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Spherical fuzzy and soft topology: some applications

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Abstract

A generalized soft set model that is more accurate, useful, and realistic is the spherical fuzzy soft set. So, the fuzzy soft topological models in use can be extended to create spherical fuzzy soft topological spaces, which are valuable for expressing unreliable data in real-world applications. Subbase, separation axioms, compactness, and connectedness are all defined in this work. To examine these notions' features, we also investigate their forefathers. The application of a decision-making algorithm is then demonstrated, and a numerical example is used to describe how it can be used.

Keywords: Sfs-set, Sfs-topology, Sfs-subspace, Sfs-separation axioms, Sfs-connectedness and Sfs-compactness.

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1. Introduction

Classical mathematical approaches are insufficient to tackle everyday issues, and they are also insufficient to fulfill new requirements. Theories like fuzzy set (f_s) theory [26] and soft set (s_s) theory [4, 20] have been developed to solve these problems. (fs) theory is one of the most effective methods for dealing with multi-attribute decision-making problems. To handle the issues of finding reliable data that is both sufficient and correct due to the social economics ambiguity and imprecision. (f_s) [14, 26], intuitionistic fuzzy sets (If_s) [8–10], Pythagorean (PYf_s) and picture fuzzy sets (Pf_s) [11, 21], and other mathematical methods are necessary because many disciplines (as physics, social sciences, and medicine) deal with unclear data. Molodtsov [20] advanced a ground-breaking strategy known as "(s_s) theory", which has a wide range of applications. As a result, various academics have created (s_s) theory approaches and operations. In [18], for example, provided several soft set concepts and operations, followed by research on inverse fuzzy soft sets [15]. Initially, fuzzy sets (f_s) proposed by Zadeh [26] were employed based on fuzziness situations. In (f_s), each domain set element ξ has only one index, the degree of membership $m(\zeta)$, that varies from 0 to 1. For the (f_s), a non-membership degree is straightforwardly identical to $1-m(\xi)$. As [9], and If_s were developed to account for uncertainties about the degree of membership. [9] presented a generalization of Zadeh's (f_s), defining the If_s with the constraint that $0 \leqslant m(\xi)+n(\xi) \leqslant 1$

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and two indexes (membership degree $m(\xi)$, and non-membership degree $n(\xi)$) are used. Many scholars have studied the If_s throughout the previous few decades, and it has been successfully used in many other sectors, including medical diagnosis and decision making [3, 5]. Numerous scholars, such as [2, 9, 10, 12, 19], have investigated the If_s and interval valued intuitionistic fuzzy sets (IVIfs), which have been widely used in many applications, including group decision making. Due to the fact that in some applications of real-life decision theory, decision makers encounter circumstances involving particular features in which the total of their membership degrees exceeds 1. In this case, If_s fails to generate any outcomes that are satisfactory. As a generalization of the If_s to address this issue, the PYf_s was created by Olgun et al. and Yager et al. [24, 25]. It guarantees that the squared sum of its membership degrees is either less than or equal to one. The If_s and PYf_s fail to deliver an appropriate result when the neutral membership degree calculates independently in real-world applications. Cuong and Kerinovich [11] advanced the idea of the picture fuzzy set Pf_s in response to these circumstances. In Pf_s, we used three indexes (membership degree $m(\zeta)$, neutral-membership degree $i(\xi)$ and a non-membership degree $n(\xi)$ with the constraint $0 \leqslant m(\xi) + i(\xi) + n(\xi) \leqslant 1$. Pf_s obviously outperforms If_s and PYf_s when it comes to dealing with fuzziness and ambiguity. In real life, we run into a number of problems that Pfs cannot resolve, such as when $m(\xi) + i(\xi) + n(\xi) > 1$. Pf_s is unable to produce satisfactory outcomes in such cases. This satisfied the criterion that their total exceeds 1 and that Pf_s is not present. The idea of fuzzy soft sets (fs_s) that are spherical in light of these circumstances, (Sfs_s) is presented as a broad generalization of picture fuzzy soft set Pfs_s [1, 6, 7, 13, 16, 17, 22, 23, 27]. Degrees of membership satisfy the criteria in Sfs_s, $0 < m^2(\xi) + i^2(\xi) + n^2(\xi) < 1$. With some basic theorems, we'll look at the subspace of Sfs-topological space, Sfs-separation axioms, Sfs-compactness, and Sfs-connectedness. The importance of expanded topological spaces is in solving some problems that are difficult to solve in the non-expanded topological space. We discuss some topological concepts in the spaces of spherical fuzzy soft topologies. Also, we study their main characterizations with the help of some application examples.

2. Preliminaries

The fundamental descriptions of (f_s), $(s_s)t$, Sf set, and Sf soft set are presented in this section.

Definition 2.1 ([26])**.** A fuzzy set B on the global set Ω is an object of the form $B = \{(\xi, \Phi_B(\xi) \mid \xi \in \Omega\}$, where $\Phi_B : \Omega \to [0,1]$ is the membership function of B, the grade of ξ in B is represented by $\Phi_B(\xi)$.

Definition 2.2 ([20])**.** If Ω is a domain set and $\eta \sqsubseteq \mathfrak{R}$ is a collection of parameters and $\mathfrak{R}$ the set of attributes, a pair $(\Phi,\eta) = \{(i, \Phi(i)) : i \in \eta\}$ is named a soft set on Ω, where $\Phi : \eta \to 2^{\Omega}$ is a set-valued mapping. $S_\eta(\Omega)$ identifies the set of all soft sets on Ω parameterized by η. A soft set (Ω,η)-(Φ,η) is the complement of (Φ,η), where $\Phi^c : \eta \to P(\Omega)$ is given by $\Phi^c(i) = \Omega$-$\Phi(i)$, $\forall i \in \eta$.

Definition 2.3 ([8])**.** Let Ω be the universe setting, then the set $B = \{(\xi, P_B(\xi), \mathcal{N}_B(\xi)) : \xi \in \Omega\}$ is called intuitionistic fuzzy set of Ω, $P_B : \Omega \to [0,1]$ and $\mathcal{N}_B : \Omega \to [0,1]$ are called degree of positive-membership of ξ in Ω and negative-membership degree of ξ in Ω successively with the condition $0 \leqslant P_B(\xi) + \mathcal{N}_B(\xi) \leqslant 1$, $\forall \xi \in \Omega$.

Definition 2.4 ([11])**.** Assume Ω is the universe setting, then the set $B = \{\xi, P_B(\xi), \mathcal{I}_B(\xi), \mathcal{N}_B(\xi) : \xi \in \Omega\}$ is called picture fuzzy set of Ω, $P_B : \Omega \to [0,1]$, $\mathcal{I}_B : \Omega \to [0,1]$ and $\mathcal{N}_B : \Omega \to [0,1]$ are named the degree of positive-membership of ξ in Ω, neutral-membership of ξ in Ω and negative-membership degree of ξ in Ω successively with the condition $0 \leqslant P_B(\xi) + \mathcal{I}_B(\xi) + \mathcal{N}_B(\xi) \leqslant 1$, $\forall \xi \in \Omega$.

Definition 2.5 ([7])**.** Assume Ω is the universe setting, the set $B = \{\xi, P_B(\xi), \mathcal{I}_B(\xi), \mathcal{N}_B(\xi) : \xi \in \Omega\}$ is called spherical fuzzy set(Sf_s) of Ω, where $P_B : \Omega \to [0,1]$, $\mathcal{I}_B : \Omega \to [0,1]$ and $\mathcal{N}_B : \Omega \to [0,1]$ are named the degree of positive-membership of ξ in Ω, neutral-membership of ξ in Ω and negative-membership degree of ξ in Ω successively with the condition $0 \leqslant (P_B(\xi))^2 + (\mathcal{I}_B(\xi))^2 + (\mathcal{N}_B(\xi))^2 \leqslant 1$, $\forall \xi \in \Omega$. For (Sf_s) $\{\xi, P_B(\xi), \mathcal{I}_B(\xi), \mathcal{N}_B(\xi) : \xi \in \Omega\}$, which is triple components $(P_B(\xi), \mathcal{I}_B(\xi), \mathcal{N}_B(\xi))$ are named (Sf_n)

and every (Sf_n) can be denoted by $i = (P_i, \mathcal{J}_i, \mathcal{N}_i)$, where $P_i, \mathcal{J}_i$ and $\mathcal{N}_i \in [0,1]$ with condition $0 \leqslant (P_i)^2 + (\mathcal{J}_i)^2 + (\mathcal{N}_i)^2 \leqslant 1$.

Definition 2.6 ([7, 10])**.** Let $B = \{\xi, P_B(\xi), \mathcal{J}_B(\xi), \mathcal{N}_B(\xi) : \xi \in \Omega\}$ and $\rho = \{\xi, P_\rho(\xi), \mathcal{J}_\rho(\xi), \mathcal{N}_\rho(\xi) : \xi \in \Omega\}$ be two (Sf_s) over Ω. Then

(1) $B \sqsubseteq \rho$ if $P_B(\xi) \leqslant P_\rho(\xi)$, $\mathcal{J}_B(\xi) \leqslant \mathcal{J}_\rho(\xi)$, and $\mathcal{N}_B(\xi) \geqslant \mathcal{N}_\rho(\xi)$;
(2) $B = \rho$ if and only if $B \sqsubseteq \rho$ and $B \sqsupseteq \rho$;
(3) $B \sqcup \rho = \{\xi, P_B(\xi) \vee P_\rho(\xi), \mathcal{J}_B(\xi) \vee \mathcal{J}_\rho(\xi), \mathcal{N}_B(\xi) \wedge \mathcal{N}_\rho(\xi) : \xi \in \Omega\}$;
(4) $B \sqcap \rho = \{\xi, P_B(\xi) \wedge P_\rho(\xi), \mathcal{J}_B(\xi) \wedge \mathcal{J}_\rho(\xi), \mathcal{N}_B(\xi) \vee \mathcal{N}_\rho(\xi) : \xi \in \Omega\}$,

where $\vee$ represent the maximum and $\wedge$ represent the minimum operations.

Example 2.7. Let $\Omega = \{\xi_1, \xi_2\}$ be the universal set. Suppose that B and ρ are two Sf_s over Ω given by $B = \{(\xi_1, 0.2, 0.3, 0.4), (\xi_2, 0.5, 0.3, 0.3)\}$, $\rho = \{(\xi_1, 0.3, 0.4, 0.2), (\xi_2, 0.6, 0.4, 0.2)\}$, then it is easy that $B \sqsubseteq \rho$ and $B \sqcup \rho = \{(\xi_1, 0.3, 0.4, 0.2), (\xi_2, 0.6, 0.4, 0.2)\}$.

Definition 2.8 ([17])**.** Let $i_j = \{P_{ij}, \mathcal{J}_{ij}, \mathcal{N}_{ij}\}$ and $i_k = \{P_{ik}, \mathcal{J}_{ik}, \mathcal{N}_{ik}\}$ be any two Sf_n. The intersection, union and complement are describes as:

(i) $i_k \sqsubseteq i_j$ if and only if $\forall\ \xi \in \Omega, P_{ik} \leqslant P_{ij}, \mathcal{J}_{ik} \leqslant \mathcal{J}_{ij}$ and $\mathcal{N}_{ik} \geqslant \mathcal{N}_{ij}$;
(ii) $i_k = i_j$ if and only if $i_k \sqsubseteq i_j$ and $i_j \sqsubseteq i_k$;
(iii) $i_k \sqcup i_j = (\max(P_{ik}, P_{ij}), \min(\mathcal{J}_{ik}, \mathcal{J}_{ij}), \min(\mathcal{N}_{ik}, \mathcal{N}_{ij}))$;
(iv) $i_k \sqcap i_j = (\min(P_{ik}, P_{ij}), \min(\mathcal{J}_{ik}, \mathcal{J}_{ij}), \max(\mathcal{N}_{ik}, \mathcal{N}_{ij}))$;
(v) $i_k^c = \{\mathcal{N}_{ik}, \mathcal{J}_{ik}, P_{ik}\}$.

Definition 2.9 ([7])**.** Let $i_j = \{P_{ij}, \mathcal{J}_{ij}, \mathcal{N}_{ij}\}$ and $i_k = \{P_{ik}, \mathcal{J}_{ik}, \mathcal{N}_{ik}\}$ be any two Sf_n and $m \geqslant 0$. Then we can note

(i) $mi_k = (\sqrt{1-(1-P_{ik}^2)^m}, (\mathcal{J}_{ik})^2, (\mathcal{N}_{ik})^2)$;
(ii) $i_k + i_j = (\sqrt{P_{ik}^2 + P_{ij}^2 - P_{ik}^2.P_{ij}^2}, \mathcal{J}_{ik}.\mathcal{J}_{ij}, \mathcal{N}_{ik}.\mathcal{N}_{ik})$;
(iii) $i_k \times i_j = (P_{ik}.P_{ij}, \mathcal{J}_{ik}.\mathcal{J}_{ij}, \sqrt{\mathcal{N}_{ik}^2 + \mathcal{N}_{ij}^2 - \mathcal{N}_{ik}^2.\mathcal{N}_{ij}^2})$;
(iv) $i_k^m = ((P_{ik})^m, (\mathcal{J}_{ik})^m, \sqrt{1-(1-\mathcal{N}_{ik}^2)^m})$.

Definition 2.10 ([23])**.** Suppose $Sf_s(\Omega)$ that the collection of all spherical fuzzy sets over Ω. A Sf_s is a pair (Φ, η), where Φ is an equation that maps η to $Sf_s(\Omega)$. For all $i \in \eta$, $\rho(i)$ is a Sf_s that $\rho(i) = \{(\xi, \{\xi, P_{B(i)}(\xi), \mathcal{J}_{B(i)}(\xi), \mathcal{N}_{B(i)}(\xi) : \xi \in \Omega\}$.

3. Particular topological structures on Sf soft topology

The purpose of this section is to motivate future research into topological structure using Sf soft topology.

Definition 3.1 ([13])**.** Suppose $Sfs_s(\Omega, \mathfrak{R})$ is the family of all spherical fuzzy soft sets over Ω and the parameter set $\mathfrak{R}$. Let $\eta, \zeta \tilde{\sqsubseteq} \mathfrak{R}$. Then a sub-collection Σ of $Sfs_s(\Omega, \mathfrak{R})$ is named spherical fuzzy soft topology (Sfs-topology) on Ω, if

(1) $\phi_{\mathfrak{R}}, \Omega_{\mathfrak{R}} \in \Sigma$;
(2) if $(\rho_1, \eta), (\rho_2, \zeta) \in \Sigma$, then $(\rho_1, \eta) \tilde{\sqcap} (\rho_2, \zeta) \in \Sigma$;
(3) if $(\rho_i, \eta_i) \in \Sigma, \forall i \in I$, then $\tilde{\sqcup}(\rho_i, \eta_i) \in \Sigma$.

Each of element of Σ is spherical fuzzy soft open set and its complement is a closed, spherical, fuzzy set. The binary $(\Omega_{\mathfrak{R}}, \Sigma)$ is named a spherical fuzzy soft topological space.

Example 3.2. Suppose that $\Omega = \{\xi_1, \xi_2, \xi_3\}$ is the universal set and that $\mathfrak{R} = \{i_1, i_2, i_3, i_4\}$ is its attribute set. Let $\eta, \zeta \sqsubseteq \mathfrak{R}$, where $\eta = \{i_1, i_2\}$ and $\zeta = \{i_1, i_2, i_3\}$. Consider the following Sfs_s

$$(\boldsymbol{æ_1}, \boldsymbol{J}) = \begin{array}{c} \\ \xi_1 \\ \xi_2 \\ \xi_3 \end{array} \begin{array}{c} \begin{array}{cc} i_1 & i_2 \end{array} \\ \begin{pmatrix} (0.5, 0.2, 0.4) & (0.7, 0.2, 0.3) \\ (0.6, 0.3, 0.5) & (0.4, 0.2, 0.6) \\ (0.9, 0.2, 0.5) & (0.9, 0.1, 0.1) \end{pmatrix} \end{array}, \quad (\boldsymbol{æ_2}, \boldsymbol{\imath}) = \begin{array}{c} \\ \xi_1 \\ \xi_2 \\ \xi_3 \end{array} \begin{array}{c} \begin{array}{ccc} i_1 & i_2 & i_3 \end{array} \\ \begin{pmatrix} (0.6, 0.3, 0.2) & (0.8, 0.3, 0.1) & (0.1, 0.2, 0.9) \\ (0.8, 0.3, 0.4) & (0.6, 0.2, 0.5) & (0.3, 0.1, 0.7) \\ (1.0, 0.0, 0.0) & (0.9, 0.2, 0.1) & (0.5, 0.2, 0.3) \end{pmatrix} \end{array}.$$

Then the following $\Sigma = \{\phi_{\mathfrak{R}}, \Omega_{\mathfrak{R}}, (\rho_1, \eta), (\rho_2, \zeta)\}$ is a Sfs-topology on Ω.

Definition 3.3. Let (Ω_η, Σ_1) and (Ω_η, Σ_2) be spherical fuzzy soft topological spaces.

(1) If $\Sigma_2 \sqsupseteq \Sigma_1$, we say Σ_2 is Sf soft finer than Σ_1.
(2) If $\Sigma_2 \sqsupset \Sigma_1$, we say Σ_2 is Sf soft strictly finer than Σ_1.
(3) If $\Sigma_2 \sqsupseteq \Sigma_1$ or $\Sigma_1 \sqsupseteq \Sigma_2$, then Σ_1 is comparable with Σ_2.

Definition 3.4. Let Ω represent the universal set and $\mathfrak{R}$ represent the set of parameters.

(1) Let Σ be the family of all Sfs_s, which is defined on Ω. Then Σ is named the Sf discrete topology on Ω and (Ω_η, Σ) is named a Sfs discrete space over Ω.
(2) $\Sigma = \{\phi, \tilde{\Omega}\}$ is called Sfs indiscrete topology on Ω and (Ω_η, Σ) is named a Sfs indiscrete space over Ω.

Definition 3.5. Let (Ω_η, Σ) be a Sfs topological space on Ω and let Z be a subset of Ω that is not empty. Then $\Sigma_Z = \{(Z_\Phi, \eta) : (\Phi, \eta) \in \Sigma\}$ is named the Sfs topology on Z and (Z_η, Σ_Z) is named a Sfs subspace of (Ω_η, Σ), where $(Z_\Phi, \eta) = \tilde{Z_\eta} \tilde{\sqcap} (\Phi, \eta)$. It is simple for us to confirm that Σ_Z is a Sfs topology on Z.

Example 3.6. Any discrete topological space that has Sfs as a subspace is a discrete topological space with Sfs.
Also, any Sfs subspace of a Sfs indiscrete topological space is a Sfs indiscrete topological space.

3.1. *Axioms of separation*

A Sfs point is defined as (Φ, η).

Definition 3.7. A Sfs point is defined as (Φ, η), denoted by i_Φ, if for the element $i \in \eta$, $\Phi(i) \neq \tilde{0}$ and $\Phi(\acute{i}) = \tilde{0}$, $\forall \acute{i} \in \eta$-$\{i\}$.

Definition 3.8. The complement of a Sfs point i_Φ is a Sfs point i^c_Φ such that $i^c_\Phi = (i_\Phi)^c$.

Example 3.9. Suppose $\Omega = \{\xi_1, \xi_2, \xi_3\}$ and $\eta = \{i_1, i_2\}$. Then $i_{2\Phi} = \{(\xi_1, 0.6, 0.4, 0.1), (\xi_2, 0.5, 0.3, 0.2), (\xi_3, 0.4, 0.2, 0.1)\}$ is a Sfs point whose complement is $i^c_{2\Phi} = \{(\xi_1, 0.4, 0.6, 0.8), (\xi_2, 0.5, 0.7, 0.8), (\xi_3, 0.6, 0.8, 0.9)\}$.

Definition 3.10. A Sfs point i_Φ is named in a Sfs set (Φ, η), denoted by $i_\Phi \in (\Phi, \eta)$ if for the element $i \in \eta$, $\Phi(i) \leqslant \eta(i)$.

Definition 3.11. The following are the definitions of the separation axioms.

Σ_0**:** Let (Ω_η, Σ) be a Sfs topological space over Ω and $i_1, i_2 \in \tilde{\Omega_\eta}$ such that $i_1 \neq i_2$. If there are any open Sfs sets (Φ, η) and (Ψ, η) that $i_1 \in (\Phi, \eta)$ and $i_2 \notin (\Phi, \eta)$ or $i_2 \in (\Psi, \eta)$ and $i_1 \notin (\Psi, \eta)$, then (Ω_η, Σ) is named a Sfs Σ_0-space.

Σ_1**:** Let (Ω_η, Σ) be a Sfs topological space over Ω and $i_1, i_2 \in \tilde{\Omega_\eta}$ such that $i_1 \neq i_2$. If there are any open Sfs sets (Φ, η) and (Ψ, η) that $i_1 \in (\Phi, \eta)$, $i_2 \notin (\Phi, \eta)$ and $i_2 \in (\Psi, \eta)$, $i_1 \notin (\Psi, \eta)$, then (Ω_η, Σ) is named a Sfs Σ_1-space.

Σ_2**:** Let (Ω_η, Σ) be a Sfs topological space over Ω and $i_1, i_2 \in \tilde{\Omega_\eta}$ such that $i_1 \neq i_2$. If there are any open Sfs sets (Φ, η) and (Ψ, η) that $i_1 \in (\Phi, \eta)$ and $i_2 \in (\Psi, \eta)$ and $(\Phi, \eta) \tilde{\sqcap} (\Psi, \eta) = \phi_\eta$, then (Ω_η, Σ) is named a Sfs Σ_2-space.

Regular: Let (Ω_η, Σ) be a Sfs topological space over Ω, (Ψ, η) be a Sfs closed set in Ω and $i_1 \in \tilde{\Omega_\eta}$ such that $i_1 \notin (\Psi, \eta)$. If there exist Sfs_s open (Φ_1, η) and (Φ_2, η) such that $i_1 \in (\Phi_1, \eta)$, $(\Psi, \eta)\tilde{\sqsubseteq}(\Phi_2, \eta)$ and $(\Phi_1, \eta)\tilde{\sqcap}(\Phi_2, \eta) = \phi_\eta$, then $(\Omega_{\mathfrak{R}}, \Sigma)$ is named a Sfs regular space.

Σ_3**:** Let (Ω_η, Σ) be a Sfs topological space on Ω. Then (Ω_η, Σ) is named a Sfs Σ_3-space if it is Sfs regular and a Sfs Σ_1-space.

Normal: Let (Ω_η, Σ) be a Sfs topological space over Ω, (Ψ_1, η), (Ψ_2, η) a Sfs closed sets on Ω that $(\Psi_1, \eta)\tilde{\sqcap}(\Psi_2, \eta) = \phi_\eta$. If there exist Sfs open sets (Φ_1, η) and (Φ_2, η) such that $(\Psi_1, \eta)\tilde{\sqsubseteq}(\Phi_1, \eta)$, $(\Psi_2, \eta)\tilde{\sqsubseteq}(\Phi_2, \eta)$ and $(\Phi_1, \eta)\tilde{\sqcap}(\Phi_2, \eta) = \phi_\eta$, hence (Ω_η, Σ) is named a Sfs normal space.

Σ_4**:** Let (Ω_η, Σ) be a Sfs topological space on Ω. Then (Ω_η, Σ) is called a Sfs Σ_4-space if it is a Sfs normal and a Sfs Σ_1-space.

Proposition 3.12. *Let (Ω_η, Σ) be a Sfs topological space and Ω and Z be a non empty subset of Ω.*

(1) *If (Ω_η, Σ) is Sfs Σ_0-space, (Z_η, Σ_Z) is a Sfs Σ_0-space.*
(2) *If (Ω_η, Σ) is Sfs Σ_1-space, (Z_η, Σ_Z) is a Sfs Σ_1-space.*
(3) *If (Ω_η, Σ) is Sfs Σ_2-space, (Z_η, Σ_Z) is a Sfs Σ_2-space.*
(4) *If (Ω_η, Σ) is Sfs regular space, (Z_η, Σ_Z) is a Sfs regular space.*

Proof. Here, we present evidence for (1). Similar evidence can be shown for the others.

Let $i_1, i_2 \in \tilde{Z_\eta}$ and $i_1 \neq i_2$. If (Ω_η, Σ) is Sfs Σ_0-space, then $\exists$ Sfs open sets (Φ_1, η) and (Φ_2, η) that $i_1 \in (\Phi_1, \eta)$, $i_2 \notin (\Phi_1, \eta)$ or $i_2 \in (\Phi_2, \eta)$, $i_1 \notin (\Phi_2, \eta)$. Then $i_1 \in \tilde{Z_\eta}\tilde{\sqcap}(\Phi_1, \eta) = (Z_{\Phi_1}, \eta)$, $i_2 \notin (Z_{\Phi_1}, \eta)$ or $i_2 \in (Z_{\Phi_2}, \eta)$ and $i_1 \notin (Z_{\Phi_2}, \eta)$. Then (Z_η, Σ_Z) is a Sfs Σ_0-space. □

Proposition 3.13.

(1) *Every Sfs Σ_1-space is a Sfs Σ_0-space.*
(2) *Every Sfs Σ_2-space is a Sfs Σ_1-space.*
(3) *Every Sfs Σ_3-space is a Sfs Σ_2-space.*
(4) *Every Sfs Σ_4-space is a Sfs Σ_3-space.*

Proof. Here, we present evidence for (1). Similar evidence can be shown for the others. Suppose that (Ω_η, Σ) is Sfs topological space over Ω and $e_1, e_2 \in \tilde{\Omega_\eta}$ and $i_1 \neq i_2$. At (Ω_η, Σ) is Sfs Σ_1-space, notice that exist Sfs open sets (Φ_1, η) and (Φ_2, η) that $i_1 \in (\Phi_1, \eta)$, $i_2 \notin (\Phi_1, \eta)$, and $i_2 \in (\Phi_2, \eta)$, $i_1 \notin (\Phi_2, \eta)$. Obviously, we have $i_1 \in (\Phi_1, \eta)$, $i_2 \notin (\Phi_1, \eta)$ or $i_2 \in (\Phi_2, \eta)$, $i_1 \notin (\Phi_2, \eta)$. Thus (Ω_η, Σ) is Sfs Σ_0-space. □

4. Compactness

Through this section, we're debating and expanding on the definition of spherical fuzzy soft compactness ($Sfsc$).

Definition 4.1. Suppose $(\Omega_{\eta_1}, \Sigma_1)$ and $(\Omega_{\eta_2}, \Sigma_2)$ two Sfs topological spaces.

(1) A Sfs function $f : (\Omega_{\eta_1}, \Sigma_1) \to (\Omega_{\eta_2}, \Sigma_2)$ is named Sfs continuous if for all $(F, D) \in \Sigma_2, f^{-1}(F, D) \in \Sigma_1$.
(2) A Sfs function $f : (\Omega_{\eta_1}, \Sigma_1) \to (\Omega_{\eta_2}, \Sigma_2)$ is named Sfs open if for each $(\Phi, \eta) \in \Sigma_1$, $f(\Phi, \eta) \in \Sigma_2$.

Definition 4.2. A collection Σ of Sfs_s is a cover of a Sfs_s (Φ, η) if $(\Phi, \eta)\tilde{\sqsubseteq}\tilde{\sqcup}\{(\Phi_n, \eta) : (\Phi_n, \eta) \in \Sigma, n \in I\}$. It is a Sfs open cover if every member of Σ is a Sfs_s open. A subfamily of Σ that is also a cover is referred to as a subcover of Σ.

Definition 4.3. Assume (Ω_η, Σ) that a Sfs topological space and (Φ, η) exist. If each Sfs open cover of (Φ, η) has a finite subcover, then $Sfs(\Phi, \eta)$ is named compact. Also, Sfs topological space (Ω_η, Σ) is compact if each Sfs open cover of $\tilde{\Omega_\eta}$ has a finite subcover.

Example 4.4. A Sfs topological space (Ω_η, Σ) is compact if Ω is finite.

Example 4.5. Suppose $(\Omega_{\eta_1}, \Sigma_1)$ and $(\Omega_{\eta_2}, \Sigma_2)$ are two Sfs topological spaces and $\Sigma_1 \sqsubseteq \Sigma_2$. Hence, Sfs topological space $(\Omega_{\eta_1}, \Sigma_1)$ is compact if $(\Omega_{\eta_2}, \Sigma_2)$ is compact.

Proposition 4.6. *Suppose (F, D) that a Sfs closed set in Sfs compact space (Ω_η, Σ). Then (F, D) is compact also.*

Proof. Let (Φ_n, η) be any open covering of (F, D). Then $\tilde{\Omega_\eta} \tilde{\sqsubseteq} (\sqcup_{\tilde{n} \in I} (\Phi_n, \eta)) \sqcup (F, D)^c$; that is, (Φ_n, η) together with Sfs_s open $(F, D)^c$ is open covering of $\tilde{\Omega_\eta}$. Hence, there exists a finite subcovering $(\Phi_1, \eta), (\Phi_2, \eta), \ldots, (\Phi_n, \eta), (F, D)^c$. Hence, $\tilde{\Omega_\eta} \tilde{\sqsubseteq} (\Phi_1, \eta) \tilde{\sqcup} (\Phi_2, \eta) \tilde{\sqcup} \cdots \tilde{\sqcup} (\Phi_n, \eta) \tilde{\sqcup} (F, D)^c$. Therefore $(F, D) \tilde{\sqsubseteq} (\Phi_1, \eta) \tilde{\sqcup} (\Phi_2, \eta) \tilde{\sqcup} \cdots \tilde{\sqcup} (\Phi_n, \eta) \tilde{\sqcup} (F, D)^c$ that obviously implies $(F, D) \tilde{\sqsubseteq} (\Phi_1, \eta) \tilde{\sqcup} (\Phi_2, \eta) \tilde{\sqcup} \cdots \tilde{\sqcup} (\Phi_n, \eta)$, since $(F, D) \tilde{\sqcap} (F, D)^c = \phi_\eta$. As a result, it has a finite subcovering and is compact. □

Proposition 4.7. *Let (F, D) be a Sfs compact set in Sfs Σ_2-space (Ω_η, Σ). Then (F, D) is closed.*

Proof. Let $i_1 \in (F, D)^c$. For every $i_2 \in (F, D)$, we have $i_1 \neq i_2$, so there exist disjoint Sfs open sets (Φ_{i_2}, η) and (H_{i_2}, η), so that $i_1 \in (\Phi_{i_2}, \eta)$ and $i_2 \in (H_{i_2}, \eta)$, hence, $\{(H_{i_2}, \eta) : i_2 \in (F, D)\}$ is an Sfs open cover of (F, D). Suppose $\{(H_{i_{2_1}}, \eta), (H_{i_{2_2}}, \eta), \ldots, (H_{i_{2_n}}, \eta)\}$ that is a finite subcover. Then $\tilde{\sqcap}_{i=1}^{n} (\Phi_{i_{2_i}}, \eta)$ is an open set containing i_1 and contained in $(F, D)^c$. Thus $(F, D)^c$ is Sfs open and (F, D) is closed. □

Theorem 4.8. *Suppose $(\Omega_{\eta_1}, \Sigma_1)$ and $(\Omega_{\eta_2}, \Sigma_2)$ that tow Sfs topological spaces and $f : (\Omega_{\eta_1}, \Sigma_1) \to (\Omega_{\eta_2}, \Sigma_2)$ is continuous and onto Sfs function. If $(\Omega_{\eta_1}, \Sigma_1)$ is Sfs compact, then $(\Omega_{\eta_2}, \Sigma_2)$ is Sfs compact.*

Proof. If (Φ_n, η) is any open covering of $\tilde{\Omega_{\eta_2}}$, that is $\Omega_{\eta_2} \tilde{\sqsubseteq} \tilde{\sqcup}_{n \in I} (\Phi_n, \eta)$, then $f^{-1}(\tilde{\Omega_{\eta_2}}) \tilde{\sqsubseteq} f^{-1}(\tilde{\sqcup}_{n \in I} (\Phi_n, \eta))$ and $\tilde{\Omega_{\eta_1}} \tilde{\sqsubseteq} \tilde{\sqcup}_{n \in I} f^{-1}(\Phi_n, \eta)$. So $f^{-1}(\Phi_n, \eta)$ is an open covering of $\tilde{\Omega_{\eta_1}}$. As (Ω, Σ) is compact, there exists $\Omega_{\eta_1} \tilde{\sqsubseteq} f^{-1}((\Phi_1, \eta)) \tilde{\sqcup} f^{-1}((\Phi_2, \eta)) \tilde{\sqcup} \cdots \tilde{\sqcup} f^{-1}((\Phi_n, \eta))$. From Definition 4.2 we have $\tilde{\Omega_{\eta_2}} = f(\Omega_{\eta_2}) \tilde{\sqsubseteq} f(f^{-1}((\Phi_1, \eta)) \tilde{\sqcup} f^{-1}((\Phi_2, \eta)) \tilde{\sqcup} \cdots \tilde{\sqcup} f^{-1}((\Phi_n, \eta))) = f(f^{-1}((\Phi_1, \eta)) \tilde{\sqcup} f^{-1}((\Phi_2, \eta)) \tilde{\sqcup} \cdots \tilde{\sqcup} f^{-1}((\Phi_n, \eta))) = (\Phi_1, \eta)) \tilde{\sqcup} f^{-1}((\Phi_2, \eta)) \tilde{\sqcup} \cdots \tilde{\sqcup} f^{-1}((\Phi_n, \eta)$. So we have $\tilde{\Omega_{\eta_2}} \sqsubseteq (\Phi_1, \eta)) \tilde{\sqcup} f^{-1}((\Phi_2, \eta)) \tilde{\sqcup} \cdots \tilde{\sqcup} f^{-1}((\Phi_n, \eta)$ that is $\tilde{\Omega_{\eta_2}}$ is covered by a finite number of (Φ_n, η). So $(\Omega_{\eta_2}, \Sigma_2)$ is Sfs compact. □

Definition 4.9. Suppose $(\Omega_{\eta_1}, \Sigma_1)$ and $(\Omega_{\eta_2}, \Sigma_2)$ are two Sfs topological spaces. A Sfs function $f : (\Omega_{\eta_1}, \Sigma_1) \to (\Omega_{\eta_2}, \Sigma_2)$ is named Sfs closed if $f(\Phi, \eta)$ is a Sfs closed set in $(\Omega_{\eta_2}, \Sigma_2)$, for each Sfs closed set $f(\Phi, \eta)$ in $(\Omega_{\eta_1}, \Sigma_1)$.

Theorem 4.10. *Suppose $(\Omega_{\eta_1}, \Sigma_1)$ is a Sfs topological space and $(\Omega_{\eta_2}, \Sigma_2)$ is a Sfs soft Σ_2-space. Sfs function f is closed if Sfs function $f : (\Omega_{\eta_1}, \Sigma_1) \to (\Omega_{\eta_2}, \Sigma_2)$ is continuous.*

Proof. Let (F, D) be a Sfs closed in $(\Omega_{\eta_1}, \Sigma_1)$. From Proposition 4.6 we have (F, D) is compact. Because of Sfs function f is continuous, Sfs set $f(F, D)$ is compact in $(\Omega_{\eta_2}, \Sigma_2)$. As $(\Omega_{\eta_2}, \Sigma_2)$ is a Sfs soft Σ_2-space, Sfs set $f(F, D)$ is closed. Hence Sfs function f is closed. □

Definition 4.11. If the null Sfs set is not formed by the intersection of the members of each finite subfamily of Σ of Sfs sets the family Σ satisfies the finite intersection property.

Theorem 4.12. *If and only if every family of Sfs closed sets with the finite intersection property has a non null intersection, the Sfs topological space is compact.*

Proof.

$\Rightarrow$: Let Σ be any family of Sfs closed such that $\tilde{\sqcap}\{(\Phi_n, \eta) : (\Phi_n, \eta) \in \Sigma, n \in I\} = \phi_\eta$. Consider $\Upsilon = \{(\Phi_n, \eta)^c : (\Phi_n, \eta) \in \Sigma, n \in I\}$. Hence Υ is a Sfs open cover of $\tilde{\Omega_\eta}$. As Sfs topological space is compact, there is a finite subcovering $(\Phi_1, \eta)^c, (\Phi_2, \eta)^c, \ldots, (\Phi_n, \eta)^c$, then $\tilde{\sqcap}_{n=1}^{m} (\Phi_n, \eta) = \tilde{\Omega_\eta}$-$\tilde{\sqcup}_{n=1}^{m} (\Phi_n, \eta)^c = \tilde{\Omega_\eta}$-$\Omega_\eta = \phi_\eta$. Then Σ can not have finite intersection property.

$\Leftarrow$: Let a Sfs topological space is not compact. Hence, any Sfs open cover of $\tilde{\Omega_\eta}$ has not a finite subcover. Let $\{(\Phi_n, \eta) : n \in I\}$ be Sfs open cover of $\tilde{\Omega_\eta}$. So $\tilde{\sqcup}_{n=1}^{m} (\Phi_n, \eta) \neq \tilde{\Omega_\eta}$. Therefore $\tilde{\sqcap}_{n=1}^{m} (\Phi_n, \eta)^c \neq \phi_\eta$. Thus, $\{(\Phi_n, \eta)^c : n = 1, \ldots, m\}$ have a finite intersection property. From hypothesis, $\tilde{\sqcap}(\Phi_n, \eta)^c \neq \tilde{\phi_\eta}$ and so $\tilde{\sqcup}_{n=1}^{m} (\Phi_n, \eta) \neq \tilde{\Omega_\eta}$. That is a contradiction. Hence, Sfs topological space is compact. □

5. Connectedness

Through this section, we're debating and expanding on the definition of spherical fuzzy soft Connectedness.

Definition 5.1. Let (Ω_η, Σ) be a Sfs topological space on Ω. A Sfs separation of $\tilde{\Omega_\eta}$ is a pair (Φ, η), (Ψ, η) of no- null Sfs open sets such that $\tilde{\Omega_\eta} = (\Phi, \eta)\tilde{\sqcup}(\Psi, \eta)$, $(\Phi, \eta)\tilde{\sqcap}(\Psi, \eta) = \phi$.

Definition 5.2. A Sfs topological space (Ω_η, Σ) is said to be a Sfs connected if there does not exist a Sfs separation of Ω_η. Otherwise, (Ω_η, Σ) is named a Sfs disconnected.

Example 5.3. Consider the Sfs topological space (Ω_η, Σ) that is given in Example 3.2. Since $(\Phi_1, \eta)\tilde{\sqcap}(\Phi_2, \eta) \neq \phi_\eta$, $(\Phi_1, \eta)\tilde{\sqcap}(\Phi_3, \eta) \neq \phi_\eta$, $(\Phi_2, \eta)\tilde{\sqcap}(\Phi_3, \eta) \neq \phi_\eta$ and also $(\Phi_1, \eta)\tilde{\sqcup}(\Phi_2, \eta) \neq \Omega_\eta$, $(\Phi_1, \eta)\tilde{\sqcup}(\Phi_3, \eta) \neq \Omega_\eta$ and $(\Phi_2, \eta)\tilde{\sqcup}(\Phi_3, \eta) \neq \Omega_\eta$, Sfs topological space (Ω_η, Σ) is Sfs disconnected.

Theorem 5.4. *Sfs topological space (Ω_η, Σ) is Sfs connected if and only if the only Sfs sets in $\mathrm{Sfs}(\Omega_\eta)$ that are both Sfs open and Sfs closed are ϕ and $\tilde{\Omega}$.*

Proof. Let (Ω_η, Σ) be connected Sfs. On the contrary suppose that (Φ, η) is a Sfs open and Sfs closed that is distinct from ϕ_η, $\tilde{\Omega}_\eta$. It is obvious that, $(\Phi, \eta)^c$ is a Sfs open set distinct from ϕ_η, $\tilde{\Omega}_\eta$. Also $(\Phi, \eta)\tilde{\sqcup}(\Phi, \eta)^c = \tilde{\Omega}_\eta$ and $(\Phi, \eta)\tilde{\sqcap}(\Phi, \eta)^c = \phi_\eta$. So, we have $(\Phi, \eta), (\Phi, \eta)^c$ is a Sfs separation of $\tilde{\Omega}_\eta$. This contradicts itself, thus the only Sfs sets both cloesd and open in Ω_η are ϕ_η and $\tilde{\Omega}_\eta$.

Conversely, let (Φ, η), (Ψ, η) be a Sfs separation of $\tilde{\Omega}_\eta$. Let $(\Phi, \eta) = \tilde{\Omega}_\eta$. Then $(\Psi, \eta) = \phi_\eta$. This contradicts itself. Then $(\Phi, \eta) \neq \tilde{\Omega}_\eta$. Therefore $(\Phi, \eta) = (\Psi, \eta)^c$. That is (Φ, η) is Sfs closed and open distinct from ϕ_η, $\tilde{\Omega}_\eta$. This is a contradiction and (Ω_η, Σ) is soft connected. □

Example 5.5. Due to the fact that the only Sfs sets in $\mathrm{Sfs}(\Omega_\eta)$ that are Sfs closed and open are ϕ_η, $\tilde{\Omega}_\eta$, Sfs, indiscrete topological space (Ω_η, Σ) is Sfs connected.

Example 5.6. Sfs discrete topological space (Ω_η, Σ) is Sfs disconnected. Because for at least one Sfs set (Φ, η) in $\mathrm{Sfs}(\Omega_\eta)$, Sfs set (Φ, η) is Sfs closed and open.

Corollary 5.7. *Let (Ω_η, Σ) be Sfs topological space on Ω. Consequently, the subsequent claims are similar.*

1. *(Ω_η, Σ) is Sfs connected.*
2. *No-null Sfs open sets (Φ, η), (Ψ, η) and $\tilde{\Omega_\eta} = (\Phi, \eta)\tilde{\sqcup}(\Psi, \eta)$ but $(\Phi, \eta)\tilde{\sqcap}(\Psi, \eta) \neq \phi_\eta$.*
3. *The only Sfs sets in $\mathrm{Sfs}(\Omega_\eta)$ that are both Sfs open and Sfs closed in Ω_η are ϕ and $\tilde{\Omega}$.*
4. *If $\tilde{\Omega_\eta} = (\Phi, \eta)\tilde{\sqcup}(\Psi, \eta)$ and $(\Phi, \eta)\tilde{\sqcap}(\Psi, \eta) = \phi_\eta$, then $(\Phi, \eta) = \phi_\eta$ or $(\Psi, \eta) = \phi_\eta$.*
5. *If $\tilde{\Omega_\eta} = (\Phi, \eta)\tilde{\sqcup}(\Psi, \eta)$ and $(\Phi, \eta)\tilde{\sqcap}(\Psi, \eta) = \phi_\eta$, then $(\Phi, \eta) = \tilde{\Omega_\eta}$ or $(\Psi, \eta) = \tilde{\Omega_\eta}$.*

6. Conclusion

Because topological space is devoid of geometric shapes, it is used to quantify things that are difficult to quantify, such as intelligence, beauty, and goodness. Because the problems of daily life are not clear, we are working to present new spaces that suit the reality we are living. We discuss some of these ideas' fundamental features. In our study, we thereby presented the partial emptying of the Sfs topology as well as the Sfs separation axioms, Sfs compactness, and Sfs connectedness. We list some of these notions' fundamental attributes. We also provided an application of these properties through the examples shown. This enables us to resign and offer applications to this expanding structures in several industries.

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