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Conflict distance-based variable precision Pythagorean fuzzy rough set in Pythagorean fuzzy decision systems with applications in decision making

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Abstract

Real-life decision-making problems are hard to handle by any single uncertainty method because of the complex and uncertain nature of the physical world and the human limitations in understanding it. Therefore, we naturally consider combining the benefits of various uncertainty theories to create a more effective hybrid soft decision-making method. Based on this idea, we use the variable precision rough sets (VPRSs) and Pythagorean fuzzy sets approach to build a new Pythagorean fuzzy rough set (PFRS) model. Since the information system is Pythagorean fuzzy, we use the Pythagorean fuzzy similarity measure to define the new type of distance based on conflict. Then, we merge this notion with the VPRSs to form a variable precision PFRS model and study its properties. We also propose an algorithm for attribute reduction based on this model and apply it to a case study to test its feasibility and performance. The results demonstrate that our model enhances the classification capability of previous models, and it achieves accurate classification by deriving the decision rules.

Keywords: Pythagorean fuzzy set, rough set, conflict distance, variable precision rough set.

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1. Introduction

Rough set (RS) theory is a mathematical technique that can deal with imprecise, uncertain, and vague knowledge. Pawlak [14] proposed this theory, and it has many applications in various scientific domains, such as finding knowledge, mining data, analyzing decisions, and recognizing patterns. The classical

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RS theory uses equivalence relationships to reason with data. However, it is hard to meet the strict conditions of this relationship in real-world applications. Moreover, the binary relationship in the domain of discourse is often fuzzy or similar rather than equivalent. Therefore, Dubois and Prade [6] proposed the fuzzy RS theory with the combination of RS and fuzzy set (FS) theories. By accounting for the variability of various types of information, this new theory can more accurately represent the reality of the physical world. It has quickly become a hot topic of RS theory and boosted its development. Pythagorean fuzzy sets (PFSs) [16, 17] can capture more information and express subtle uncertainties of the real world than intuitionistic fuzzy sets (IFSs) [3] and traditional FSs because they account for the membership degree (MD) and non-membership degree (NMD) of an object's to a set. Therefore, much work has been done with PFSs in existing literature [1, 2, 10, 12, 13]. By combining PFSs and RSs, a new model called the Pythagorean fuzzy RS (PFRS) [20] is created. This model has great theoretical and practical significance and has attracted much academic attention. But this model is made by solving a granulation process. The relationship between PFS theory and Decision-theoretic rough sets (DTRSs) is studied by Mandal and Ranadive [9]. They used Pythagorean fuzzy numbers (PFNs) to obtain loss functions and proposed group decision-making (GDM) based on Pythagorean fuzzy DTRS (PFDTRS). In [11], Mandal and Ranadive propose a new method for dealing with multi-criteria decision-making (MCDM) problems that involve Pythagorean fuzzy information (PFI) with different levels of granularity. The method combines two extensions of classical rough sets: multi-granulation rough sets (MGRSs) and DTRSs. The method uses the Pythagorean fuzzy inclusion measure to construct four multi-granulation Pythagorean fuzzy DTRS (MG-PF-DTRS) models and analyzes their properties and uncertainty measures. The method also provides a way to handle incomplete multi-source information systems (IMSISs) by computing Pythagorean fuzzy similarity degrees and Pythagorean fuzzy decision-making objects. The paper illustrates the application of the method to a mutual funds investment problem. In [21], Zhang and Ma studied PFDTRS based on Pythagorean fuzzy covering. A new rough set model that uses PFSs to handle uncertainty in covering information systems studied in [18]. The study also presents a method to find attribute reduction based on the discernibility matrix of the rough set model. The paper applies the method to a medical decision problem and shows its advantages over existing methods. A method for solving multi-criteria group decision-making (MCGDM) problems with uncertain information using multi-granularity PFRS over two universes and grey relational analysis is proposed in [15]. This method applies to medical decision problems that involve Pythagorean fuzzy information on different levels of granularity. The main focus of the current research on the Pythagorean fuzzy RS (PFRS) is to develop various models and study their properties. As mentioned before, most of the existing literature focuses on the theoretical aspects and the model building of PFRSs, while there are few applications of them to real-world decision-making problems and the corresponding attribute reduction methods. No attention to variable precision RSs with PFNs is made. However, the study in [7, 8] shows that the variable precision RSs model is vital in attribute reduction methods. Based on this observation and taking into account the noise data with the benefits of PFSs and RSs, here we combine variable precision RSs with PFSs and create a new type of rough set model called the variable precision PFRS (VPPFRS) model in the Pythagorean fuzzy decision system (PFDS). Few other concepts on fuzzy systems can be found in [23–28, 28].

The structure of this paper is as follows. Section 2 reviews the basic concepts of PFSs and PFI systems. In Section 3, we propose a VPPFRS model based on the distance of conflict and study its properties. We also develop a new attribute reduction algorithm. In Section 1, we apply our method to a real-world problem and demonstrate its effectiveness and rationality. Section 6 summarizes our work.

2. Preliminaries

Here, we recall some of the most fundamental ideas and terms employed throughout the article.

Definition 2.1 ([17]). A PFS $\mathcal{P}$ over a universe of discourse $\mathcal{U}$ is given by

$$\mathcal{P} = \{(u, \langle \mu_{\mathcal{P}}(u), \nu_{\mathcal{P}}(u)\rangle) \mid u \in \mathcal{U}\},$$

where $\mu_{\mathcal{P}} : \mathcal{U} \longrightarrow [0,1]$ and $\nu_{\mathcal{P}} : \mathcal{U} \longrightarrow [0,1]$ are MD and NMD of u to $\mathcal{P}$, respectively, and satisfy

$$0 \leqslant \mu_{\mathcal{P}}^2(u) + \nu_{\mathcal{P}}^2(u) \leqslant 1, \ \forall u \in \mathcal{U}.$$

In Definition 2.1, the hesitancy degree is represented by $\pi_{\mathcal{P}}(u) = \sqrt{1 - \mu_{\mathcal{P}}^2(u) - \nu_{\mathcal{P}}^2(u)}$. According to Zhang and Xu [19], a Pythagorean fuzzy number (PFN) is denoted by $p = \langle \mu_{\mathcal{P}}, \nu_{\mathcal{P}} \rangle$.

Definition 2.2 ([19])**.** The score and accuracy functions of any PFN $p = \langle \mu, \nu \rangle$ are defined by $\mathcal{S}(p) = \mu_{\mathcal{P}}^2 - \nu_{\mathcal{P}}^2$ and $\mathcal{H}(p) = \mu_{\mathcal{P}}^2 + \nu_{\mathcal{P}}^2$. It is clear that $\mathcal{S}(p) \in [-1,1]$ and $\mathcal{H}(p) \in [0,1]$.

For the ranking of PFNs, we adopt Zhang and Xu [19] concept.

Definition 2.3. For any PFNs, p_1 and p_2,

(1) if $\mathcal{S}(p_1) > \mathcal{S}(p_2)$, then p_1 is bigger than p_2, represented by $p_1 \succ p_2$;
(2) if $\mathcal{S}(p_1) = \mathcal{S}(p_2)$, then
   (I) if $\mathcal{H}(p_1) = \mathcal{H}(p_2)$, then p_1 is equivalent to p_2, represented by $p_1 \sim p_2$;
   (II) if $\mathcal{H}(p_1) > \mathcal{H}(p_2)$, then p_1 is bigger than p_2, represented by $p_1 \succ p_2$;

In this paper, we use the notations "$\prec$" and "$\preccurlyeq$" for inferior and equivalent or inferior relationships, respectively.

3. Conflict distance-based VPPFRS in PFDS

This section defines the conflict distance-based VPPFRS in the PFDS. First, we introduce the concept of PFDs as follows.

Definition 3.1. A quadruple $IS = (\mathcal{U}, \mathcal{A}, \mathcal{V}, f)$ represents a PFIS, where U stated before, $\mathcal{A}(\neq \emptyset)$ set of attributes, and $\mathcal{V}$ is a set of PFNs. The information function f is a map from $\mathcal{U} \times \mathcal{A}$ onto $\mathcal{V}$, such that $f(u,a) \in \mathcal{V}$, $\forall u \in \mathcal{U}$ and $a \in \mathcal{A}$, where $f(u,a)$ are PFNs, denoted by $f(u,a) = \langle \mu_a(u), \nu_a(u) \rangle$. If $\mathcal{A} = \mathcal{C} \cup \mathcal{D}$ and $\mathcal{C} \cap \mathcal{D} = \emptyset$ with $\mathcal{C}$ being the condition attribute set and $\mathcal{D}$ the decision attribute set, then $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ is called a PFDS and the decision classes generated by $\mathcal{D}$ are denoted by $\Omega_{\mathcal{D}} = \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_t\}$.

Based on Definition 3.1, we introduce the concept of conflict distance based on PFNs called Pythagorean fuzzy conflict distance (PFCD) in the following.

Definition 3.2. Let $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ be a PFDS. For all u_i, u_j $(i,j \in \{1,2,\ldots,n\})$, $\forall b \in \mathcal{B} \subseteq \mathcal{C}$, the PFCD ξ_{ij}^b of u_i and u_j w.r.t. the attribute b is obtained in the following way:

$$\xi_{ij}^b = \frac{\left|\mathcal{S}_i^b(p_i) - \mathcal{S}_j^b(p_j)\right| + \left|\delta_i^b(p_i) - \delta_j^b(p_j)\right| + \left|\sigma_i^b(p_i) - \sigma_j^b(p_j)\right|}{4},$$

where, $p_i^b = \langle \mu_i^b, \nu_i^b \rangle$, $p_j^b = \langle \mu_j^b, \nu_j^b \rangle$, $\mathcal{S}_i^b(p_i) = \mu_i^b - \nu_i^b$, $\mathcal{S}_j^b(p_j) = \mu_j^b - \nu_j^b$, $\pi_i^b = \sqrt{1 - (\mu_i^b)^2 - (\nu_i^b)^2}$, $\pi_j^b = \sqrt{1 - (\mu_j^b)^2 - (\nu_j^b)^2}$, $\delta_i^b(p_i) = \mu_i^b + \pi_i^b \mu_i^b$, $\delta_j^b(p_j) = \mu_j^b + \pi_j^b \mu_j^b$, $\sigma_i^b(p_i) = \nu_i^b + \pi_i^b \nu_i^b$, and $\sigma_j^b(p_j) = \nu_j^b + \pi_j^b \nu_j^b$.

It is evident from Definition 3.2 that the core, support, opposition, and hesitant degrees of the PFNs for u_i and u_j concerning b are discernible.

Definition 3.3. Let $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ be a PFDS. For any real number $\eta \in [0,1]$, and attribute set $\mathcal{B} = (b_1, b_2, \ldots, b_l) \subseteq \mathcal{C}$, we define the $\eta - \mathcal{B}$ neighborhood bases of each u_i $(i \in \{1,2,\ldots,n)$ as

$$[u_i]_{\mathcal{B}}^{\eta} = \{u_j \in \mathcal{U} \mid \xi_{ij}^b \leqslant \eta, \forall b \in \mathcal{B}\},$$

where ξ_{ij}^b $(i,j \in \{1,2,\ldots,n\})$ defined in Definition 3.2.

Definition 3.4. Let $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ be a PFDS. For any $\mathcal{X} \subseteq \mathcal{U}$, $\eta \in [0,1]$, and $0.5 < \vartheta \leqslant 1$, the PFCD-based (η,ϑ)-lower and (η,ϑ)-upper approximations of $\mathcal{X}$ with respect to the the attribute set $\mathcal{B} = (b_1, b_2, \ldots, b_l) \subseteq \mathcal{C}$ are denoted by $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})$ and $\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})$, respectively, in which

$$\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}) = \bigcup \left\{ u \in \mathcal{U} \mid \frac{|[u]_{\mathcal{B}}^{\eta} \cap \mathcal{X}|}{|[u]_{\mathcal{B}}^{\eta}|} \geqslant \vartheta \right\}, \qquad \overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}) = \bigcup \left\{ u \in \mathcal{U} \mid \frac{|[u]_{\mathcal{B}}^{\eta} \cap \mathcal{X}|}{|[u]_{\mathcal{B}}^{\eta}|} \geqslant 1 - \vartheta \right\},$$

where $[u_i]_{\mathcal{B}}^{\eta} = \{u_j \in \mathcal{U} \mid \xi_{ij}^{b} \leqslant \eta, \forall b \in \mathcal{B}\}$ is defined as Definition 3.3.

We call the pair $(\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}), \overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}))$ a PFCD-based variable precision Pythagorean fuzzy rough (PFCD-VPPFRS) set of $\mathcal{X}$ with respect to $\mathcal{B}$.

From Definition 3.4, we have following Theorems.

Theorem 3.5. *For any PFDS* $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$, *the following holds*

$$\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}) \subseteq \mathcal{X} \subseteq \overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}).$$

Proof. Suppose $u \in \underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})$. Then $\exists$, $[u]_{\mathcal{B}}^{\eta} \subseteq \mathcal{X}$ and $u \in [u]_{\mathcal{B}}^{\eta}$. Thus, $u \in \mathcal{X}$. Hence, $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}) \subseteq \mathcal{X}$. Again, for $u \in \mathcal{X}$, we have $[u]_{\mathcal{B}}^{\eta} \cap \mathcal{X} \neq \emptyset$. Therefore, we get $u \in \overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})$. Hence, $\mathcal{X} \subseteq \overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})$. □

Theorem 3.6. *For any PFDS* $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ *and for* $\mathcal{X} \subseteq \mathcal{U}$, $\mathcal{B}_1 \subseteq \mathcal{B}_2 \subseteq \mathcal{C}$, $\eta \in [0,1]$, $0.5 < \vartheta \leqslant 1$, *the following are hold.*

(1) $\underline{Apr}_{\mathcal{B}_2}^{\eta,\vartheta}(\mathcal{X}) \subseteq \underline{Apr}_{\mathcal{B}_1}^{\eta,\vartheta}(\mathcal{X})$;

(2) $\overline{Apr}_{\mathcal{B}_2}^{\eta,\vartheta}(\mathcal{X}) \subseteq \overline{Apr}_{\mathcal{B}_1}^{\eta,\vartheta}(\mathcal{X})$.

Proof.

(1) Suppose $u \in \underline{Apr}_{\mathcal{B}_1}^{\eta,\vartheta}(\mathcal{X})$. Then $\exists$ $[u]_{\mathcal{B}_1}^{\eta} \subseteq \mathcal{X}$. Since, $\mathcal{B}_1 \subseteq \mathcal{B}_2 \subseteq \mathcal{C}$, we get $[u]_{\mathcal{B}_2}^{\eta} \subseteq \left|[u]_{\mathcal{B}_1}^{\eta}\right|$. Hence, $u \in \underline{Apr}_{\mathcal{B}_2}^{\eta,\vartheta}(\mathcal{X})$. Thus, $\underline{Apr}_{\mathcal{B}_2}^{\eta,\vartheta}(\mathcal{X}) \subseteq \underline{Apr}_{\mathcal{B}_1}^{\eta,\vartheta}(\mathcal{X})$.

(2) Suppose $u \in \overline{Apr}_{\mathcal{B}_2}^{\eta,\vartheta}(\mathcal{X})$. Then $\exists$ $[u]_{\mathcal{B}_2}^{\eta} \cap \mathcal{X} \neq \emptyset$. Since, $\mathcal{B}_1 \subseteq \mathcal{B}_2 \subseteq \mathcal{C}$, we get $[u]_{\mathcal{B}_1}^{\eta} \cap \mathcal{X} \neq \emptyset$. Hence, $u \in \overline{Apr}_{\mathcal{B}_1}^{\eta,\vartheta}(\mathcal{X})$. Thus, $\overline{Apr}_{\mathcal{B}_2}^{\eta,\vartheta}(\mathcal{X}) \subseteq \overline{Apr}_{\mathcal{B}_1}^{\eta,\vartheta}(\mathcal{X})$.

□

Theorem 3.7. *For any PFDS* $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ *and for* $\mathcal{X} \subseteq \mathcal{U}$, $\mathcal{B} \subseteq \mathcal{C}$, $0 \leqslant \eta_1 \leqslant \eta_2 \leqslant 1$, $0.5 < \vartheta \leqslant 1$, *the following are hold.*

(1) $\underline{Apr}_{\mathcal{B}}^{\eta_2,\vartheta}(\mathcal{X}) \subseteq \underline{Apr}_{\mathcal{B}}^{\eta_1,\vartheta}(\mathcal{X})$;

(2) $\overline{Apr}_{\mathcal{B}}^{\eta_1,\vartheta}(\mathcal{X}) \subseteq \overline{Apr}_{\mathcal{B}}^{\eta_2,\vartheta}(\mathcal{X})$.

Proof.

(1) Suppose $u \in \underline{Apr}_{\mathcal{B}}^{\eta_2,\vartheta}(\mathcal{X})$. Then $\exists$ $[u]_{\mathcal{B}}^{\eta_2} \subseteq \mathcal{X}$. Since, $0 \leqslant \eta_1 \leqslant \eta_2 \leqslant 1$, so $[u]_{\mathcal{B}}^{\eta_1} \subseteq \left|[u]_{\mathcal{B}}^{\eta_1}\right|$. Hence, $u \in \underline{Apr}_{\mathcal{B}}^{\eta_1,\vartheta}(\mathcal{X})$. Thus, $\underline{Apr}_{\mathcal{B}}^{\eta_2,\vartheta}(\mathcal{X}) \subseteq \underline{Apr}_{\mathcal{B}}^{\eta_1,\vartheta}(\mathcal{X})$.

(2) Suppose $u \in \overline{Apr}_{\mathcal{B}}^{\eta_1,\vartheta}(\mathcal{X})$. Then $\exists$ $[u]_{\mathcal{B}}^{\eta_1} \cap \mathcal{X} \neq \emptyset$. Since, $0 \leqslant \eta_1 \leqslant \eta_2 \leqslant 1$, so $\exists$ $[u]_{\mathcal{B}}^{\eta_2} \cap \mathcal{X} \neq \emptyset$. Thus, $u \in \overline{Apr}_{\mathcal{B}}^{\eta_2,\vartheta}(\mathcal{X})$. Hence, $\overline{Apr}_{\mathcal{B}}^{\eta_1,\vartheta}(\mathcal{X}) \subseteq \overline{Apr}_{\mathcal{B}}^{\eta_2,\vartheta}(\mathcal{X})$.

□

Theorem 3.8. *For any PFDS* $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ *and for* $\mathcal{X} \subseteq \mathcal{U}$, $\mathcal{B} \subseteq \mathcal{C}$, $\eta \in [0,1]$, $0.5 < \vartheta_1 \leqslant \vartheta_2 \leqslant 1$, *the following are hold.*

(1) $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta_1}(\mathcal{X}) \subseteq \underline{Apr}_{\mathcal{B}}^{\eta,\vartheta_2}(\mathcal{X})$;
(2) $\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta_1}(\mathcal{X}) \supseteq \overline{Apr}_{\mathcal{B}}^{\eta,\vartheta_2}(\mathcal{X})$.

Proof. Like the proof of Theorem 3.7, this one follows a similar pattern.

According to Definition 3.4, given a threshold value η, one may obtain $[u]_{\mathcal{B}}^{\eta}$ and then compute the (η,ϑ)-lower and (η,ϑ)-upper approximations of $\mathcal{X}$. In simpler terms, the set $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})$ may be defined as a collection of element sets from the universe $\mathcal{U}$ that can be categorized with certainty based on provided confidence threshold values η and ϑ. The (η,ϑ)-upper approximation $\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})$ of the set $\mathcal{X}$ is a representation that takes into account the specified threshold values η and ϑ. It allows for classifying all elements in the universe $\mathcal{U}$ into sets that are likely to belong to the set $\mathcal{X}$.

For any $\mathcal{X} \subseteq \mathcal{U}$, the three regions like positive, boundary, and negative regions of $\mathcal{X}$ w.r.t. $\mathcal{B} \subseteq \mathcal{C}$ are calculated by the following way:

$$\begin{aligned}
POS_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{X}) &= \underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}),\\
BND_{\mathcal{B}}^{(\eta,\alpha)}(\mathcal{X}) &= \overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}) - \underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X}),\\
NEG_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{X}) &= \bigcup \left\{ u \in \mathcal{U} \mid \frac{\left|[u]_{\mathcal{B}}^{\eta} \cap \mathcal{X}\right|}{\left|[u]_{\mathcal{B}}^{\eta}\right|} \leqslant 1-\vartheta \right\}.
\end{aligned}$$

From a semantic perspective, items located inside the positive zone can be categorized with a higher degree of certainty into a certain judgment class. Typically, a larger positive zone is associated with a smaller border region, indicating a reduced presence of ambiguous or unclear items. Furthermore, in the context of a classification task, a reduction in ambiguity is directly associated with an increase in accuracy. In this connection, in the following we defined the approximation accuracy and the quality of classification in PFCD-based VPPFRS is defined as follows:

$$\begin{aligned}
\tau(\mathcal{X}) &= \frac{\left|\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})\right|}{\left|\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})\right|},\\
\zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D}) &= \frac{\left|\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{X})\right|}{|\mathcal{U}|}.
\end{aligned} \tag{3.1}$$

□

For attribute reduction, we have the following definition from equation 3.1.

Definition 3.9. Let $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$ be a PFDS. For $\mathcal{B} \subseteq \mathcal{C} \subseteq \mathcal{A}$, $\eta \in [0,1]$, and $\vartheta \in (0.5, 1]$, if $\zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D}) = \zeta_{\mathcal{C}}^{(\eta,\vartheta)}(\mathcal{D})$, then $\mathcal{B}$ is called a reduction of the given PFDS concerning η and ϑ.

Definition 3.10. In a PFDS $(\mathcal{U}, \mathcal{A} = \mathcal{C} \cup \mathcal{D}, \mathcal{V}, f)$, given $\eta \in [0,1]$, $\vartheta \in (0.5,1]$, $\mathcal{B} \subseteq \mathcal{C}$, and $a \in \mathcal{B}$, the degree of dependence of the attribute a with respect to η is denoted by $\rho_{\mathcal{B}}^{(\eta,\vartheta)}(a, \mathcal{D})$ and defined in the following way:

$$\rho_{\mathcal{B}}^{(\eta,\vartheta)}(a, \mathcal{D}) = \zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D}) - \zeta_{\mathcal{B}-\{a\}}^{(\eta,\vartheta)}(\mathcal{D}). \tag{3.2}$$

4. Algorithm of attribute reduction

In this section, we present the following attribute reduction procedure of the proposed PFCD-VPPFRS model based on the degree of similarity in the positive areas. The details of procedure are given in Algorithm 1.

Algorithm 1 The PFCD-VPPFRS-based attribute reduction

Input: $IS = (\mathcal{U}, \mathcal{A}, \mathcal{V}, f)$, η and ϑ.
Output: Reduction $\mathcal{B}$.

Step 1: Let $\mathcal{B} = \mathcal{C}$.

Step 2: By equation (3.1), obtain $\zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D})$.

Step 3: By equation (3.2), obtain $\rho_{\mathcal{B}}^{(\eta,\vartheta)}(a, \mathcal{D})$ for any $a \in \mathcal{B}$.

Step 4: If $\zeta_{\mathcal{B}-\{a\}}^{(\eta,\vartheta)}(\mathcal{D}) \geqslant \zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D})$ and $\rho_{\mathcal{B}}^{(\eta,\vartheta)}(a, \mathcal{D})$ is the least, then let $\mathcal{B} = \mathcal{B} - \{a\}$ and follow Step 3. If $\zeta_{\mathcal{B}-\{a\}}^{(\eta,\vartheta)}(\mathcal{D}) < \zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D})$ and $\rho_{\mathcal{B}}^{(\eta,\vartheta)}(a, \mathcal{D})$ are equal to each other, then follow Step 5.

Step 5: Reduction $\mathcal{B}$.

5. An example with comparative study

Here, we consider a case study selection of green suppliers for electronics manufacturing as an example and then illustrate how our model is superior and rational.

5.1. *An example: a case study*

In real life, we often face complex and uncertain situations that are hard to comprehend. To deal with this, we sometimes use Pythagorean fuzzy numbers to represent the values of different attributes. These values form a PFDS. Let us assume $IS = (\mathcal{U}, \mathcal{A}, \mathcal{V}, f)$ be a PFDS of chose of green supplier for manufacturing of electronics. The details of PFDS are shown in Table 1, where $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ is a set of green supplier and $\mathcal{C} = \{a_1, a_2, a_3, a_4, a_5\}$ is a set of criteria such that a_1, a_2, a_3, a_4, and a_5 indicate environmental protection capability, product quality, technology capability, product cost and warranties, and claim policies, respectively. Let $\Omega_{\mathcal{D}} = \{\mathcal{D}\}$ be the decision making criteria set. The information that influences set $\mathcal{V}$ comes from both the statistical data and the decision-makers experience knowledge.

Table 1: The PFDS for choosing green suppliers.

	a_1	a_2	a_3	a_4	a_5	$\mathcal{D}$
u_1	$\langle 0.7, 0.4\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.9, 0.3\rangle$	$\langle 0.8, 0.3\rangle$	1
u_2	$\langle 0.8, 0.4\rangle$	$\langle 0.7, 0.5\rangle$	$\langle 0.7, 0.4\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.9, 0.2\rangle$	1
u_3	$\langle 0.8, 0.4\rangle$	$\langle 0.4, 0.8\rangle$	$\langle 0.7, 0.4\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.9, 0.2\rangle$	2
u_4	$\langle 0.9, 0.4\rangle$	$\langle 0.9, 0.4\rangle$	$\langle 0.8, 0.3\rangle$	$\langle 0.9, 0.4\rangle$	$\langle 0.9, 0.1\rangle$	1
u_5	$\langle 0.6, 0.5\rangle$	$\langle 0.7, 0.3\rangle$	$\langle 0.5, 0.1\rangle$	$\langle 0.5, 0.6\rangle$	$\langle 0.7, 0.1\rangle$	1
u_6	$\langle 0.7, 0.4\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.8, 0.4\rangle$	$\langle 0.6, 0.4\rangle$	2
u_7	$\langle 0.5, 0.6\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.6, 0.3\rangle$	2

Based on Definition 3.3 and for $\eta = 0.1$, $\mathcal{U}$ can be divided with respect to criteria set $\mathcal{C}$ as follows:

$$\mathcal{U}/\mathcal{C} = \{\mathcal{X}_1^{0.1}, \mathcal{X}_2^{0.1}, \mathcal{X}_3^{0.1}, \mathcal{X}_4^{0.1}\},$$

where $\mathcal{X}_1^{0.1} = \{u_1\}$, $\mathcal{X}_2^{0.1} = \{u_2, u_4\}$, $\mathcal{X}_3^{0.1} = \{u_3, u_5, u_6\}$, and $\mathcal{X}_4^{0.1} = \{u_7\}$. With respect to decision making criteria set $\Omega_{\mathcal{D}}$, $\mathcal{U}$ can be divided as follows: $\mathcal{U}/\Omega_{\mathcal{D}} = \{\mathcal{Y}_1, \mathcal{Y}_2\}$, where $\mathcal{Y}_1 = \{u_1, u_2, u_4, u_5\}$ and $\mathcal{Y}_2 = \{u_3, u_6, u_7\}$.

Based on Definition 3.4 and $\vartheta = 1$, the lower and upper approximations of $\mathcal{Y}_1$ and $\mathcal{Y}_2$ are calculated as follows: $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_1) = \{u_1, u_2, u_4\}$, $\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_1) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_2) = \{u_7\}$, $\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_1) =$

$\{u_3, u_5, u_6, u_7\}$. By expression (3.1), we have $\zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D}) = 0.571$. Again, if $\vartheta = 0.65$, then based on Definition 3.4, the lower and upper approximations of $\mathcal{Y}_1$ and $\mathcal{Y}_2$ are calculated as follows: $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_1) = \{u_1, u_2, u_4\}$, $\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_1) = \{u_1, u_2, u_4\}$, $\underline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_2) = \{u_3, u_5, u_6, u_7\}$, $\overline{Apr}_{\mathcal{B}}^{\eta,\vartheta}(\mathcal{Y}_1) = \{u_3, u_5, u_6, u_7\}$, and by expression (3.1), $\zeta_{\mathcal{B}}^{(\eta,\vartheta)}(\mathcal{D}) = 1$. Finally, by Algorithm 1, the reduction $\mathcal{B} = \{a_2, a_4\}$, when $\eta = 0.1$ and $\vartheta = 0.65$. Using reduction $\mathcal{B}$, we can create rules that make decisions based on probabilities. These rules are shown in Table 2.

Table 2: The decision rules based on probabilities with respect to $\{a_2, a_4\}$.

Rules	Number of support	Confidence(%)
$a_2 \geqslant \langle 0.9, 0.4\rangle,\ a_4 \geqslant \langle 0.8, 0.4\rangle \rightarrow^{100\%} \mathcal{D} = 1$	3	100
$a_2 \leqslant \langle 0.7, 0.3\rangle,\ a_4 \geqslant \langle 0.8, 0.1\rangle \rightarrow^{65.6\%} \mathcal{D} = 2$	3	65.6
$a_2 = \langle 0.5, 0.4\rangle,\ a_4 = \langle 0.5, 0.4\rangle \rightarrow^{100\%} \mathcal{D} = 2$	1	100

Table 2 demonstrates that all seven things may be accurately categorized. In other words, the categorization accuracy is 100%. The manufacturing of electronics firms prioritizes delivery capabilities and influence when evaluating and selecting suppliers by reducing suppliers to only a_2 and a_4. Suppliers are considered contractible if their product quality is greater than or equal to $\langle 0.9, 0.4\rangle$ and their warranties and claim policies are greater than or equal to $\langle 0.8, 0.4\rangle$. Suppliers should not be considered if their product quality is less than or equal to $\langle 0.7, 0.3\rangle$ and their warranties and claim policies are less than or equal to $\langle 0.8, 0.1\rangle$. Suppliers are significantly worse if their product quality equals $\langle 0.5, 0.4\rangle$ and their warranties and claim policies are equal to $\langle 0.5, 0.4\rangle$. In this case, they should definitely not be chosen.

5.2. *Superiority and rationality analysis*

Using the attribute reduction Algorithm 1, we can show the results of our proposed model and compare it with the methods from the literature [18]. Table 3 illustrates how our model is superior and rational.

Table 3: The decision rules based on probabilities with respect to $\{a_2, a_4\}$ by method proposed in [18].

Rules	Number of support	Confidence(%)
$a_2 \geqslant \langle 0.9, 0.4\rangle,\ a_4 \geqslant \langle 0.8, 0.4\rangle \rightarrow^{100\%} \mathcal{D} = 1$	3	100
$a_2 = \langle 0.5, 0.4\rangle,\ a_4 = \langle 0.5, 0.4\rangle \rightarrow^{100\%} \mathcal{D} = 2$	1	100

Table 3 shows the number and quality of objects that are correctly classified by the methods from the literature [18]. Their quality values are 59.4% and 72.8%, respectively. Our proposed model in this paper achieves much higher classification performance than these methods, as shown by our results.

By changing the model's parameters η and ϑ, we can enhance its classification ability based on the previous calculation and analysis. This way, the model can handle faults better, accurately classify objects, and discover useful knowledge from the decision-making information table. Then, we can extract decision rules from the knowledge.

By comparing studies in [4, 5, 7, 8, 18, 22], we have the following differences and advantages.

(1) In [4, 5, 7, 8, 22], considering information systems are based on IFNs. This paper considers information systems based on PFNs, which are more general than those based on IFNs discussed in [16, 17, 19].

(2) Our attribute reduction algorithm is simpler, more effective, and more accurate than the attribute reduction algorithm proposed in [18], which is shown in Section 5.2.

6. Conclusions

Decision-making is the core of management, and it involves various problems in real life. However, the real world is complex and uncertain, and humans have limitations. Therefore, using only one method of uncertainty is not enough to solve real-life decision-making problems. We can use different methods of uncertainty to create a strong and effective soft computing method for these problems. This paper uses PFSs and RSs to build a hybrid model based on conflict distance. By changing the parameter ϑ, the model can handle faults better and work well with decision-making problems with PFNs as decision values. This also expands the use of PFS theory and RS theory.

We will focus on decision-making problems that have PFNs as decision values. These problems have a lot of noise data and preference information in their decision-making information systems. We will build the corresponding dominance Pythagorean fuzzy variable precision rough sets for these problems.

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