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Further properties of soft somewhere dense continuous functions and soft Baire spaces

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Abstract

In this paper, we first explore more properties of soft somewhere dense continuous functions. Then, we discuss the preservation of soft Baire property and soft Baire category. We give some concrete examples to illustrate how our findings extend some conclusions and connections made in the literature.

Keywords: Soft continuous, soft somewhat continuous, soft semicontinuous, soft somewhere dense continuous, fuzzy Baire space, soft Baire space.

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1. Introduction

In 1999, the theory of soft sets was given by the Russian mathematician Molodtsov [45] as a tool for modeling mathematical problems that deal with uncertainties. According to Molodtsov, there are numerous applications for soft set theory. In the present theory, there is no restricted condition to describe the objects, as a result, researchers are free to choose the type of parameters that they demand, significantly simplifying decision-making and allowing the method quite efficient in the lack of incomplete information. Alternative theories like fuzzy set theory, rough set theory, and vague set theory, might be thought of as mathematical techniques for dealing with uncertainty, nevertheless. Maji et al. [44] investigated a (deep) conceptual framework of soft set theory in their study. They developed numerous operators specifically for soft set theory. A number of other mathematical structures, including soft ring theory [1],

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soft group theory [2], soft σ-algebra [18], soft algebra [23], etc have been investigated in soft context after Maji's work.

Shabir and Naz [50], and Çağman et al. [30] individually defined the idea of soft (standard) topology in 2011. Then various methods of extending soft topologies have been introduced, see [4, 15, 21, 29, 47]. Soft continuity of functions was defined by Nazmul and Samanta [46] in 2013. Then, in the literature, multiple generalized forms of soft continuous and soft open functions showed up. Namely: soft bi-continuity [16], soft $\mathcal{U}$-continuous functions [20], soft somewhat continuous functions [24], soft β-continuous functions [53], soft semicontinuous functions [43], soft semi-open functions [43], soft β-open functions [53], soft somewhat open functions [24], Soft complete continuity [6], etc. In 2018, Al-shami [11] defined the notion of soft somewhere dense sets and explored main properties. Then, many soft notions were established depending on soft somewhere dense sets such as soft interior and closure operators [36], soft continuous functions [14] and soft separation axioms [13]. Some findings that show the divergences between classical and soft topological spaces were displayed in [12].

Correspondingly, the concepts of compactness [10, 28], connectedness [42], and separability [49], extremally disconnectedness [26], submaximality [7], separation axioms [17, 34, 35, 51], bases [3], Caliber and chain conditions [5], etc, have been studied in soft settings. In addition, the maximal or minimal soft topologies with regard to some of the latter soft topological properties have been investigated in [8, 22].

The Baire category theorem is an important criteria in topology and analysis. In general topology, the theory of Baire spaces has been widely researched in [32, 33, 37–39]. In fuzzy settings, Thangaraj and Anjalmose [52] examined Baire spaces in 2013. Riaz and Zain [48] investigated the notion of soft Baire metric spaces in 2017. According to our knowledge, very few is known about Baire spaces in soft set theoretic approach. In particular, the topic of soft Baire spaces being preserved under soft functions. One could deduce from Theorem 2.38 in [11] that soft Baire spaces can be kept under soft open soft continuous images. While soft open soft continuous preimages are preserved under soft Baire space without imposing any additional requirements, this claim may be false. Recently, authors in [25, 27] have studied the soft Baire invariance under the functions that are somewhat soft continuous, soft semicontinuous, soft somewhat open, and soft somewhere dense open. In this paper, we continue in the same direction and provide better results after studying more attributes of some categories of soft functions.

2. Definitions and notations

Certain fundamental concepts and notations that will be utilized in the continuation are presented in this section. From now on, we refer to Z as our universal set and $\widehat{\mathcal{T}}$ as a collection of parameters.

Definition 2.1 ([45])**.** A soft set in $\widetilde{Z}$ is the pair $(R, \mathcal{T}) = \{(\theta, R(\theta)) : \theta \in \mathcal{T}\}$, where R is a set-valued mapping from a $\mathcal{T} \subseteq \widehat{\mathcal{T}}$ into the power set 2^Z of Z. The system of all soft sets in $\widetilde{Z}$ parameterized by $\mathcal{T}$ is represented by $S(Z_{\mathcal{T}})$.

Definition 2.2 ([45])**.** Let $(R, \mathcal{T})$ be a soft subset in $\widetilde{Z}$. Then $(R, \mathcal{T})$ is said to be

(i) null if $R(\theta) = \emptyset$, $\forall \theta \in \mathcal{T}$;
(ii) absolute if $R(\theta) = Z$, $\forall \theta \in \mathcal{T}$.

$\widetilde{Z}$ and $\widetilde{\Phi}$, respectively, represent the soft absolute and null sets. Evidently, $\widetilde{Z}^c = \widetilde{\Phi}$ and $\widetilde{\Phi}^c = \widetilde{Z}$.

Definition 2.3 ([19, 46])**.** A soft set $(R, \mathcal{T})$ in $\widetilde{Z}$ is said to be a soft point, denoted by z_θ, if $\theta \in \mathcal{T}$ and $x \in X$ such that $R(\theta) = \{z\}$ and $R(\vartheta) = \emptyset$ for every $\vartheta \neq \theta$, $\vartheta \in \mathcal{T}$. An expression $z_\theta \sqsubseteq (R, \mathcal{T})$ means that $z \in R(\theta)$.

Definition 2.4 ([9])**.** The soft complement $(R, \mathcal{T})^c$ of is a soft set $(R, \mathcal{T})$ in $\widetilde{Z}$ is a soft set $(R^c, \mathcal{T})$, where $R^c : \mathcal{T} \to 2^Z$ is given by $R^c(\theta) = Z \backslash R(\theta)$ for every $\theta \in \mathcal{T}$.

Definition 2.5 ([44])**.** Let $\Theta_1, \Theta_2 \subseteq \widehat{\mathcal{T}}$. Then, $(S, \mathcal{T}_1)$ is a soft subset of (T, Θ_2) (written by $(S, \Theta_1) \sqsubseteq (T, \Theta_2)$) if $\Theta_1 \subseteq \Theta_2$ and $R(\theta) \subseteq S(\theta)$ for any $\theta \in \Theta_1$. We say $(S, \Theta_1) = (T, \Theta_2)$ if $(S, \Theta_1) \sqsubseteq (T, \Theta_2)$ and $(T, \Theta_2) \sqsubseteq (S, \Theta_1)$.

Definition 2.6 ([9])**.** Let $\{(R_\lambda, \mathcal{T}) : \lambda \in \Lambda\}$ be an indexed collection of soft sets in $\widetilde{Z}$ with an index set Λ. Then

(i) the soft set $(S, \mathcal{T})$ is the intersection of all $(R_\lambda, \mathcal{T})$, where $S(\theta) = \bigcap_{\lambda\in\Lambda} F_\lambda(\theta)$ for every $\theta \in \mathcal{T}$. $(S, \mathcal{T})$ is denoted by $(S, \mathcal{T}) = \bigsqcap_{\lambda\in\Lambda}(R_\lambda, \mathcal{T})$;

(ii) the soft set $(S, \mathcal{T})$ is the union of all $(R_\lambda, \mathcal{T})$, where $S(\theta) = \bigcup_{\lambda\in\Lambda} F_\lambda(\theta)$ for every $\theta \in \mathcal{T}$. $(S, \mathcal{T})$ is denoted by $(S, \mathcal{T}) = \bigsqcup_{\lambda\in\Lambda}(R_\lambda, \mathcal{T})$;

(iii) the symmetric difference of $(R_{\lambda_1}, \mathcal{T})$ and $(R_{\lambda_2}, \mathcal{T})$ is defined by

$$(R_{\lambda_1}, \mathcal{T})\widetilde{\Delta}(R_{\lambda_2}, \mathcal{T}) = ((R_{\lambda_1}, \mathcal{T})\backslash(R_{\lambda_2}, \mathcal{T}))\bigsqcup((R_{\lambda_2}, \mathcal{T})\backslash(R_{\lambda_1}, \mathcal{T})).$$

Definition 2.7 ([50])**.** A subclass Ψ of $S(Z_\mathcal{T})$ is said to be a soft topology on Z if

(c1) Ψ includes both $\widetilde{\Phi}$ and $\widetilde{Z}$;
(c2) Ψ is closed under finite intersections; and
(c3) Ψ is closed under any unions.

The 3-tuple $(Z, \Psi, \mathcal{T})$ is said to be a soft topological space on Z. The members of Ψ are soft open sets, and their complements are soft closed sets. By Ψ^c we mean the set of all soft closed sets.

Definition 2.8 ([50])**.** For a soft subset $(R, \mathcal{T})$ of $(Z, \Psi, \mathcal{T})$, the soft interior of $(R, \mathcal{T})$ is denoted by $\mathrm{Int}_Z((R, \mathcal{T}))$ (or simply $\mathrm{Int}((R, \mathcal{T}))$) and defined by

$$\mathrm{Int}((R, \mathcal{T})) = \widetilde{\bigcup}\{(G, \mathcal{T}) : (G, \mathcal{T})\widetilde{\subseteq}(R, \mathcal{T}), (G, \mathcal{T}) \in \Psi\}.$$

The soft interior of $(R, \mathcal{T})$ is denoted by $\mathrm{Cl}_Z((R, \mathcal{T}))$ (or simply $\mathrm{Cl}((R, \mathcal{T}))$) and defined by

$$\mathrm{Cl}((R, \mathcal{T})) = \widetilde{\bigcap}\{(F, \mathcal{T}) : (F, \mathcal{T})\widetilde{\supseteq}(R, \mathcal{T}), (G, \mathcal{T}) \in \Psi^c\}.$$

Henceforward, $(Z, \Psi, \mathcal{T})$ means a soft topological space.

Definition 2.9 ([8, 22])**.** Let $\mathcal{F}\widetilde{\subseteq}S(Z_\mathcal{T})$ and let $\{(Z, \Psi_\lambda, \mathcal{T}) : \lambda \in \Lambda\}$ be an indexed family of soft topological space on Z with an arbitrary index Λ such that $\mathcal{F}\widetilde{\subseteq}\Theta_\lambda$. Then $\widetilde{\bigcap}_{\lambda\in\Lambda}\Theta_\lambda$ is called the soft topology on Z generated by $\mathcal{F}$.

Definition 2.10 ([30])**.** Given a soft topological space $(Z, \Psi, \mathcal{T})$. If any member of Ψ is represent as a union of some members of a collection $\mathcal{B}$, then $\mathcal{B}\widetilde{\subseteq}\Psi$ is referred to as a soft base for Ψ.

Remark 2.11 ([40]). If $(S, \mathcal{T})$ is a soft subset of $(Z, \Psi, \mathcal{T})$, then

$$\mathrm{Cl}((S, \mathcal{T})^c) = (\mathrm{Int}((S, \mathcal{T})))^c \text{ and } \mathrm{Int}((S, \mathcal{T})^c) = (\mathrm{Cl}((S, \mathcal{T})))^c.$$

Definition 2.12 ([50])**.** Let $(Y, \mathcal{T}) \neq \widetilde{\Phi}$ be a soft subset of $(Z, \Psi, \mathcal{T})$. The relative soft topology on Y is defined by

$$\Psi_Y := \{(S, \mathcal{T}) \sqcap (Y, \mathcal{T}) : (S, \mathcal{T}) \sqsubseteq \Psi\}.$$

The 3-tuple $(Y, \Psi_Y, \mathcal{T})$ is called a soft subspace of $(Z, \Psi, \mathcal{T})$.

Definition 2.13. Let $(S, \mathcal{T})$ be a soft subset of $(Z, \Psi, \mathcal{T})$. Then $(S, \mathcal{T})$ is called soft dense [48] if $\mathrm{Cl}((S, \mathcal{T})) = \widetilde{Z}$; soft codense if $\mathrm{Int}((S, \mathcal{T})) = \widetilde{\Phi}$; soft nowhere dense [48] if $\mathrm{Int}(\mathrm{Cl}((S, \mathcal{T}))) = \widetilde{\Phi}$; soft meager [48] if $(S, \mathcal{T}) = \widetilde{\bigcup}_{n=1}^{\infty}(R_n, \mathcal{T})$, where each $(R_n, \mathcal{T})$ is soft nowhere dense; soft semiopen [31] if $(S, \mathcal{T}) \sqsubseteq \mathrm{Cl}(\mathrm{Int}((S, \mathcal{T})))$; soft β-open [53] if $(S, \mathcal{T}) \sqsubseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}((S, \mathcal{T}))))$; soft somewhat open [24] (briefly soft *sw*-open) if $\mathrm{Int}((S, \mathcal{T})) \neq \widetilde{\Phi}$ or $(S, \mathcal{T}) = \widetilde{\Phi}$; and soft somewhere dense [11] (briefly soft SD-set) if $\mathrm{Int}(\mathrm{Cl}((S, \mathcal{T}))) \neq \widetilde{\Phi}$.

The set of all soft dense sets in $(Z, \Psi, \mathfrak{T})$ is denoted by $\mathcal{D}(Z, \Psi, \mathfrak{T})$ or simply by $\mathcal{D}_\Psi$.

Definition 2.14. Let $(Z, \Psi, \mathfrak{T})$ and $(Y, \Upsilon, \mathfrak{T}')$ be soft topological spaces. A soft function $g : (Z, \Psi, \mathfrak{T}) \to (Y, \Upsilon, \mathfrak{T}')$ is said to be

- (i) soft open [46] (resp., soft *sw*-open [24], soft SD-open [14], soft semiopen [43]) if for every soft open set $(G, \mathfrak{T})$ in $(Z, \Psi, \mathfrak{T})$, $g((G, \mathfrak{T}))$ is a soft open (resp., soft *sw*-open, soft soft SD-open, soft semiopen) set in $(Y, \Upsilon, \mathfrak{T}')$.
- (ii) soft continuous [46] (resp., soft *sw*-continuous [24], soft SD-continuous [14], soft semicontinuous [43], soft β-continuous [53]) if for every soft open set $(H, \mathfrak{T}')$ in $(Y, \Upsilon, \mathfrak{T}')$, $g^{-1}((H, \mathfrak{T}'))$ is a soft open (resp., soft *sw*-open, soft SD-set, soft semiopen, β-open) set in $(Z, \Psi, \mathfrak{T})$.
- (iii) soft homeomorphism [46] (resp. soft *sw*-homomorphism) if g is one to one, onto, soft continuous, and soft open (resp. soft *sw*-continuous, and soft *sw*-open).

We direct the reader to [41] for a concept of soft functions defined on the collection of all soft sets. From now on, when we refer to "function," we mean "soft function."

As a link between the soft types of continuous functions previously developed, the following diagram is provided in [24].

$$\begin{array}{ccc} \text{soft semicontinuous} & \longrightarrow & \text{soft } \beta\text{-continuous} \\ \downarrow & & \downarrow \\ \text{soft } sw\text{-continuous} & \longrightarrow & \text{soft } SD\text{-continuous} \end{array}$$

The above arrows mean inclusions and their reverses are false, generally, see Examples 4.3-4.5 in [24].

3. Some properties of soft SD-continuous functions

Theorem 3.1. *Let $(Z, \Psi, \mathfrak{T})$ and $(Y, \Upsilon, \mathfrak{T}')$ be soft topological spaces. The following properties are equivalent for a function $f : (X, \Psi, E) \to (Y, \Upsilon, E')$:*

(1) *g is soft SD-continuous;*

(2) *for every $(W, \mathfrak{T}') \in \Upsilon$ with $g^{-1}((W, \mathfrak{T}')) \neq \widetilde{\Phi}$, there exists $\widetilde{\Phi} \neq (V, \mathfrak{T}) \in \Psi$ such that $(V, \mathfrak{T}) \sqsubseteq \mathrm{Cl}(g^{-1}((W, \mathfrak{T}')))$;*

(3) *for every $(W, \mathfrak{T}') \in \Upsilon^c$ with $g^{-1}((R, \mathfrak{T}')) \neq \widetilde{Z}$, there exists $\widetilde{Z} \neq (V, \mathfrak{T}) \in \Psi^c$ such that $\mathrm{Int}(g^{-1}((R, \mathfrak{T}'))) \sqsubseteq (T, \mathfrak{T})$;*

(4) *for every $(D, \mathfrak{T}) \in \Psi \widetilde{\sqcap} \mathcal{D}_\Psi$, then $R((D, \mathfrak{T}))$ is soft dense in $g(\widetilde{Z})$.*

Proof.

(1) $\Longrightarrow$ (2) Instantly comes from definition of soft SD-continuity.

(2) $\Longrightarrow$ (3) Let $(R, \mathfrak{T}') \in \Upsilon^c$ such that $g^{-1}((R, \mathfrak{T}')) \neq \widetilde{Z}$. Then $\widetilde{Y} \setminus (R, \mathfrak{T}') \in \Upsilon$ with $g^{-1}(\widetilde{Y} \setminus (R, \mathfrak{T}') \neq \widetilde{\Phi}$. By (2), there exists a soft open set $(V, \mathfrak{T})$ in $\widetilde{Z}$ such that $\widetilde{\Phi} \neq (V, \mathfrak{T}) \sqsubseteq \mathrm{Cl}(g^{-1}(\widetilde{Y} \setminus (R, \mathfrak{T}')) = \widetilde{Z} \setminus \mathrm{Int}(g^{-1}((R, \mathfrak{T}')))$. This implies that $\mathrm{Int}(g^{-1}((R, \mathfrak{T}'))) \sqsubseteq \widetilde{Z} \setminus (V, \mathfrak{T}) \neq \widetilde{Z}$. If $(T, \mathfrak{T}) = \widetilde{Z} \setminus (V, \mathfrak{T})$, then $(T, \mathfrak{T}) \neq \widetilde{Z}$ that fulfills the needed criterion.

(3) $\Longrightarrow$ (4) Let $(D, \mathfrak{T}) \in \Psi \widetilde{\sqcap} \mathcal{D}_\Psi$. We have to show that $g((D, \mathfrak{T}))$ is soft dense over $g(\widetilde{Z})$. If not, then there exists $(R, \mathfrak{T}') \in \Upsilon^c$ such that $g((D, \mathfrak{T})) \sqsubseteq (R, \mathfrak{T}') \neq g(\widetilde{Z})$. Therefore $(D, \mathfrak{T}) \sqsubseteq g^{-1}((R, \mathfrak{T}'))$. By (3), there exist a soft closed set $(T, \mathfrak{T})$ in $\widetilde{Z}$ such that $(D, \mathfrak{T}) \sqsubseteq \mathrm{Int}(g^{-1}((R, \mathfrak{T}'))) \sqsubseteq (T, \mathfrak{T}) \neq \widetilde{Z}$. This contradicts that $(D, \mathfrak{T}) \in \mathcal{D}_\Psi$. Thus (4) holds.

(4) $\Longrightarrow$ (1) Let $(X,\mathcal{T}') \in \Upsilon$. If $g^{-1}((X,\mathcal{T}')) = \widetilde{\Phi}$, it is trivially a soft SD-set. Suppose that $g^{-1}((X,\mathcal{T}')) \neq \widetilde{\Phi}$ and $g^{-1}((X,\mathcal{T}'))$ is not a soft SD-set. That is $\mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')))) = \widetilde{\Phi}$. Therefore $\mathrm{Cl}(\mathrm{Int}(\widetilde{Z}\backslash g^{-1}((X,\mathcal{T}'))) = \widetilde{Z}$. By (4), $g(\widetilde{Z}\backslash g^{-1}((X,\mathcal{T}')))$ is soft dense in $g(\widetilde{Z})$, i.e., $\mathrm{Cl}(g(\widetilde{Z}\backslash g^{-1}((X,\mathcal{T}')))) = g(\widetilde{Z})$. This yields that $\mathrm{Cl}(g(\widetilde{Z})\backslash(X,\mathcal{T}')) = g(\widetilde{Z})\backslash(X,\mathcal{T}') = g(\widetilde{Z})$ and so $(X,\mathcal{T}') = \widetilde{\Phi}$. Contradiction to the choice of $(X,\mathcal{T}')$. It follows that $\mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')))$ must not be null. Thus $g^{-1}((X,\mathcal{T}'))$ is a soft SD-set in $\widetilde{Z}$. □

Theorem 3.2. *The following are equivalent for a one to one and onto function* $g : (X,\Psi,E) \to (Y,\Upsilon,\mathcal{T}')$*:*

(1) g *is soft* SD*-continuous;*

(2) *for every soft (closed) nowhere dense set* $(N,\mathcal{T})$ *in* $\widetilde{Z}$, $g((N,\mathcal{T}))$ *is soft codense in* $\widetilde{Y}$.

Proof.

(1) $\Longrightarrow$ (2) Assume $(N,\mathcal{T})$ is soft (closed) nowhere dense in $\widetilde{Z}$. We need to show that $g((N,\mathcal{T}))$ is soft codense in $\widetilde{Y}$. Suppose otherwise, then there is $\widetilde{\Phi} \neq (X,\mathcal{T}') \in \Upsilon$ such that $(X,\mathcal{T}') \sqsubseteq g((N,\mathcal{T}))$ and so $g^{-1}((X,\mathcal{T}')) \sqsubseteq g^{-1}(g((N,\mathcal{T}))) = (N,\mathcal{T})$. By (1), $\widetilde{\Phi} \neq \mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}'))) \sqsubseteq \mathrm{Int}(\mathrm{Cl}(((N,\mathcal{T}))) = \mathrm{Int}(((N,\mathcal{T})))$. This proves that $(N,\mathcal{T})$ is not soft (closed) nowhere dense in $\widetilde{Z}$, which is contradiction. Hence (2) is established.

(2) $\Longrightarrow$ (1) Let $(X,\mathcal{T}') \in \Upsilon$. If $g^{-1}((X,\mathcal{T}')) = \widetilde{\Phi}$, so $g^{-1}((X,\mathcal{T}'))$ is a soft SD-set by the definition. Let $g^{-1}((X,\mathcal{T}')) \neq \widetilde{\Phi}$. If $g^{-1}((X,\mathcal{T}'))$ is not a soft SD-set, then $g^{-1}((X,\mathcal{T}'))$ is soft nowhere dense in $\widetilde{Z}$. By (2), $g(g^{-1}((X,\mathcal{T}')))$ is soft codense in $\widetilde{Y}$. That is, $\widetilde{\Phi} = \mathrm{Int}(g(g^{-1}((X,\mathcal{T}')))) = (X,\mathcal{T}')$. This cannot be done. Hence g is soft SD-continuous. □

Lemma 3.3. *Let* $(S,\mathcal{T}) \neq \widetilde{\Phi}$ *be a soft* β*-open subset of* (X,Ψ,E). *Then* $\mathrm{Int}(\mathrm{Cl}((S,\mathcal{T}))) \neq \widetilde{\Phi}$.

Proof. Straightforward. □

Theorem 3.4. *Suppose* $g : (Z,\Psi,\mathcal{T}) \to (Y,\Upsilon,\mathcal{T}')$ *is an onto function. Then* g *is soft* SD*-continuous if and only* $g^{-1}((R,\mathcal{T}'))$ *is a soft* SD*-set for every soft sw-soft open set* $(R,\mathcal{T}')$ *in* $\widetilde{Y}$.

Proof. Let g be soft SD-continuous. Let $(R,\mathcal{T}')$ be a soft *sw*-open set in $\widetilde{Y}$. If $(R,\mathcal{T}') = \widetilde{\Phi}$, then $\widetilde{\Phi} = g^{-1}((R,\mathcal{T}'))$ is clearly a soft SD-set. Let $(R,\mathcal{T}') \neq \widetilde{\Phi}$. Then there is $\widetilde{\Phi} \neq (X,\mathcal{T}') \in \Upsilon$ such that $(X,\mathcal{T}') \sqsubseteq (R,\mathcal{T}')$. Therefore $g^{-1}((X,\mathcal{T}')) \sqsubseteq g^{-1}((R,\mathcal{T}'))$. By assumption,

$$\widetilde{\Phi} \neq \mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')))) \sqsubseteq \mathrm{Int}(\mathrm{Cl}(g^{-1}((R,\mathcal{T}')))).$$

This proves that $g^{-1}((R,\mathcal{T}'))$ is a soft SD-set.

Conversely, if $(X,\mathcal{T}')$ is a soft open set in $\widetilde{Y}$, then it is soft *sw*-open. By assumption, $g^{-1}((X,\mathcal{T}'))$ is a soft SD-set. Hence g is soft SD-continuous. □

Theorem 3.5. *Assume* $g : (Z,\Psi,\mathcal{T}) \to (Y,\Upsilon,\mathcal{T}')$ *is a one to one, onto, and soft semiopen function. Then the next conditions are equivalent:*

(1) g *is soft* SD*-continuous;*

(2) *for every soft (closed) nowhere dense set* $(N,\mathcal{T})$ *in* $\widetilde{Z}$, $g((N,\mathcal{T}))$ *is a soft nowhere dense set in* $\widetilde{Y}$*;*

(3) *for every soft* SD*-set* $(R,\mathcal{T}')$ *in* $\widetilde{Y}$, $g^{-1}((R,\mathcal{T}'))$ *is a soft* SD*-set in* $\widetilde{Z}$*;*

(4) g *is soft* β*-continuous.*

Proof.

(1) $\Longrightarrow$ (2) Suppose $(N,\mathcal{T})$is soft closed nowhere dense in$\widetilde{Z}$. By soft semiopenness of g, $\mathrm{Int}(\mathrm{Cl}(g((N,\mathcal{T}))))\sqsubseteq g((N,\mathcal{T}))$ and so $\mathrm{Int}(\mathrm{Cl}(g((N,\mathcal{T})))) = \mathrm{Int}(g((N,\mathcal{T})))$. By Theorem 3.2, $\mathrm{Int}(g((N,\mathcal{T}))) = \widetilde{\Phi}$. Thus $\mathrm{Int}(\mathrm{Cl}(g((N,\mathcal{T})))) = \widetilde{\Phi}$. Hence $g((N,\mathcal{T}))$ is soft nowhere dense in $\widetilde{Y}$.

(2)$\Longleftrightarrow$(3) Suppose (3) is not true. One can find a soft SD-set $(R,\mathcal{T}')$ in $\widetilde{Y}$ such that $g^{-1}((R,\mathcal{T}'))$ is not a soft SD-set, that is to say $g^{-1}((R,\mathcal{T}'))$ is soft nowhere dense in $\widetilde{Z}$. By (2), $g(g^{-1}((R,\mathcal{T}'))) = (R,\mathcal{T}')$ is soft nowhere dense, i.e., $(R,\mathcal{T}')$ is not a soft SD-set. This is contradiction. Hence (3) must be true. The converse can be proved similarly.

(2) $\Longrightarrow$ (4) Let $(X,\mathcal{T}') \in \Upsilon$. We show that $g^{-1}((X,\mathcal{T}'))$ is β-open in $\widetilde{Z}$. Let $z_\theta \notin \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')))))$. Then we can find $(S,\mathcal{T}) \in \Psi$ containing z_θ such that $\mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')))) \sqcap (S,\mathcal{T}) = \widetilde{\Phi}$ and so

$$\widetilde{\Phi} = \mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')))) \bigsqcap \mathrm{Int}(\mathrm{Cl}((S,\mathcal{T}))) \sqsupseteq \mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')) \bigsqcap (S,\mathcal{T}))).$$

Therefore $g^{-1}((X,\mathcal{T}')) \sqcap (S,\mathcal{T})$ is soft nowhere dense in $\widetilde{Z}$. By (2),

$$g(g^{-1}((X,\mathcal{T}')) \bigsqcap (S,\mathcal{T})) = (X,\mathcal{T}') \bigsqcap g((S,\mathcal{T}))$$

is soft nowhere dense in $\widetilde{Y}$. This implies that

$$\mathrm{Int}((X,\mathcal{T}') \bigsqcap g((S,\mathcal{T}))) = (X,\mathcal{T}') \bigsqcap \mathrm{Int}(g((S,\mathcal{T}))) = \widetilde{\Phi}$$

and so $(X,\mathcal{T}') \sqcap \mathrm{Cl}(\mathrm{Int}(g((S,\mathcal{T})))) = \widetilde{\Phi}$. Since g is soft semiopen, then

$$g((S,\mathcal{T})) \sqsubseteq \mathrm{Cl}(\mathrm{Int}(g((S,\mathcal{T})))).$$

Therefore $(X,\mathcal{T}') \sqcap g((S,\mathcal{T})) = \widetilde{\Phi}$ and then $g^{-1}((X,\mathcal{T}')) \sqcap (S,\mathcal{T}) = \widetilde{\Phi}$. Thus $z_\theta \notin g^{-1}((X,\mathcal{T}'))$. This yields that $g^{-1}((X,\mathcal{T}')) \sqsubseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(g^{-1}((X,\mathcal{T}')))))$, which establishes that, g is soft β-continuous.

(4) $\Longrightarrow$ (1) Let $(W,\mathcal{T}')$ be a soft open set in $\widetilde{Y}$. If $(W,\mathcal{T}') = \widetilde{\Phi}$, clearly its inverse is a soft SD-set. Suppose that $(W,\mathcal{T}') \neq \widetilde{\Phi}$. By (4), $g^{-1}((W,\mathcal{T}')) \sqsubseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(g^{-1}((W,\mathcal{T}')))))$. Then Lemma 3.3 verifies that $\mathrm{Int}(\mathrm{Cl}(g^{-1}((W,\mathcal{T}')))) \neq \widetilde{\Phi}$. Thus g is soft SD-continuous. □

Proposition 3.6. *Suppose* $g : (Z,\Psi,\mathcal{T}) \to (Y,\Upsilon,\mathcal{T}')$ *is a one to one, onto, soft semiopen, soft SD-continuous function and* $(M,\mathcal{T})$ *is a soft meager set in* $\widetilde{Z}$. *Then* $g((M,\mathcal{T}))$ *is soft meager in* $\widetilde{Y}$.

Proof. If $(M,\mathcal{T})$ is a soft meager set in $\widetilde{Z}$, so we can have $(M,\mathcal{T}) = \bigsqcup_{i=1}^{\infty}(N_i,\mathcal{T})$ such that $(N_i,\mathcal{T})$ are soft nowhere set in $\widetilde{Z}$ for $i = 1,2,\ldots$. Therefore

$$g((M,\mathcal{T})) = f\left(\bigsqcup_{i=1}^{\infty}(N_i,\mathcal{T})\right) = \bigsqcup_{i=1}^{\infty} g((N_i,\mathcal{T})).$$

By Theorem 3.5 (2), $g((N_i,\mathcal{T}))$ is soft nowhere dense for every i. Hence $g((M,\mathcal{T}))$ is soft meager in $\widetilde{Y}$. □

Theorem 3.7. *Let* $f : (X,\Psi,E) \to (Y,\Upsilon,E')$ *be a function. If g is soft semiopen, then the next conditions are equivalent:*

(1) *g is soft SD-continuous;*

(2) *for every* $(D,\mathcal{T}) \in \Psi\widetilde{\cap}\mathcal{D}_\Psi$, $\mathrm{Int}(g((D,\mathcal{T})))$ *is soft dense in* $g(\widetilde{Z})$.

Proof.

(1) $\Longrightarrow$ (2) Suppose $(D,\mathcal{T}) \in \Psi\widetilde{\cap}\mathcal{D}_\Psi$. By Theorem 3.1 (4), $g((D,\mathcal{T}))$ is soft dense in $g(\widetilde{Z})$. As g is soft semiopen, then $g((D,\mathcal{T})) \sqsubseteq \mathrm{Cl}(\mathrm{Int}(g((D,\mathcal{T}))))$ and thus $g(\widetilde{Z}) = \mathrm{Cl}(g((D,\mathcal{T}))) = \mathrm{Cl}(\mathrm{Int}(g((D,\mathcal{T}))))$ by Lemma 3.16 in [24]. Hence $\mathrm{Int}(g((D,\mathcal{T})))$ is soft dense in $g(\widetilde{Z})$.

(2) $\Longrightarrow$ (1) Straightforward (from Theorem 3.1 (4) $\Longrightarrow$ (1)). □

4. Preservation of soft Baire property and category

We first define the terms soft sets with Baire property and soft Baire topological spaces, then investigate them in depth.

Definition 4.1. A set $(R, \mathcal{T})$ in $\widetilde{Z}$ is said to have the soft Baire property if it is can be represented as $(R, \mathcal{T}) = (S, \mathcal{T})\widetilde{\Delta}(M, \mathcal{T})$, where $(S, \mathcal{T}) \in \Psi$ and $(M, \mathcal{T})$ is a soft meager set.

Definition 4.2 ([48])**.** Let $(Z, \Psi, \mathcal{T})$ be a soft topological space. Then $(Z, \Psi, \mathcal{T})$ is called a soft Baire space if $\widetilde{\bigcap}_{n=1}^{\infty}(G_n, \mathcal{T}) \in \mathcal{D}_\Psi$ for each collection $\{(G_n, \mathcal{T}) : (G_n, \mathcal{T}) \in \Psi\widetilde{\cap}\mathcal{D}_\Psi, n \in \mathbb{N}\}$.

The above definition is equivalent to say, soft Baire is a soft topological space in which every soft open set that is not null is not soft meager.

Lemma 4.3. *Every soft semiopen subset $(R, \mathcal{T})$ of $(Z, \Psi, \mathcal{T})$ has the soft Baire property.*

Proof. If $(R, \mathcal{T})$ is a soft semiopen set, by Theorem 3.1 in [31], there is $(S, \mathcal{T}) \in \Psi$ such that $(S, \mathcal{T}) \sqsubseteq (R, \mathcal{T}) \sqsubseteq \mathrm{Cl}((S, \mathcal{T}))$. Clearly, $(R, \mathcal{T}) = (S, \mathcal{T}) \bigsqcup ((R, \mathcal{T}) \setminus (S, \mathcal{T}))$. Since $(S, \mathcal{T})$ is soft open, then $\mathrm{Cl}((R, \mathcal{T})) \setminus (S, \mathcal{T})$ is a soft nowhere dense set. But $(R, \mathcal{T}) \setminus (S, \mathcal{T}) \sqsubseteq \mathrm{Cl}((R, \mathcal{T})) \setminus (S, \mathcal{T})$, so $(R, \mathcal{T}) \setminus (S, \mathcal{T})$ is soft nowhere dense. Set $(N, \mathcal{T}) = (R, \mathcal{T}) \setminus (S, \mathcal{T})$. Therefore $(R, \mathcal{T}) = (S, \mathcal{T}) \bigsqcup (N, \mathcal{T}) = (S, \mathcal{T})\widetilde{\Delta}(N, \mathcal{T})$ as $(S, \mathcal{T}) \bigsqcap (N, \mathcal{T}) = \widetilde{\Phi}$. Hence $(R, \mathcal{T})$ has the soft Baire property. □

Theorem 4.4. *Let $g : (Z, \Psi, \mathcal{T}) \to (Y, \Upsilon, \mathcal{T}')$ be a one to one, onto, soft semiopen, and soft SD-continuous function. If $(T, \mathcal{T})$ is a soft set in $\widetilde{Z}$ and has the soft Baire property, then $g((T, \mathcal{T}))$ has the soft Baire property in $\widetilde{Y}$.*

Proof. Let $(T, \mathcal{T}) \sqsubseteq (Z, \Psi, \mathcal{T})$ be a set of soft Baire property. Then $(T, \mathcal{T}) = (S, \mathcal{T})\widetilde{\Delta}\ (N, \mathcal{T})$ for some soft open $(S, \mathcal{T})$ and soft meager $(N, \mathcal{T})$ subsets in $\widetilde{Z}$. Now, $g((T, \mathcal{T})) = g((S, \mathcal{T}))\widetilde{\Delta}g((N, \mathcal{T}))$. By Proposition 3.6, $g((N, \mathcal{T}))$ is soft meager. It is enough to show that $g((S, \mathcal{T}))$ has the soft Baire property. Since $(S, \mathcal{T})$ is a soft open set and the function g is soft semiopen, so $g((S, \mathcal{T}))$ a soft semiopen set in $\widetilde{Y}$, by Lemma 4.3, $g((S, \mathcal{T}))$ has the soft Baire property. Thus $g((R, \mathcal{T}))$ has the soft Baire property. □

Theorem 4.5. *Let $g : (Z, \Psi, \mathcal{T}) \to (Y, \Upsilon, \mathcal{T}')$ be a one to one, onto, and soft semiopen soft SD-continuous function. Then $(Z, \Psi, \mathcal{T})$ is a soft Baire space.*

Proof. Suppose $(S, \mathcal{T}) \in \Psi$, which is also soft meager. By Proposition 3.6, $g((S, \mathcal{T}))$ is a soft meager set in $\widetilde{Y}$. But g is soft semiopen, so $g((S, \mathcal{T})) \neq \widetilde{\Phi}$ is soft semiopen in $\widetilde{Y}$. Therefore, $\mathrm{Int}(g((S, \mathcal{T}))) \neq \widetilde{\Phi}$, and this is opposite to the hypothesis that $(Y, \Upsilon, \mathcal{T}')$ is soft Baire. Hence $(Z, \Psi, \mathcal{T})$ is a soft Baire space. □

From Theorems 3.5 and 4.5, one can have following.

Corollary 4.6. *Let $g : (Z, \Psi, \mathcal{T}) \to (Y, \Upsilon, \mathcal{T}')$ be a one to one, onto, soft semiopen, and soft β-continuous function. If $(Y, \Upsilon, \mathcal{T}')$ is a soft Baire space, then $(Z, \Psi, \mathcal{T})$ is soft Baire.*

We remark that the "one to one" condition in Theorem 4.5 can be weakened to "σ-fiber-complete".

Definition 4.7 ([25])**.** A soft function $f : (X, \Psi, E) \to (Y, \Upsilon, E')$is called σ-fiber complete if for every countable collection $\{(K_n, \mathcal{T})\}_{n \geqslant 1}$ of soft open sets that is centered in $\widetilde{Z}$, $\bigsqcap(K_n, \mathcal{T}) \neq \widetilde{\Phi}$, there is $z_{\theta'} \sqsubseteq \widetilde{Y}$ with $g^{-1}(z_{\theta'}) \bigsqcap (K_n, \mathcal{T}) \neq \widetilde{\Phi}$ for every $n \geqslant 1$.

Theorem 4.8. *Let $g : (Z, \Psi, \mathcal{T}) \to (Y, \Upsilon, \mathcal{T}')$ be an onto, soft semiopen, soft SD-continuous, σ-fiber-complete soft function. If $(Y, \Upsilon, \mathcal{T}')$ is a soft Baire space, then $(Z, \Psi, \mathcal{T})$ is soft Baire.*

Proof. Let $\{(G_n, \mathcal{T}) : (G_n, \mathcal{T}) \in \Psi \widetilde{\sqcap} \mathcal{D}_\Psi, n \in \mathbb{N}\}$. We have to check that $\sqcap_{n\in\mathbb{N}}(D_n, \mathcal{T}) \in \mathcal{D}_\Psi$. Let $\widetilde{\Phi} \neq (S, \mathcal{T}) \in \Psi$. Since g is soft semiopen, then $g((S, \mathcal{T}))$ is a non-null soft semiopen set in $\widetilde{Y}$. Therefore, $\mathrm{Int}(g((S, \mathcal{T}))) \neq \widetilde{\Phi}$. Set $(U, \mathcal{T}) = \mathrm{Int}(g((S, \mathcal{T})))$. By Theorem 3.7, $\mathrm{Int}(g((D_n, \mathcal{T}))) \in \mathcal{D}_\Upsilon$, for every n, and clearly $\{\mathrm{Int}(g((D_n, \mathcal{T})))\}_{n\in\mathbb{N}}$ is a countable collection. Since $(Y, \Upsilon, \mathcal{T}')$ is a soft Baire space, then $\sqcap_{n\in\mathbb{N}} \mathrm{Int}(g((D_n, \mathcal{T}))) \in \mathcal{D}_\Upsilon$. This means that

$$\bigsqcap_{n\in\mathbb{N}} \mathrm{Int}(g((D_n, \mathcal{T}))) \bigsqcap (U, \mathcal{T}) \neq \widetilde{\Phi}.$$

Let

$$z_{\theta'} \sqsubseteq \bigsqcap_{n\in\mathbb{N}} \mathrm{Int}(g((D_n, \mathcal{T}))) \bigsqcap (U, \mathcal{T}).$$

This implies that

$$\{z_{\theta'}\} \bigsqcap g((D_n, \mathcal{T})) \bigsqcap (U, \mathcal{T}) \neq \widetilde{\Phi}$$

and therefore

$$g^{-1}\{z_{\theta'}\} \bigsqcap (D_n, \mathcal{T}) \bigsqcap (S, \mathcal{T}) \neq \widetilde{\Phi} \text{ for every } n.$$

By σ-fiber-completeness of g, $\sqcap_{n\in\mathbb{N}}(D_n, \mathcal{T}) \sqcap (S, \mathcal{T}) \neq \widetilde{\Phi}$, which means that $\sqcap_{n\in\mathbb{N}}(D_n, \mathcal{T})$ is soft dense in $\widetilde{Z}$. This proves that $(Z, \Psi, \mathcal{T})$ is a soft Baire space. □

The next example shows that the condition "soft semiopenness of a function g" in Theorems 4.5 and 4.8 cannot be removed:

Example 4.9. Let $X = Y = \mathbb{R}$ and let $\mathcal{T} = \{\theta\}$. Suppose Ψ is a soft topology generated by $\{(\theta, B(\theta)) : B(\theta) = (r, \infty); r \in \mathbb{R}\}$ (called the soft right order topology) and $\Upsilon = \{(\theta, S(\theta)) : S(\theta) = \emptyset \text{ or } \mathbb{R}\backslash S(\theta) \text{ is finite}\}$ is the finite complement soft topology on Y. If $g : (Z, \Psi, \mathcal{T}) \to (Y, \Upsilon, E)$ is the identity soft function, we assert that g is soft SD-continuous but not soft semiopen. On the other hand, (Y, Υ, E) is a soft Baire space but $(Z, \Psi, \mathcal{T})$ is not. Clearly g is one to one and onto (consequently g satisfies countable fiber-completeness). We start to prove that g is soft SD-continuous. Given any soft open set $(G, \mathcal{T})$ in $\widetilde{Y}$, then it has the form $(G, \mathcal{T}) = \{(\theta, (-\infty, x_1))\} \bigsqcup \{(\theta, (x_1, x_2))\} \bigsqcup \cdots \bigsqcup \{(\theta, (x_n, \infty))\}$ for some $x_1, x_2, \ldots, x_n \in \mathbb{R}$. It follows that $g^{-1}((G, \mathcal{T}))$ always contains some basic soft open set in $\widetilde{Z}$, and each soft open set in $\widetilde{Z}$ is soft dense, so $\mathrm{Int}(\mathrm{Cl}(g^{-1}((G, \mathcal{T})))) \neq \widetilde{\Phi}$. Thus g is soft SD-continuous. Now, take the soft open set $(W, \mathcal{T}) = \{(\theta, (0, \infty)) : \theta \in \mathcal{T}\}$ in $\widetilde{Z}$, then $(W, \mathcal{T}) = g((W, \mathcal{T})) \not\sqsubseteq \mathrm{Cl}(\mathrm{Int}(g((W, \mathcal{T})))) = \widetilde{\Phi}$. Therefore g is not soft semiopen. Obviously, all finite soft sets in $\widetilde{Y}$ are soft nowhere dense. Then (Y, Υ, E) a soft Baire space since it cannot be represented as a countable union of finite sets. While $\widetilde{Z} = \bigsqcup_{m\in\mathbb{N}}(N_m, \mathcal{T})$, where $(N_m, \mathcal{T}) = \{(\theta, (-\infty, m)) : \theta \in \mathcal{T}\}$ and, for every m, $\mathrm{Int}(\mathrm{Cl}((N_m, \mathcal{T}))) = \widetilde{\Phi}$. Thus $(Z, \Psi, \mathcal{T})$ cannot be soft Baire.

Here, we shall notify the reader that following result is given in [25].

Theorem 4.10. *If* $g : (Z, \Psi, \mathcal{T}) \to (Y, \Upsilon, \mathcal{T}')$ *is a soft sw-homeomorphism and* $(Y, \Upsilon, \mathcal{T}')$ *is a soft Baire space, then* $(Z, \Psi, \mathcal{T})$ *is a soft Baire space.*

The following example justifies that Theorems 4.5 and 4.8 are respectively natural generalizations of Theorems 4.5 and 4.7 proved in [25].

Example 4.11. Let $\mathbb{Q}$ be the set of rational numbers, $X = Y = \mathbb{R}$, and $\mathcal{T} = \{\theta\}$. If Ψ is the soft topology on Z with the soft base $\{(\theta, B(\theta)) : B(\theta) = (r, s); r, s \in \mathbb{R}; r < s\}$ and $\Upsilon = \{(W, \mathcal{T}) : (W, \mathcal{T}) = (G, \mathcal{T}) \bigsqcup ((V, \mathcal{T}) \backslash (\mathbb{Q}, \mathcal{T})) : \text{ where } (G, \mathcal{T}), (V, \mathcal{T}) \sqsubseteq \Psi\}$ is a soft topology on Y, then the identity function $g : (Z, \Psi, \mathcal{T}) \to (Y, \Upsilon, E)$ is soft semiopen and soft SD-continuous. On the other hand, g cannot be a soft *sw*-homeomorphism. Clearly g is soft semiopen. To check g for soft SD-continuity, let $(H, \mathcal{T})$ be a soft open set in $\widetilde{Y}$. The preimage of $(H, \mathfrak{T})$ includes either a soft set $\{(\theta, (r, s))\}$, $\{(\theta, (r, s) \cap \mathbb{Q}^c)\}$ or their unions, and in all cases, $\mathrm{Int}(\mathrm{Cl}((H, \mathfrak{T}))) \neq \widetilde{\Phi}$. Now, we shall show that g cannot be a soft *sw*-homeomorphism. Let $(\mathbb{Q}, \mathfrak{T})^c$ be the soft open set in $\widetilde{Y}$. Evidently, $\mathrm{Int}(g^{-1}((\mathbb{Q}, \mathfrak{T})^c)) = \widetilde{\Phi}$. Therefore g is not soft *sw*-continuous and hence it is not a soft *sw*-homeomorphism. On the other hand, by soft Baire category theorem in [48], both $(Z, \Psi, \mathfrak{T})$ and (Y, Υ, E) are soft Baire spaces.

5. Conclusion

This paper contributes to the area of soft topology introduced in [50]. We have given some characterizations of soft $\mathcal{SD}$-continuous functions in terms of some other types of generalized soft continuous and open functions. Then, we have defined soft sets that have Baire property and studied the main properties. We determine some conditions under which the image and preimage of soft sets with Baire property and soft Baire spaces are preserved, respectively. To show our findings we extended some results and relationships from the literature and demonstrated some examples.

As future work, we plan to study further properties of soft somewhere dense sets and soft sets with Baire property.

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