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Applications of neutrosophic soft open sets in decision making via operation approach

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Abstract

Enterprise resource planning (ERP) has a significant impact on modern businesses by enhancing productivity, automation, and streamlining of business processes, even accounting. Manufacturers can assure proper functioning and timely client demand using ERP software. Coordination, procurement control, inventory control, and dispatch of commodities are all features of supply chain management. Manufacturers may design better logistics plans with this capability, which will substantially aid them in lowering operational and administrative expenses. In this article, we instigate the idea of neutrosophic soft γ- open sets ($\mathcal{N}_{\mathcal{S}\gamma}$-open sets) by employing the operation γ on the family of neutrosophic soft open sets written symbolically as τ_u in neutrosophic soft topological spaces. Additionally, by employing the operation on τ_u, we bring forth new notions namely $\mathcal{N}_{\mathcal{S}\gamma}$-closure, $\mathcal{N}_{\mathcal{S}\gamma}$-interior, $\mathcal{N}_{\mathcal{S}\gamma}$-regular space, $\mathcal{N}_{\mathcal{S}\gamma}$-regular operation and obtain their characteristics in neutrosophic soft topological spaces. With the $\mathcal{N}_{\mathcal{S}\gamma}$ open sets, we discuss a methodology for overcoming the challenge of selecting the best ERP for a business firm.

Keywords: Topological space, operation, $\mathcal{N}_{\mathcal{S}\gamma}$-open sets, $\mathcal{N}_{\mathcal{S}\gamma}$-closure, $\mathcal{N}_{\mathcal{S}\gamma}$-regular operation, decision making.

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1. Introduction

ERP is a software solution designed to improve the productivity of the business that it is incorporated into by digitizing, integrating, sufficiently automating, and streamlining its entire workflow. A good ERP should completely rule out data misinterpretations and communication gaps in all the internal and external communications of the business by unifying the storage of all its data in a common repository. Choosing the right ERP for their enterprise is a critical job for the enterprisers, and a solution for this problem is discussed in this paper using the newly developed concept.

The fuzzy set theory was instigated by Zadeh [24] in 1965. It has become a highly significant tool for solving problems with uncertainties. Molodtsov [15] proposed soft set theory in 1999, which deals with uncertainty. He developed the fundamental principles of this new theory in his work and effectively applied it to various fields like optimization, algebraic structures, clustering, lattice, topology, data analysis, game theory, medical diagnosis, operations research, and decision-making under uncertainty.

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Atanassov [3] amended the fuzzy set ideology and instigated the theory of intuitionistic fuzzy sets. It contains non-membership and membership values. It is incapable of dealing with the uncertain and conflicting information found in value systems. As a generalization of crisp sets, fuzzy set theory and intuitionistic fuzzy set theory, Smarandache [19, 20] proposed the novel concept, Neutrosophic set ($\mathscr{N}\mathscr{S}$). Fuzzy logic, intuitionistic fuzzy logic, and paraconsistent logic are all generalized in the new philosophical discipline known as neurosophy. Neutrosophic logic acts as a mathematical kit for problems involving incomplete, indeterminant and inconsistent knowledge. Neutrosophic sets and logic are applied in various fields like information systems, semantic web services, relational database systems, financial dataset detection, analysis of the new economy's growth and fall, etc.

In 2003, Maji et al. [13] established a theoretical approach to soft set theory and defined the operations like intersection (AND), union (OR) of two soft sets and justified some propositions on soft set operations. The notion of soft topological spaces, which are built over an initial universe with a predetermined set of attributes was developed by Shabir and Naz [18]. They demonstrated that a soft topological space yields a parameterized collection of topological space. As an extension of the soft set, Smarandache[21–23] brought forth the concept of hypersoft set, indeterm soft set, indeterm hypersoft set and tree soft set. Maji [14] by integrating the idea of soft set and neutrosophic set, defined neutrosophic soft sets ($\mathscr{N}_{\mathscr{S}}\mathscr{S}$s) and provided an application of neutrosophic soft set in decision-making problems, which was later refined by Deli and Broumi [8]. Bera and Mahapatra [5, 6] constructed a topological structure on neutrosophic soft sets and studied its structural characterizations and discussed the concepts related to topological space such as closure, interior, boundary, neighborhood, base, subspace, separation axioms, connectedness, compactness and neutrosophic soft continuous mappings along with specific illustrations and proofs.

Kasahara [12] proposed the idea of an operation approach to topological spaces and defined α-closed graphs of functions by genera1izing the idea of almost-strong1y-closed, strong1y-closed and closed graph of a function. Jankovic [10] investigated the mappings with α and strongly-closed graphs. Following this, Ogata [16] introduced γ-open sets utilizing the operation γ on open sets and related continuity concepts in topological spaces.

In 2017, Kalaivani et al. [11] and Benchalli et al. [4] brought forth the theory of operation approach in soft topological spaces. El-Sheikh and El-Sayed [9] in the year 2020, extended the conception of γ-operation in fuzzy soft ideal topological spaces. Asaad et al. [1] put forth the idea of γ operation on Supra Topology and defined supra γ-regular and supra open operations and analyzed some of their characteristics. Roy and Noiri [17] investigated the features of $\gamma\mu$ open sets by defining operation on generalized topological spaces. The study of bioperations on soft topological spaces was initialized by Asaad et al. [2] in 2021. They contemplated the properties of soft (γ,γ')-open sets, soft (γ,γ')-g closed sets and soft $(\gamma,\gamma')-T_{1/2}$ spaces. Das et al. [7] examined the characteristics of operation on generalized fuzzy topological spaces.

In this paper, we define the operation γ on neutrosophic soft open sets and introduce neutrosophic soft γ-open sets in neutrosophic soft topological spaces. Also, we analyze the properties of closure and interior operators by utilizing neutrosophic soft γ-closed and neutrosophic soft γ-open sets, respectively. Finally, we implement the notion of neutrosophic soft γ-open sets for decision-making problems.

2. Preliminaries

This module is concerned with some important definitions associated to neutrosophic set ($\mathscr{N}\mathscr{S}$), neutrosophic soft set ($\mathscr{N}_{\mathscr{S}}\mathscr{S}$) and neutrosophic soft topological spaces ($\mathscr{N}_{\mathscr{S}}\mathscr{T}\mathscr{S}$s)

Definition 2.1 ([20]). For the universal set $\mathscr{U}$ and the values $T;\ I;\ F\ :\ \mathscr{U} \to]^{-}0, 1^{+}[$ and $^{-}0 \leqslant T_{\mathscr{L}}(k)+ I_{\mathscr{L}}(k) + F_{\mathscr{L}}(k) \leqslant\ 3^{+}$, an $\mathscr{N}\mathscr{S}$ is defined as:

$$\mathscr{L} = \{< k,\ T_{\mathscr{L}}(k),\ I_{\mathscr{L}}(k), F_{\mathscr{L}}(k) >: k \in \mathscr{U}\}.$$

Definition 2.2 ([14])**.** For the universal set $\mathscr{U}$ and the values $T_{f_{\mathscr{P}(\omega)}}(k)$, $I_{f_{\mathscr{P}(\omega)}}(k)$, $F_{f_{\mathscr{P}(\omega)}}(k) \in [0,1]$ are the "truth", "indeterminacy", and "falsity" functions of $f_{\mathscr{P}(\omega)}$, respectively, where $f_{\mathscr{P}}$ is defined from the set of parameters (Ω) to $\mathscr{P}(\mathscr{U})$. Then the $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ is defined as

$$\mathscr{P} = \left\{ \left(\omega, \left\{ < k,\ T_{f_{\mathscr{P}(\omega)}}(k),\ I_{f_{\mathscr{P}(\omega)}}(k), F_{f_{\mathscr{P}(\omega)}}(k) >: k \in \mathscr{U} \right\}\right) : \omega \in \Omega \right\}.$$

Definition 2.3 ([8])**.** For the universal set $\mathscr{U}$ and two $\mathscr{N}_{\mathcal{S}}\mathcal{S}$s $(\mathscr{H}, \Omega)$ and $(\mathscr{G}, \Omega)$ over $\mathscr{U}$,

1. $(\mathscr{H}, \Omega) \subseteq (\mathscr{G}, \Omega)$ if $T_{\mathscr{H}(\omega)}(k) \leqslant T_{\mathscr{G}(\omega)}(k)$, $I_{\mathscr{H}(\omega)}(k) \geqslant I_{\mathscr{G}(\omega)}(k)$, $F_{\mathscr{H}(\omega)}(k) \geqslant F_{\mathscr{G}(\omega)}(k)$, $\forall\, \omega \in \Omega$, $k \in \mathscr{U}$;
2. $\mathscr{H}^c = \left\{ \left(\omega, \left\{ < k,\ F_{f_{\mathscr{H}(\omega)}}(k), 1 - I_{f_{\mathscr{H}(\omega)}}(k), T_{f_{\mathscr{H}(\omega)}}(k) >: k \in \mathscr{U} \right\}\right) : \omega \in \Omega \right\}$;
3. $\mathscr{H}$ is termed as a null $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ if $T_{f_{\mathscr{H}(\omega)}}(k) = 0$, $I_{f_{\mathscr{H}(\omega)}}(k) = 1$, $F_{f_{\mathscr{H}(\omega)}}(k) = 1\ \forall\, k \in \mathscr{U}$ and $\omega \in \Omega$ which is symbolically written as ϕ_u;
4. $\mathscr{H}$ is termed as a absolute $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ if $T_{f_{\mathscr{H}(\omega)}}(k) = 1$, $I_{f_{\mathscr{H}(\omega)}}(k) = 0$, $F_{f_{\mathscr{H}(\omega)}}(k) = 0\ \forall\, k \in \mathscr{U}$ and $\omega \in \Omega$ which is symbolically written as 1_u;
5. if $\mathscr{H} \cup \mathscr{G} = \mathscr{P}$, then $\mathscr{P} = \left\{ \left(\omega, \left\{ < k,\ T_{f_{\mathscr{P}(\omega)}}(k),\ I_{f_{\mathscr{P}(\omega)}}(k), F_{f_{\mathscr{P}(\omega)}}(k) >: k \in \mathscr{U} \right\}\right) : \omega \in \Omega \right\}$, where $T_{f_{\mathscr{P}(\omega)}}(k) = \max\left(T_{f_{\mathscr{H}(\omega)}}(k), T_{f_{\mathscr{G}(\omega)}}(k)\right)$, $I_{f_{\mathscr{P}(\omega)}}(k) = \min\left(I_{f_{\mathscr{H}(\omega)}}(k), I_{f_{\mathscr{G}(\omega)}}(k)\right)$, $F_{f_{\mathscr{P}(\omega)}}(k) = \min\left(F_{f_{\mathscr{H}(\omega)}}(k), F_{f_{\mathscr{G}(\omega)}}(k)\right)$;
6. if $\mathscr{H} \cap \mathscr{G} = \mathscr{Q}$, then $\mathscr{Q} = \left\{ \left(\omega, \left\{ < k,\ T_{f_{\mathscr{Q}(\omega)}}(k),\ I_{f_{\mathscr{Q}(\omega)}}(k), F_{f_{\mathscr{Q}(\omega)}}(k) >: k \in \mathscr{U} \right\}\right) : \omega \in \Omega \right\}$, where $T_{f_{\mathscr{Q}(\omega)}}(k) = \min\left(T_{f_{\mathscr{H}(\omega)}}(k), T_{f_{\mathscr{G}(\omega)}}(k)\right)$, $I_{f_{\mathscr{Q}(\omega)}}(k) = \max\left(I_{f_{\mathscr{H}(\omega)}}(k), I_{f_{\mathscr{G}(\omega)}}(k)\right)$, $F_{f_{\mathscr{Q}(\omega)}}(k) = \max\left(F_{f_{\mathscr{H}(\omega)}}(k), F_{f_{\mathscr{G}(\omega)}}(k)\right)$.

Definition 2.4 ([5])**.** If $\mathscr{N}_{\mathcal{S}}\mathcal{S}(\mathscr{U}, \Omega)$ is the family of all $\mathscr{N}_{\mathcal{S}}\mathcal{S}$s over $\mathscr{U}$ via parameters in Ω and $\tau_u \subset \mathscr{N}_{\mathcal{S}}\mathcal{S}(\mathscr{U}, \Omega)$, then τ_u is termed as an $\mathscr{N}_{\mathcal{S}}$ topology on $(\mathscr{U}, \Omega)$ provided the following constraints hold:

1. $\phi_u,\ 1_u \in \tau_u$;
2. for $H_1, H_2 \in \tau_u \Rightarrow H_1 \cap H_2 \in \tau_u$;
3. for $\cup_{i \in J} U_i \in \tau_u$, for every $\{U_i : i \in J\} \subseteq \tau_u$.

Then the triplet $(\mathscr{U}, \Omega, \tau_u)$ is termed as an $\mathscr{N}_{\mathcal{S}}\mathcal{T}\mathcal{S}$. Every member of τ_u is termed as neutrosophic soft open set ($\mathscr{N}_{\mathcal{S}}\mathcal{O}\mathcal{S}$). And $(\mathscr{N}_{\mathcal{S}}\mathcal{O}\mathcal{S})^c$ is termed as neutrosophic soft closed set ($\mathscr{N}_{\mathcal{S}}\mathcal{C}\mathcal{S}$).

Definition 2.5 ([5])**.** An $\mathscr{N}_{\mathcal{S}}$ point in an $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ $\mathscr{P}$ is defined as an element $(\omega, f_{\mathscr{P}}(\omega))$ of $\mathscr{P}$, for $\omega \in \Omega$ and is denoted by $\omega_{\mathscr{P}}$, if $f_{\mathscr{P}}(\omega) \notin \phi_u$ and $f_{\mathscr{P}}(\omega') \in \phi_u\ \forall\, \omega' \in \Omega \backslash \{\omega\}$. An $\mathscr{N}_{\mathcal{S}}$ point $\omega_{\mathscr{P}}$ belongs to an $\mathscr{N}_{\mathcal{S}}\mathcal{S}$, say $\mathscr{M}$, if for the element $\omega \in \Omega$, $f_{\mathscr{P}}(\omega) \leqslant f_{\mathscr{M}}(\omega)$.

Definition 2.6 ([5])**.** Let $(\mathscr{U}, \Omega, \tau_u)$ be an $\mathscr{N}_{\mathcal{S}}\mathcal{T}\mathcal{S}$ over $(\mathscr{U}, \Omega)$ and $\mathscr{P} \in \mathscr{N}_{\mathcal{S}}\mathcal{S}(\mathscr{U}, \Omega))$ be arbitrary. Then the closure of $\mathscr{P}$ is the intersection of all closed neutrosophic soft supersets of $\mathscr{P}$.

3. Operation approach on neutrosophic soft open sets

Definition 3.1. Let $(\mathscr{U}, \Omega, \tau_u)$ be an $\mathscr{N}_{\mathcal{S}}\mathcal{T}\mathcal{S}$. A mapping γ from τ_u into the $\mathscr{N}_{\mathcal{S}}$ power set $P(\mathscr{U})$ of $\mathscr{U}$ is known as an operation if $(\mathscr{H}, \Omega) \subseteq \gamma(\mathscr{H}, \Omega)\ \forall\, (\mathscr{H}, \Omega) \in \tau_u$, where $\gamma(\mathscr{H}, \Omega)$ is the value of $(\mathscr{H}, \Omega)$ under the operation γ.

Example 3.2. Let $\mathscr{U} = \{k_1, k_2, k_3\}$ and the attributes $\Omega = \{\omega_1, \omega_2\}$. Then the family of $\mathscr{N_S S}$s are

$$(\mathscr{M}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.9,0.4,0.3}, \frac{k_2}{0.5,0.3,0.5}, \frac{k_3}{0.4,0.1,0.3} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.7,0.1,0.4}, \frac{k_2}{0.6,0.3,0.2} \frac{k_3}{0.6,0.1,0.5} \right\} > \right\},$$
$$(\mathscr{N}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.7,0.4,0.5}, \frac{k_2}{0.4,0.5,0.5}, \frac{k_3}{0.3,0.3,0.4} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.6,0.2,0.4}, \frac{k_2}{0.5,0.4,0.3}, \frac{k_3}{0.4,0.6,0.5} \right\} > \right\},$$
$$(\mathscr{O}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.5,0.8,0.6}, \frac{k_2}{0.3,0.9,0.7}, \frac{k_3}{0.2,0.6,0.5} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.4,0.6,0.5}, \frac{k_2}{0.4,0.6,0.4}, \frac{k_3}{0.1,0.7,0.6} \right\} > \right\},$$
$$(\mathscr{P}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.8,0.3,0.4}, \frac{k_2}{0.5,0.4,0.3}, \frac{k_3}{0.7,0.1,0.2} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.7,0.1,0.3}, \frac{k_2}{0.6,0.2,0.1}, \frac{k_3}{0.7,0.4,0.3} \right\} > \right\}.$$

Then the subfamily $\tau_u = \{\phi_u, 1_u, (\mathscr{N}, \Omega), (\mathscr{P}, \Omega)\}$ forms an $\mathscr{N_S}$ topology. Define the operation $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L}, \Omega) = \begin{cases} (\mathscr{L}, \Omega), & \text{if } \omega_{2\mathscr{P}} \in (\mathscr{L}, \Omega), \\ 1_u, & \text{if } \omega_{2\mathscr{P}} \notin (\mathscr{L}, \Omega), \end{cases} \qquad \forall (\mathscr{L}, \Omega) \in \tau_u.$$

Then, γ is an operation on τ_u since $(\mathscr{L}, \Omega) \subseteq \gamma(\mathscr{L}, \Omega)$, $\forall (\mathscr{L}, \Omega) \in \tau_u$.

Definition 3.3. an $\mathscr{N_S S}$ $(\mathscr{L}, \Omega)$ over $\mathscr{U}$ via parameters in Ω with an operation γ on τ_u is known as a neutrosophic soft γ-open set ($\mathscr{N_S}_\gamma\mathscr{OS}$) if $\forall\ \omega_{i\mathscr{L}} \in (\mathscr{L}, \Omega)$, $\exists$ an $\mathscr{N_S OS}$ $(\mathscr{H}, \Omega)$ such that $\omega_{i\mathscr{L}} \in (\mathscr{H}, \Omega)$ and $\gamma(\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)$, where $\omega_i \in \Omega$. The collection of all neutrosophic soft γ-open sets in $(\mathscr{U}, \Omega, \tau_u)$ is written symbolically as τ_{u_γ}. We call $(\mathscr{N_S}_\gamma\mathscr{OS})^c$ as neutrosophic soft γ-closed set ($\mathscr{N_S}_\gamma\mathscr{CS}$).

Example 3.4. Consider the Example 3.2, let $\gamma : \tau_u \to P(\mathscr{U})$ be a mapping defined by $\gamma(\mathscr{L}, \Omega) = cl(\mathscr{L}, \Omega)$, $\forall (\mathscr{L}, \Omega) \in \tau_u$. Here $\tau_{u_\gamma} = \{\phi_u, 1_u\}$.

Remark 3.5. Every $\mathscr{N_S}_\gamma\mathscr{OS}$ is $\mathscr{N_S OS}$ as it is clear from the Definition 3.3.

Theorem 3.6. *Arbitrary union of $\mathscr{N_S}_\gamma\mathscr{OS}$s is $\mathscr{N_S}_\gamma\mathscr{O}$.*

Proof. Consider $\left\{(\mathscr{L}, \Omega)_{\alpha_i} : \alpha_i \in \Delta\right\}$ to be the collection of $\mathscr{N_S}_\gamma\mathscr{OS}$s in an $\mathscr{N_S TS}$ $(\mathscr{U}, \Omega, \tau_u)$. Let $\omega_{\alpha_i\mathscr{L}} \in \bigcup_{\alpha_i\in\Delta} (\mathscr{L}, \Omega)_{\alpha_i}$. Then $\omega_{\alpha_i\mathscr{L}} \in (\mathscr{L}, \Omega)_{\alpha_i}$ for some $\alpha_i \in \Delta$. Since $(\mathscr{L}, \Omega)_{\alpha_i}$ is $\mathscr{N_S}_\gamma\mathscr{O}$, by the Definition 3.3, $\exists$ an $\mathscr{N_S OS}$ $(\mathscr{H}, \Omega)$ in $(\mathscr{U}, \Omega, \tau_u)$ such that $\omega_{\alpha_i\mathscr{L}} \in (\mathscr{H}, \Omega)$ and $\gamma(\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)_{\alpha_i} \subseteq \bigcup_{\alpha_i\in\Delta} (\mathscr{L}, \Omega)_{\alpha_i}$. Therefore, $\bigcup_{\alpha_i\in\Delta} (\mathscr{L}, \Omega)_{\alpha_i}$ is $\mathscr{N_S}_\gamma\mathscr{O}$ in $\mathscr{U}$. $\square$

Remark 3.7. Intersection of any two $\mathscr{N_S}_\gamma\mathscr{OS}$s is not necessarily $\mathscr{N_S}_\gamma\mathscr{O}$, which is verified by the following example.

Example 3.8. Let $\mathscr{U} = \{k_1, k_2, k_3\}$ and the attributes $\Omega = \{\omega_1, \omega_2\}$. Then the family of $\mathscr{N_S S}$s are

$$(\mathscr{R}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.5,0.6,0.7}, \frac{k_2}{0.5,0.5,0.4}, \frac{k_3}{0.4,0.7,0.3} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.3,0.8,0.8}, \frac{k_2}{0.1,0.6,0.7}, \frac{k_3}{0.2,0.4,0.5} \right\} > \right\},$$
$$(\mathscr{S}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.3,0.6,0.9}, \frac{k_2}{0.2,0.5,0.6}, \frac{k_3}{0.2,0.9,0.7} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.4,0.8,0.6}, \frac{k_2}{0.3,0.6,0.5}, \frac{k_3}{0.5,0.4,0.4} \right\} > \right\},$$
$$(\mathscr{P}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.3,0.6,0.9}, \frac{k_2}{0.2,0.5,0.6}, \frac{k_3}{0.2,0.9,0.7} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.3,0.8,0.8}, \frac{k_2}{0.1,0.6,0.7}, \frac{k_3}{0.2,0.4,0.5} \right\} > \right\},$$
$$(\mathscr{T}, \Omega) = \left\{ <\omega_1, \left\{ \frac{k_1}{0.5,0.6,0.7}, \frac{k_2}{0.5,0.5,0.4}, \frac{k_3}{0.4,0.7,0.3} \right\} >, <\omega_2, \left\{ \frac{k_1}{0.4,0.8,0.6}, \frac{k_2}{0.3,0.6,0.5}, \frac{k_3}{0.5,0.4,0.4} \right\} > \right\}.$$

Then $\tau_u = \{\phi_u, 1_u, (\mathscr{R}, \Omega), (\mathscr{S}, \Omega), (\mathscr{P}, \Omega), (\mathscr{T}, \Omega)\}$ forms an $\mathscr{N_S}$ topology. Define the operation $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L}, \Omega) = \begin{cases} (\mathscr{L}, \Omega), & \text{if } \omega_{i\mathscr{T}} \in (\mathscr{L}, \Omega), \\ 1_u, & \text{if } \omega_{i\mathscr{T}} \notin (\mathscr{L}, \Omega), \end{cases} \qquad \forall (\mathscr{L}, \Omega) \in \tau_u.$$

Then, $\tau_{u\gamma} = \{\phi_u, 1_u, (\mathscr{R}, \Omega), (\mathscr{S}, \Omega), (\mathscr{T}, \Omega)\}$. Here $(\mathscr{R}, \Omega) \cap (\mathscr{S}, \Omega) = (\mathscr{P}, \Omega) \notin \tau_{u\gamma}$.

Definition 3.9. An $\mathscr{N_S TS}$ $(\mathscr{U}, \Omega, \tau_u)$ is termed to be an $\mathscr{N_S}_\gamma$-regular space if for each $\mathscr{N_S}$ point $\omega_{i\mathscr{F}} \in \tilde{\mathscr{U}}$, where $\tilde{\mathscr{U}}$ is the collection of $\mathscr{N_S}$ points in τ_u and for each $\mathscr{N_S OS}$ $(\mathscr{H}, \Omega)$ containing $\omega_{i\mathscr{F}}$, there exists an $\mathscr{N_S OS}$ $(\mathscr{K}, \Omega)$ containing $\omega_{i\mathscr{F}}$ in $(\mathscr{U}, \Omega, \tau_u)$ such that $\gamma(\mathscr{K}, \Omega) \subseteq (\mathscr{H}, \Omega)$.

Example 3.10. Consider the collection of $\mathscr{N}_{\mathcal{S}}\mathcal{S}$s in Example 3.2, the subfamily $\tau_u = \{\phi_u, 1_u, (\mathscr{M}, \Omega), (\mathscr{N}, \Omega), (\mathscr{O}, \Omega)\}$ forms an $\mathscr{N}_{\mathcal{S}}$ topology. Define the operation $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L}, \Omega) = \begin{cases} (\mathscr{L}, \Omega), & \text{if } \omega_{1\mathscr{F}} \in (\mathscr{L}, \Omega), \\ cl(\mathscr{L}, \Omega), & \text{if } \omega_{1\mathscr{F}} \notin (\mathscr{L}, \Omega), \end{cases} \qquad \forall (\mathscr{L}, \Omega) \in \tau_u,$$

where $\omega_{1\mathscr{F}} = \left\{\frac{k_1}{0.4,0.9,0.7}, \frac{k_2}{0.3,0.9,0.7}, \frac{k_3}{0.1,0.8,0.6}\right\}$. Here the $\mathscr{N}_{\mathcal{S}}\mathscr{T}\mathcal{S}$ $(\mathscr{U}, \Omega, \tau_u)$ is an $\mathscr{N}_{\mathcal{S}\gamma}$-regular space.

Remark 3.11. Every $\mathscr{N}_{\mathcal{S}}\mathscr{T}\mathcal{S}$ need not to be an $\mathscr{N}_{\mathcal{S}\gamma}$-regular space.

Example 3.12. Consider the Example 3.10, define the operation $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L}, \Omega) = \begin{cases} (\mathscr{L}, \Omega), & \text{if } \omega_{2\mathscr{F}} \in (\mathscr{L}, \Omega), \\ 1_u, & \text{if } \omega_{2\mathscr{F}} \notin (\mathscr{L}, \Omega), \end{cases} \qquad \forall (\mathscr{L}, \Omega) \in \tau_u,$$

where $\omega_{2\mathscr{F}} = \left\{\frac{k_1}{0.5,0.7,0.5}, \frac{k_2}{0.4,0.6,0.7}, \frac{k_3}{0.3,0.8,0.7}\right\}$. Then $\gamma(\mathscr{L}, \Omega) = (\mathscr{L}, \Omega)$ for $(\mathscr{L}, \Omega) = (\mathscr{M}, \Omega)$ and $(\mathscr{N}, \Omega)$. $\gamma(\mathscr{O}, \Omega) = 1_u$.

Here the $\mathscr{N}_{\mathcal{S}}\mathscr{T}\mathcal{S}$ $(\mathscr{U}, \Omega, \tau_u)$ is not an $\mathscr{N}_{\mathcal{S}\gamma}$-regular space, since for $\omega_{i\mathscr{O}} \in \tilde{\mathscr{U}} (i = 1, 2)$ and $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{O}, \Omega)$ containing $\omega_{i\mathscr{O}}$, there dose not exist $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{K}, \Omega)$ containing $\omega_{i\mathscr{O}} \ni \gamma(\mathscr{K}, \Omega) \subseteq (\mathscr{O}, \Omega)$ as $\omega_{i\mathscr{O}} \in (\mathscr{M}, \Omega), (\mathscr{N}, \Omega), (\mathscr{O}, \Omega)$ and $(\mathscr{O}, \Omega) \subseteq (\mathscr{N}, \Omega) \subseteq (\mathscr{M}, \Omega)$.

Theorem 3.13. *Let $(\mathscr{U}, \Omega, \tau_u)$ be an $\mathscr{N}_{\mathcal{S}}\mathscr{T}\mathcal{S}$ with the operation $\gamma : \tau_u \to P(\mathscr{U})$. Then the following are equivalent:*

1. $\tau_u = \tau_{u\gamma}$;
2. $(\mathscr{U}, \Omega, \tau_u)$ *is an* $\mathscr{N}_{\mathcal{S}\gamma}$*-regular space;*
3. *given* $\omega_{i\mathscr{F}} \in \tilde{\mathscr{U}}$ *and* $\forall$ $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ $(\mathscr{L}, \Omega)$ *containing* $\omega_{i\mathscr{F}}$*, there exists an* $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$ $(\mathscr{H}, \Omega)$ *such that* $\omega_{i\mathscr{F}} \in (\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)$.

Proof.

1 $\Rightarrow$ 2: Assume that $\tau_u = \tau_{u\gamma}$. Then for each $\omega_{i\mathscr{F}} \in \tilde{\mathscr{U}}$ and for every $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{H}, \Omega)$ containing $\omega_{i\mathscr{F}}$, there exists an $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{K}, \Omega)$ containing $\omega_{i\mathscr{F}}$ such that $\gamma(\mathscr{K}, \Omega) \subseteq (\mathscr{H}, \Omega)$, since $\tau_u = \tau_{u\gamma}$. Therefore $(\mathscr{U}, \Omega, \tau_u)$ is an $\mathscr{N}_{\mathcal{S}\gamma}$-regular space.

2 $\Rightarrow$ 3: Consider $\omega_{i\mathscr{F}} \in \tilde{\mathscr{U}}$ and an $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{L}, \Omega)$ containing $\omega_{i\mathscr{F}}$. By 2, there exists an $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{H}, \Omega)$ containing $\omega_{i\mathscr{F}}$ such that $\gamma(\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)$. Since γ is an operation on τ_u, $(\mathscr{H}, \Omega) \subseteq \gamma(\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)$. As $(\mathscr{H}, \Omega)$ is an $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ containing $\omega_{i\mathscr{F}}$, again by 2, there exists an $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{K}, \Omega)$ containing $\omega_{i\mathscr{F}}$ such that $\gamma(\mathscr{K}, \Omega) \subseteq (\mathscr{H}, \Omega)$. This implies that $(\mathscr{H}, \Omega)$ is an $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$. Hence $(\mathscr{H}, \Omega)$ is an $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$ such that $\omega_{i\mathscr{F}} \in (\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)$.

3 $\Rightarrow$ 1: Let $(\mathscr{L}, \Omega)$ be an $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ containing $\omega_{i\mathscr{F}}$. Then by assumption, there exists an $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$ $(\mathscr{H}, \Omega)$ such that $\omega_{i\mathscr{F}} \in (\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)$. By Definition 3.3, $\exists$ an $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{K}, \Omega) \ni \omega_{i\mathscr{F}} \in (\mathscr{K}, \Omega)$ and $\gamma(\mathscr{K}, \Omega) \subseteq (\mathscr{H}, \Omega) \subseteq (\mathscr{L}, \Omega)$. Therefore $(\mathscr{L}, \Omega)$ is an $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$. Hence $\tau_u \subseteq \tau_{u\gamma}$. By Remark 3.5, we have $\tau_{u\gamma} \subseteq \tau_u$. Therefore $\tau_u = \tau_{u\gamma}$. □

Remark 3.14. If the space is not an $\mathscr{N}_{\mathcal{S}\gamma}$-regular space, then $\tau_u \neq \tau_{u\gamma}$, which is evident from the Example 3.12, since $\tau_u = \{\phi_u, 1_u, (\mathscr{M}, \Omega), (\mathscr{N}, \Omega), (\mathscr{O}, \Omega)\}$ and $\tau_{u_\gamma} = \{\phi_u, 1_u, (\mathscr{M}, \Omega), (\mathscr{N}, \Omega)\}$

Definition 3.15. An operation $\gamma : \tau_u \to P(\mathscr{U})$ is termed as $\mathscr{N}_{\mathcal{S}\gamma}$-regular if $\forall$ $\omega_{i\mathscr{F}} \in \tilde{\mathscr{U}}$ and $\forall$ pairs of $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$s $(\mathscr{L}, \Omega)$ and $(\mathcal{S}, \Omega)$ containing $\omega_{i\mathscr{F}}$, $\exists$ an $\mathscr{N}_{\mathcal{S}}\mathscr{O}\mathcal{S}$ $(\mathscr{J}, \Omega)$ containing $\omega_{i\mathscr{F}}$ such that $\gamma(\mathscr{L}, \Omega) \cap \gamma(\mathcal{S}, \Omega) \supseteq \gamma(\mathscr{J}, \Omega)$.

Example 3.16. Consider $\mathscr{U} = \{k_1, k_2, k_3\}$, $\Omega = \{\omega_1, \omega_2\}$ with the $\mathscr{N}_{\mathscr{S}}\mathscr{S}$s

$$(\mathscr{M},\Omega) = \left\{ <\omega_1, \left\{\frac{k_1}{1.0,0.5,0.4}, \frac{k_2}{0.6,0.6,0.6}, \frac{k_3}{0.5,0.6,0.4}\right\}>, <\omega_2, \left\{\frac{k_1}{0.8,0.4,0.5}, \frac{k_2}{0.7,0.7,0.3}, \frac{k_3}{0.7,0.5,0.6}\right\}> \right\},$$
$$(\mathscr{N},\Omega) = \left\{ <\omega_1, \left\{\frac{k_1}{0.8,0.5,0.6}, \frac{k_2}{0.5,0.7,0.6}, \frac{k_3}{0.4,0.7,0.5}\right\}>, <\omega_2, \left\{\frac{k_1}{0.7,0.6,0.5}, \frac{k_2}{0.6,0.8,0.4}, \frac{k_3}{0.5,0.8,0.6}\right\}> \right\},$$
$$(\mathscr{O},\Omega) = \left\{ <\omega_1, \left\{\frac{k_1}{0.6,0.6,0.7}, \frac{k_2}{0.4,0.8,0.8}, \frac{k_3}{0.3,0.8,0.6}\right\}>, <\omega_2, \left\{\frac{k_1}{0.5,0.8,0.6}, \frac{k_2}{0.5,0.9,0.5}, \frac{k_3}{0.2,0.9,0.7}\right\}> \right\}.$$

Then the subfamily $\tau_u = \{\phi_u, 1_u, (\mathscr{M},\Omega), (\mathscr{N},\Omega), (\mathscr{O},\Omega)\}$ forms an $\mathscr{N}_{\mathscr{S}}\mathscr{T}\mathscr{S}$. Define $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L},\Omega) = \begin{cases} (\mathscr{L},\Omega) \cup \{\omega_{1\mathscr{F}}\}, & \text{if } \omega_{2\mathscr{N}} \notin (\mathscr{L},\Omega), \\ (\mathscr{L},\Omega), & \text{if } \omega_{2\mathscr{N}} \in (\mathscr{L},\Omega), \end{cases} \quad \forall (\mathscr{L},\Omega) \in \tau_u,$$

where $\omega_{1\mathscr{F}} = \left\{\frac{k_1}{0.4,0.9,0.7}, \frac{k_2}{0.3,0.9,0.7}, \frac{k_3}{0.1,0.8,0.6}\right\}$. Here, γ is an $\mathscr{N}_{\mathscr{S}\gamma}$-regular operation.

Example 3.17. Consider the Example 3.16, define $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L},\Omega) = \begin{cases} (\mathscr{L},\Omega) \cup \{\omega_{2\mathscr{M}}\}, & \text{if } \omega_{2\mathscr{N}} \notin (\mathscr{L},\Omega), \\ (\mathscr{L},\Omega), & \text{if } \omega_{2\mathscr{N}} \in (\mathscr{L},\Omega), \end{cases} \quad \forall (\mathscr{L},\Omega) \in \tau_u.$$

Since $\omega_{2\mathscr{N}} \in (\mathscr{M},\Omega)$ and $\omega_{2\mathscr{N}} \in (\mathscr{N},\Omega), \gamma(\mathscr{M},\Omega) = (\mathscr{M},\Omega)$ and $\gamma(\mathscr{N},\Omega) = (\mathscr{N},\Omega)$. Since $\omega_{2\mathscr{N}} \notin (\mathscr{O},\Omega)$,

$$\gamma(\mathscr{O},\Omega) = \left\{ <\omega_1, \left\{\frac{k_1}{0.6,0.6,0.7}, \frac{k_2}{0.4,0.8,0.8}, \frac{k_3}{0.3,0.8,0.6}\right\}>, <\omega_2, \left\{\frac{k_1}{0.8,0.4,0.5}, \frac{k_2}{0.7,0.7,0.3}, \frac{k_3}{0.7,0.5,0.6}\right\}> \right\},$$

$\tau_{u_\gamma} = \{\phi_u, 1_u, (\mathscr{M},\Omega), (\mathscr{N},\Omega)\}$. Here γ is not an $\mathscr{N}_{\mathscr{S}\gamma}$-regular operation, since for $\omega_{i\mathscr{O}} \in \tilde{\mathscr{U}} (i = 1,2)$ and $\omega_{i\mathscr{O}} \in (\mathscr{M},\Omega), (\mathscr{N},\Omega), (\mathscr{O},\Omega)$ for $(i = 1,2)$, consider the $\mathscr{N}_{\mathscr{S}}\mathscr{O}\mathscr{S}$s $(\mathscr{O},\Omega)$ and $(\mathscr{N},\Omega)$ containing $\omega_{i\mathscr{O}}$ there does not exist $\mathscr{N}_{\mathscr{S}}\mathscr{O}\mathscr{S}$ in τ_u, say $(\mathscr{K},\Omega)$ containing $\omega_{i\mathscr{O}}$ such that $\gamma(\mathscr{K},\Omega) \subseteq \gamma(\mathscr{O},\Omega) \cap \gamma(\mathscr{N},\Omega) = \{< \omega_1, \left\{\frac{k_1}{0.6,0.6,0.7}, \frac{k_2}{0.4,0.8,0.8}, \frac{k_3}{0.3,0.8,0.6}\right\} >, < \omega_2, \left\{\frac{k_1}{0.7,0.6,0.5}, \frac{k_2}{0.6,0.8,0.4}, \frac{k_3}{0.5,0.8,0.6}\right\} >\}$.

Theorem 3.18. *Let γ be an $\mathscr{N}_{\mathscr{S}\gamma}$-regular operation on τ_u. If $(\mathscr{L},\Omega)$ and (S,Ω) are $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}\mathscr{S}$s of $(\mathscr{U},\Omega,\tau_u)$, then $(\mathscr{L},\Omega) \cap (S,\Omega)$ is $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}$.*

Proof. Let $(\mathscr{L},\Omega)$ and (S,Ω) be $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}\mathscr{S}$s of $(\mathscr{U},\Omega,\tau_u)$. Consider $(\mathscr{J},\Omega) = (\mathscr{L},\Omega) \cap (\mathscr{S},\Omega)$. Let $\omega_{i\mathscr{F}} \in (\mathscr{J},\Omega)$ implies $\omega_{i\mathscr{F}} \in (\mathscr{L},\Omega)$ and $\omega_{i\mathscr{F}} \in (\mathscr{S},\Omega)$. Since $(\mathscr{L},\Omega)$ and $(\mathscr{S},\Omega)$ are $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}\mathscr{S}$s, $\exists$ $\mathscr{N}_{\mathscr{S}}\mathscr{O}\mathscr{S}$s $(\mathscr{H},\Omega)$ and $(\mathscr{K},\Omega)$ containing $\omega_{i\mathscr{F}} \ni \gamma(\mathscr{H},\Omega) \subseteq (\mathscr{L},\Omega)$ and $\gamma(\mathscr{K},\Omega) \subseteq (\mathscr{S},\Omega)$. Since the operation γ is $\mathscr{N}_{\mathscr{S}\gamma}$-regular, $\exists$ an $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}\mathscr{S}$ $(\mathscr{J},\Omega)$ containing $\omega_{i\mathscr{F}} \ni \gamma(\mathscr{J},\Omega) \subseteq \gamma(\mathscr{H},\Omega) \cap \gamma(\mathscr{K},\Omega) \subseteq (\mathscr{L},\Omega) \cap (\mathscr{S},\Omega)$. Therefore $(\mathscr{L},\Omega) \cap (\mathscr{S},\Omega)$ is an $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}\mathscr{S}$. □

Remark 3.19. If γ is an $\mathscr{N}_{\mathscr{S}\gamma}$-regular operation on τ_u, then τ_{u_γ} forms an $\mathscr{N}_{\mathscr{S}}$ topology on $(\mathscr{U},\Omega,\tau_u)$.

Proof. It is evident from Theorems 3.6 and 3.18 □

Definition 3.20. Let $(\mathscr{U},\Omega,\tau_u)$ be an $\mathscr{N}_{\mathscr{S}}\mathscr{T}\mathscr{S}$ and $(\mathscr{L},\Omega)$ be any arbitrary $\mathscr{N}_{\mathscr{S}}\mathscr{S}$. Then neutrosophic soft γ-closure of an $\mathscr{N}_{\mathscr{S}}\mathscr{S}$ $(\mathscr{L},\Omega)$ is the intersection of all $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{C}\mathscr{S}$s containing $(\mathscr{L},\Omega)$, i.e., $\gamma_{\mathscr{N}_{\mathscr{S}}} - cl(\mathscr{L},\Omega) = \cap\{(\mathscr{F},\Omega) : (\mathscr{L},\Omega) \subseteq (\mathscr{F},\Omega)$, where $(\mathscr{F},\Omega)$ is an $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{C}\mathscr{S}$ in $\mathscr{U}.\}$

Example 3.21. Consider Example 3.16, Define $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L},\Omega) = \begin{cases} (\mathscr{L},\Omega) \cup \{\omega_{2\mathscr{M}}\}, & \text{if } \omega_{2\mathscr{N}} \notin (\mathscr{L},\Omega), \\ (\mathscr{L},\Omega), & \text{if } \omega_{2\mathscr{N}} \in (\mathscr{L},\Omega), \end{cases} \quad \forall (\mathscr{L},\Omega) \in \tau_u.$$

Then $\gamma(\mathscr{L},\Omega) = (\mathscr{L},\Omega)$ for $(\mathscr{L},\Omega) = (\mathscr{M},\Omega)$ and $(\mathscr{N},\Omega)$.

$$\gamma(\mathscr{O},\Omega) = \left\{ <\omega_1, \left\{\frac{k_1}{0.6,0.6,0.7}, \frac{k_2}{0.4,0.8,0.8}, \frac{k_3}{0.3,0.8,0.6}\right\}>, <\omega_2, \left\{\frac{k_1}{0.8,0.4,0.5}, \frac{k_2}{0.7,0.7,0.3}, \frac{k_3}{0.7,0.5,0.6}\right\}> \right\}.$$

$\tau_{u_\gamma} = \{\phi_u,\ 1_u,\ (\mathscr{M},\Omega),(\mathscr{N},\Omega)\}$. $\tau^c_{u_\gamma} = \{\phi_u,\ 1_u,\ (\mathscr{M},\Omega)^c,(\mathscr{N},\Omega)^c\}$, where

$$(\mathscr{M},\Omega)^c = \left\{ <\omega_1,\left\{\frac{k_1}{0.4,0.5,1.0},\frac{k_2}{0.6,0.4,0.6},\frac{k_3}{0.4,0.4,0.5}\right\}>,<\omega_2,\left\{\frac{k_1}{0.5,0.6,0.8},\frac{k_2}{0.3,0.3,0.7},\frac{k_3}{0.6,0.5,0.7}\right\}> \right\},$$
$$(\mathscr{N},\Omega)^c = \left\{ <\omega_1,\left\{\frac{k_1}{0.6,0.5,0.8},\frac{k_2}{0.6,0.3,0.5},\frac{k_3}{0.5,0.3,0.4}\right\}>,<\omega_2,\left\{\frac{k_1}{0.5,0.4,0.7},\frac{k_2}{0.4,0.2,0.6},\frac{k_3}{0.6,0.2,0.5}\right\}> \right\}.$$

Clearly $(\mathscr{M},\Omega)^c \subset (\mathscr{N},\Omega)^c$. Let $(\mathscr{P},\Omega)$ be an arbitrary $\mathscr{N}_\mathscr{S}\mathscr{S}$ defined as,

$$(\mathscr{P},\Omega) = \left\{ <\omega_1,\left\{\frac{k_1}{0.5,0.6,0.9},\frac{k_2}{0.5,0.4,0.7},\frac{k_3}{0.4,0.5,0.6}\right\}>,<\omega_2,\left\{\frac{k_1}{0.4,0.5,0.8},\frac{k_2}{0.3,0.5,0.7},\frac{k_3}{0.3,0.7,0.8}\right\}> \right\}.$$

Here $\gamma_{\mathscr{N}_\mathscr{S}}$ - $\mathrm{cl}(\mathscr{P},\Omega) = (\mathscr{N},\Omega)^c$.

Proposition 3.22. *Let $(\mathscr{L},\Omega)$ and $(\mathscr{S},\Omega)$ be two $\mathscr{N}_\mathscr{S}\mathscr{S}$s in $(\mathscr{U},\Omega,\tau_u)$. Then*

(i) $(\mathscr{L},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$;

(ii) $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$ *is the smallest* $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{C}\mathscr{S}$ *containing* $(\mathscr{L},\Omega)$;

(iii) $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\phi) = \phi$ *and* $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(1_u) = 1_u$;

(iv) $(\mathscr{L},\Omega)$ *is* $\mathscr{N}_{\mathscr{S}_\gamma}$*-closed if and only if* $(\mathscr{L},\Omega) = \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$;

(v) *if* $(\mathscr{L},\Omega) \subset (\mathscr{S},\Omega)$, *then* $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{L},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{S},\Omega)$;

(vi) $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,((\mathscr{L},\Omega)\cup(\mathscr{S},\Omega)) \supset \gamma_{\mathscr{N}_\mathscr{S}}\ \text{-}\mathrm{cl}(\mathscr{L},\Omega)\cup\gamma_{\mathscr{N}_\mathscr{S}}\ \text{-}\mathrm{cl}(\mathscr{S},\Omega)$;

(vii) $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,((\mathscr{L},\Omega)\cap(\mathscr{S},\Omega)) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)\cap\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{S},\Omega)$;

(viii) $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)) = \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$.

Proof. (i) and (iii) are immediate.

(ii): To prove $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$ is an $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{C}\mathscr{S}$, it is enough to prove $(\gamma_{\mathscr{N}_\mathscr{S}} - \mathrm{cl}(\mathscr{L},\Omega))^c$ is $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{O}\mathscr{S}$ in $\mathscr{U}$. Let $\omega_{i\mathscr{F}} \in (\gamma_{\mathscr{N}_\mathscr{S}} - \mathrm{cl}(\mathscr{L},\Omega))^c$. Then $\omega_{i\mathscr{F}} \notin \gamma_{\mathscr{N}_\mathscr{S}} - \mathrm{cl}(\mathscr{L},\Omega)$, i.e., $\omega_{i\mathscr{F}} \notin (\mathscr{J},\Omega)$, for at least one $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{C}\mathscr{S}$ containing $(\mathscr{L},\Omega)$. This implies $\omega_{i\mathscr{F}} \in (\mathscr{J},\Omega)^c$ and $(\mathscr{J},\Omega)^c$ is an $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{O}\mathscr{S}$. By the Definition 3.3, $\forall$ $\omega_{i\mathscr{F}} \in (\mathscr{J},\Omega)^c$, $\exists$ an $\mathscr{N}_\mathscr{S}\mathscr{O}\mathscr{S}$ $(\mathscr{H},\Omega)$ such that $\omega_{i\mathscr{F}} \in (\mathscr{H},\Omega)$ and $\gamma(\mathscr{H},\Omega) \subseteq (\mathscr{J},\Omega)^c$. Since $(\mathscr{J},\Omega) \supseteq \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$, $(\mathscr{J},\Omega)^c \subseteq (\gamma_{\mathscr{N}_\mathscr{S}} - \mathrm{cl}(\mathscr{L},\Omega))^c$, which implies $\gamma(\mathscr{H},\Omega) \subseteq (\mathscr{J},\Omega)^c \subseteq (\gamma_{\mathscr{N}_\mathscr{S}} - \mathrm{cl}(\mathscr{L},\Omega))^c$. Therefore, $(\gamma_{\mathscr{N}_\mathscr{S}} - \mathrm{cl}(\mathscr{L},\Omega))^c$ is an $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{O}\mathscr{S}$ in $\mathscr{U}$.

(iv): If $(\mathscr{L},\Omega) = \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$, then by (ii), $(\mathscr{L},\Omega)$ is $\mathscr{N}_{\mathscr{S}_\gamma}$-closed. Now, to prove $(\mathscr{L},\Omega) = \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$ when $(\mathscr{L},\Omega)$ is $\mathscr{N}_{\mathscr{S}_\gamma}$-closed. Assume that $(\mathscr{L},\Omega)$ is $\mathscr{N}_{\mathscr{S}_\gamma}$-closed. Let $\omega_{i\mathscr{F}} \in \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$. Then $\omega_{i\mathscr{F}} \in (\mathscr{J},\Omega)$, $\forall$ $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{C}\mathscr{S}$ containing $(\mathscr{L},\Omega)$. Since $(\mathscr{L},\Omega)$ is also an $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{C}\mathscr{S}$ containing $(\mathscr{L},\Omega)$, $\omega_{i\mathscr{F}} \in (\mathscr{L},\Omega)$, implies that $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega) \subseteq (\mathscr{L},\Omega)$. By (i), $(\mathscr{L},\Omega) \subseteq \gamma_{\mathscr{N}_\mathscr{S}}\ \text{-}\mathrm{cl}(\mathscr{L},\Omega)$. Hence $(\mathscr{L},\Omega) = \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$.

(v): $(\mathscr{L},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$ and $(\mathscr{S},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{S},\Omega) \implies (\mathscr{L},\Omega) \subset (\mathscr{S},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{S},\Omega)$. Since $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{L},\Omega)$ is the smallest $\mathscr{N}_{\mathscr{S}_\gamma}\mathscr{C}\mathscr{S}$ containing $(\mathscr{L},\Omega)$, $(\mathscr{L},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{L},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{S},\Omega)$. Hence $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{L},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}(\mathscr{S},\Omega)$.

(vi): $(\mathscr{L},\Omega) \subset (\mathscr{L},\Omega)\cup(\mathscr{S},\Omega)$ and $(\mathscr{S},\Omega) \subset (\mathscr{L},\Omega)\cup(\mathscr{S},\Omega)$. By (v), $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{L},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}((\mathscr{L},\Omega)\cup(\mathscr{S},\Omega))$ and $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{S},\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,((\mathscr{L},\Omega)\cup(\mathscr{S},\Omega)) \implies \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{L},\Omega)\cup\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(S,\Omega) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}((\mathscr{L},\Omega)\cup(\mathscr{S},\Omega))$.

(vii): $(\mathscr{L},\Omega)\cap(\mathscr{S},\Omega) \subset (\mathscr{L},\Omega)$ and $(\mathscr{L},\Omega)\cap(\mathscr{S},\Omega) \subset (\mathscr{S},\Omega)$. By (v), $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,((\mathscr{L},\Omega)\cap(\mathscr{S},\Omega)) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{L},\Omega)$ and $\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,((\mathscr{L},\Omega)\cap(\mathscr{S},\Omega)) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{S},\Omega) \implies \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,((\mathscr{L},\Omega)\cap(\mathscr{S},\Omega)) \subset \gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{L},\Omega)\cap\gamma_{\mathscr{N}_\mathscr{S}}\text{-}\mathrm{cl}\,(\mathscr{S},\Omega)$.

(viii): Obvious from (ii) and (iv). □

Definition 3.23. Let $(\mathscr{U},\Omega,\tau_u)$ be an $\mathscr{N_S T S}$ and $(\mathscr{L},\Omega)$ be any arbitrary $\mathscr{N_S S}$. Then neutrosophic soft γ-interior of an $\mathscr{N_S S}$ $(\mathscr{L},\Omega)$ is the union of all $\mathscr{N_S}_\gamma\mathscr{O S}$s contained in $(\mathscr{L},\Omega)$, i.e., $\gamma_{\mathscr{N_S}}\text{-int}(\mathscr{L},\Omega) = \cup\{(\mathscr{G},\Omega) : (\mathscr{G},\Omega) \subseteq (\mathscr{L},\Omega)$, where $(\mathscr{G},\Omega)$ is an $\mathscr{N_S}_\gamma\mathscr{O S}$ in $\mathscr{U}$ $\}$.

Example 3.24. Consider Example 3.21, $\tau_{u_\gamma} = \{\phi_u,\ 1_u,\ (\mathscr{M},\Omega), (\mathscr{N},\Omega)\}$. Let $(\mathscr{P},\Omega)$ be an arbitrary $\mathscr{N_S S}$ defined as

$$(\mathscr{P},\Omega) = \left\{ <\omega_1, \left\{\frac{k_1}{0.8,0.3,0.2}, \frac{k_2}{0.5,0.4,0.2}, \frac{k_3}{0.4,0.2,0.4}\right\}>, <\omega_2, \left\{\frac{k_1}{0.7,0.1,0.3}, \frac{k_2}{0.6,0.3,0.1}, \frac{k_3}{0.8,0.4,0.3}\right\}> \right\}.$$

Then $\gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{P},\Omega) = (\mathscr{N},\Omega)$.

Proposition 3.25. *Let $(\mathscr{L},\Omega)$ and $(\mathscr{S},\Omega)$ be two $\mathscr{N_S S}$s of $\mathscr{U}$. Then*

(i) $\gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{L},\Omega) \subseteq (\mathscr{L},\Omega)$ *and* $\gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{L},\Omega)$ *is the largest* $\mathscr{N_S}_\gamma\mathscr{O S}$ *contained in* $(\mathscr{L},\Omega)$*;*

(ii) $\gamma_{\mathscr{N_S}}\text{-int}\,(\phi) = \phi$ *and* $\gamma_{\mathscr{N_S}}\text{-int}\,(1_u) = 1_u$*;*

(iii) $(\mathscr{L},\Omega)$ *is* $\mathscr{N_S}_\gamma\mathscr{O}$ *if and only if* $(\mathscr{L},\Omega) = \gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{L},\Omega)$*;*

(iv) *if* $(\mathscr{L},\Omega) \subseteq (\mathscr{S},\Omega)$*, then* $\gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{L},\Omega) \subseteq \gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{S},\Omega)$*;*

(v) $\gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{L},\Omega) \cup \gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{S},\Omega) \subset \gamma_{\mathscr{N_S}}\text{-int}\,((\mathscr{L},\Omega) \cup (\mathscr{S},\Omega))$*;*

(vi) $\gamma_{\mathscr{N_S}}\text{-int}\,((\mathscr{L},\Omega) \cap (\mathscr{S},\Omega)) \subset \gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{L},\Omega) \cap \gamma_{\mathscr{N_S}}\text{-int}\,(\mathscr{S},\Omega)$*;*

(vii) $\gamma_{\mathscr{N_S}}\text{-int}(\gamma_{\mathscr{N_S}}\text{-int}(\mathscr{L},\Omega)) = \gamma_{\mathscr{N_S}}\text{-int}(\mathscr{L},\Omega)$.

4. Application of neutrosophic soft γ-open sets in decision making

Neutrosophic soft set has numerous applications in daily life problems involving uncertainties. Here, we use the idea of Neutrosophic soft set for modelling one such problem of decision-making.

Definition 4.1. Comparison matrix is a matrix whose rows and columns are named by the object names $\vartheta_1, \vartheta_2, \ldots, \vartheta_n$ and parameters $\zeta_1, \zeta_2, \ldots, \zeta_m$, respectively. The entries c_{ij} are computed by

$$c_{ij} = \begin{cases} 0, & \text{if } T_{\vartheta_i}\ (\zeta_j\ ) = 0,\ I_{\vartheta_i}\ (\zeta_j\ ) = 1,\ F_{\vartheta_i}\ (\zeta_j\ ) = 1, \\ l + p - q, & \text{otherwise,} \end{cases}$$

where 'l' is the integer, counted by 'number of times $T_{\vartheta_i}\ (\zeta_j\ ) \geqslant T_{\vartheta_h}\ (\zeta_j)$', for $\vartheta_i \neq \vartheta_h$, $\forall \vartheta_h \in \mathscr{U}$, 'p' is the integer, counted by 'number of times $I_{\vartheta_i}\ (\zeta_j\ ) \geqslant I_{\vartheta_h}\ (\zeta_j\ )$', for $\vartheta_i \neq \vartheta_h$, $\forall \vartheta_h \in \mathscr{U}$ and 'q' is the integer, counted by 'number of times $F_{\vartheta_i}\ (\zeta_j\ ) \geqslant F_{\vartheta_h}\ (\zeta_j\ )$', for $\vartheta_i \neq \vartheta_h$, $\forall \vartheta_h \in \mathscr{U}$.

Definition 4.2 ([14])**.** 'Score of an object' ϑ_i is computed as $S_i = \sum_j c_{ij}$.

4.1. Decision making with neutrosophic soft γ-open sets

As enterprises strive for more efficient operations across the board, enterprise resource planning (ERP) software is becoming an increasingly sought-after solution for enhancing procedures at the business application level. In a single platform, ERP software unifies several back-office applications, business processes, and workflows. It provides unique benefits like improved data sharing, improved data quality, and accuracy, as well as enhanced administrative visibility.

It doesn't necessarily follow that an ERP platform is a suitable choice for one's business simply because widely recognized or highly reviewed. Choosing the right ERP is a critical job for enterprise runners.

This application aims to achieve the main target of determining the suitable ERP for the effective operation of one's firm and a particular concern. There is plenty of ERP software available in the market. Each software appeal to different business and has its pros and cons. An ERP which operates well in human resource sectors need not duly operates the same for the manufacturing or financial sector. If a person is running a garment manufacturing firm and deciding to buy the best ERP, in his business sector, manufacturing and inventory management are the key features that he must be mainly concerned about. He might reach experts for their opinion or search for widely recognized or highly reviewed products in the market, but that might not fully satisfy the key requirements of his firm. The methodology proposed in this section mainly focuses on the key requirements of one's firm.

To carry out this application, we, first, enumerate the four different types of ERP available in the markets based on the opinion given by the experts. Second, we set up some features of ERP; each ERP is expressed by a neutrosophic soft set (primary) such that the initial universal set is all four different types of the ERP and the set of parameters is the features of ERP. Third, we will put together all suggested ERP models to initiate a neutrosophic soft topological structure. This structure is framed using the $\mathscr{N}_{\mathcal{S}}$ union and $\mathscr{N}_{\mathcal{S}}$ intersection operation and the absolute $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ and null $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ are always in the structure. Fourth, we set up an operation according to the person's requirement and compute the $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$s . The union of the primary $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ is taken and a comparison matrix is determined. Finally, the score of the ERPs is found.

Algorithm 4.3. Algorithm of determining the optimum solution utilizing the $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$s .

Step-1: Input the data into $\mathscr{N}_{\mathcal{S}}\mathcal{S}$ (which we call as primary $\mathscr{N}_{\mathcal{S}}\mathcal{S}$s).
Step-2: Frame the $\mathscr{N}_{\mathcal{S}}\mathscr{T}\mathcal{S}$.
Step-3: Define a suitable operation for the given problem.
Step-4: Determine $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$s .
Step-5: Consider the $\mathscr{N}_{\mathcal{S}\gamma}\mathscr{O}\mathcal{S}$ which is the union of the primary $\mathscr{N}_{\mathcal{S}}\mathcal{S}$s contained in the collection $\tau_{u\gamma}$.
Step-6: Find the comparison table.
Step-7: Compute the score S_i.
Step-8: The optimum alternative is selected by finding the maximum score.

For a numerical example, consider the problem of selecting the right ERP for a garment firm among the top ERP's in the market. Let $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \dots, \vartheta_m$ denote the elements in the universal set and $\zeta_1, \zeta_2, \zeta_3, \dots, \zeta_n$ denote the parameters.

Suppose that $\mathscr{U} = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}$ be the set of four different ERP softwares (SAP S/4 HANA, ORACLE cloud, MS Dynamics, and Acumatica) and $\Omega = \zeta_1$ =Inventory, ζ_2 = Manufacturing, ζ_3 = Order Management, ζ_4 =Service, ζ_5 = Customer Relationship Management, ζ_6= Integration, and Extensibility be a set of parameters.

Step-1: Input the primary $\mathscr{N}_{\mathcal{S}}\mathcal{S}$s as Tables 1-4.

Table 1: $(\mathscr{A}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Table 2: $(\mathscr{B}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Table 3: $(\mathscr{C}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_2	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Table 4: $(\mathscr{D}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_2	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Step-2: Frame the $\mathscr{N}_{\mathscr{S}}\mathscr{T}\mathscr{S}$ as

$$\begin{aligned}\tau_u = \{&\phi_u,\ 1_u, (\mathscr{A},\Omega), (\mathscr{B},\Omega), (\mathscr{C},\Omega), (\mathscr{D},\Omega), (\mathscr{F},\Omega), (\mathscr{G},\Omega),\ (\mathscr{H},\Omega),\\ &(\mathscr{I},\Omega),\ (\mathscr{J},\Omega), (\mathscr{K},\Omega),\ (\mathscr{L},\Omega), (\mathscr{M},\Omega),\ (\mathscr{N},\Omega), (\mathscr{O},\Omega),\ (\mathscr{P},\Omega)\},\end{aligned}$$

where we have Tables 5-15.

Table 5: $(\mathscr{F}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Table 6: $(\mathscr{G}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Table 7: $(\mathscr{H}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Table 8: $(\mathscr{I}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_2	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Table 9: $(\mathscr{J}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Table 10: $(\mathscr{K}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Table 11: $(\mathscr{L}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Table 12: $(\mathscr{M}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Table 13: $(\mathscr{N}, \Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Table 14: $(\mathscr{O},\Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Table 15: $(\mathscr{P},\Omega)$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	1, 0, 0	0.79, 0.4, 0.2	0.96, 0.1, 0.1	.78, 0.3, 0.4	0.9, 0.1, 0.1	0.71, 0.4, 0.5
ϑ_4	1, 0, 0	0.76, 0.5, 0.6	0.82, 0.4, 0.4	0.98, 0.1, 0.1	0.71, 0.2, 0.2	0.96, 0.1, 0.1

Step-3: If the person is concerned more about the manufacturing, where as other attributes are his secondary concerns, define $\gamma : \tau_u \to P(\mathscr{U})$ as

$$\gamma(\mathscr{L},\Omega) = \begin{cases} cl(\mathscr{L},\Omega), & \text{if } \zeta_{2\mathscr{L}} \in \zeta_{1F}, \\ (\mathscr{L},\Omega), & \text{otherwise}, \end{cases} \quad \forall (\mathscr{L},\Omega) \in \tau_u,$$

where $\zeta_{1F} = \left\{ \frac{\vartheta_1}{0.8,0.2,0.2}, \frac{\vartheta_2}{0.8,0.2,0.2}, \frac{\vartheta_3}{0.8,0.2,0.2}, \frac{\vartheta_4}{0.8,0.2,0.2} \right\}$.

Step-4: Determine $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}\mathscr{S}$s as

$$\tau_{u_\gamma} = \{\phi_u, 1_u, (\mathscr{A},\Omega), (\mathscr{B},\Omega), (\mathscr{F},\Omega), (\mathscr{G},\Omega), (\mathscr{H},\Omega), (\mathscr{J},\Omega), (\mathscr{K},\Omega), (\mathscr{L},\Omega), (\mathscr{M},\Omega), (\mathscr{N},\Omega), (\mathscr{O},\Omega), (\mathscr{P},\Omega)\}.$$

Step-5: Consider the $\mathscr{N}_{\mathscr{S}\gamma}\mathscr{O}\mathscr{S}$, which is the union of the primary $\mathscr{N}_{\mathscr{S}}\mathscr{S}$s contained in the collection $\tau_{u\gamma}$ as in Table 16.

Table 16: $(\mathscr{F},\Omega)$ = union of primary $\mathscr{N}_{\mathscr{S}}\mathscr{S}$s in $\tau_{u\gamma}$.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	1, 0, 0	0.92, 0.1, 0.1	0.92, 0.2, 0.1	0.65, 0.4, 0.6	0.7, 0.4, 0.3	0.86, 0.2, 0.3
ϑ_2	0.9, 0.1, 0.2	0.92, 0.2, 0.2	0.9, 0.3, 0.2	0.84, 0.2, 0.3	0.9, 0.1, 0.1	0.71, 0.3, 0.4
ϑ_3	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1
ϑ_4	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1	0, 1, 1

Step-6: Find the comparison table as

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	3	3	3	2	2	3
ϑ_2	2	3	2	3	3	2
ϑ_3	0	0	0	0	0	0
ϑ_4	0	0	0	0	0	0

Step-7: Compute the score S_i as

	Score
ϑ_1	16
ϑ_2	15
ϑ_3	0
ϑ_4	0

Step-8: The optimum alternative is selected by finding the maximum score. Clearly maximum score is 16, scored by the ERP ϑ_1.

Comparison matrix for $\mathcal{N}_{\mathcal{S}}\mathcal{S}(\mathscr{P}, \boldsymbol{\Omega})$ by Maji's approach is presented in Table 17, where (P, Ω) is the union of all primary $\mathcal{N}_{\mathcal{S}}\mathcal{S}$s in τ_u.

Table 17: Comparison matrix for $\mathcal{N}_{\mathcal{S}}\mathcal{S}(\mathscr{P}, \boldsymbol{\Omega})$ by Maji's approach.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
ϑ_1	3	3	3	0	0	2
ϑ_2	0	2	1	2	3	1
ϑ_3	3	1	2	1	3	1
ϑ_4	3	0	0	3	1	3

Score of ϑ_i by Maji's approach for the $\mathcal{N}_{\mathcal{S}}\mathcal{S}$ $(\mathscr{P},\Omega)$ is as

	Score
ϑ_1	10
ϑ_2	9
ϑ_3	11
ϑ_4	10

Therefore, by Maji's approach, based on the score the person could choose the ERP ϑ_3. But ERP ϑ_3 has the truth membership 0.79 for the manufacturing feature (ζ_2) which is less than the truth membership of ERP ϑ_1. Since the person is more concerned about the manufacturing feature of the ERP, he might get disappointed in choosing the ERP ϑ_3. But whereas if he chose the ERP ϑ_1, he would be satisfied as per his primary and secondary attribute requirements.

5. Conclusion

To conclude this paper explicitly, a new concept called neutrosophic soft γ-open sets has been found in a new way by defining an operation on neutrosophic soft open sets. Its operations like union and intersection are discussed with illustrations. Some fundamental operators like closure and interior concerning neutrosophic soft γ-sets are investigated and their basic properties are analyzed. Finally, using the proposed algorithm, an optimum decision with respect to the requirement of a person using neutrosophic soft γ-open sets is found. In the future research, neutrosophic soft operations can be extended to neutrosophic hypersoft, indeterm soft and tree soft set operations. Futher, this study can be extended by developing python programme for the proposed model.

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