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A topological tool to develop novel rough set

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Abstract

Here, we introduce a new rough set model-building topological method. This concept is based on "somewhat open sets," one of the popular generalizations of open sets. First, we create a few topologies using different kinds of $\mathcal{M}_{\xi}$-adhesion neighborhoods. Then, we create new kinds of rough approximations and accuracy metrics with respect to somewhat closed and somewhat open sets. We examine their key characteristics and demonstrate that the monotonic requirement is maintained by the accuracy and roughness metrics. Their ability to be compared is one of their special qualities. We demonstrate that our method is more accurate than those resulting from open, α-open, and semi-open sets by comparing it with the previous approaches. We also evaluate the applicability of the technique in a heart failure problem. Lastly, we evaluate the benefits and drawbacks of our approach and make some recommendations for further research.

Keywords: Topology, somewhat open set, $\mathcal{M}_{\xi}$-adhesion neighborhood space, lower/upper approximation, accuracy, rough set.

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1. Introduction

To deal with unclear knowledge, Pawlak [37] suggested rough set theory as a nonstatistical technique. Rough set theory uses classifications (upper/lower approximations) and accuracy measures to describe each subset. Calculations are made using a boundary region, also referred to as the difference between the upper and lower approximations, we may establish whether the subset is exact or not. Without knowing its size, the set's approximations provide some details about the shape of the boundary region. The set's accuracy measure provides an answer to the question of how thorough our knowledge is by displaying the boundary region size without describing its structure. As far as we know, rough set theory starts with an equivalency relation, which seems to be a strict requirement that restricts the rough set's applicability. To remedy this unreasonableness, some extensions under other relations were proposed, such as those in [51, 52]. New neighborhood types, including minimal right (left) [5, 6], union (intersection) [1], maximal [20], remote [47], P_j [35], E_j [13], C_j-neighborhoods [8], and most recently S_j-neighborhoods [12], have been introduced for a variety of reasons, including improving the set's accuracy values. The ideas are specified using rough sets based on the knowledge we have of them. For instance, if two sets with different

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items have the same upper and/or lower approximations, we can claim that they are roughly equivalent. Instead of comparing sets based on their elements, we can compare them based on their closure and interior points, these ideas pertain to the topological spaces. Skowron [46] and Wiweger [49] investigated rough set theory in this direction in light of topological concepts. In order to expand the key ideas in rough set theory, Lashin et al. [33] built a topology from binary relations. Rough approximations and topology were used by Abu-Donia [2] to present a number of knowledge bases. The problem of insufficient attribute values was addressed by Salama [43] using topological concepts. Kondo [30] discussed a few methods for building topologies out of approximation space covers. In [11], the authors used topological spaces generated by the system of N_j-neighborhoods to investigate separation axioms. El-Bably and Al-shami [22] demonstrated a few methods for constructing a topology out of various kinds of neighborhoods. They also talked about how generalized nanotopology might be used in medicine. Numerous works, including [3, 14, 18, 24, 27, 31, 32, 34, 44, 45, 53], focused on topology and rough set theory's link. Various topological extensions, include bitopology [39] and minimum structure [19, 23], were covered in this line of research. In [38, 39], hybridization of rough sets with various techniques for reducing uncertainty, including soft and fuzzy sets, was examined. One of the main fields of topological research is near open sets. The basic topological notions of compactness, connectedness, and separation axioms are redefined using them. With the aid of some near open sets, new types of topological approximations in the cases of fore-set and after-set discussed in [1]. Five different near open set types were used by Amer et al. [17] to create novel topological approximations. By developing novel topological approximations using $\delta\beta$-open sets and $\wedge_\beta$-sets, Hosny [25] showed that her approaches were more accurate than those of Amer et al. Salama [42] established higher order sets as a novel family of nearly closed and open sets by repeated cycles of closure and interior operators. Al-Shami [9, 10] has recently taken advantage of one of the expansions of open sets known as somewhere dense and somewhat open sets to enhance rough subset approximations and the measurements of accuracy. This paper contributes to this technique by introducing new rough set models depending on the concept of topology known as "somewhat open sets." It makes sense to wonder why these models were introduced. There are actually four primary reasons to research these models. First, the approximations and the measurements of accuracy provided in the written literature should be improved. This point was demonstrated using several comparisons that demonstrate that our method outperforms those reported in [1, 17, 41]. Second, to keep the majority of Pawlak's approximations' attributes that the earlier approximations have eliminated, as shown in Propositions 3.6 and 3.7. Third, as demonstrated in Proposition 3.10 and Corollary 3.11, to keep the monotonic property for the accuracy and roughness measurements without additional restrictions. Because they are specified using interior and closure operators, both of which compete for set size, the types of approximations and the measures of accuracy inspired from the other generalizations are not guaranteed to have this desired attribute. Lastly, we can contrast between the various types of "ξso-approximations" and "ξso-accuracy measures" (as examined in Proposition 3.22 and Corollary 3.23). The structure of this document is as follows. The concepts and a few topological space and rough set characteristics that aid in understanding this study is discussed in Section 2. Section 3, the main portion, is broken down into three subsections. In the first subsection, we describe and investigate new categories of approximations and the measures of accuracy using somewhat open and somewhat closed sets. In the second subsection, we contrast the adopted technique with the ones that came before it in terms of approximations and the measures of accuracy. In the third subsection, we apply our method to a medical problem. In Section 4, we examine the benefits of our strategy and outline its shortcomings in comparison to earlier approaches. In Section 6, we conclude with some recommendations for additional research.

2. Preliminaries

In this part, we will go over the essential definitions and findings of topology and rough set theory that we will need for this study.

Definition 2.1 ([37])**.** Suppose we are given equivalence relation Ω in a finite set $\aleph \neq \phi$. We associate two subsets:

$\overline{\Omega}(\Psi) = \bigcup\{Y \in \aleph/\Omega : Y \text{ and } \Psi \text{ has an intersection that is not empty}\}$,

$\underline{\Omega}(\Psi) = \bigcup\{Y \in \aleph/\Omega : Y \text{ is a subset of } \Psi\}$ called the upper and lower approximation ofΨ, for each $\Psi \subseteq \aleph$.

From this point forward, unless otherwise stated, we assume $\aleph$ to be a non-empty finite set. The next outcome gives a description of the main characteristics of these approximations.

Proposition 2.2 ([37]). *Consider the equivalence relation Ω in $\aleph$ and $\Psi, \Upsilon \subseteq \aleph$. The following characteristics are met.*

(1) $\underline{\Omega}(\Psi) \subseteq \Psi$;
(2) $\Psi \subseteq \overline{\Omega}(\Psi)$;
(3) $\underline{\Omega}(\phi) = \phi$;
(4) $\overline{\Omega}(\phi) = \phi$;
(5) $\underline{\Omega}(\aleph) = \aleph$;
(6) $\overline{\Omega}(\aleph) = \aleph$;
(7) *if* $\Psi \subseteq \Upsilon$, *hence* $\underline{\Omega}(\Psi) \subseteq \underline{\Omega}(\Upsilon)$;
(8) *if* $\Psi \subseteq \Upsilon$, *hence* $\overline{\Omega}(\Psi) \subseteq \overline{\Omega}(\Upsilon)$;
(9) $\underline{\Omega}(\Psi \cap \Upsilon) = \underline{\Psi} \cap \underline{\Upsilon}$;
(10) $\overline{\Omega}(\Psi \cap \Upsilon) \subseteq \overline{\Psi} \cap \overline{\Upsilon}$;
(11) $\underline{\Omega}(\Psi) \cup \underline{\Omega}(\Upsilon) \subseteq \underline{\Omega}(\Psi \cup \Upsilon)$;
(12) $\overline{\Omega}(\Psi \cup \Upsilon) = \overline{\Omega}(\Psi) \cup \overline{\Omega}(\Upsilon)$;
(13) $\underline{\Omega}(\Psi^c) = (\overline{\Omega}(\Psi))^c$;
(14) $\overline{\Omega}(\Psi^c) = (\underline{\Omega}(\Psi))^c$;
(15) $\underline{\Omega}(\underline{\Omega}(\Psi)) = \underline{\Omega}(\Psi)$;
(16) $\overline{\Omega}(\overline{\Omega}(\Psi)) = \overline{\Omega}(\Psi)$;
(17) $\underline{\Omega}((\underline{\Omega}(\Psi))^c) = (\overline{\Omega}(\Psi))^c$;
(18) $\overline{\Omega}(\overline{(\overline{\Omega}(\Psi))^c}) = (\overline{\Omega}(\Psi))^c$;
(19) *for each* $Y \in \aleph/\Omega \Rightarrow \underline{\Omega}(Y) = Y$;
(20) *for each* $Y \in \aleph/\Omega \Rightarrow \overline{\Omega}(Y) = Y$.

Definition 2.3 ([1, 5, 6, 51, 53]). In an arbitrary relation Ω on $\aleph$, where $\xi = \xi_i, i \in \{1, 2, \ldots, 8\}$, the ξ-neighborhoods of an $\mu \in \aleph$ (denoted by $M_\xi(\mu)$) are defined as follows.

(1) $M_{\xi_1}(\mu) = \{\nu \in \aleph : \mu\zeta\nu\}$.
(2) $M_{\xi_2}(\mu) = \{\nu \in \aleph : \nu\zeta\mu\}$.
(3) $M_{\xi_3}(\mu) = \begin{cases} \cap_{\mu \in m_{\xi_1}(\nu)} m_{\xi_1}(\nu), & \exists m_{\xi_1}(\nu) \text{ containing } \mu, \\ \phi, & \text{otherwise.} \end{cases}$
(4) $M_{\xi_4}(\mu) = \begin{cases} \cap_{\mu \in m_{\xi_2}(\nu)} m_{\xi_2}(\nu), & \exists m_{\xi_2}(\nu) \text{ containing } \mu, \\ \phi & \text{otherwise.} \end{cases}$.
(5) $M_{\xi_5}(\mu) = M_{\xi_1}(\mu) \cap M_{\xi_2}(\mu)$.
(6) $M_{\xi_6}(\mu) = M_{\xi_1}(\mu) \cup M_{\xi_2}(\mu)$.
(7) $M_{\xi_7}(\mu) = M_{\xi_3}(\mu) \cap M_{\xi_4}(\mu)$.
(8) $M_{\xi_8}(\mu) = M_{\xi_3}(\mu) \cup M_{\xi_4}(\mu)$.

From now on, we consider $\xi = \xi_i, i \in \{1, 2, \ldots, 8\}$, unless otherwise specified.

Definition 2.4 ([1]). Let Ω represent any relation on $\aleph$, and let δ_ξ represent a map from $\aleph$ to $2^\aleph$ connecting each $\mu \in \aleph$ to its ξ-neighborhood in $2^\aleph$. The triple $(\aleph, \Omega, \delta_\xi)$ is referred to as a ξ-neighborhood space (shorthand, ξ-NS).

The following theorem offers one of the significant and intriguing ways to create topological spaces using the neighborhood idea. More comments between the concepts of the space of topology and theory of rough set is also made possible by this.

Theorem 2.5 ([50]). *A class $\omega_\xi = \{Y \subseteq \aleph : M_\xi(\mu) \subseteq Y, \forall \mu \in Y\}$ constitutes a topology on $\aleph$ for every ξ if $(\aleph, \Omega, \delta_\xi)$ is a ξ-NS.*

Definition 2.6 ([1]). The members of ω_ξ are named ξ-open sets and the complement of ξ-open sets are ξ-closed sets. And we symbolize the family of all ξ-closed sets by the symbols Γ_ξ.

The rough approximation meanings that follow are topologically appealing.

Definition 2.7 ([1]). The formulations for the ξ-lower and ξ-upper approximations of a set Ψ in a ξ-NS $(\aleph, \Omega, \delta_\xi)$ are as, respectively, $\underline{\Omega}_\xi(\Psi) = \bigcup\{Y \in \omega_\xi : Y \text{ is a subset of } \Psi\}$ and $\overline{\Omega}_\xi(\Psi) = \bigcap\{X \in \Gamma_\xi : \Psi \text{ is a subset of } X\}$.

In a topological structure $(\aleph, \omega_\xi)$, $\underline{\Omega}_\xi(\Psi)$ and $\overline{\Omega}_\xi(\Psi)$ are obviously, the interior and closure of Ψ. So, we write $\underline{\Omega}_\xi(\Psi) = \text{int}_\xi(\Psi)$ and $\overline{\Omega}_\xi(\Psi) = \text{cl}_\xi(\Psi)$.

Definition 2.8 ([1]). In a ξ-NS$(\aleph, \Omega, \delta_\xi)$, the ξ-positive, ξ-negative, ξ-boundary regions, ξ-accuracy, and ξ-roughness measure of a set Ψ are formulated as, respectively, $POS_\xi(\Psi) = \underline{\Omega}_\xi(\Psi)$, $NEG_\xi(\Psi) = \aleph \backslash \overline{\Omega}_\xi(\Psi)$, $B_\xi(\Psi) = \overline{\Omega}_\xi(\Psi) \backslash \underline{\Omega}_\xi(\Psi)$, $A_\xi(\Psi) = \frac{|\underline{\Omega}_\xi(\Psi)|}{|\overline{\Omega}_\xi(\Psi)|}$ such that $\overline{\Omega}_\xi(\Psi) \neq \phi$, and $R_\xi(\Psi) = 1 - A_\xi(\Psi)$. It is evident that for any $\Psi \subseteq \aleph$, $R_\xi(\Psi) \in [0, 1]$.

Definition 2.9 ([7, 16]). In a topological structure $(\aleph, \delta)$, a set Ψ is named:

(1) α-open if $\Psi \subseteq \text{int}(\text{cl}(\text{int}(\Psi)))$;
(2) semi-open if $\Psi \subseteq \text{cl}(\text{int}(\Psi))$;
(3) somewhat open if $\text{int}(\Psi) \neq \phi$;
(4) somewhere dense if $\text{int}(\text{cl}(\Psi)) \neq \phi$;
(5) α-closed if Ψ^c is α-open;
(6) semi-closed if Ψ^c is semi-open;
(7) somewhat closed if Ψ^c is somewhat open;
(8) closed somewhere dense if Ψ^c is somewhere dense.

Definition 2.10 ([17, 41]). A subset Ψ of a ξ-NS $(\aleph, \Omega, \delta_\xi)$ is named $\xi\alpha$-open (resp. ξ-semi-open) if $\Psi \subseteq \text{int}_\xi(\text{cl}_\xi(\text{int}_\xi(\Psi)))$ (resp. $\Psi \subseteq \text{cl}_\xi(\text{int}_\xi(\Psi))$). The complement of Ψ is named $\xi\alpha$-closed (resp. ξ-semi-closed).

Remark 2.11. The classes of ξ-semi-closed, ξ-semi-open, $\xi\alpha$-open, and $\xi\alpha$-closed are denoted by $\text{semiC}(\Gamma_\xi)$, $\text{semiO}(\omega_\xi)$, $\alpha O(\omega_\xi)$, and $\alpha C(\Gamma_\xi)$, successively.

Definition 2.12 ([17, 41]). The $\xi\jmath$-lower and $\xi\jmath$-upper approximations of a set in a ξ-NS$(\aleph, \Omega, \delta_\xi)$ are defined, successively, for every $\jmath \in \{\alpha, \text{semi}\}$, $\underline{\Omega^\jmath}_\xi(\Psi) = \bigcup\{Y \in \jmath O(\omega_\xi) : Y \subseteq \Psi\} = \jmath\text{int}_\xi(\Psi)$ and $\overline{\Omega^\jmath}_\xi(\Psi) = \bigcap\{X \in \jmath C(\Gamma_\xi) : \Psi \subseteq X\} = \jmath\text{cl}_\xi(\Psi)$.

From this point on, if nothing else is said, we assume that $\jmath \in \{\alpha, \text{semi}\}$.

Definition 2.13 ([17, 41]). In a ξ-NS$(\aleph, \Omega, \delta_\xi)$, the $\xi\jmath$-positive, $\xi\jmath$-negative, $\xi\jmath$-boundary regions, $\xi\jmath$-accuracy, and $\xi\jmath$-roughness measure of a set Ψ are formulated as, successively, $POS^\jmath_\xi(\Psi) = \underline{\Omega}^\jmath_\xi(\Psi)$, $NEG^\jmath_\xi(\Psi) = \aleph \backslash \overline{\Omega}^\jmath_\xi(\Psi)$, $B^\jmath_\xi(\Psi) = \overline{\Omega}^\jmath_\xi(\Psi) \backslash \underline{\Omega}^\jmath_\xi(\Psi)$, $A^\jmath_\xi(\Psi) = \frac{|\underline{\Omega}^\jmath_\xi(\Psi)|}{|\overline{\Omega}^\jmath_\xi(\Psi)|}$ such that $\overline{\Omega}^\jmath_\xi(\Psi) \neq \phi$, and $R^\jmath_\xi(\Psi) = 1 - A^\jmath_\xi(\Psi)$. It is evident that for any $\Psi \subseteq \aleph$, $R^\jmath_\xi(\Psi) \in [0, 1]$.

Definition 2.14 ([16]). Regarding a subset Ψ of $(\aleph, \omega)$:

(1) the union of all somewhat open subgroups of Ψ is called the *sw*-interior of Ψ (also known as swint(Ω));
(2) the intersection of all supersets of Ψ that are somewhat closed is *sw*-closure of Ψ (shortly, swcl(Ω)).

As it is coming, when computing $POS^{\jmath}_{\xi}(\Psi), NEG^{\jmath}_{\xi}(\Psi), B^{\jmath}_{\xi}(\Psi), \underline{\Omega}^{\jmath}_{\xi}(\Psi), \overline{\Omega}^{\jmath}_{\xi}(\Psi), M_{\xi}(\mu)$, and $A^{\jmath}_{\xi}(\Psi)$ of two different ξ-NS$(\aleph, \Omega_1, \delta_{\xi})$ and ξ-NS$(\aleph, \Omega_2, \delta_{\xi})$, we write $(POS^{\jmath}_{1\xi}(\Psi), NEG^{\jmath}_{1\xi}(\Psi), B^{\jmath}_{1\xi}(\Psi), \underline{\Omega}^{\jmath}_{1\xi}(\Psi), \overline{\Omega}^{\jmath}_{1\xi}(\Psi)$, $M_{1\xi}(\mu), M_{2\xi}(\mu)$, and $A^{\jmath}_{1\xi}(\Psi))$ and $(POS^{\jmath}_{2\xi}(\Psi), NEG^{\jmath}_{2\xi}(\Psi), B^{\jmath}_{2\xi}(\Psi), \underline{\Omega}^{\jmath}_{2\xi}(\Psi), \overline{\Omega}^{\jmath}_{2\xi}(\Psi), A^{\jmath}_{2\xi}(\Psi))$.

Proposition 2.15 ([12])**.** *Suppose that ξ-NS$(\aleph, \Omega_1, \delta_{\xi})$ and $(\aleph, \Omega_2)$ are two ξ-NSs, with $\Omega_1 \subseteq \Omega_2$. Then, $M_{1\xi}(\mu) \subseteq M_{2\xi}(\mu)$ for every $\mu \in \aleph$ and $\xi \in \{\xi_1, \xi_2, \xi_5, \xi_6\}$.*

On the same approach of ξ-neighborhood we can deal with ξ-adhesion neighborhood which is an expansion of the neighborhood and its definition as indicated below.

Definition 2.16 ([18])**.** Suppose that Ω is a binary relation on $\aleph$. The ξ-adhesion neighborhood of element $\mu \in \aleph$ are listed below defined where, $\xi = \xi_i, i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$:

(1) $\mathcal{M}_{\xi_1}(\mu) = \{\nu \in \Psi : M_{\xi_1}(\mu) = M_{\xi_1}(\nu)\}$;
(2) $\mathcal{M}_{\xi_2}(\mu) = \{\nu \in \Psi : M_{\xi_2}(\mu) = M_{\xi_2}(\nu)\}$;
(3) $\mathcal{M}_{\xi_3}(\mu) = \{\nu \in \Psi : \bigcap_{\mu \in M_{\xi_1}(\nu)} M_{\xi_1}(\nu) = \bigcap_{\nu \in M_{\xi_1}(\mu)} M_{\xi_1}(\mu)\}$;
(4) $\mathcal{M}_{\xi_4}(\mu) = \{\nu \in \Psi : \bigcap_{\mu \in M_{\xi_2}(\nu)} M_{\xi_2}(\nu) = \bigcap_{\nu \in M_{\xi_2}(\mu)} M_{\xi_2}(\mu)\}$;
(5) $\mathcal{M}_{\xi_5}(\mu) = \mathcal{M}_{\xi_1}(\mu) \cap \mathcal{M}_{\xi_2}(\mu)$;
(6) $\mathcal{M}_{\xi_6}(\mu) = \mathcal{M}_{\xi_1}(\mu) \cup \mathcal{M}_{\xi_2}(\mu)$;
(7) $\mathcal{M}_{\xi_7}(\mu) = \mathcal{M}_{\xi_3}(\mu) \cap \mathcal{M}_{\xi_4}(\mu)$;
(8) $\mathcal{M}_{\xi_8}(\mu) = \mathcal{M}_{\xi_3}(\mu) \cup \mathcal{M}_{\xi_4}(\mu)$.

We refer to $(\aleph, \Omega, \delta_{\xi})$ as a ξ-adhesion neighborhood space.

3. New ξ-adhesion neighborhood, rough set models

In this part, Based on somewhat open and somewhat closed sets concepts, we create novel rough approximations and accuracy measurements, that are generalizations of open sets. We demonstrate their key features and show that our technique provides higher measures of accuracy and approximations than open, α-open, and semi-open sets [1, 17, 41]. In addition, We contrast the approximations produced by our method, demonstrating that the measures of accuracy provided in the instances of $\xi \in \{\xi_5, \xi_7\}$ are the best. Finally, we give a medical example of how approximations and accuracy measures might be improved by using somewhat open sets.

Theorem 3.1. *A class $\omega_{\xi} = \{Y \subseteq \aleph : \mathcal{M}_{\xi}(\mu) \subseteq Y, \forall \mu \in Y\}$ constitutes a topology on $\aleph$ for every ξ if $(\aleph, \Omega, \delta_{\xi})$ is a ξ-adhesion-NS.*

Definition 3.2. The members of ω_{ξ} are named ξ-open sets and the complement of ξ-open sets are ξ-closed sets. And we symbolize the family of all ξ-closed sets by the symbols Γ_{ξ}.

The following definitions of the rough approximations have a topological relish.

Definition 3.3. The formulations for the ξ-lower and ξ-upper approximations of a set Ψ in a ξ-adhesion-NS $(\aleph, \Omega, \delta_{\xi})$ are as, respectively, $\underline{\Omega}_{\xi}(\Psi) = \bigcup\{Y \in \omega_{\xi} : Y \text{ is a subset of } \Psi\}$ and $\overline{\Omega}_{\xi}(\Psi) = \bigcap\{X \in \Gamma_{\xi} : \Psi \text{ is a subset of } X\}$.

Definition 3.4. If $\text{int}_{\xi}(\Psi) \neq \phi$, a subset Ψ of a ξ-adhesion-NS $(\aleph, \Omega, \delta_{\xi})$ is called ξ-somewhat open and the complement of Ψ is known as ξ-somewhat closed.

$so(\omega_{\xi})$ and $sc(\omega_{\xi})$ denote the classes of ξ-somewhat open and ξ-somewhat closed sets, respectively.

Definition 3.5. ξso-lower/upper approximation of Ψ of ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$ denotes as $\underline{\Omega}_\xi^{so}$, $\overline{\Omega}_\xi^{so}$ which are explained as $\underline{\Omega}_\xi^{so}(\Psi) = \bigcup\{Y \in so(\omega_\xi) : Y \subseteq \Psi\}$ and $\overline{\Omega}_\xi^{so}(\Psi) = \bigcap\{X \in sc(\omega_\xi) : \Psi \subseteq X\}$. In the next two results, we clarify the key characteristics of the ξso-lower and ξso-upper approximations.

Proposition 3.6. *If Ψ, Υ are a subset of ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$, then, the following properties are met.*

(1) $\underline{\Omega}_\xi^{so}(\Psi) \subseteq \Psi$;
(2) $\underline{\Omega}_\xi^{so}(\phi) = \phi$;
(3) $\underline{\Omega}_\xi^{so}(\aleph) = \aleph$;
(4) *if* $\Psi \subseteq \Upsilon$, *then* $\underline{\Omega}_\xi^{so}(\Psi) \subseteq \underline{\Omega}_\xi^{so}(\Upsilon)$;
(5) $\underline{\Omega}_\xi^{so}(\Psi \cap \Upsilon) \subseteq \underline{\Omega}_\xi^{so}(\Psi) \cap \underline{\Omega}_\xi^{so}(\Upsilon)$;
(6) $\underline{\Omega}_\xi^{so}(\Psi) \cup \underline{\Omega}_\xi^{so}(\Upsilon) \subseteq \underline{\Omega}_\xi^{so}(\Psi \cup \Upsilon)$;
(7) $\underline{\Omega}_\xi^{so}(\Psi^c) = (\overline{\Omega}_\xi^{so}(\Psi))^c$;
(8) $\underline{\Omega}_\xi^{so}(\underline{\Omega}_\xi^{so}(\Psi)) = \underline{\Omega}_\xi^{so}(\Psi)$.

Proof. The evidence stems from the characteristics of a counterpart to ξso-near lower approximation $\underline{\Omega}_\xi^{so}$ called a sw-interior operator. □

Proposition 3.7. *If Ψ, Υ is a subset of ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$, then, the following properties are met.*

(1) $\Psi \subseteq \overline{\Omega}_\xi^{so}(\Psi)$;
(2) $\overline{\Omega}_\xi^{so}(\phi) = \phi$;
(3) $\overline{\Omega}_\xi^{so}(\aleph) = \aleph$;
(4) *if* $\Psi \subseteq \Upsilon$, *then* $\overline{\Omega}_\xi^{so}(\Psi) \subseteq \overline{\Omega}_\xi^{so}(\Upsilon)$;
(5) $\overline{\Omega}_\xi^{so}(\Psi \cap \Upsilon) \subseteq \overline{\Omega}_\xi^{so}(\Psi) \cap \overline{\Omega}_\xi^{so}(\Upsilon)$;
(6) $\overline{\Omega}_\xi^{so}(\Psi) \cup \overline{\Omega}_\xi^{so}(\Upsilon) \subseteq \overline{\Omega}_\xi^{so}(\Psi \cup \Upsilon)$;
(7) $\overline{\Omega}_\xi^{so}(\Psi^c) = (\underline{\Omega}_\xi^{so}(\Psi))^c$;
(8) $\overline{\Omega}_\xi^{so}(\overline{\Omega}_\xi^{so}(\Psi)) = \overline{\Omega}_\xi^{so}(\Psi)$.

Proof. The evidence stems from the characteristics of a counterpart to ξso-near lower approximation $\overline{\Omega}_\xi^{so}$ called a sw-closure operator. □

The following example validates this matter in the case of $\xi = \xi_1$, proving that the inclusion relations of 1 and (4)-(6) of Propositions 3.6 and 3.7 are appropriate.

Example 3.8. Let $(\aleph, \Omega, \delta_\xi)$ be a ξ-adhesion-NS, where $\xi = \{(fy, fy), (fz, fz), (fu, fv), (fu, fy), (fy, fv)\}$ is a relation on the universe $\aleph = \{fu, fv, fy, fz\}$, at hence, $\mathcal{M}_{\xi_1}(fu) = \mathcal{M}_{\xi_1}(fy) = \{fu, fy\}$, $\mathcal{M}_{\xi_1}(fv) = \{fv\}$ and $\mathcal{M}_{\xi_1}(fz) = \{fz\}$. A topology created from ξ_1-adhesion neighborhoods on $\aleph$ is $\omega_{\xi_1} = \{\aleph, \phi, \{fv\}, \{fz\}, \{fv, fz\}, \{fu, fy\}, \{fv, fy, fu\}, \{fu, fy, fz\}\}$ according to Theorem 3.1. Let $Y = \{fy\}$, $X = \{fu, fv\}$, $\Psi = \{fu, fz\}$, $\Upsilon = \{fu, fy\}$, and $\Sigma = \{fv, fz\}$. We obtain via calculation $\underline{\Omega}_{\xi_1}^{so}(Y) = \phi$, $\overline{\Omega}_{\xi_1}^{so}(Y) = \{fu, fy\}$, $\underline{\Omega}_{\xi_1}^{so}(X) = \overline{\Omega}_{\xi_1}^{so}(X) = X$, $\underline{\Omega}_{\xi_1}^{so}(\Psi) = \overline{\Omega}_\xi^{so}(\Psi) = \Psi$, $\underline{\Omega}_\xi^{so}(\Upsilon) = \phi$, $\overline{\Omega}_{\xi_1}^{so}(\Upsilon) = \Upsilon$, $\underline{\Omega}_{\xi_1}^{so}(\Sigma) = \Sigma$, and $\overline{\Omega}_{\xi_1}^{so}(\Sigma) = \aleph$.

We now observe the following:

(1) $Y \nsubseteq \underline{\Omega}_{\xi_1}^{so}(Y)$, $\overline{\Omega}_{\xi_1}^{so}(Y) \nsubseteq Y$;
(2) $\underline{\Omega}_{\xi_1}^{so}(Y) \subseteq \underline{\Omega}_{\xi_1}^{so}(X)$, but $Y \nsubseteq X$. Also, $\overline{\Omega}_{\xi_1}^{so}(X) \subseteq \overline{\Omega}_{\xi_1}^{so}(\Sigma)$, but $X \nsubseteq \Sigma$;
(3) $\underline{\Omega}_{\xi_1}^{so}(X) \cap \underline{\Omega}_{\xi_1}^{so}(\Psi)=\{fu\} \nsubseteq \underline{\Omega}_{\xi_1}^{so}(X \cap \Psi)=\phi$, also, $\overline{\Omega}_{\xi_1}^{so}(\Upsilon) \cap \overline{\Omega}_{\xi_1}^{so}(\Sigma)=\Upsilon \nsubseteq \overline{\Omega}_{\xi_1}^{so}(\Upsilon \cap \Sigma)=\phi$;
(4) $\underline{\Omega}_{\xi_1}^{so}(Y \cup X) = Y \cup X = \{fu, fv, fy\} \nsubseteq \underline{\Omega}_{\xi_1}^{so}(Y) \cup \underline{\Omega}_{\xi_1}^{so}(X) = \{fu, fv\}$, also, $\overline{\Omega}_{\xi_1}^{so}(\{fv\} \cup \{fz\}) = \aleph \nsubseteq \overline{\Omega}_{\xi_1}^{so}(\{fv\}) \cup \overline{\Omega}_{\xi_1}^{so}(\{fz\}) = \{fv, fz\}$.

Remark 3.9. In the presence of limited intersection, the class of somewhat open sets is closed if $(\aleph, \omega_\xi)$ is a hyperconnected space, which indicates that it forms a topology and the equivalence relations in Propositions 3.6 and 3.7, respectively, (5) and (6), are satisfied. The approximations named using dense sets [9] under substantially hyperconnected spaces maintain these characteristics. This suggests that even in a weaker environment, our strategy maintains all Pawlak features.

Proposition 3.10. *If $(\aleph, \Omega_1, \delta_\xi)$ and $(\aleph, \Omega_2, \delta_\xi)$ are two ξ-adhesion-NSs, where $\Omega_1 \subseteq \Omega_2$ and $\xi \in \{\xi_1, \xi_2, \xi_5, \xi_6\}$. At hence, $\omega_{2_\xi} \subseteq \omega_{1_\xi}$.*

Proof. Let Y, a subset of $\aleph$, be a member of ω_{2_ξ}, and $\xi \in \{\xi_1, \xi_2, \xi_5, \xi_6\}$. Then, $\mathcal{M}_{2\xi}(\mu) \subseteq Y$ for every $\mu \in Y$. Since $\Omega_1 \subseteq \Omega_2$, it follows from Proposition 2.15 that $\mathcal{M}_{1\xi}(\mu) \subseteq \mathcal{M}_{2\xi}(\mu)$. This implies that Y is a member in ω_{1_ξ}. Then, $\omega_{2_\xi} \subseteq \omega_{1_\xi}$. □

Corollary 3.11. *If $(\aleph, \Omega_1, \delta_\xi)$ and $(\aleph, \Omega_2, \delta_\xi)$ are two ξ-adhesion-NSs, where $\Omega_1 \subseteq \Omega_2$ and $\xi \in \{\xi_1, \xi_2, \xi_5, \xi_6\}$. At hence, the class of somewhat open sets in $(\aleph, \omega_{2_\xi})$ is a subset of the class of somewhat open sets in $(\aleph, \omega_{1_\xi})$.*

Definition 3.12. In a ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$, the ξ so-accuracy measure and ξ so-roughness measure of a set Ψ are defined, respectively, by $A^{so}_\xi(\Psi) = \frac{|\underline{\Omega}^{so}_\xi(\Psi)|}{|\overline{\Omega}^{so}_\xi(\Psi)|}$ such that $|\overline{\Omega}^{so}_\xi(\Psi)| \neq 0$ and $R^{so}_\xi(\Psi) = 1 - A^{so}_\xi(\Psi)$. Clearly, $A^{so}_\xi(\Psi)$, $R^{so}_\xi(\Psi) \in [0, 1]$ for each $\Psi \subseteq \aleph$.

We demonstrate the monotonicity of the A^{so}_ξ-accuracy and A^{so}_ξ-roughness measurements in the next two outcomes.

Proposition 3.13. *If $(\aleph, \Omega_1, \delta_\xi)$ and $(\aleph, \Omega_2, \delta_\xi)$ are two ξ-adhesion-NSs, where $\Omega_1 \subseteq \Omega_2$ and $\xi \in \{\xi_1, \xi_2, \xi_5, \xi_6\}$. At hence, $A^{so}_{1_\xi}(\Psi) \geqslant A^{so}_{2_\xi}(\Psi)$ for each set Ψ.*

Proof. Since $\underline{\Omega}^{so}_\xi(\Psi) = \text{swint}_\xi(\Psi)$ and $\overline{\Omega}^{so}_\xi(\Psi) = \text{swcl}_\xi(\Psi)$, from Corollary 3.11, it follows that $|\underline{\Omega}^{so}_{2_\xi}(\Psi)| \leqslant |\underline{\Omega}^{so}_{1_\xi}(\Psi)|$ and $\frac{1}{|\overline{\Omega}^{so}_{2_\xi}(\Psi)|} \leqslant \frac{1}{|\overline{\Omega}^{so}_{1_\xi}(\Psi)|}$. So, $\frac{|\underline{\Omega}^{so}_{2_\xi}(\Psi)|}{|\overline{\Omega}^{so}_{2_\xi}(\Psi)|} \leqslant \frac{|\underline{\Omega}^{so}_{1_\xi}(\Psi)|}{|\overline{\Omega}^{so}_{1_\xi}(\Psi)|}$ this implies $A^{so}_{1_\xi}(\Psi) \geqslant A^{so}_{2_\xi}(\Psi)$. □

Corollary 3.14. *If $(\aleph, \Omega_1, \delta_\xi)$ and $(\aleph, \Omega_2, \delta_\xi)$ are two ξ-adhesion-NSs, where $\Omega_1 \subseteq \Omega_2$ and $\xi \in \{\xi_1, \xi_2, \xi_5, \xi_6\}$, at hence, $R^{so}_{1_\xi}(\Psi) \leqslant R^{so}_{2_\xi}(\Psi)$ for each set Ψ.*

Definition 3.15. A subset Ψ of a ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$ is named ξso-exact if $\underline{\Omega}^{so}_\xi(\Psi) = \overline{\Omega}^{so}_\xi(\Psi) = \Psi$. If not, it is referred to as a ξso-rough set.

We easily remark that a $\xi\alpha$-exact (ξ semi-exact) set is a ξso-exact set due to the well-known correlations between these two sets, but the opposite as the accompanying example shows, this is not always the case.

Example 3.16. Let $\Psi = \{fu, fz\}$ be a set in ξ_1-adhesion-NS $(\aleph, \Omega, \delta_{\xi_1})$ seen in Example 3.8. As previously demonstrated, $\underline{\Omega}^{so}_{\xi_1}(\Psi) = \overline{\Omega}^{so}_{\xi_1}(\Psi) = \Psi$. At hence, Ψ is $\xi_1$so-exact set. And, $\underline{\Omega}^{semi}_{\xi_1}(\Psi) = \underline{\Omega}^{\alpha}_{\xi_1}(\Psi) = \{fz\} \neq \overline{\Omega}^{semi}_{\xi}(\Psi) = \overline{\Omega}^{\alpha}_{\xi_1}(\Psi) = \Psi$; so, Ψ is neither a $\xi_1\alpha$-exact set nor a $\xi_1$semi-exact set.

Proposition 3.17. *A set Ψ is ξso-exact in a ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$ iff $B^{so}_\xi(\Psi) = \phi$.*

Proof. If Ψ is ξso-exact set, then $B^{so}_\xi(\Psi) = \overline{\Omega}^{so}_{\xi_1}(\Psi) \setminus \underline{\Omega}^{so}_{\xi_1}(\Psi) = \overline{\Omega}^{so}_{\xi_1}(\Psi) \setminus \overline{\Omega}^{so}_{\xi_1}(\Psi) = \phi$. Conversely, if $B^{so}_\xi(\Psi) = \phi$, then, $\overline{\Omega}^{so}_{\xi_1}(\Psi) \setminus \underline{\Omega}^{so}_{\xi_1}(\Psi) = \phi$ this implies $\overline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_1}(\Psi)$, $\underline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_1}(\Psi)$ and hence, $\overline{\Omega}^{so}_{\xi_1}(\Psi) = \underline{\Omega}^{so}_{\xi_1}(\Psi)$. So, Ψ is ξso-exact. □

Definition 3.18. In a ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$, the ξso-boundary, ξso-positive, and ξso-negative regions of a set Ψ, respectively defined by $B^{so}_\xi(\Psi) = \overline{\Omega}^{so}_\xi(\Psi) \setminus \underline{\Omega}^{so}_\xi(\Psi)$, $POS^{so}_\xi(\Psi) = \underline{\Omega}^{so}_\xi(\Psi)$, $NEG^{so}_\xi(\Psi) = \aleph \setminus \overline{\Omega}^{so}_\xi(\Psi)$.

Proposition 3.13 provides the proof for the following proposition.

Proposition 3.19. *Let* $(\aleph, \Omega_1, \delta_\xi)$ *and* $(\aleph, \Omega_2, \delta_\xi)$ *are two* ξ*-adhesion-NSs, with* $\Omega_1 \subseteq \Omega_2$ *and* $\xi \in \{\xi_1, \xi_2, \xi_5, \xi_6\}$*. Thus, for each non-empty set, we obtain the results shown below:*

(1) $B^{so}_{1\xi}(\Psi) \subseteq B^{so}_{2\xi}(\Psi)$;
(2) $NEG^{so}_{\xi}(2\Psi) \subseteq NEG^{so}_{2\xi}(\Psi)$.

Proposition 3.20. *If* ω_1 *and* ω_2 *are two topologies on* $\aleph$*, such that* $\omega_1 \subseteq \omega_2$*, then,* $so(\omega_1) \subseteq so(\omega_2)$ *and* $sc(\omega_1) \subseteq sc(\omega_2)$*.*

Proof. Let Y be a subset of $\aleph$ and a set in $so(\omega_1)$. Then, $int_{\omega_1}(Y) \neq \phi$. By supposition $\omega_1 \subseteq \omega_2$, we get $int_{\omega_2}(Y) \neq \phi$. So, $Y \in so(\omega_2)$. At hence, $so(\omega_1) \subseteq so(\omega_2)$. Similar to that, it may be demonstrated that $sc(\omega_1) \subseteq sc(\omega_2)$. □

Corollary 3.21. *If* ω_1 *and* ω_2 *are two topologies on* $\aleph$*, such that* $\omega_1 \subseteq \omega_2$*, then,* $\text{swint}_{\omega_1}(\Psi) \subseteq \text{swint}_{\omega_2}(\Psi)$ *and* $\text{swcl}_{\omega_2}(\Psi) \subseteq \text{swcl}_{\omega_1}(\Psi)$ *foe all* $\Psi \subseteq \aleph$*.*

We can now demonstrate the following two results, which are a distinguishing feature of accuracy measures and approximations derived from somewhat open sets. They mainly show that the higher the number of given topologies, the better the accuracy measures.

Proposition 3.22. *If* $(\aleph, \Omega, \delta_\xi)$ *is* ξ*-adhesion-NS and* $\Psi \subseteq \aleph$*, then*

(1) $\underline{\Omega}^{so}_{\xi_6}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_5}(\Psi)$;
(2) $\underline{\Omega}^{so}_{\xi_6}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_2}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_5}(\Psi)$;
(3) $\underline{\Omega}^{so}_{\xi_8}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_3}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_7}(\Psi)$;
(4) $\underline{\Omega}^{so}_{\xi_8}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_4}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_7}(\Psi)$;
(5) $\overline{\Omega}^{so}_{\xi_5}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_6}(\Psi)$;
(6) $\overline{\Omega}^{so}_{\xi_5}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_2}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_6}(\Psi)$;
(7) $\overline{\Omega}^{so}_{\xi_7}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_3}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_8}(\Psi)$;
(8) $\overline{\Omega}^{so}_{\xi_7}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_4}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_8}(\Psi)$.

Proof. For (1), let $\mu \in \underline{\Omega}^{so}_{\xi_6}(\Psi)$. Then $\exists\, Y \in so(\omega_{\xi_6})$, where $\mu \in Y \subseteq \Psi$. Since $\omega_6 \subseteq \omega_1$, then $so(\omega_{\xi_6}) \subseteq so(\omega_{\xi_1})$ from Proposition 3.20. At hence, $\mu \in \underline{\Omega}^{so}_{\xi_1}(\Psi)$ and $\underline{\Omega}^{so}_{\xi_6}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_1}(\Psi)$. In a similar way, we demonstrate that $\underline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_5}(\Psi)$.

To demonstrate (5), allow $\mu \in \overline{\Omega}^{so}_{\xi_5}(\Psi)$. Then every somewhat closed set in ω_5 that contains μ has a non-empty intersection with Ψ. Because of $sc(\omega_1) \subseteq sc(\omega_5)$, every somewhat closed set in ω_1 containing μ has a non-empty intersection with Ψ. At hence, $\mu \in \overline{\Omega}^{so}_{\xi_1}(\Psi)$. Thus, $\overline{\Omega}^{so}_{\xi_5}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_1}(\Psi)$. Similarly, we prove that $\overline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_6}$. Similar justifications are used to support the remaining cases. □

Corollary 3.23. *If* Ψ *is a subset of* ξ*-adhesion-NS* $(\aleph, \Omega, \delta_\xi)$*, then*

(1) $A^{so}_{\xi_6}(\Psi) \leqslant A^{so}_{\xi_1}(\Psi) \leqslant A^{so}_{\xi_5}(\Psi)$;
(2) $A^{so}_{\xi_6}(\Psi) \leqslant A^{so}_{\xi_2}(\Psi) \leqslant A^{so}_{\xi_5}(\Psi)$;
(3) $A^{so}_{\xi_8}(\Psi) \leqslant A^{so}_{\xi_3}(\Psi) \leqslant A^{so}_{\xi_7}(\Psi)$;
(4) $A^{so}_{\xi_8}(\Psi) \leqslant A^{so}_{\xi_4}(\Psi) \leqslant A^{so}_{\xi_7}(\Psi)$.

Proof. We provide evidence for (1). The evidence in the other cases is similar. Since $\underline{\Omega}^{so}_{\xi_6}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \underline{\Omega}^{so}_{\xi_5}(\Psi)$, then $|\,\underline{\Omega}^{so}_{\xi_6}(\Psi)\,| \leqslant |\,\underline{\Omega}^{so}_{\xi_1}(\Psi)\,| \leqslant |\,\underline{\Omega}^{so}_{\xi_5}(\Psi)\,|$. Since $\overline{\Omega}^{so}_{\xi_5}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_1}(\Psi) \subseteq \overline{\Omega}^{so}_{\xi_6}(\Psi)$, we have $|\,\overline{\Omega}^{so}_{\xi_5}(\Psi)\,| \leqslant |\,\overline{\Omega}^{so}_{\xi_1}(\Psi)\,| \leqslant |\,\overline{\Omega}^{so}_{\xi_6}(\Psi)\,|$. So, $\frac{1}{|\overline{\Omega}^{so}_{\xi_6}(\Psi)|} \leqslant \frac{1}{|\overline{\Omega}^{so}_{\xi_1}(\Psi)|} \leqslant \frac{1}{|\overline{\Omega}^{so}_{\xi_5}(\Psi)|}$, and hence $\frac{|\underline{\Omega}^{so}_{\xi_6}(\Psi)|}{|\overline{\Omega}^{so}_{\xi_6}(\Psi)|} \leqslant \frac{|\underline{\Omega}^{so}_{\xi_1}(\Psi)|}{|\overline{\Omega}^{so}_{\xi_1}(\Psi)|} \leqslant \frac{|\underline{\Omega}^{so}_{\xi_5}(\Psi)|}{|\overline{\Omega}^{so}_{\xi_5}(\Psi)|}$. The proof is so finished. □

ξ-adhesion-NS $(\aleph, \Omega, \delta_\xi)$ stated in Example 3.8 is taken into consideration in order to verify the outcomes in the aforementioned proposition and corollary. We begin by computing the various N_ξ-adhesion neighborhoods categories in Table 1.

Table 1: N_ξ-adhesion neighborhoods of all element in $\aleph$.

	fu	fv	fy	fz
$\mathcal{M}m_{\xi_1}$	$\{fu, fy\}$	$\{fv\}$	$\{fu, fy\}$	$\{fz\}$
$\mathcal{M}m_{\xi_2}$	$\{fu\}$	$\{fv, fy\}$	$\{fv, fy\}$	$\{fz\}$
$\mathcal{M}m_{\xi_3}$	$\{fu, fy\}$	$\{fv\}$	$\{fu, fy\}$	$\{fz\}$
$\mathcal{M}m_{\xi_4}$	$\{fu\}$	$\{fv, fy\}$	$\{fv, fy\}$	$\{fz\}$
$\mathcal{M}m_{\xi_5}$	$\{fu\}$	$\{fv\}$	$\{fy\}$	$\{fz\}$
$\mathcal{M}m_{\xi_6}$	$\{fu, fy\}$	$\{fv, fy\}$	$\{fu, fv, fy\}$	$\{fz\}$
$\mathcal{M}m_{\xi_7}$	$\{fu\}$	$\{fv\}$	$\{fy\}$	$\{fz\}$
$\mathcal{M}m_{\xi_8}$	$\{fu, fy\}$	$\{fv, fy\}$	$\{fu, fv, fy\}$	$\{fz\}$

The topologies derived from these neighborhoods are then determined by using Theorem 2.5 as follows:

$$\begin{aligned}
\omega_{\xi_1} &= \{\aleph, \phi, \{fv\}, \{fz\}, \{fv, fz\}, \{fu, fy\}, \{fv, fy, fu\}, \{fu, fy, fz\}\},\\
\omega_{\xi_2} &= \{\phi, \aleph, \{fu\}, \{fz\}, \{fu, fz\}, \{fv, fy\}, \{fu, fv, fy\}, \{fv, fy, fz\}\},\\
\omega_{\xi_3} &= \{\phi, \aleph, \{fv\}, \{fz\}, \{fu, fy\}, \{fv, fz\}, \{fu, fv, fy\}, \{fu, fy, fz\}\},\\
\omega_{\xi_4} &= \{\phi, \aleph, \{fu\}, \{fz\}, \{fu, fz\}, \{fv, fy\}, \{fu, fv, fy\}, \{fv, fy, fz\}\},\\
\omega_{\xi_5} &= \wp(\aleph),\\
\omega_{\xi_6} &= \{\phi, \aleph, \{fz\}, \{fu, fv, fy\}\},\\
\omega_{\xi_7} &= \wp(\aleph),\\
\omega_{\xi_8} &= \{\phi, \aleph, \{fz\}, \{fu, fv, fy\}\}.
\end{aligned}$$

4. Comparison of our method to earlier approaches

In this part, we contrast our strategy with the earlier ones debuted in [1, 17, 41]. The writers of [1] used interior and closure topological operators to approximate a subset, while the authors of [17, 41] estimated a subset using generalizations of interior and closure topological operators such as α-interior and α-closure, as well as semi-interior and semi-closure topological operators. In this section, we demonstrate that our approach outperforms techniques In terms of approximations and accuracy measures, approaches induced from open sets as provided in [1] and approaches induced from α-open and semi-open sets as described in [17, 41] are comparable. We start with the next two findings, which display the degree of approximations and accuracy values in accordance with some open set generalizations.

Theorem 4.1. *If* $(\aleph, \Omega, \delta_\xi)$ *is a* ξ*-adhesion-NS and* $\Psi \subseteq \aleph$, *then* $\underline{\Omega}_\xi(\Psi) \subseteq \underline{\Omega}^{\jmath}_\xi(\Psi) \subseteq \underline{\Omega}^{so}_\xi(\Psi) \subseteq \Omega \subseteq \overline{\Omega}^{so}_\xi(\Psi) \subseteq \overline{\Omega}^{\jmath}_\xi(\Psi) \subseteq \overline{\Omega}_\xi(\Psi)$, *where* $\jmath \in \{\alpha o, \mathrm{semio}\}$.

Proof. The class of α-open (semi-open) subgroups of (α, ω_ξ) has a topology, which is already known. At hence, $\forall\ \Psi \subseteq \aleph$, we have $\underline{\Omega}_\xi(\Psi) \subseteq \underline{\Omega}^{\jmath}_\xi(\Psi)$. Moreover, the classes of α-open and semi-open subsets are contained in the class of somewhat open subsets of (Ψ, ω_ξ). Then, $\underline{\Omega}^{\jmath}_\xi(\Psi) \subseteq \underline{\Omega}^{so}_\xi(\Psi)$. $\underline{\Omega}^{so}_\xi(\Psi) \subseteq \Omega$ is derived from Proposition 3.6 in this sentence. At hence, $\underline{\Omega}_\xi(\Psi) \subseteq \underline{\Omega}^{\jmath}_\xi(\Psi) \subseteq \underline{\Omega}^{so}_\xi(\Psi) \subseteq \Omega$. Likewise, we can demonstrate $\Omega \subseteq \overline{\Omega}^{so}_\xi(\Psi) \subseteq \overline{\Omega}^{\jmath}_\xi(\Psi) \subseteq \overline{\Omega}_\xi(\Psi)$. □

Proposition 4.2. *Any subset of a* ξ*-adhesion-NS* $(\aleph, \Omega, \delta_\xi)$ *and* $\jmath \in \{\alpha, \mathrm{semi}\}$ *satisfies the next two results.*

(1) $B_{\xi}^{so}(\Psi) \subseteq B_{\xi}^{\jmath}(\Psi) \subseteq B_{\xi}(\Psi)$.

(2) $A_{\xi}(\Psi) \leqslant A_{\xi}^{\jmath}(\Psi) \leqslant A_{\xi}^{so}(\Psi)$.

Proof.

(1) Theorem 4.1 provides the evidence.

(2) Theorem 4.1 states that $\underline{\Omega}_{\xi}^{\jmath}(\Psi) \subseteq \underline{\Omega}_{\xi}^{so}(\Psi)$ and $\overline{\Omega}_{\xi}^{so}(\Psi) \subseteq \overline{\Omega}_{\xi}^{\jmath}(\Psi)$. This implies that $\mid \underline{\Omega}_{\xi}^{\jmath}(\Psi) \mid \leqslant \mid \underline{\Omega}_{\xi}^{so}(\Psi) \mid$ and $\mid \overline{\Omega}_{\xi}^{so}(\Psi) \mid \leqslant \mid \overline{\Omega}_{\xi}^{\jmath}(\Psi) \mid$. So, $\mid \underline{\Omega}_{\xi}^{\jmath}(\Psi) \mid \times \mid \overline{\Omega}_{\xi}^{so}(\Psi) \mid \leqslant \mid \underline{\Omega}_{\xi}^{so}(\Psi) \mid \times \mid \overline{\Omega}_{\xi}^{\jmath}(\Psi) \mid$. Thus, we get $\frac{|\underline{\Omega}_{\xi}^{\jmath}(\Psi)|}{|\overline{\Omega}_{\xi}^{\jmath}(\Psi)|} \leqslant \frac{|\underline{\Omega}_{\xi}^{so}(\Psi)|}{|\overline{\Omega}_{\xi}^{so}(\Psi)|}$. Also, we get $\frac{|\underline{\Omega}_{\xi}(\Psi)|}{|\overline{\Omega}_{\xi}(\Psi)|} \leqslant \frac{|\underline{\Omega}_{\xi}^{\jmath}(\Psi)|}{|\overline{\Omega}_{\xi}^{\jmath}(\Psi)|}$. At hence, we get also $\frac{|\underline{\Omega}_{\xi}(\Psi)|}{|\overline{\Omega}_{\xi}(\Psi)|} \leqslant \frac{|\underline{\Omega}_{\xi}^{\jmath}(\Psi)|}{|\overline{\Omega}_{\xi}^{\jmath}(\Psi)|} \leqslant \frac{|\underline{\Omega}_{\xi}^{so}(\Psi)|}{|\overline{\Omega}_{\xi}^{so}(\Psi)|}$. As a result, the proof is finished. □

In the case of α-open and semi-open sets, we provide the following illustration to demonstrate that compared to the methods suggested in [1] and [17, 41], our technique provides higher accuracy measures and approximations. We actually show case $\xi = \xi_1$ for the sake of economy.

Example 4.3. Let $(\aleph, \Omega, \delta_{\xi})$ be a ξ-adhesion-NS given in Example 3.8. Then, $\omega_{\xi_1} = \{\aleph, \phi, \{fv\}, \{fz\}, \{fv, fz\}, \{fu, fy\}, \{fv, fy, fu\}, \{fu, fy, fz\}\}$. We will be satisfied with the class of semi-open sets because it includes the family of α-open sets. $\text{semio}(\omega_{\xi_1}) = \{\aleph, \phi, \{fv\}, \{fz\}, \{fv, fz\}, \{fu, fy\}, \{fv, fy, fu\}, \{fu, fy, fz\}\}$ and $\text{so}(\omega_{\xi_1}) = \{\aleph, \phi, \{fv\}, \{fz\}, \{fu, fv\}, \{fu, fy\}, \{fu, fz\}, \{fv, fy\}, \{fv, fz\}, \{fy, fz\}, \{fu, fv, fy\}, \{fu, fv, fz\}, \{fv, fy, fz\}, \{fu, fy, fz\}\}$.

Table 4 show some accuracy measures derived from three distinct methods, including (1) open and closed subsets of ξ_1-adhesion neighborhood topology; (2) semi-open and semi-closed subsets of ξ_1-adhesion neighborhood topology; and (3) somewhat open and somewhat closed subsets of ξ_1-adhesion neighborhood topology. Compared to the other two approaches, our method obviously reduces the size of boundary regions and improves subset accuracy measurements. Because the class of somewhat open sets is larger than the classes of open and semi-open sets, the ξso-lower approximation is maximized while the ξso-upper approximation is minimized. As a result, accuracy measures are increasing. Lastly, if the generated topology is hyperconnected, the two classes of somewhat open and semi-open sets coincide, implying that our and semi-open approaches give equal approximations and accuracy measures.

Table 2: Comparison of the situations of ξ_1, $\xi_1$semi, and $\xi_1$so.

	ω_{ξ_1}			$\text{semiO}(\omega_{\xi_1})$			$\text{so}(\omega_{\xi_1})$		
	$\underline{\Omega}_{\xi_1}$	$\overline{\Omega}_{\xi_1}$	A_{ξ_1}	$\underline{\Omega}_{\xi_1}^{semi}$	$\overline{\Omega}_{\xi_1}^{semi}$	$A_{\xi_1}^{semi}$	$\underline{\Omega}_{\xi_1}^{so}$	$\overline{\Omega}_{\xi_1}^{so}$	$A_{\xi_1}^{so}$
$\{fu\}$	ϕ	$\{fu, fy\}$	0	ϕ	$\{fu, fy\}$	0	ϕ	$\{fu\}$	0
$\{fv\}$	$\{fv\}$	$\{fv\}$	1	$\{fv\}$	$\{fv\}$	1	$\{fv\}$	$\{fv\}$	1
$\{fy\}$	ϕ	$\{fu, fy\}$	0	ϕ	$\{fu, fy\}$	0	ϕ	$\{fy\}$	0
$\{fz\}$	$\{fz\}$	$\{fz\}$	1	$\{fz\}$	$\{fz\}$	1	$\{fz\}$	$\{fz\}$	1
$\{fu, fv\}$	$\{fv\}$	$\{fu, fv, fy\}$	$\frac{1}{3}$	$\{fv\}$	$\{fu, fv, fy\}$	$\frac{1}{3}$	$\{fu, fv\}$	$\{fu, fv\}$	1
$\{fu, fy\}$	$\{fu, fy\}$	$\{fu, fy\}$	1	$\{fu, fy\}$	$\{fu, fy\}$	1	$\{fu, fy\}$	$\{fu, fy\}$	1
$\{fu, fz\}$	$\{fz\}$	$\{fu, fy, fz\}$	$\frac{1}{3}$	$\{fz\}$	$\{fu, fy, fz\}$	$\frac{1}{3}$	$\{fu, fz\}$	$\{fu, fz\}$	1
$\{fv, fy\}$	$\{fv\}$	$\{fu, fv, fy\}$	$\frac{1}{3}$	$\{fv\}$	$\{fu, fv, fy\}$	$\frac{1}{3}$	$\{fv, fy\}$	$\{fv, fy\}$	1
$\{fv, fz\}$	$\{fv, fz\}$	$\{fv, fz\}$	1	$\{fv, fz\}$	$\{fv, fz\}$	1	$\{fv, fz\}$	$\{fv, fz\}$	1
$\{fy, fz\}$	$\{fz\}$	$\{fu, fy, fz\}$	$\frac{1}{3}$	$\{fz\}$	$\{fu, fy, fz\}$	$\frac{1}{3}$	$\{fy, fz\}$	$\{fy, fz\}$	1
$\{fu, fv, fy\}$	$\{fu, fv, fy\}$	$\{fu, fv, fy\}$	1	$\{fu, fv, fy\}$	$\{fu, fv, fy\}$	1	$\{fu, fv, fy\}$	$\{fu, fv, fy\}$	1
$\{fu, fv, fz\}$	$\{fv, fz\}$	$\aleph$	$\frac{1}{2}$	$\{fv, fz\}$	$\aleph$	$\frac{1}{2}$	$\{fu, fv, fz\}$	$\aleph$	$\frac{3}{4}$
$\{fv, fy, fz\}$	$\{fv, fz\}$	$\aleph$	$\frac{1}{2}$	$\{fv, fz\}$	$\aleph$	$\frac{1}{2}$	$\{fv, fy, fz\}$	$\aleph$	$\frac{3}{4}$
$\{fu, fy, fz\}$	$\{fu, fy, fz\}$	$\{fu, fy, fz\}$	1	$\{fu, fy, fz\}$	$\{fu, fy, fz\}$	1	$\{fu, fy, fz\}$	$\{fu, fy, fz\}$	1
$\aleph$	$\aleph$	$\aleph$	1	$\aleph$	$\aleph$	1	ϕ	ϕ	1

5. Medical example: heart failure

As an introduction to discussing the experimental findings, Dickstein et al. [19] investigation on five heart disease symptoms for twelve individuals is presented in this part. The research was conducted at Al-Azhar University. The remaining 5 records, which had similar presenting symptoms, a thorough medical history, a thorough physical examination, extensive blood tests, a resting ECG, and a conventional echo assessment completed, were provided to this institution. The remaining 25 records were used for training. Only five patients' information system data were used to describe the heart failure problem in Table 3 due to comparable patients. The columns show the signs of a diagnosis of heart failure, with a $\checkmark$ sign denoting that the patient has symptoms and a $\star$ sign denoting that the patient has none [19] (condition attributes), where $\mathcal{H}_1$ is the breathlessness, $\mathcal{H}_2$ is the orthopnea, $\mathcal{H}_3$ is the paroxysmal nocturnal dyspnea, $\mathcal{H}_4$ reduced exercise tolerance, and $\mathcal{H}_5$ is the ankle swelling. Heart failure is the choice represented by attribute D. The rows in Table 3, $\hbar = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$, represents the patients.

Table 3: Original Heart failure information system.

$\hbar$	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$	$\hbar_5$
$\mathcal{H}_1$	$\checkmark$	$\star$	$\checkmark$	$\star$	$\checkmark$
$\mathcal{H}_2$	$\checkmark$	$\star$	$\checkmark$	$\star$	$\star$
$\mathcal{H}_3$	$\checkmark$	$\star$	$\checkmark$	$\star$	$\star$
$\mathcal{H}_4$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$\mathcal{H}_5$	$\star$	$\checkmark$	$\checkmark$	$\star$	$\checkmark$
D	$\checkmark$	$\star$	$\checkmark$	$\star$	$\star$

We begin by converting the description attributes $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$, $\mathcal{H}_4$, and $\mathcal{H}_5$ into qualitative terms as shown in Table 4, which express similarities between symptoms patients where the degree of similarity $\beta(u,v) = \frac{\sum_{i=1}^{n}(a_i(u)=a_i(v))}{n}$, where n is the number of condition attributes.

Table 4: Similarities between symptoms of five of patients.

	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$	$\hbar_5$
$\hbar_1$	1	0.2	0.8	0.4	0.4
$\hbar_2$	0.2	1	0.4	0.8	0.8
$\hbar_3$	0.8	0.4	1	0.2	0.6
$\hbar_4$	0.4	0.8	0.2	1	0.6
$\hbar_5$	0.4	0.8	0.6	0.6	1

Now, we establish a relationship in each situation depending on the needs of the system's experts. For instance, let $u\Omega v \Leftrightarrow \beta(u,v) > 0.8$, where $\beta(u,v)$ is the total of the symptoms that are comparable between u and v divided by the number of symptoms. It is important to note that the proposed connection $>$ and number 0.8 have been modified in light of system experts' opinions. There is only one type of $\mathcal{M}_\xi$-adhesion neighborhood because of the stated relation Ω, which is an equivalence relation. It should be remembered that connection Ω need not always be an equivalence. Table 4 contains the following information: $\Omega(\hbar_1) = \{\hbar_1\}$, $\Omega(\hbar_2) = \{\hbar_2\}$, $\Omega(\hbar_3) = \{\hbar_3\}$, $\Omega(\hbar_4) = \{\hbar_4\}$, $\Omega(\hbar_5) = \{\hbar_5\}$. The topology ω_ξ induced by the basis $\{\mathcal{M}_\xi(\hbar) : \hbar \in \aleph\}$ is the topology generated by $\mathcal{M}_\xi$-adhesion neighborhoods. We investigate $\Psi = \{\hbar_2, \hbar_4, \hbar_5\}$, which is the set of patients who do not have Heart failure, to validate the benefits of the followed technique in increasing the approximations and accuracy measures compared with the techniques presented in [16, 39]. We determined the following approximations and accuracy measures: $\underline{\Omega}_\xi^{so}(\Psi) = \overline{\Omega}_\xi^{so}(\Psi) = \Psi$, and $A_\xi^{so} = 1$. Only when the generative topology is the discrete topology do we observe that our approach is equal to the other ways; in all other situations, our best method is used.

6. Conclusion

It is commonly recognized that topological ideas offer an essential tool for understanding rough set theory. In this paper, we examine new kinds of rough set models using a topological method called ”somewhat open and somewhat closed sets”. The primary characteristics of the presented models have been examined, and their distinctive features have been discussed. We have compared our model to earlier versions as well as performed some comparisons between the various types of our models. Also, we have provided a medical case to assess the effectiveness of our strategy. Our focus in this work has been on key points such as a novel rough approximations and accuracy measurements by using New ξ-adhesion neighborhood, comparison of our method to earlier approaches, and some of applications. We investigate novel approximations of the rough set theory with novel and distinct neighborhoods in our upcoming work.

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