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A memory state-feedback controller via random pocket dropouts for multi-agent systems with external disturbance

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Abstract

This study explores the challenge of achieving consensus in multi-agent systems (MASs) when facing the random packet dropouts and disturbances. It employs memory state-feedback control (MSFC) in the context of undirected graphs and specified leader agents. The analysis focuses on mean square consensus, considering MASs within strongly connected networks or networks with undirected spanning trees. The MSFC approach is developed to ensure asymptotic consensus despite packet dropouts and also to reduce the impact of disturbances. Specifically, the consensus analysis leverages the Lyapunov-Krasovskii functional (LKF) framework, and the necessary conditions for implementing the proposed MSFC are established using linear matrix inequalities (LMIs). The system, augmented with an H_∞ attenuation level, is guaranteed to achieve asymptotic mean-square stability according to the provided criteria. In conclusion, two examples are provided to illustrate the effectiveness and practicality of the proposed control mechanism.

Keywords: Memory state-feedback control, multi-agent systems, Kronecker product, leader-following consensus, linear matrix inequality.

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1. Introduction

The mechanism of agent-based system is a renowned new approach to develop a thought, create, and implement software systems in artificial intelligence research. In general, agents are refined computer programs that work independently in support of their users across open and dispersed settings to tackle

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the arising variety of complicated issues. However, from the view of application, it necessitates that numerous agents can work collaboratively [12, 19].

Multi-agent systems (MASs) are roughly a connected network of software agents that communicate to derive issues that are beyond any problem solver's capabilities or understanding. MASs have gotten a lot of attention during the last couple of decades because of their numerous applications in sectors such as vehicle formulation, robotics, and mobile networks [8, 9, 11]. MASs provide several advantages over traditional control systems, including flexibility, reliability, cost reduction, improved system efficiency, and the provision of additional capabilities such as resilience and reusability [10, 13]. The reason for the widespread interest in MASs is that they are viewed as a technical instrument for analyzing and developing models and rules of interaction in complex human-centered systems [2, 25, 37]. Very huge number of MASs applications require a set of agents to establish an agreement (consensus) on the parameters of specific variables and so addressing the consensus problem for MASs is quite useful [6, 17, 32].

Often referred to as "agreement dynamics", consensus dynamics draws concept from both systems theory and graph theory. The agreement or consensus issue is currently a hot topic in MASs research [7, 29]. This refers to the process by which all communication agents arrive at the same value by drawing the knowledge of their immediate neighbors. Physiological systems, gene networks [21], large-scale energy systems, and vehicle fleets on land, in space, or in the air are examples of agent networks that communicate information to establish an agreement. The distributed controller allows all associated agents to attain a shared goal. Many scholars studied the two forms of consensus, namely, the leader following [18, 39] and leaderless [26, 34] consensus, along with numerous applications, including collaboration among agents.

In many real-world scenarios, control packet loss occurs due to actuator problems, communication disruption, congestion, and so on [24]. Since the occurrence of random packet dropout in control is unavoidable, it may cause system instability [16, 20]. As a result, it is critical to consider the consequences of controlling random packet dropout. There are two forms of packet dropouts: random packet dropout and deterministic packet dropout. For instance, in [1], the authors investigated the leaderless consensus of delayed sampled-data control for MASs with random packet losses. In [15], the authors discussed robust consensus of reliable control scheme nonlinear MASs with probabilistic time delay.

Agent systems and random pocket dropouts in consensus controllers have been extensively used to mitigate the impact of external disruptions. Some fruitful outcomes have been recorded. For instance, [14] has found a leader-following consensus of non-fragile H_∞ approaches for MASs when the topologies are changing. Recently, several systems have used feedback control to analyze the dynamic performance of closed-loop systems. In [30], the authors discussed state-feedback control (SFC) of MASs in the communication channels under data packet dropout through markovian approach. H_∞ control of MASs under time-delayed signal condition with unknown leader states and switching graph has been discussed in [33]. In [28], the authors derived an adaptive SFC scheme for the output consensus of MASs. However, according to the author's understanding, SFC and MSFC combined with stochastic variables and disturbances for MASs has not been discussed in the existing literature. The current study seeks to fill this need. Motivated by the facts stated above, this work uses MSFC to examine the leader-following consensus of MASs with random packet dropout and external disturbance. This study primarily discusses the consensus of MASs with constant transmission delay using the MSFC scheme by building the suitable LKF.

- Given the existing constraints in the utilization of current research findings, it is evident that a more comprehensive investigation is needed to address the practical implementation of MASs incorporating MSFC.
- Previous research has not thoroughly examined the interplay between random packet dropout, external disturbances, transmission delays, and the development of MSFC design.

This study explores the challenge of achieving consensus in MASs when faced with random packet dropouts and disturbances. It employs MSFC in the context of undirected graphs and specified leader agents. The analysis focuses on mean square consensus, considering MASs within strongly connected networks or networks with undirected spanning trees. The MSFC approach is developed to ensure asymptotic consensus despite packet dropouts and also to reduce the impact of disturbances. Specifically, the consensus analysis leverages the LKF framework, and the necessary conditions for implementing the proposed MSFC are established using LMIs. The system, augmented with an H_∞ attenuation level, is guaranteed to achieve asymptotic mean-square stability according to the provided criteria. In conclusion, two examples are provided to illustrate the effectiveness and practicality of the proposed control mechanism.

The following are the major contributions:

- In most previous research, undirected topologies are represented by random packet dropouts but in our study, it is modeled along with a transmission delay, random packet dropouts, and an external disturbance.
- Beside, the asymptotic consensus for MASs under transmission delay, random packet dropouts and disturbances has been studied using LMI, LKF, graph theory techniques, and the free-weight matrix approach, making this paper more advanced than prior studies [27].
- In addition, adequate criteria are established to ensure that all followers may asymptotically follow the leader in the sense of mean square using a specified H_∞ performance index.
- The current approaches in [27] have been compared to confirm the efficacy of the suggested method, which is shown in the numerical simulation.

Notation: There is a positive real value for the time delay γ. $R^{n\times q}$ and R^n stand for the space of real matrices $n \times q$ and the n-dimensional Euclidean space, respectively. The symmetric matrix and the Kronecker product are represented by $\star$ and $\otimes$, respectively.

2. Basic concepts of graph theory

Consider $G = (\sigma, \varepsilon, \mathcal{W})$ as a weighted graph, $\varepsilon \subseteq \{\sigma \times \sigma\}$ is an edge set. $\mathcal{W} = [a_{ij}]_{N\times N}$ is the adjacent weighted matrix, where the existence of the edge $(\mathfrak{z}_i, \mathfrak{z}_j)$ implying a_{ij} must be zero. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is described by $l_{ij} = -a_{ij}$, $i \neq j$ and $l_{ii} = \sum_{j=1,j\neq i}^{N} l_{ij}$ $(i, j = 1, 2, \dots, N)$. For additional information on communication topologies, please refer to [31].

3. System formulation

Consider the follower system

$$\begin{cases} \dot{\wp}_i(t) = \mathcal{A}\wp_i(t) + \mathcal{B}u_i(t) + \mathcal{D}\omega_i(t), \\ y_i(t) = \mathcal{C}\wp_i(t), \end{cases}$$

where $\wp_i(t) = [\wp_{i1}(t), \wp_{i2}(t), \dots, \wp_{in}(t)] \in R^n$ and $u_i(t) \in R^q$ are the state and control inputs, respectively. $\omega_i(t) \in R^m$ is the external disturbance. $y_i(t) \in R^\mu$ is the system output. $\mathcal{A} = (a_{rs}) \in R^{n\times n}$, $\mathcal{B} = (b_{rs}) \in R^{n\times q}$, $\mathcal{D} = (d_{rs}) \in R^{n\times m}$ and $\mathcal{C} = (c_{rs}) \in R^{\mu\times n}$ are constants and well-known matrices.

Consider the leader system described as

$$\begin{cases} \dot{\wp}_0(t) = \mathcal{A}\wp_0(t), \\ y_0(t) = \mathcal{C}\wp_0(t), \end{cases}$$

here the leader agent state is $\wp_0(t) \in R^n$. Consider $\eta_i(t) = \wp_i(t) - \wp_0(t)$ and $\bar{y}_i(t) = y_i(t) - y_0(t)$, the consensus error is described as

$$\begin{cases} \dot{\eta}_i(t) = \mathcal{A}\eta_i(t) + \mathcal{B}u_i(t) + \mathcal{D}\omega_i(t), \\ \bar{y}_i(t) = \mathcal{C}\eta_i(t). \end{cases} \tag{3.1}$$

The compact version of the consensus error system (3.1) is

$$\begin{cases} \dot{\eta}(t) = (I \otimes \mathcal{A})\eta(t) + (I \otimes \mathcal{B})U(t) + (I \otimes \mathcal{D})W(t), \\ \bar{y}(t) = (I \otimes \mathcal{C})\eta(t), \end{cases} \tag{3.2}$$

where $\eta(t) = [\eta_1(t), \eta_2(t), \dots, \eta_N(t)]^T$, $U(t) = [u_1(t), u_2(t), \dots, u_N(t)]^T$, $W(t) = [\omega_1(t), \omega_2(t), \dots, \omega_N(t)]^T$. On the other hand, the control input $u_i(t)$ is specified in this manuscript as

$$\begin{aligned} u_i(t) &= \mathcal{K}_1 \mathbb{F}(t)\left[\sum_{j=1}^{N} a_{ij}(\wp_j(t) - \wp_i(t)) - \mathfrak{I}_i(\wp_i(t) - \wp_0(t))\right] \\ &\quad + \mathcal{K}_2(1 - \mathbb{F}(t))\left[\sum_{j=1}^{N} a_{ij}(\wp_j(t-\gamma) - \wp_i(t-\gamma)) - \mathfrak{I}_i(\wp_i(t) - \wp_0(t))\right], \end{aligned} \tag{3.3}$$

where $\mathcal{K}_1$, $\mathcal{K}_2$ are control gain matrices.

$$\mathbb{F}(t) = \begin{cases} 1, & \text{successfully signal transmitted,} \\ 0, & \text{otherwise,} \end{cases}$$

and $\mathbb{F}(t)$ satisfies Bernoulli distributed white sequence with $\Pr\{\mathbb{F}(t) = 1\} = \varepsilon\{\mathbb{F}(t)\} = \mathbb{F}$ and $\Pr\{\mathbb{F}(t) = 0\} = 1 - \varepsilon\{\mathbb{F}(t)\} = 1 - \mathbb{F}$.

Remark 3.1. It is important to point out that the stochastic variable $\mathbb{F}(t)$ of the coupling scheme for the MASs consensus problem was examined to couple the proportional sample data control and the in-memory sample data control in [27, 38]. Inspired by the previous studies, in this article, we suggest the coupling approach in SFC, which is coupled with both SFC and MSFC.

Remark 3.2. Instability in a control system occurs for many causes in real-world engineering, but the most common ones are transmission delay, random packet dropouts, and stochastic disturbance. This means that neither transmission delay nor random packet dropouts are taken into account in the previous literature, despite their importance to the MASs consensus process. Therefore, the model used here is more accurate.

Define $\Lambda = \mathrm{diag}\{\mathfrak{I}_1, \mathfrak{I}_2, \dots, \mathfrak{I}_N\}$. Combining equations (3.2) and (3.3), the error system is stated as

$$\begin{aligned} \dot{\eta}(t) &= (I \otimes \mathcal{A})\eta(t) - \mathbb{F}(t)\Big[(L \otimes \mathcal{B}\mathcal{K}_1)\eta(t) + (\Lambda \otimes \mathcal{B}\mathcal{K}_1)\eta(t)\Big] \\ &\quad - (1 - \mathbb{F}(t))\Big[(L \otimes \mathcal{B}\mathcal{K}_2)\eta(t-\gamma) + (\Lambda \otimes \mathcal{B}\mathcal{K}_2)\eta(t)\Big] + (I \otimes \mathcal{D})W(t). \end{aligned} \tag{3.4}$$

Definition 3.3 ([15]). The system (3.4) attains asymptotic consensus of mean-square if for all agent $\wp_i \ni$: the system satisfies $\lim_{t\to\infty} \varepsilon\{\|\wp_i(t) - \wp_j(t)\|^2\} = 0$.

The following Lemmas can be used to achieve sufficient conditions with appropriate LF.

Lemma 3.4 ([22]). *For a given matrix $\mathcal{M} > 0$, differentiable function $\mathfrak{R} : [\mu,\ \delta] \to R^n$, satisfies*

$$\int_{\mu}^{\delta} \dot{\mathfrak{R}}^T(\sigma)\mathcal{M}\dot{\mathfrak{R}}(\sigma)d\sigma \geqslant \frac{1}{6(\delta-\mu)}\sigma^T \begin{bmatrix} 22\mathcal{M} & 10\mathcal{M} & -32\mathcal{M} \\ \star & 16\mathcal{M} & -26\mathcal{M} \\ \star & \star & 12\mathcal{M} \end{bmatrix} \sigma,$$

where $\sigma^T = \Big[\mathfrak{R}^T(\delta)\ \ \mathfrak{R}^T(\gamma)\ \ \frac{1}{\delta-\mu}\int_{\mu}^{\delta} \mathfrak{R}^T(\sigma)d\sigma\Big]$.

Lemma 3.5 ([35]). *For a given matrix $\mathcal{M} > 0 \in R^{4n\times 4n}$, a continuous differentiable function $\mathfrak{R} : [\mu, \ \delta] \to R^n$, vectors $\mathcal{Z}_\ell(i = 1,2,3)$, and any appropriate matrices $\mathcal{W}_\iota(\iota = 1,2,3)$, the following inequality holds:*

$$-\int_\mu^\delta\int_\vartheta^\delta \dot{\mathfrak{R}}^T(\sigma)\mathcal{M}\mathfrak{R}(\sigma)d\sigma d\vartheta \leqslant \frac{\delta_\mu^2}{2}\mathcal{Z}_1^T\mathcal{W}_1\mathcal{M}^{-1}\mathcal{W}_1^T\mathcal{Z}_1 + \frac{\delta_\mu^4}{36}\mathcal{Z}_2^T\mathcal{W}_2\mathcal{M}^{-1}\mathcal{W}_2^T\mathcal{Z}_2$$
$$+\frac{\delta_\mu^6}{600}\mathcal{Z}_3^T\mathcal{W}_3\mathcal{M}^{-1}\mathcal{W}_3^T\mathcal{Z}_3 + 2\sum_{\ell=1}^3\mathcal{Z}_\ell\mathcal{W}_\ell\Pi_\iota,$$

where

$$\delta_\mu = \delta - \mu,$$
$$\Pi_1 = \delta_\mu\mathfrak{R}(\delta) - \int_\mu^\delta \mathfrak{R}(\sigma)d\sigma,$$
$$\Pi_2 = \frac{\delta_\mu^2}{3}\mathfrak{R}(\delta) + \frac{2\delta_\mu}{3}\int_\mu^\delta\mathfrak{R}(\sigma)d\sigma - 2\int_\mu^\delta\int_\vartheta^\delta\mathfrak{R}(\sigma)d\sigma d\vartheta,$$
$$\Pi_3 = \frac{\delta_\mu^3}{10}\mathfrak{R}(\delta) - \frac{2\delta_\mu^2}{10}\int_\mu^\delta\mathfrak{R}(\sigma)d\sigma + \frac{12\delta_\mu}{5}\int_\mu^\delta\int_\vartheta^\delta\mathfrak{R}(\sigma)d\sigma d\vartheta - 6\int_\mu^\delta\int_\vartheta^\delta\int_\varepsilon^\delta\mathfrak{R}(\sigma)d\sigma d\vartheta d\varepsilon.$$

The primary purpose of this research is to acquire the mean-square asymptotic stable of MASs (3.4) via MSFC with an upper bound on the communication delay, as shown below.

Problem 3.6. For the required MFSC-states $\eta(t)$ to be asymptotically stable, given an error of MASs (3.4), the following conditions must hold:

- The consensus of MASs (3.4) is mean square asymptotic when $W(t) = 0$.
- With the zero initial condition and $\beta > 0$, the following inequality holds:

$$\int_0^{+\infty} y^T(s)y(s)ds \leqslant \beta^2\int_0^{+\infty} W^T(s)W(s)ds.$$

4. Main results

This section contains the main results for asymptotic consensus criteria for MASs (3.4) using MSFC. We express block matrices $\psi_r^T = [0_{n,(r-1)n} \quad I_n \quad 0_{n,(7-r)n}]$ $(r = 1,2,\ldots,7)$, and the other symbols are stated as

$$\xi^T(t) = [\eta^T(t) \ \ \dot{\eta}^T(t-\gamma) \ \ \frac{1}{\gamma}\int_{t-\gamma}^t \eta^T(s)ds]^T, \quad \zeta^T(t) = \left[\eta^T(t) \ \ \eta^T(t-\gamma) \ \ \dot{\eta}^T(t) \ \ \Xi_1^T \ \ \Xi_2^T \ \ \Xi_3^T \ \ W(t)\right]^T,$$
$$\Xi_1 = \int_{t-\gamma}^t \eta^T(s)ds, \qquad \Xi_2 = \int_{t-\gamma}^t\int_\theta^t \eta^T(s)dsd\theta, \quad \Xi_3 = \int_{t-\gamma}^t\int_\theta^t\int_\sigma^t \eta^T(s)dsd\theta d\sigma.$$

4.1. Multi-agent systems with disturbance

Theorem 4.1. *The system (3.4) is asymptotically mean-square consensus for some positive constant ρ, γ and given gain matrices $\mathcal{K}_1$, $\mathcal{K}_2$ if there exists positive matrices P_r $(r = 1,2,3,4)$, any matrices Q_ℓ $(\ell = 1,2,3)$ with appropriate dimensions, such that*

$$\Psi = \begin{bmatrix} \Psi_{11} & \mathcal{G} \\ \star & \mathcal{P}\end{bmatrix} < 0,$$

where

$$\Psi_{11} = \mathrm{Sym}\{\psi_1(I\otimes P_1)\psi_3^T - \psi_1\frac{1}{6\gamma}10(I\otimes P_3)\psi_2^T + \psi_1\frac{1}{6\gamma^2}32(I\otimes P_3)\psi_4^T + \psi_2\frac{1}{6\gamma^2}26(I\otimes P_3)\psi_4^T\}$$

$$
\begin{aligned}
&+\psi_1(I\otimes P_2)\psi_1^T-\psi_2(I\otimes P_2)\psi_2^T+\psi_3\gamma(I\otimes P_3)\psi_3^T-\psi_1\frac{1}{6\gamma}22(I\otimes P_3)\psi_1^T-\psi_2\frac{1}{6\gamma}16(I\otimes P_3)\psi_2^T\\
&-\psi_4\frac{1}{6\gamma^3}58(I\otimes P_3)\psi_4^T+\mathrm{Sym}\{\sum_{r=1}^{3}\mu_{1r}^T(I\otimes G_r)\Gamma_r\}+\mathrm{Sym}\{\varpi\times(-\psi_3+(I\otimes\mathcal{A})\psi_1-\mathbb{F}(L\otimes\mathcal{B}\mathcal{K}_1)\psi_1\\
&-\mathbb{F}(\Lambda\otimes\mathcal{B}\mathcal{K}_1)\psi_1-(1-\mathbb{F})(L\otimes\mathcal{B}\mathcal{K}_2)\psi_2-(1-\mathbb{F})(\Lambda\otimes\mathcal{B}\mathcal{K}_2)\psi_1+(I\otimes\mathcal{D})\psi_7\}\\
&+\psi_1(I\otimes\mathcal{C})(I\otimes\mathcal{C})^T\psi_1^T-\beta^2\psi_7\psi_7^T,\ \varpi=\psi_1(I\otimes Q_1)^T+\psi_2(I\otimes Q_2)^T+\psi_3(I\otimes Q_3)^T,
\end{aligned}
$$

$$\mathcal{P}=\mathrm{diag}\{2(I\otimes P_4),\ 36(I\otimes P_4),\ 600(I\otimes P_4)\};\ \mathcal{G}=\{\gamma\ \mu_{11}^T(I\otimes G_1),\ \gamma\ \mu_{12}^T(I\otimes G_2),\ \gamma\ \mu_{13}^T(I\otimes G_3)\},$$

$$\Xi_1=\gamma\,\psi_1-\psi_3,\ \Xi_2=\frac{\gamma^2}{3}\psi_1+\frac{2\gamma}{3}\psi_3-2\psi_4,\ \Xi_3=\frac{\gamma^3}{10}\psi_1-\frac{3\gamma^2}{10}\psi_3+\frac{12\gamma}{5}\psi_4-6\psi_5.$$

Proof. Let us construct the LKF as

$$V(t)=\sum_{i=1}^{3}V_i(t), \tag{4.1}$$

where

$$
\begin{aligned}
V_1(t)&=\eta^T(t)(I\otimes P_1)\eta(t)+\int_{t-\gamma}^{t}\eta^T(s)(I\otimes P_2)\eta(s)ds,\\
V_2(t)&=\int_{-\gamma}^{0}\int_{t+\theta}^{t}\dot\eta^T(s)(I\otimes P_3)\dot\eta(s)dsd\theta,\\
V_3(t)&=\int_{t-\gamma}^{t}\int_{\theta}^{t}\int_{\sigma}^{t}\dot\eta^T(s)(I\otimes P_4)\dot\eta(s)dsd\theta d\sigma.
\end{aligned}
$$

Based on (4.1), we get

$$\varepsilon\{\mathcal{L}V_1(t)\}=2\eta^T(t)(I\otimes P_1)\dot\eta(t)+\eta^T(t)(I\otimes P_2)\eta(t)-\eta^T(t-\gamma)(I\otimes P_2)\eta(t-\gamma), \tag{4.2}$$

$$\varepsilon\{\mathcal{L}V_2(t)\}=\dot\eta^T(t)\gamma(I\otimes P_3)\dot\eta(t)-\int_{t-\gamma}^{t}\dot\eta^T(s)(I\otimes P_3)\dot\eta(s)ds, \tag{4.3}$$

$$\varepsilon\{\mathcal{L}V_3(t)\}=\frac{\gamma^2}{2}\dot\eta^T(t)(I\otimes P_4)\dot\eta(t)-\int_{t-\gamma}^{t}\int_{\theta}^{t}\dot\eta^T(s)(I\otimes P_4)\dot\eta(s)dsd\theta. \tag{4.4}$$

Based on Lemma 3.4, we have

$$-\int_{t-\gamma}^{t}\dot\eta^T(s)(I\otimes P_3)\dot\eta(s)ds\leqslant\xi^T(t)\begin{bmatrix}22(I\otimes P_3)&10(I\otimes P_3)&-32(I\otimes P_3)\\ \star&16(I\otimes P_3)&-26(I\otimes P_3)^T\\ \star&\star&12(I\otimes P_3)\end{bmatrix}\xi(t), \tag{4.5}$$

where $\xi^T(t)=[\eta^T(t)\quad \eta^T(t-\gamma)\quad \frac{1}{\gamma}\int_{t-\gamma}^{t}\eta^T(s)ds]$. Applying Lemma 3.5, we have

$$
\begin{aligned}
&-\int_{t-\gamma}^{t}\int_{\theta}^{t}\dot\eta^T(s)(I\otimes P_4)\dot\eta(s)dsd\theta\\
&\quad\leqslant\zeta^T(t)\Big\{\frac{\gamma^2}{2}\mu_{11}^T(I\otimes G_1)(I\otimes P_4)^{-1}(I\otimes G_1)^T\mu_{11}+\frac{\gamma^4}{36}\mu_{12}^T(I\otimes G_2)(I\otimes P_4)^{-1}(I\otimes G_2)^T\mu_{12}\\
&\quad+\frac{\gamma^6}{600}\mu_{13}^T(I\otimes G_3)(I\otimes P_4)^{-1}(I\otimes G_3)^T\mu_{13}+2\sum_{r}^{3}\mu_{1r}(I\otimes G_r)\Xi_r\Big\}\zeta(t).
\end{aligned} \tag{4.6}
$$

Moreover, for any matrices Q_ℓ $(1,2,3)$, it follows that,

$$\varepsilon\Big\{0 = 2\varpi^T(t)\{-\dot{\eta}(t) + (I\otimes \mathcal{A})\eta(t) - \mathbb{F}(t)[(L\otimes \mathcal{B}\mathcal{K}_1)\eta(t) + (\Lambda\otimes \mathcal{B}\mathcal{K}_1)\eta(t)] \\ - (1-\mathbb{F}(t))[(L\otimes \mathcal{B}\mathcal{K}_2)\eta(t-\gamma) + (\Lambda\otimes \mathcal{B}\mathcal{K}_2)\eta(t)] + (I\otimes \mathcal{D})W(t)\}\Big\}, \tag{4.7}$$

with $\varpi(t) = \eta^T(t)(I\otimes Q_1)^T + \dot{\eta}^T(t)(I\otimes Q_2)^T + \eta^T(t-\gamma)(I\otimes Q_3)^T$. Hence, on collecting equations (4.2)-(4.7), we get

$$\varepsilon\{\mathcal{L}V(t)\} \leqslant \zeta^T(t)\Psi\zeta(t) - y^T(t)y(t) + \beta W^T(t)W(t).$$

Clearly it is evident that $\Psi < 0$. Moreover

$$\varepsilon\{\mathcal{L}\mathbb{V}(t)\} \leqslant \varepsilon\{-y^T(t)y(t) + \beta^2 W^T(t)W(t)\}.$$

Integrating on both sides from 0 to ∞, we get

$$V(\infty) - V(0) \leqslant \varepsilon\Big\{\int_0^\infty \Big(-y^T(t)y(t) + \beta^2 W^T(t)W(t)\Big)dt\Big\}.$$

For any nonzero $W(t) \in L_2[0,\infty)$, we have

$$\varepsilon\int_0^\infty y^T(t)y(t)dt \leqslant \beta^2\int_0^\infty W^T(t)W(t)dt,$$

under the initial condition $V(0) = 0$. Suppose $W(t) = 0$, $\exists$ a scalar $\beta \ni$:

$$\varepsilon\{\mathcal{L}V(t)\} \leqslant -\varepsilon\{\|\eta(t)\|^2\} \ \text{ for } \ \|\eta(t)\| \neq 0.$$

Thus, the MASs (3.4) is asymptotically mean-square consensus under H_∞ performance index. This ends the proof. □

Remark 4.2. The dynamic performance of MASs (3.4) must be improved. To do this, the forms of $\mu_{1\iota}(\iota = 1,2,3)$ can be chosen as either $\zeta(t)$ or by applying Theorem 4.1. The two scenarios are as follows:

Case 1: $\mu_{11} = \mathrm{col}[\psi_1,\ \psi_2,\ \psi_3]$, $\mu_{12} = \psi_5$, $\mu_{13} = \psi_6$ with the free matrices G_1, G_2, and G_3 are implemented in Theorem 4.1.

Case 2: $\mu_{11} = \mathrm{col}[\psi_1,\ \psi_2]$, $\mu_{12} = \psi_4$, $\mu_{13} = \psi_5$ with the free matrices G_1, G_2, and G_3 are implemented in Theorem 4.1.

The control gain matrices for the MSFC method (3.4) to attain asymptotically mean square consensus between leader and follower system (3.4) will be derived using Theorems 4.1 and 4.3.

Theorem 4.3. *The system* (3.4) *attains asymptotic mean square consensus for some positive constant* γ *if there exists positive matrices* $\tilde{P}_r$ $(r = 1,2,3,4)$ *and any matrices* $\tilde{Q}_\ell$ $(\ell = 1,2,3)$ *with appropriate dimensions, such that*

$$\tilde{\Psi} = \begin{bmatrix} \tilde{\Psi}_{11} & \tilde{\mathcal{G}} & \tilde{\mathcal{Y}} \\ \star & \tilde{\mathcal{P}} & 0 \\ \star & \star & -I \end{bmatrix} < 0, \tag{4.8}$$

where

$$\tilde{\Psi}_{11} = \mathrm{Sym}\{\psi_1(I\otimes\tilde{P}_1)\psi_3^T - \psi_1\frac{1}{6\gamma}10(I\otimes\tilde{P}_3)\psi_2^T + \psi_1\frac{1}{6\gamma^2}32(I\otimes\tilde{P}_3)\psi_4^T + \psi_2\frac{1}{6\gamma^2}26(I\otimes\tilde{P}_3)\psi_4^T\}$$

$$+\psi_1(I\otimes\tilde{P}_2)\psi_1^T-\psi_2(I\otimes\tilde{P}_2)\psi_2^T+\psi_3\gamma(I\otimes\tilde{P}_3)\psi_3^T-\psi_1\frac{1}{6\gamma}22(I\otimes\tilde{P}_3)\psi_1^T-\psi_2\frac{1}{6\gamma}16(I\otimes\tilde{P}_3)\psi_2^T$$
$$-\psi_4\frac{1}{6\gamma^3}58(I\otimes\tilde{P}_3)\psi_4^T+\mathrm{Sym}\{\sum_{r=1}^{3}\mu_{1r}^T(I\otimes\tilde{G}_r)\Gamma_r\}+\mathrm{Sym}\{\varpi\times(-\psi_3+(I\otimes\mathcal{A}Q)\psi_1$$
$$-\mathbb{F}(L\otimes\mathcal{B}\mathcal{H}_1)\psi_1-\mathbb{F}(\Lambda\otimes\mathcal{B}\mathcal{H}_1)\psi_1-(1-\mathbb{F})(L\otimes\mathcal{B}\mathcal{H}_2)\psi_2-(1-\mathbb{F})(\Lambda\otimes\mathcal{B}\mathcal{H}_2)\psi_1+(I\otimes\mathcal{D})\psi_7\}$$
$$-\beta^2\psi_7\psi_7^T,\ \tilde{\varpi}=\psi_1+\theta_1\psi_2+\theta_2\psi_3,$$
$$\tilde{\mathcal{P}}=\mathrm{diag}\{2(I\otimes\tilde{P}_4),\ 36(I\otimes\tilde{P}_4),\ 600(I\otimes\tilde{P}_4)\};\ \tilde{\mathcal{G}}=\{\gamma\,\mu_{11}^T(I\otimes\tilde{G}_1),\ \gamma\,\mu_{12}^T(I\otimes\tilde{G}_2),\ \gamma\,\mu_{13}^T(I\otimes\tilde{G}_3)\},$$
$$\Xi_1=\gamma\,\psi_1-\psi_3,\ \Xi_2=\frac{\gamma^2}{3}\psi_1+\frac{2\gamma}{3}\psi_3-2\psi_4,\ \Xi_3=\frac{\gamma^3}{10}\psi_1-\frac{3\gamma^2}{10}\psi_3+\frac{12\gamma}{5}\psi_4-6\psi_5,$$
$$\tilde{\mathcal{Y}}=[(I\otimes CQ^T)\ \underbrace{0_n\cdots 0_n}_{8\text{ times}}]^T.$$

In addition, the gain matrices are determined by $\mathcal{K}_m=\mathcal{H}_mQ^{-1}$, $m=1,2$.

Proof. We define, $Q_1=Q^{-1}$, $Q_2=\theta_1Q^{-1}$, $Q_3=\theta_2Q^{-1}$, $Q^TP_{\mathfrak{K}}Q=\tilde{P}_{\mathfrak{K}}$, $(\mathfrak{K}=1,2,3,4)$, and $\mathcal{H}_m=\mathcal{K}_mQ$. Then, the LMI (4.8) is pre and post multiplied on both the sides with $\mathrm{diag}\{\underbrace{(I\otimes Q),\ldots,(I\otimes Q)}_{8\text{ times}},I\}$ and $\tilde{\Psi}$ is expressed as in Theorem 4.1. Thus the proof concludes. □

4.2. Multi-agent systems without disturbance

Let us consider the error system without disturbance which is described as

$$\dot{\eta}(t)=(I\otimes\mathcal{A})\eta(t)-\mathbb{F}(t)\Big[(L\otimes\mathcal{B}\mathcal{K}_1)\eta(t)+(\Lambda\otimes\mathcal{B}\mathcal{K}_1)\eta(t)\Big]$$
$$-(1-\mathbb{F}(t))\Big[(L\otimes\mathcal{B}\mathcal{K}_2)\eta(t-\gamma)+(\Lambda\otimes\mathcal{B}\mathcal{K}_2)\eta(t)\Big]. \tag{4.9}$$

Theorem 4.4. *The system (4.9) attains asymptotically mean square consensus for some positive constant* γ *if there exists positive matrices* $\tilde{P}_r$ $(r=1,2,3,4)$ *and any matrices* $\tilde{Q}_r$ $(r=1,2,3)$ *with appropriate dimensions, such that*

$$\tilde{\Psi}=\begin{bmatrix}\tilde{\Psi}_{11} & \tilde{\mathcal{G}}\\ \star & \tilde{\mathcal{P}}\end{bmatrix}<0, \tag{4.10}$$

where

$$\tilde{\Psi}_{11}=\mathrm{Sym}\{\psi_1(I\otimes\tilde{P}_1)\psi_3^T-\psi_1\frac{1}{6\gamma}10(I\otimes\tilde{P}_3)\psi_2^T+\psi_1\frac{1}{6\gamma^2}32(I\otimes\tilde{P}_3)\psi_4^T+\psi_2\frac{1}{6\gamma^2}26(I\otimes\tilde{P}_3)\psi_4^T\}$$
$$+\psi_1(I\otimes\tilde{P}_2)\psi_1^T-\psi_2(I\otimes\tilde{P}_2)\psi_2^T+\psi_3\gamma(I\otimes\tilde{P}_3)\psi_3^T-\psi_1\frac{1}{6\gamma}22(I\otimes\tilde{P}_3)\psi_1^T-\psi_2\frac{1}{6\gamma}16(I\otimes\tilde{P}_3)\psi_2^T$$
$$-\psi_4\frac{1}{6\gamma^3}58(I\otimes\tilde{P}_3)\psi_4^T+\mathrm{Sym}\{\sum_{r=1}^{3}\mu_{1r}^T(I\otimes\tilde{G}_r)\Gamma_r\}+\mathrm{Sym}\{\varpi\times(-\psi_3+(I\otimes\mathcal{A})\psi_1$$
$$-\mathbb{F}(L\otimes\mathcal{B}\mathcal{K}_1)\psi_1-\mathbb{F}(\Lambda\otimes\mathcal{B}\mathcal{K}_1)\psi_1-(1-\mathbb{F})(L\otimes\mathcal{B}\mathcal{K}_2)\psi_2-(1-\mathbb{F})(\Lambda\otimes\mathcal{B}\mathcal{K}_2)\psi_1\}$$
$$\tilde{\varpi}=\psi_1+\theta_1\psi_2+\theta_2\psi_3,$$
$$\tilde{\mathcal{P}}=\mathrm{diag}\{2(I\otimes\tilde{P}_4),\ 36(I\otimes\tilde{P}_4),\ 600(I\otimes\tilde{P}_4)\};\ \tilde{\mathcal{G}}=\{\gamma\,\mu_{11}^T(I\otimes\tilde{G}_1),\ \gamma\,\mu_{12}^T(I\otimes\tilde{G}_2),\ \gamma\,\mu_{13}^T(I\otimes\tilde{G}_3)\},$$
$$\Xi_1=\gamma\,\psi_1-\psi_3,\ \Xi_2=\frac{\gamma^2}{3}\psi_1+\frac{2\gamma}{3}\psi_3-2\psi_4,\ \Xi_3=\frac{\gamma^3}{10}\psi_1-\frac{3\gamma^2}{10}\psi_3+\frac{12\gamma}{5}\psi_4-6\psi_5.$$

In addition, the gain matrices are determined by $\mathcal{K}_m=\mathcal{H}_mQ^{-1}$, $m=1,2$.

Proof. We define, $Q_1 = Q^{-1}$, $Q_2 = \theta_1 Q^{-1}$, $Q_3 = \theta_2 Q^{-1}$, $Q^T P_{\mathfrak{K}} Q = \tilde{P}_{\mathfrak{K}}$, $(\mathfrak{K} = 1,2,3,4)$, and $\mathcal{H}_m = \mathcal{K}_m Q$. Then, the LMI (4.10) is pre and post multiplied on both the sides with $\text{diag}\{\underbrace{(I \otimes Q), \ldots, (I \otimes Q)}_{7 \text{ times}}\}$ and $\tilde{\Psi}$ is expressed as in Theorem 4.3. Thus the proof concludes. □

Remark 4.5. Unlike the results in [1, 15], random packet dropouts are studied in this study under a directed graph coupled with transmission delay and external disturbance. The MASs consensus in (3.4) through MSFC with random packet loss highlights the interaction between the transmission delay γ, the random packet loss $\mathbb{F}$, and the control gain matrices $\mathcal{K}_1$ and $\mathcal{K}_2$.

Remark 4.6. The algorithm presented below outlines the process of constructing the control gain matrix using the LMI condition as defined in Theorem 4.3.

Algorithm 4.7 (Grid search algorithm).

1. **function** LYAPUNOV FUNCTION $(\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \theta_1, \theta_2)\triangleright$
   where $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, and $\mathcal{E}$ are system parameters with positive constants γ, θ_1, θ_2.
2. **if** the matrices $\tilde{P}_r$, $(r = 1, \ldots, 4) > 0$,
   any matrices $\tilde{Q}_\ell$, $(\ell = 1, \ldots, 3)$ exists
   and for a given upper bound, LMI (4.8) true **then**
3. Compute, the control gain matrices $\mathcal{K}_m = \mathcal{H}_m Q^{-1}$, $m = 1, 2$
4. **else**
5. **repeat** with a superior upper limit
6. **end if**
7. $N = 0$ to End time
8. **Input:** In Examples, the system values are shown
9. **for** $i = 1$ to N **do**
10. Derive the system by R-K fourth order method
11. **end for**
12. **end function**

5. Numerical examples

The validation of the proposed results are carried out in two examples.

Example 5.1. Consider the network of agents with one leader and four followers, which is described as

$$\dot{\wp}_i(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \wp_i(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_i(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega_i(t), \ i = 1,2,3,4, \qquad y_i(t) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \wp_i(t).$$

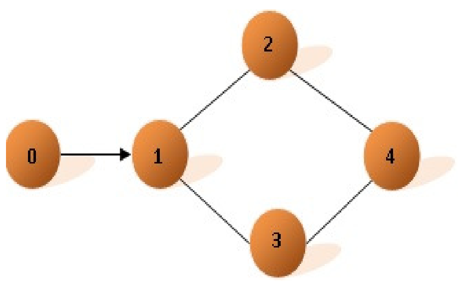

Figure 1: Topology structure of MASs.

The leaders' adjacency matrix and Laplacian matrix are identified as:

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}.$$

In case 1, assume $\omega(t) = 0.01 * \cos t$, we have chosen parameters as $\theta_1 = 0.01$, $\theta_2 = 0.07$, $\mathbb{F} = 0.5$, and $\gamma = 0.05$. The following gain matrices are generated using MATLAB LMI control toolbox by solving the LMI (4.8) in Theorem 4.3, and their gain matrices are derived as

$$\mathcal{K}_1 = \begin{bmatrix} -1.4133 & 0.2715 \end{bmatrix}, \qquad \mathcal{K}_2 = \begin{bmatrix} 1.6281 & -0.2604 \end{bmatrix}.$$

In case 2, assume $\omega(t) = 0.01 * \cos t$, we have chosen parameters as $\theta_1 = 0.01$, $\theta_2 = 0.07$, $\mathbb{F} = 0.5$, and $\gamma = 0.07$. The following gain matrices are generated using MATLAB LMI control toolbox by solving the LMI (4.8) in Theorem 4.3, and their gain matrices are derived as

$$\mathcal{K}_1 = \begin{bmatrix} -1.2851 & 0.5281 \end{bmatrix}, \qquad \mathcal{K}_2 = \begin{bmatrix} 1.8432 & -0.1706 \end{bmatrix}.$$

Table 1: The comparison table of transmission delay γ.

Method	γ
[27]	0.001
Theorem 4.3 (Based on Remark 4.2 Case 1)	0.05
Theorem 4.3 (Based on Remark 4.2 Case 2)	0.07

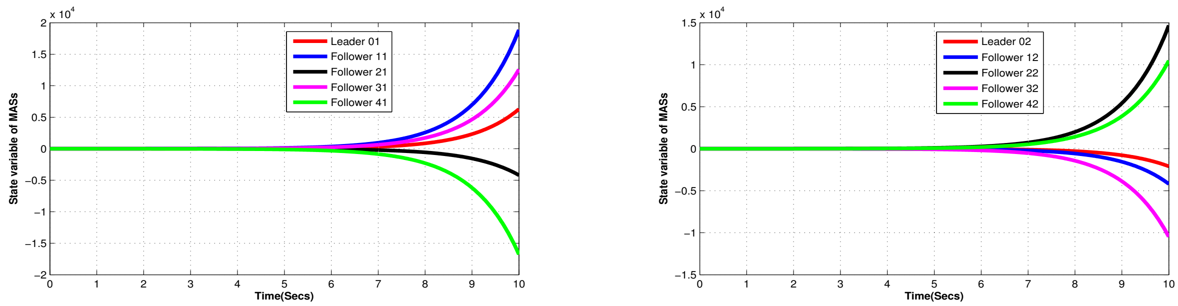

Figure 2: The state evolution of MASs with $u_i(t) = 0$.

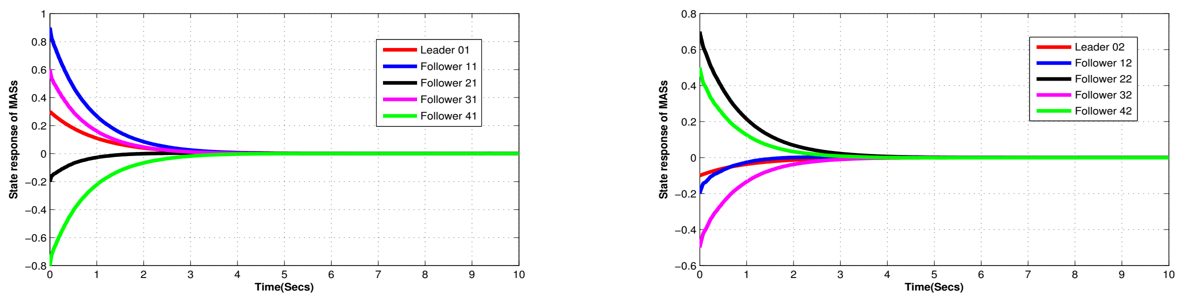

Figure 3: The plot of MASs $\wp_{i1}(t)$, $\wp_{i2}(t)$ $(i = 0, 1, \dots, 4)$.

For the present circumstances, the agents are chosen to be of $\wp_0(0) = [0.3, -0.1]^T$, $\wp_1(0) = [0.9, -0.2]^T$, $\wp_2(0) = [-0.2, 0.7]^T$, $\wp_3(0) = [0.6, -0.5]^T$, and $\wp_4(0) = [-0.8, 0.5]^T$, the consensus of the network is plotted in Fig. 1. Fig. 3 shows the closed-loop system's (3.4) state responses in the presence of control input (3.3). Using the MFSC approach, Fig. 4 shows the progression of the leader and four followers. Fig. 5 depicts the stochastic variable $\mathbb{F}(t)$. Additionally, Fig. 2 shows how the system's (3.4) state responds in the absence of a control input (3.3). The comparison table of transmission delay is presented in Table 1.

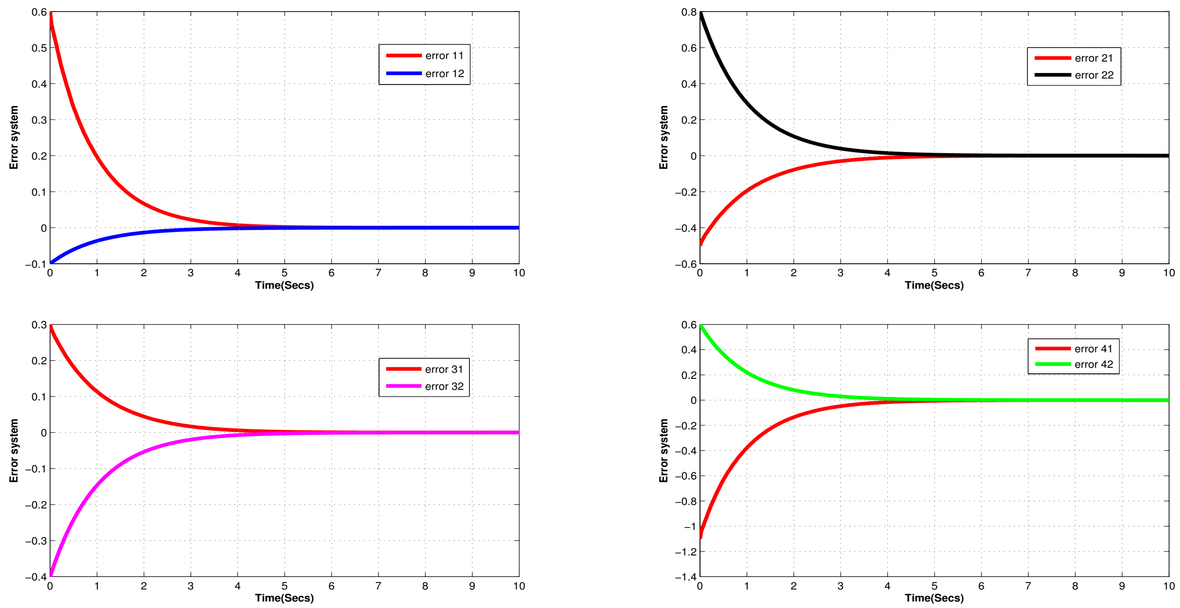

Figure 4: The plot of error $\mathfrak{R}_i(t)$ under a MSFC.

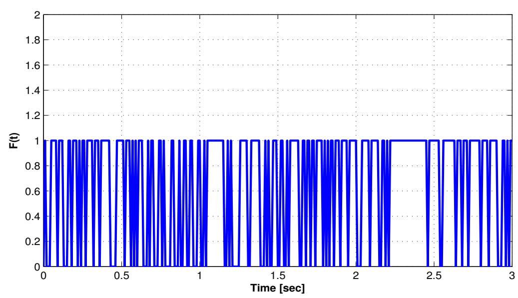

Figure 5: Stochastic Variable of $\mathbb{F}(t)$.

Example 5.2. Consider the network of agents with one leader and four followers, which is described as

$$\dot{\wp}_i(t) = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \wp_i(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_i(t).$$

The leaders' adjacency matrix and Laplacian matrix are identified in Fig. 1,

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In case 1, we chose the parameters as $\theta_1 = 0.02$, $\theta_2 = 0.06$, $\mathbb{F} = 0.5$, and $\gamma = 0.15$. The following gain matrices are generated using MATLAB LMI control toolbox by solving the LMI (4.10) in Theorem 4.4, and their gain matrices are derived as

$$\mathcal{K}_1 = \begin{bmatrix} -1.2153 & 0.5414 & 0.7234 \end{bmatrix}, \quad \mathcal{K}_2 = \begin{bmatrix} 0.2316 & -0.6525 & -0.9163 \end{bmatrix}.$$

In case 2, we chose the parameters as $\theta_1 = 0.02$, $\theta_2 = 0.06$, $\mathbb{F} = 0.5$, and $\gamma = 0.21$. The following gain matrices are generated using MATLAB LMI control toolbox by solving the LMI (4.10) in Theorem 4.4, and their gain matrices are derived as

$$\mathcal{K}_1 = \begin{bmatrix} -0.5427 & 0.7714 & 0.8293 \end{bmatrix}, \quad \mathcal{K}_2 = \begin{bmatrix} 0.5169 & -0.3682 & -0.4283 \end{bmatrix}.$$

Table 2: The comparison table of transmission delay γ.

Method	γ
[27]	0.001
Theorem 4.4 (Based on Remark 4.2 Case 1)	0.15
Theorem 4.4 (Based on Remark 4.2 Case 2)	0.21

For the present circumstances, the agents are chosen to be of $\wp_0(0) = [-0.6, -0.7, 0.1]^T$, $\wp_1(0) = [0.4, -0.3, 0.2]^T$, $\wp_2(0) = [-0.5, 0.5, 0.3]^T$, $\wp_3(0) = [0.7, 0.2, -0.4]^T$, and $\wp_4(0) = [-0.1, 0.8, 0.5]^T$, the consensus of the network is plotted in Fig. 1. Fig. 7 shows the closed-loop system's (4.9) state responses in the presence of control input (3.3). Using the MFSC approach, Fig. 8 shows the progression of the leader and four followers. Fig. 9 depicts the stochastic variable $\mathbb{F}(t)$. Additionally, Fig. 6 shows how the system's (4.9) state responds in the absence of a control input (3.3). The comparison table of transmission delay is presented in Table 2.

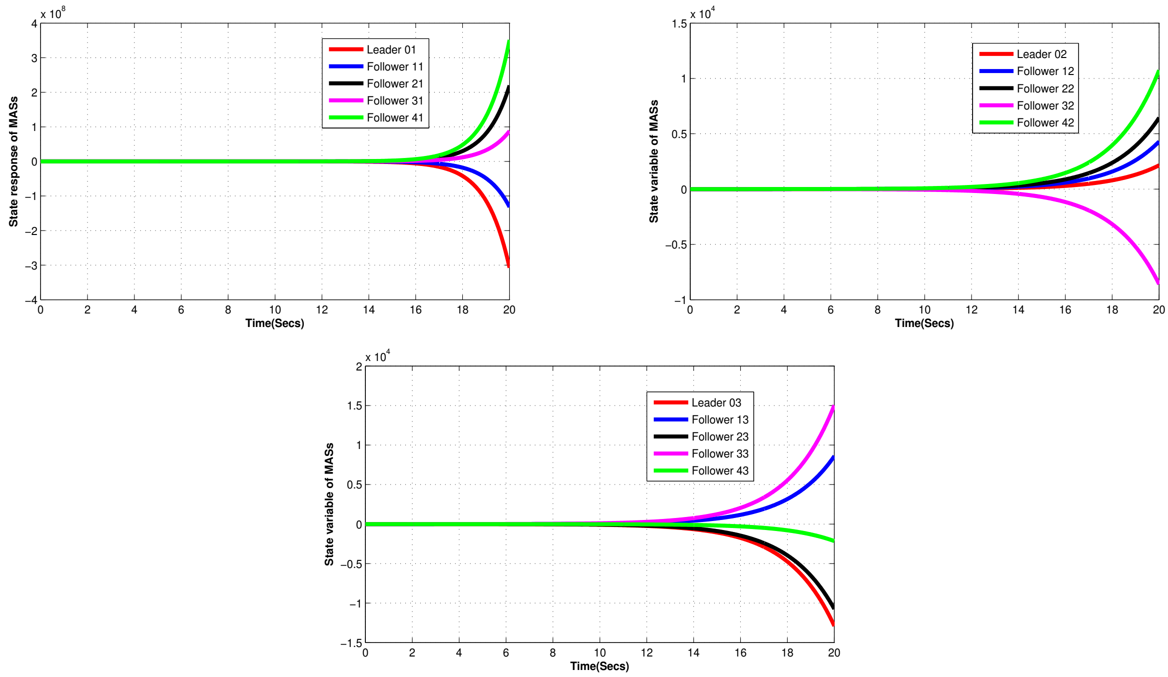

Figure 6: The state responses of MASs with $u_i(t) = 0$.

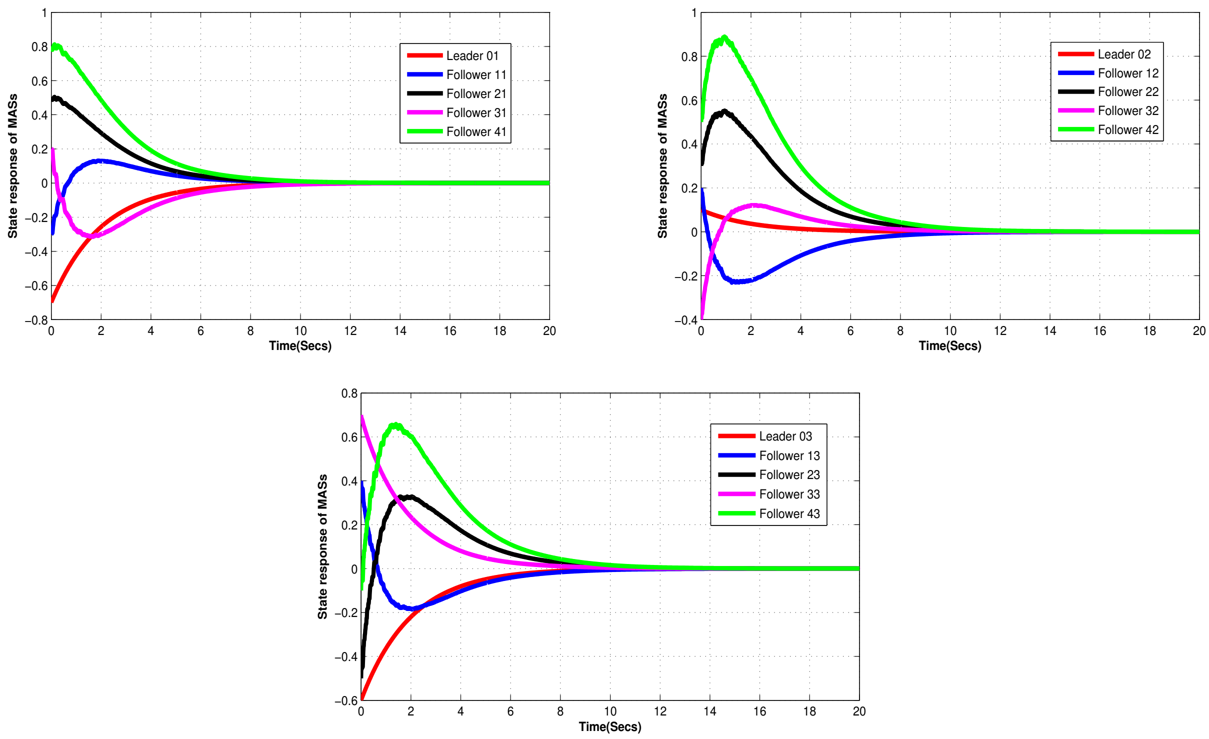

Figure 7: The plot of MASs $\wp_{i1}(t)$, $\wp_{i2}(t)$, $\wp_{i3}(t)$ $(i = 0, 1, \ldots, 4)$.

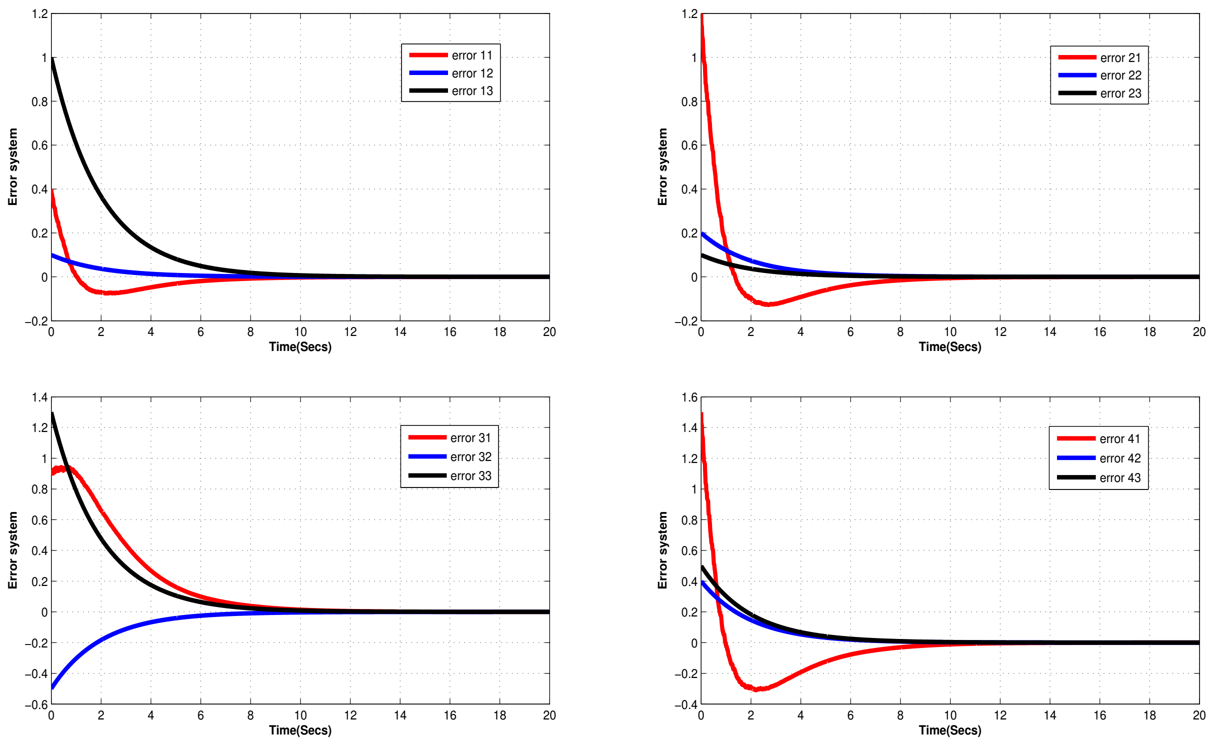

Figure 8: The plot of error $\mathfrak{R}_i(t)$ under a MSFC.

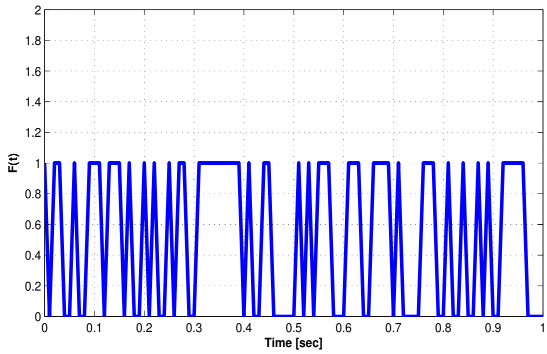

Figure 9: Stochastic Variable of $\mathbb{F}(t)$.

6. Conclusion

A novel MSFC consensus for MASs with transmission delay and disturbance under the communication of undirected graphs has been examined in this article. MSF control protocols for the aforementioned MASs have been proposed based on graph theory concepts. Then, using the LKF method, some innovative delay-dependent conditions were discovered, and it was demonstrated that these criteria ensured that the closed-loop system would eventually reach the mean-square consensus. In the meantime, these circumstances are incorporated to construct the needed MSFC, which is deployed in the specified LMI. Finally, a simulation example has been expressed to validate the suggested control technique.

In [4, 5, 36], the authors delved into a discussion concerning the implementation of a SDC approach and its impulsive effects in the context of cyber attacks. Subsequently, in [3], the authors provided a detailed explanation of T-S fuzzy systems in conjunction with SMC techniques. Drawing inspiration from the insights presented in [3–5, 36], the proposed control methodology is poised to find applications in T-S fuzzy fractional MASs through an innovative approach. This research introduces a novel method that will be employed to investigate the application of H_∞ SMC for T-S fuzzy MASs in the presence of cyber attacks in future work.

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