



## On the solution of linear and nonlinear partial differential equations: applications of local fractional Sumudu variational method

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### Abstract

In this paper, the local fractional Sumudu variational iteration method is being used to investigate the solutions of partial differential equations containing the local fractional derivatives. The present technique is the combination of the local fractional Sumudu transform and fractional variational iteration method. Three illustrative examples are given to demonstrate the efficiency of the method. ©2016 all rights reserved.

*Keywords:* Modified Riemann-Liouville derivative, local fractional derivative, fractional variational iteration method, fractional differential equations, local fractional Sumudu transform.

*2010 MSC:* 26A33, 35A22, 33E12, 35R11, 65L10.

### 1. Introduction

Fractional calculus and local fractional calculus are very popular in the fields of physics and engineering ([2, 11, 13, 17, 23]). Most of the phenomena are represented in terms of partial fractional differential equations. The solutions of these equations are very tempting due to process of finding and level of reliability they holds. Another attracting thing is that the number of techniques is presented in literature for solving such equations ([1, 3–5, 7, 10, 12, 14, 15]). The variational iteration method ([8, 9]) is the most popular method that enriched the solutions of partial differential equations due

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Received 2016-06-14

to its easy handling and flexibility ([16, 19, 20, 25, 26]). Nowadays the combination of variational iteration method and other analytical methods is attracting the researchers and earthen out more and more interesting ways to solve these types of equations ([6, 24]). The local fractional Sumudu variational iteration method is very new and fresh technique for these equations. The local fractional variational iteration method is derived by the combination of the local fractional Sumudu transform [18] and the fractional variational iteration method. This technique is being popular day by day due to its freshness and easy implementation with high level of efficiency.

In this article, our aim is to use the fractional variational iteration method and the local fractional Sumudu transform to develop an iteration formula to solve the linear and nonlinear local fractional partial differential equations with the local fractional derivatives.

## 2. Definitions and mathematical preliminaries

**Definition 2.1** ([21, 22]). The partition of interval  $[a, b]$  is denoted as  $(t_j, t_{j+1}), j = 0, \dots, N - 1, t_0 = a$  and  $t_N = b$  with  $\Delta t_j = t_{j+1} - t_j$  and  $\Delta t = \max\{\Delta t_0, \Delta t_1, \dots\}$ . Local fractional integral of  $f(x)$  in the interval  $[a, b]$  is given by

$${}_a I_b^{(\alpha)} f(x) = \frac{1}{\Gamma(1 + \alpha)} \int_a^b f(t) (dt)^\alpha = \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j) (\Delta t_j)_\alpha.$$

**Definition 2.2** ([21, 22]). The local fractional derivative of  $f(x) \in C_\alpha(a, b)$  of order  $\alpha$  at  $x = x_0$  is defined as

$$f^{(\alpha)}(x_0) = \frac{d^\alpha}{dx^\alpha} f(x)|_{x=x_0} = \lim_{x \rightarrow x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha},$$

where  $\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(\alpha + 1) f(x) - f(x_0)$ .

**Definition 2.3.** The local fractional Sumudu transform of  $f(t)$  of order  $\alpha$  is defined as [18]

$$S_\alpha [f(t)] = G_\alpha(u) = \frac{1}{\Gamma(1 + \alpha)} \int_0^\infty E_\alpha(-u^\alpha t^\alpha) \frac{f(t)}{z^\alpha} (dt)^\alpha, \quad 0 < \alpha \leq 1,$$

where  $x \in C$  and  $E_\alpha(x)$  is Mittag Leffler function  $\sum_{k=0}^\infty \left(\frac{x^k}{\alpha k!}\right)$ . And the inverse Sumudu transform is defined as

$$S_\alpha^{-1} \{f(u)\} = f(t), \quad 0 < \alpha \leq 1.$$

Using the above definition of the local fractional Sumudu transform the following result can easily be obtained [18]

$$S_\alpha \{f^\alpha(t)\} = \frac{G_\alpha(u) - f(0)}{u^\alpha},$$

$$S_\alpha \{f^{n\alpha}(t)\} = \frac{1}{u^{n\alpha}} \left[ G_\alpha(u) - \sum_{k=0}^{n-1} u^{k\alpha} f^{(k\alpha)}(0) \right].$$

## 3. The local fractional Sumudu variational iteration method

This section introduces the idea of fractional Sumudu variational method for the following fractional partial differential equation

$$D_{0+}^{m\alpha} v(x, t) = L_\alpha(v(x, t)) + N_\alpha(v(x, t)) + f(x, t), \tag{3.1}$$

where  $0 < \alpha \leq 1, x, t \geq 0, L_\alpha$  is a linear operator,  $N_\alpha$  is nonlinear operator, and  $f$  is a source term.

Taking local fractional Sumudu transform of (3.1) both the sides w.r.t. ‘ $t$ ’,

$$\frac{1}{u^{m\alpha}} \left[ G_\alpha(u) - \sum_{k=0}^{m-1} u^{k\alpha} v^{(k\alpha)}(x, 0) \right] = S_\alpha [L_\alpha(v(x, t)) + N_\alpha(v(x, t)) + f(x, t)].$$

For an algebraic equation the iterative formula can be constructed as

$$x_{n+1} = x_n + \lambda^\alpha f(x_n). \tag{3.2}$$

The optimality condition for the extreme  $\frac{\delta x_{n+1}}{\delta x_n} = 0$ , leads to

$$\lambda^\alpha = -\frac{1}{f'(x_n)},$$

where  $\delta$  is the classical variational operator.

By the formula (3.2), we get the iterative formula for equation (3.1) as follows,

$$G_\alpha v_{n+1}(x, u) = G_\alpha v_n(x, u) + \lambda^\alpha \left\{ \frac{1}{u^{m\alpha}} \left[ G_\alpha v_n(x, u) - \sum_{k=0}^{m-1} u^{k\alpha} v^{(k\alpha)}(x, 0) \right] - S_\alpha [L_\alpha(v_n(x, t)) + N_\alpha(v_n(x, t)) + f(x, t)] \right\}. \tag{3.3}$$

Put  $\lambda^\alpha = -u^{m\alpha}$ , the Lagrange multiplier in (3.3)

$$G_\alpha v_{n+1}(x, u) = + \left\{ \left[ \sum_{k=0}^{m-1} u^{k\alpha} v^{(k\alpha)}(x, 0) \right] + u^{m\alpha} S_\alpha [L_\alpha(v_n(x, t)) + N_\alpha(v_n(x, t)) + f(x, t)] \right\}.$$

Taking the inverse fractional Sumudu transform, we arrived at

$$v_{n+1}(x, t) = S_\alpha^{-1} \left\{ \left[ \sum_{k=0}^{m-1} u^{k\alpha} v^{(k\alpha)}(x, 0) \right] + u^{m\alpha} S_\alpha [L_\alpha(v_n(x, t)) + N_\alpha(v_n(x, t)) + f(x, t)] \right\}. \tag{3.4}$$

This is the iterative formula for equation (3.1).

**Example 3.1.** The generalized linear fractional partial differential equation for  $0 \leq x \leq 1, 0 \leq t \leq 1, 0 < \alpha \leq 1, 0 < \beta \leq 1$  is as follows

$$\frac{\partial^\alpha}{\partial x^\alpha} v(x, t) + x^\beta \frac{\partial^\beta v(x, t)}{\partial x^\beta} + \frac{\partial^{2\beta} v(x, t)}{\partial x^{2\beta}} = 2(t^\alpha + x^{2\beta} + 1), \tag{3.5}$$

with initial condition

$$v(x, 0) = x^{2\beta}.$$

*Proof.* By using the iterative formula (3.4) derived above, we get the following iterative formula for equation (3.5) as

$$v_{n+1}(x, t) = S_\alpha^{-1} \left[ \sum_{k=0}^{n-1} [u^{k\alpha} v^{k\alpha}(x, 0)] + u^\alpha S_\alpha \left( 2(t^\alpha + x^{2\beta} + 1) - x^\beta \frac{\partial^\beta v_n}{\partial x^\beta} - \frac{\partial^{2\beta} v_n}{\partial x^{2\beta}} \right) \right]. \tag{3.6}$$

Let us start from the initial iteration, which is given as

$$v_0(x, t) = S_\alpha^{-1} \left[ \sum_{k=0}^{n-1} [u^{k\alpha} v^{k\alpha}(x, 0)] + u^\alpha S_\alpha (2(t^\alpha + x^{2\beta} + 1)) \right].$$

Solving the second term of above equation by taking the local fractional Sumudu transform and using the initial condition,

$$v_0(x, t) = x^{2\beta} + S_\alpha^{-1} [u^\alpha (2u^\alpha \Gamma(1 + \alpha) + x^{2\beta} + 1)]. \tag{3.7}$$

After taking the local fractional Sumudu transform in (3.7), we get the initial iteration  $v_0(x, t)$  as

$$v_0(x, t) = x^{2\beta} + \frac{2t^{2\alpha}}{\Gamma(1 + 2\alpha)} \Gamma(1 + \alpha) + x^{2\beta} \frac{2t^\alpha}{\Gamma(1 + \alpha)} + \frac{t^\alpha}{\Gamma(1 + \alpha)}.$$

Now for  $v_1(x, t)$ , we have the iterative formula by using (3.6) with  $n=0$  as follows

$$v_1(x, t) = S_\alpha^{-1} \left[ \sum_{k=0}^{m-1} [u^{k\alpha} v^{k\alpha}(x, 0)] + u^\alpha S_\alpha \left( 2(t^\alpha + x^{2\beta} + 1) - x^\beta \frac{\partial^\beta v_0}{\partial x^\beta} - \frac{\partial^{2\beta} v_0}{\partial x^{2\beta}} \right) \right]. \tag{3.8}$$

Or we can write equation (3.8) as

$$v_1(x, t) = v_0(x, t) + S_\alpha^{-1} \left[ u^\alpha S_\alpha \left( -x^\beta \frac{\partial^\beta v_0}{\partial x^\beta} - \frac{\partial^{2\beta} v_0}{\partial x^{2\beta}} \right) \right]. \tag{3.9}$$

Putting the value of  $v_0(x, t)$  in (3.9) and solving the derivative terms we get

$$v_1(x, t) = x^{2\beta} + \frac{2t^{2\alpha}}{\Gamma(1 + 2\alpha)} \Gamma(1 + \alpha) + x^{2\beta} \frac{2t^\alpha}{\Gamma(1 + \alpha)} + \frac{2t^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} \left[ u^\alpha S_\alpha \left( -x^\beta \left( 2\beta x^\beta + 4\beta x^\beta \frac{t^\alpha}{\Gamma(1 + \alpha)} \right) - \left( 2\beta^2 + 4\beta^2 \frac{t^\alpha}{\Gamma(1 + \alpha)} \right) \right) \right].$$

Applying the local fractional Sumudu transform, we have

$$v_1(x, t) = x^{2\beta} + \frac{2t^{2\alpha}}{\Gamma(1 + 2\alpha)} \Gamma(1 + \alpha) + x^{2\beta} \frac{2t^\alpha}{\Gamma(1 + \alpha)} + \frac{2t^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} [u^\alpha (-x^\beta (2\beta x^\beta + 4\beta x^\beta u^\alpha) - (2\beta^2 + 4\beta^2 u^\alpha))].$$

Now taking the inverse local fractional Sumudu transform we get  $v_1(x, t)$  as follows:

$$v_1(x, t) = x^{2\beta} + \frac{2t^{2\alpha}}{\Gamma(1 + 2\alpha)} \Gamma(1 + \alpha) + x^{2\beta} \frac{2t^\alpha}{\Gamma(1 + \alpha)} + \frac{2t^\alpha}{\Gamma(1 + \alpha)} + \left[ \left( -x^\beta \left( 2\beta x^\beta \frac{t^\alpha}{\Gamma(1 + \alpha)} + 4\beta x^\beta \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)} \right) - \left( 2\beta^2 \frac{t^\alpha}{\Gamma(1 + \alpha)} + 4\beta^2 \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)} \right) \right) \right].$$

Similarly we can find

$$v_2(x, t) = x^{2\beta} + \frac{2t^{2\alpha}}{\Gamma(1 + 2\alpha)} \Gamma(1 + \alpha) + x^{2\beta} \frac{2t^\alpha}{\Gamma(1 + \alpha)} + \frac{2t^\alpha}{\Gamma(1 + \alpha)} + \left[ \left( -x^\beta \left( 2\beta x^\beta \frac{t^\alpha}{\Gamma(1 + \alpha)} + 4\beta x^\beta \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)} \right) - \left( 2\beta^2 \frac{t^\alpha}{\Gamma(1 + \alpha)} + 4\beta^2 \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)} \right) \right) \right] + x^{2\beta} \frac{4\beta^2 t^{2\alpha}}{\Gamma(1 + 2\alpha)} + 8\beta^2 x^{2\beta} \frac{t^{3\alpha}}{\Gamma(1 + 3\alpha)} + 4\beta^3 \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)} + 8\beta^3 \frac{t^{3\alpha}}{\Gamma(1 + 3\alpha)}.$$

Then general term of successive approximation is given by

$$v_n(x, t) = x^{2\beta} + \frac{2t^{2\alpha}}{\Gamma(1 + 2\alpha)}\Gamma(1 + \alpha) + \sum_{k=1}^n (-1)^k (2\beta)^k \frac{t^{k\alpha}}{\Gamma(1 + k\alpha)} (x^{2\beta} + \beta) + \sum_{k=0}^n (-1)^k 2^{k+1}\beta^k \frac{t^{(k+1)\alpha}}{\Gamma(1 + (k + 1)\alpha)} (x^{2\beta} + \beta).$$

□

**Example 3.2.** Consider the following local fractional nonlinear transport equation

$$\frac{\partial^\alpha}{\partial t^\alpha} v(x, t) - a(x, t) \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} v(x, t) - \frac{b(x, t)v(x, t)}{2} \frac{\partial^\alpha}{\partial x^\alpha} v(x, t) = 0, \quad t > 0, \quad 1 < \alpha \leq 2, \quad x \in \mathfrak{R}, \quad (3.10)$$

with initial condition

$$v(x, 0) = \frac{x^\alpha}{\Gamma(1 + \alpha)}, \quad \text{where } a(x, t) = 1 \text{ and } b(x, t) = 2.$$

*Proof.* By using the iterative formula derived from (3.4) above, we get the following iterative formula for equation (3.10).

$$v_{n+1}(x, t) = S_\alpha^{-1} \left[ \sum_{k=0}^{n-1} [u^{k\alpha} v^{k\alpha}(x, 0)] + u^\alpha S_\alpha \left( \frac{\partial^{2\alpha} v_n(x, t)}{\partial x^{2\alpha}} + v_n(x, t) \frac{\partial^\alpha v_n(x, t)}{\partial x^\alpha} \right) \right]. \quad (3.11)$$

The initial iteration  $v_0(x, t)$  is given as follows

$$v_0(x, t) = S_\alpha^{-1} \left[ \sum_{k=0}^{n-1} [u^{k\alpha} v^{k\alpha}(x, 0)] \right].$$

Using initial values we get

$$v_0(x) = S_\alpha^{-1} \left[ \frac{x^\alpha}{\Gamma(1 + \alpha)} \right]. \quad (3.12)$$

Taking the inverse local fractional Sumudu transform in (3.12), we get

$$v_0(x, t) = \frac{x^\alpha}{\Gamma(1 + \alpha)}.$$

Now, for  $v_1(x, t)$  we can find the iterative formula as follows by putting  $n = 0$  in (3.11)

$$\begin{aligned} v_1(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} \left[ u^\alpha S_\alpha \left( \frac{\partial^{2\alpha} v_0(x, t)}{\partial x^{2\alpha}} + v_0(x, t) \frac{\partial^\alpha v_0(x, t)}{\partial x^\alpha} \right) \right], \\ v_1(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} \left[ u^\alpha S_\alpha \left( \frac{x^\alpha}{\Gamma(1 + \alpha)} \cdot \frac{1}{\Gamma(1 + \alpha)} \right) \right], \\ v_1(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} \left[ u^\alpha \left( \frac{x^\alpha}{\Gamma(1 + \alpha)} \cdot \frac{1}{\Gamma(1 + \alpha)} \right) \right], \\ v_1(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + \frac{t^\alpha}{\Gamma(1 + \alpha)} \frac{x^\alpha}{\Gamma(1 + \alpha)}, \end{aligned}$$

$$v_1(x, t) = \frac{x^\alpha}{\Gamma(1 + \alpha)} \left( 1 + \frac{t^\alpha}{\Gamma(1 + \alpha)} \right).$$

Similarly, we can find other iterations  $v_2(x, t)$  as

$$\begin{aligned} v_2(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} \left[ u^\alpha S_\alpha \left( \frac{\partial^{2\alpha} v_1(x, t)}{\partial x^{2\alpha}} + v_1(x, t) \frac{\partial^\alpha v_1(x, t)}{\partial x^\alpha} \right) \right], \\ v_2(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} \left[ u^\alpha S_\alpha \left( \frac{x^\alpha}{\Gamma(1 + \alpha)} \left( 1 + \frac{t^\alpha}{\Gamma(1 + \alpha)} \right) \left( 1 + \frac{t^\alpha}{\Gamma(1 + \alpha)} \right) \right) \right], \\ v_2(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + S_\alpha^{-1} \left[ u^\alpha \left( \frac{x^\alpha}{\Gamma(1 + \alpha)} + 2u^\alpha \frac{x^\alpha}{\Gamma(1 + \alpha)} + \frac{u^{2\alpha} \Gamma(1 + 2\alpha)}{\Gamma(1 + \alpha) \Gamma(1 + \alpha)} \frac{x^\alpha}{\Gamma(1 + \alpha)} \right) \right], \\ v_2(x, t) &= \frac{x^\alpha}{\Gamma(1 + \alpha)} + \frac{t^\alpha}{\Gamma(1 + \alpha)} \frac{x^\alpha}{\Gamma(1 + \alpha)} + 2 \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)} \frac{x^\alpha}{\Gamma(1 + \alpha)} \\ &\quad + \frac{t^{3\alpha}}{\Gamma(1 + 3\alpha)} \frac{\Gamma(1 + 2\alpha)}{\Gamma(1 + \alpha) \Gamma(1 + \alpha)} \frac{x^\alpha}{\Gamma(1 + \alpha)}. \end{aligned}$$

□

**Example 3.3.** Consider the following local fractional nonlinear Fokker- Plank equation

$$\partial^\alpha \partial t^\alpha v(x, t) = a(x, v) \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} v(x, t) - b(x, v) \frac{\partial^\alpha}{\partial x^\alpha} v(x, t), \quad t > 0, \quad 1 < \alpha \leq 2, \quad x \in \mathfrak{R} \tag{3.13}$$

with initial condition

$$v(x, 0) = E_\alpha(x^\alpha), \quad \text{where } a(x, v) = v(x, t) \text{ and } b(x, v) = v(x, t).$$

*Proof.* By using the iterative formula derived from (3.4) above, we get the following iterative formula for (3.13) as

$$v_{n+1}(x, t) = S_\alpha^{-1} \left[ \sum_{k=0}^{n-1} [u^{k\alpha} v^{k\alpha}(x, 0)] + u^\alpha S_\alpha \left( v_n(x, t) \frac{\partial^{2\alpha} v_n(x, t)}{\partial x^{2\alpha}} - v_n(x, t) \frac{\partial^\alpha v_n(x, t)}{\partial x^\alpha} \right) \right].$$

The initial iteration is given as

$$\begin{aligned} v_0(x, t) &= S_\alpha^{-1} \left[ \sum_{k=0}^{n-1} [u^{k\alpha} v^{k\alpha}(x, 0)] \right], \\ v_0(x, t) &= S_\alpha^{-1} [E_\alpha(x^\alpha)] = E_\alpha(x^\alpha). \end{aligned}$$

By applying same calculation process used in previous examples we get the following iteration  $v_1(x, t)$ :

$$\begin{aligned} v_1(x, t) &= E_\alpha(x^\alpha) + S_\alpha^{-1} \left[ u^\alpha S_\alpha \left( v_0(x, t) \frac{\partial^{2\alpha} v_0(x, t)}{\partial x^{2\alpha}} - v_0(x, t) \frac{\partial^\alpha v_0(x, t)}{\partial x^\alpha} \right) \right], \\ v_1(x, t) &= E_\alpha(x^\alpha). \end{aligned}$$

In the same manner as above, the rest of the approximative solutions are presented as follows

$$v_2(x, t) = v_3(x, t) = \dots = v_n(x, t) = E_\alpha(x^\alpha).$$

The closed form solution can be written as follows

$$v(x, t) = \lim_{n \rightarrow \infty} v_n(x, t) = E_\alpha(x^\alpha).$$

□

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