfrac{Qb_h}{(Q-1)\beta_1+(Q+1)\mu_{2h}} = S_h^*$, and as $Q = 1-\dfrac{\mu_{2h}(\gamma_h-2\mu_{2h}R_h^*)}{\beta_1(\gamma_h-\mu_{2h}R_h^*)}$, we have $(Q-1)\beta_1+(Q+1)\mu_{2h} = (Q+W)\mu_{2h}$, so $\dfrac{Qb_h}{(Q-1)\beta_1+(Q+1)\mu_{2h}} = \dfrac{Qb_h}{(Q+W)\mu_{2h}} = S_h^*$. Then

$$
\begin{aligned}
&\frac{Qb_h}{S_h}(S_h-S_h^*)(I_h-I_h^*) - \frac{Q(\beta_1+\mu_{2h})}{S_h}(S_h-S_h^*)(S_hI_h-S_h^*I_h^*) + (\beta_1-\mu_{2h})(I_h-I_h^*)(S_h-S_h^*)\\
&\qquad = -(Q+W)\mu_{2h}\frac{(S_h-S_h^*)^2}{S_h}I_h - \frac{(S_h-S_h^*)^2}{S_h}(\beta_1-\mu_{2h})I_h^*.
\end{aligned}
$$

For

$$
\frac{Qb_h}{S_h}(S_h-S_h^*)(R_h-R_h^*) - \frac{Q\mu_{2h}}{S_h}(S_h-S_h^*)(S_hR_h-S_h^*R_h^*) - W\mu_{2h}(R_h-R_h^*)(S_h-S_h^*)
$$

$$
\begin{aligned}
&= \frac{(S_h - S_h^*)}{S_h}\left(Qb_h(R_h - R_h^*) - Q\mu_{2h}(S_h R_h - S_h^* R_h^*) - W\mu_{2h}(R_h - R_h^*)S_h\right) \\
&= \frac{(S_h - S_h^*)}{S_h}\left(Qb_h R_h - Qb_h R_h^* - Q\mu_{2h} S_h R_h + Q\mu_{2h} S_h^* R_h^* - W\mu_{2h} R_h S_h + W\mu_{2h} R_h^* S_h\right) \\
&= \frac{(S_h - S_h^*)}{S_h}\left((Qb_h - (Q+W)\mu_{2h} S_h)R_h + (W\mu_{2h} S_h + Q\mu_{2h} S_h^* - Qb_h)R_h^*\right) \\
&= \frac{(S_h - S_h^*)}{S_h}\left(\left(\frac{Qb_h}{(Q+W)\mu_{2h}} - S_h\right)(Q+W)\mu_{2h} R_h + \left(S_h - \frac{Qb_h - Q\mu_{2h} S_h^*}{W\mu_{2h}}\right)W\mu_{2h} R_h^*\right),
\end{aligned}
$$

with $S_h^* = \frac{Qb_h}{(Q+W)\mu_{2h}}$, we get $\frac{Qb_h - Q\mu_{2h} S_h^*}{W\mu_{2h}} = \frac{Qb_h}{(Q+W)\mu_{2h}} = S_h^*$. So,

$$
\begin{aligned}
&\frac{Qb_h}{S_h}(S_h - S_h^*)(R_h - R_h^*) - \frac{Q\mu_{2h}}{S_h}(S_h - S_h^*)(S_h R_h - S_h^* R_h^*) - W\mu_{2h}(R_h - R_h^*)(S_h - S_h^*) \\
&\quad = -\frac{(S_h - S_h^*)^2}{S_h}(Q+W)\mu_{2h} R_h + \frac{(S_h - S_h^*)^2}{S_h} W\mu_{2h} R_h^*.
\end{aligned}
$$

And for

$$
\begin{aligned}
&\frac{b_v}{S_v}(S_v - S_v^*)(I_v - I_v^*) - \frac{\mu_{2v}}{S_v}(S_v - S_v^*)(S_v I_v - S_v^* I_v^*) - H\mu_{2v}(I_v - I_v^*)(S_v - S_v^*) \\
&\quad = \frac{(S_v - S_v^*)}{S_v}\left(b_v(I_v - I_v^*) - \mu_{2v}(S_v I_v - S_v^* I_v^*) - H\mu_{2v}(I_v - I_v^*)S_v\right) \\
&\quad = \frac{(S_v - S_v^*)}{S_v}\left(b_v I_v - b_v I_v^* - \mu_{2v} S_v I_v + \mu_{2v} S_v^* I_v^* - H\mu_{2v} I_v S_v + H\mu_{2v} I_v^* S_v\right) \\
&\quad = \frac{(S_v - S_v^*)}{S_v}\left((b_v - (1+H)\mu_{2v} S_v)I_v + (H\mu_{2v} S_v + \mu_{2v} S_v^* - b_v)I_v^*\right) \\
&\quad = \frac{(S_v - S_v^*)}{S_v}\left(\left(\frac{b_v}{(1+H)\mu_{2v}} - S_v\right)(1+H)\mu_{2v} I_v + \left(S_v - \frac{b_v - \mu_{2v} S_v^*}{H\mu_{2v}}\right)H\mu_{2v} I_v^*\right),
\end{aligned}
$$

with $S_v^* = \frac{b_v}{(1+H)\mu_{2v}}$, we get $\frac{b_v - \mu_{2v} S_v^*}{H\mu_{2v}} = \frac{b_v}{(1+H)\mu_{2v}} = S_v^*$. Then

$$
\begin{aligned}
&\frac{b_v}{S_v}(S_v - S_v^*)(I_v - I_v^*) - \frac{\mu_{2v}}{S_v}(S_v - S_v^*)(S_v I_v - S_v^* I_v^*) - H\mu_{2v}(I_v - I_v^*)(S_v - S_v^*) \\
&\quad = -\frac{(S_v - S_v^*)^2}{S_v}(1+H)\mu_{2v} I_v + \frac{(S_v - S_v^*)^2}{S_v} H\mu_{2v} I_v^*.
\end{aligned}
$$

We replace the results in $\dot{V}_{EE}$:

$$
\begin{aligned}
\dot{V}_{EE} &= -Q(\mu_{1h} - b_h)\frac{(S_h - S_h^*)^2}{S_h} - Q\mu_{2h}(S_h + S_h^*)\frac{(S_h - S_h^*)^2}{S_h} \\
&\quad - (\mu_{1v} - b_v)\frac{(S_v - S_v^*)^2}{S_v} - \mu_{2v}\frac{(S_v + S_v^*)}{S_v}(S_v - S_v^*)^2 - H\mu_{2v}(I_v - I_v^*)^2 - W\gamma_h(R_h - R_h^*)^2\frac{I_h}{R_h R_h^*} \\
&\quad - \mu_{2h}(I_h - I_h^*)^2 - \mu_{2h} W(R_h - R_h^*)^2 - (Q+W)\mu_{2h}\frac{(S_h - S_h^*)^2}{S_h} I_h - \frac{(S_h - S_h^*)^2}{S_h}(\beta_1 - \mu_{2h})I_h^* \\
&\quad - \frac{(S_h - S_h^*)^2}{S_h}(Q+W)\mu_{2h} R_h + \frac{(S_h - S_h^*)^2}{S_h} W\mu_{2h} R_h^* - \frac{(S_v - S_v^*)^2}{S_v}(1+H)\mu_{2v} I_v + \frac{(S_v - S_v^*)^2}{S_v} H\mu_{2v} I_v^* \\
&\quad - \frac{\beta_2}{I_h I_h^*}(I_h - I_h^*)^2 S_h^* I_v^* - \frac{Q\beta_2}{S_h}(S_h - S_h^*)^2 I_v - \frac{\beta_3}{S_v}(S_v - S_v^*)^2 I_h \\
&= -Q(\mu_{1h} - b_h)\frac{(S_h - S_h^*)^2}{S_h} + \frac{(S_h - S_h^*)^2}{S_h} W\mu_{2h} R_h^* - Q\mu_{2h}(S_h + S_h^*)\frac{(S_h - S_h^*)^2}{S_h}
\end{aligned}
$$

$$
\begin{aligned}
&-(\mu_{1v}-b_v)\frac{(S_v-S_v^*)^2}{S_v}+\frac{(S_v-S_v^*)^2}{S_v}H\mu_{2v}I_v^*-\mu_{2v}\frac{(S_v+S_v^*)}{S_v}(S_v-S_v^*)^2-H\mu_{2v}(I_v-I_v^*)^2\\
&-W\gamma_h(R_h-R_h^*)^2\frac{I_h}{R_hR_h^*}-\mu_{2h}(I_h-I_h^*)^2-\mu_{2h}W(R_h-R_h^*)^2-(Q+W)\mu_{2h}\frac{(S_h-S_h^*)^2}{S_h}I_h\\
&-\frac{(S_h-S_h^*)^2}{S_h}(\beta_1-\mu_{2h})I_h^*-\frac{(S_h-S_h^*)^2}{S_h}(Q+W)\mu_{2h}R_h-\frac{(S_v-S_v^*)^2}{S_v}(1+H)\mu_{2v}I_v\\
&-\frac{\beta_2}{I_hI_h^*}(I_h-I_h^*)^2S_h^*I_v^*-\frac{Q\beta_2}{S_h}(S_h-S_h^*)^2I_v-\frac{\beta_3}{S_v}(S_v-S_v^*)^2I_h\\
=&-(Q(\mu_{1h}-b_h)-W\mu_{2h}R_h^*)\frac{(S_h-S_h^*)^2}{S_h}-Q\mu_{2h}(S_h+S_h^*)\frac{(S_h-S_h^*)^2}{S_h}\\
&-((\mu_{1v}-b_v)-H\mu_{2v}I_v^*)\frac{(S_v-S_v^*)^2}{S_v}-\mu_{2v}\frac{(S_v+S_v^*)}{S_v}(S_v-S_v^*)^2-H\mu_{2v}(I_v-I_v^*)^2\\
&-W\gamma_h(R_h-R_h^*)^2\frac{I_h}{R_hR_h^*}-\mu_{2h}(I_h-I_h^*)^2-\mu_{2h}W(R_h-R_h^*)^2-(Q+W)\mu_{2h}\frac{(S_h-S_h^*)^2}{S_h}I_h\\
&-\frac{(S_h-S_h^*)^2}{S_h}(\beta_1-\mu_{2h})I_h^*-\frac{(S_h-S_h^*)^2}{S_h}(Q+W)\mu_{2h}R_h-\frac{(S_v-S_v^*)^2}{S_v}(1+H)\mu_{2v}I_v\\
&-\frac{\beta_2}{I_hI_h^*}(I_h-I_h^*)^2S_h^*I_v^*-\frac{Q\beta_2}{S_h}(S_h-S_h^*)^2I_v-\frac{\beta_3}{S_v}(S_v-S_v^*)^2I_h\\
=&-(\frac{Q(\mu_{1h}-b_h)}{W\mu_{2h}R_h^*}-1)\frac{(S_h-S_h^*)^2}{S_h}-Q\mu_{2h}(S_h+S_h^*)\frac{(S_h-S_h^*)^2}{S_h}\\
&-(\frac{(\mu_{1v}-b_v)}{H\mu_{2v}I_v^*}-1)\frac{(S_v-S_v^*)^2}{S_v}-\mu_{2v}\frac{(S_v+S_v^*)}{S_v}(S_v-S_v^*)^2-H\mu_{2v}(I_v-I_v^*)^2-W\gamma_h(R_h-R_h^*)^2\frac{I_h}{R_hR_h^*}\\
&-\mu_{2h}(I_h-I_h^*)^2-\mu_{2h}W(R_h-R_h^*)^2-(Q+W)\mu_{2h}\frac{(S_h-S_h^*)^2}{S_h}I_h-\frac{(S_h-S_h^*)^2}{S_h}(\beta_1-\mu_{2h})I_h^*\\
&-\frac{(S_h-S_h^*)^2}{S_h}(Q+W)\mu_{2h}R_h-\frac{(S_v-S_v^*)^2}{S_v}(1+H)\mu_{2v}I_v\\
&-\frac{\beta_2}{I_hI_h^*}(I_h-I_h^*)^2S_h^*I_v^*-\frac{Q\beta_2}{S_h}(S_h-S_h^*)^2I_v-\frac{\beta_3}{S_v}(S_v-S_v^*)^2I_h.
\end{aligned}
$$

As $\frac{Q(\mu_{1h}-b_h)}{W\mu_{2h}R_h^*}=R_0$ and $\frac{\mu_{1v}-b_v}{H\mu_{2v}I_v^*}=R_0$, so

$$
\begin{aligned}
\dot{V}_{EE}=&-(R_0-1)\frac{(S_h-S_h^*)^2}{S_h}-Q\mu_{2h}(S_h+S_h^*)\frac{(S_h-S_h^*)^2}{S_h}\\
&-(R_0-1)\frac{(S_v-S_v^*)^2}{S_v}-\mu_{2v}\frac{(S_v+S_v^*)}{S_v}(S_v-S_v^*)^2-H\mu_{2v}(I_v-I_v^*)^2-W\gamma_h(R_h-R_h^*)^2\frac{I_h}{R_hR_h^*}\\
&-\mu_{2h}(I_h-I_h^*)^2-\mu_{2h}W(R_h-R_h^*)^2\\
&-(Q+W)\mu_{2h}\frac{(S_h-S_h^*)^2}{S_h}I_h-\frac{(S_h-S_h^*)^2}{S_h}(\beta_1-\mu_{2h})I_h^*-\frac{(S_h-S_h^*)^2}{S_h}(Q+W)\mu_{2h}R_h\\
&-\frac{(S_v-S_v^*)^2}{S_v}(1+H)\mu_{2v}I_v-\frac{\beta_2}{I_hI_h^*}(I_h-I_h^*)^2S_h^*I_v^*\\
&-\frac{\beta_2}{I_hI_h^*}(I_h-I_h^*)^2S_h^*I_v^*-\frac{Q\beta_2}{S_h}(S_h-S_h^*)^2I_v-\frac{\beta_3}{S_v}(S_v-S_v^*)^2I_h.
\end{aligned}
$$

With $R_0>1$, $\dot{V}_{DFE}$ is negative. Then the endemic equilibrium point E_2 of the model (2.1) is globally asymptotically stable if $R_0>1$. □

3.3.3. Numerical simulation

For the parametrs values in Table 3, after estimating and adding other values from [9, 18] to the previous parameters of the system (3.5), we present a numerical simulation of the model for $R_0 = 1.2143$.

Table 3: Parameter values

parameters	Values	Reference
b_h	7.666×10^{-5}	Assumed
b_v	0.13	[9]
μ_{1h}	8.8×10^{-5}	[9]
μ_{2h}	10^{-7}	Assumed
μ_{1v}	0.3	Assumed
μ_{2v}	4×10^{-5}	[9]
γ_h	0.035	[9]
δ_h	1.8×10^{-2}	[18]
δ_v	0.1	Assumed
β_1	10^{-5}	Assumed
β_2	0.0001	Assumed
β_3	0.004	[18]
$\wedge_h$	6.4666	(see (3.10))
$\wedge_v$	31.7988	(see (3.10))

By using the values of the Table 3 and with the initial population sizes $S_h(0) = 7000$, $I_h(0) = 40$, $R_h(0) = 360$, $S_v(0) = 220$, and $I_v(0) = 480$, the numerical simulation illustrated in Figure 4, declares that the infected human population will initially be increased over time and then decreases to attain 119.2478. Recovered human populations will increase over time. So for a reproduction number greater than 1 ($R_0 = 1.2143$), the endemic equilibrium $E_2 = (S_h^*, I_h^*, R_h^*, S_v^*, I_v^*) = (754.1196, 119.2478, 5.6429 \times 10^3, 66.4479, 73.0529)$ is stable. The population of infected mosquitoes decreases over time to attain 73.0529 as it's shown in Figure 5, which means that the malaria epidemic will persist in the population.

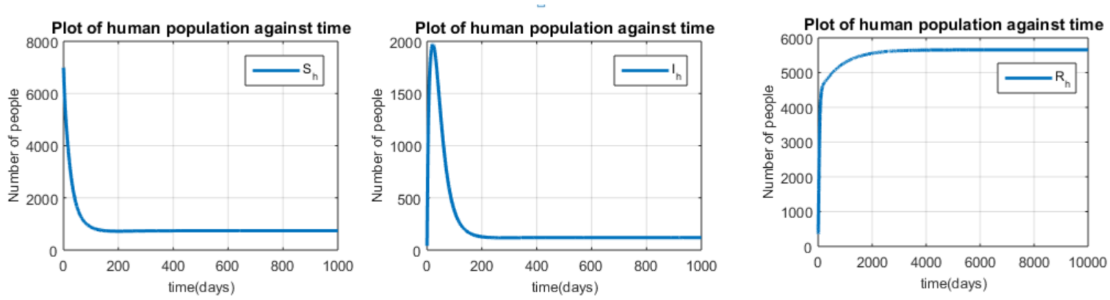

Figure 4: Simulation of the model (2.1) depending on time, shows convergence of solutions to the endemic equilibrium for human population. The parameters values in Table 3 give $R_0 = 1.2143$.

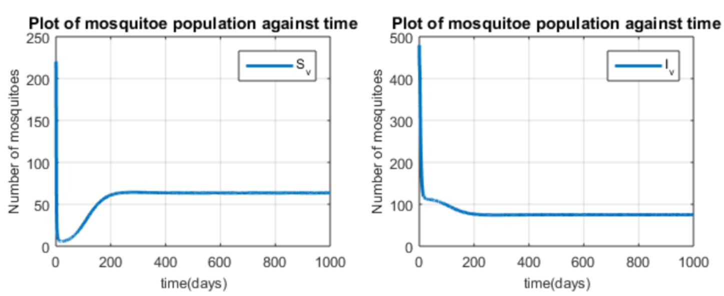

Figure 5: Simulation of the model (2.1) depending on time, shows convergence of solutions to the endemic equilibrium for mosquito population.

4. Conclusion

An ordinary differential system-based compartmental model was suggested to study the waves of malaria transmission into a safe population. Both Susceptible-Infectious-Recovered ($S_h - I_h - R_h$) group for humans and a Susceptible-Infectious ($S_v - I_v$) group for mosquitoes are included. We first examined the positivity and boundedness of the model's provided solutions to ensure that it is epidemiologically well-posed, after that we used the Lyapunov method to search for the global and local stability of disease-free equilibrium (DFE) and endemic equilibrium. Finally, it has been showed by numerical results that if the threshold R_0 is less than 1, the disease-free equilibrium is globally stable, and if not, the endemic point is. The disease can persist and spread across the population when the endemic equilibrium is generally stable and the disease-free equilibrium is unstable. Individuals who have recovered may eventually lose their immunity to the disease, which causes a delay. Our next work uses the same model, but takes into account that recovered humans may lose their immunity and become a susceptible again after a period of time.

References

[1] D. Aldila, *Dynamical analysis on a malaria model with relapse preventive treatment and saturated fumigation*, Comput. Math. Methods Med., **2022** (2022), 19 pages. 1

[2] R. M. Anderson, R. M. May, *Infectious diseases of humans: dynamics and control*, Oxford University Press, Oxford, 1992.

[3] J. L. Aron, *Mathematical modelling of immunity to malaria*, Math. Biosci., **90** (1988), 385–396. 1

[4] T. Bakary, S. Boureima, T. Sado, *A mathematical model of malaria transmission in a periodic environment*, J. Biol. Dyn., **12** (2018), 400–432. 1

[5] G. Birkhoff, G. C. Rota, *Ordinary Differential Equations*, Ginn, Boston, (1982). 3.1

[6] N. Budhwar, S. Daniel, *Stability Analysis of a Human-Mosquito Model of Malaria with Infective Immigrants*, Int. J. Math. Comput. Sci., **11** (2017), 85–89. 1

[7] N. Chitnis, *Using Mathematical Models in Controlling the Spread of Malaria*, Ph.D. Thesis, The University of Arizona, (2005).

[8] N. Chitnis, J. M. Cushing, J. M. Hyman, *Bifurcation analysis of a mathematical model for malaria transmission*, SIAM J. Appl. Math., **67** (2006), 24–45. 2, 3.2.4, 2

[9] N. Chitnis, J. M. Hyman, J. M. Cushing, *Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model*, Bull. Math. Biol., **70** (2008), 1272–1296. 1, 3.3.3, 3

[10] K. Hattaf, *Stability of fractional differential equations with new generalized hattaf fractional derivative*, Math. Prob. Eng., **2021** (2021), 1–7. 3.2.3, 3.3.2

[11] Y. Louartassi, E. El Mazoudi, N. Elalami, *A new generalization of lemma Gronwall-Bellman*, Appl. Math. Sci. (Ruse), **6** (2012), 621–628. 3.1

[12] G. Macdonald, *The Epidemiology and Control of Malaria*, Oxford University Press, Oxford, (1957). 1

[13] A. M. Niger, A. B. Gumel, *Mathematical analysis of the role of repeated exposure on malaria transmission dynamics*, Differ. Equ. Dyn. Syst., **16** (2008), 251–287. 3.1

[14] R. Ross, *The prevention of malaria*, John Murray, London, (1911). 1

[15] A. G. M. Selvam, A. J. Priya, *Analysis of a Discrete SEIR Epidemic Model*, International Journal of Emerging Technologies in Computational and Applied Science, **12** (2015), 73–76.

[16] J. Tumwiine, J. Y. T. Mugisha, L. S. Luboobi, *A mathematical model for the dynamics of malaria in a human host and mosquito vector with temporary immunity*, Appl. Math. Comput., **189** (2007), 1953–1965.

[17] J. Tumwiine, J. Y. T. Mugisha, L. S. Luboobi, *A host-vector model for malaria with infected immigrants*, J. Math. Anal. Appl., **361** (2010), 139–149.

[18] A. J. Wedajo, B. K. Bole, P. R. Koya, *Analysis of SIR mathematical model for malaria disease with the inclusion of infected immigrants*, IOSR J. Math., **14** (2018), 10–21. 1, 3.2.4, 2, 3.3.3, 3