Available online at www.isr-publications.com/jmcs
J. Math. Computer Sci. 16 (2016), 452–460
Research Article

Online: ISSN 2008-949x

Journal of Mathematics and Computer Science

Journal Homepage: www.tjmcs.com - www.isr-publications.com/jmcs

Nonlinear stabilization control of Furuta pendulum only using angle position measurements

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Abstract

In this paper, we discuss the stabilization control problem for a nonlinear mechanical system called Furuta pendulum. A new stabilizing control method that only uses the measurements of angle position is developed. This method has three successive steps. First, we present the dynamic equation of Furuta pendulum and change it into an affine nonlinear system by appropriately choosing state variables. Second, we linearize the nonlinear system around the origin and consider the nonlinear higher order term to be system's fictitious disturbance. After that, an idea of equivalent input disturbance is used to design the stabilizing controller for the nonlinear system. The effectiveness of our proposed control strategy is illustrated via a numerical example.

Keywords: Nonlinear analysis and control, Furuta pendulum, underactuated mechanical system, equivalent input disturbance.
2010 MSC: 05C12, 05C15, 05C76.

1. Introduction

In the past few years, a great deal of study efforts have been carried out on underactuated mechanical systems (UMSs), see [9, 16, 18, 20]. The control input numbers of a UMS are less than the numbers of system's degrees of freedom (DOF) [14]. This characteristic makes UMSs lightweight, flexible and low energy consumption. It is meaningful to study the control design of this kind of mechanical systems.

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Received 2016-06-27

The mechanical system that has one input and two DOFs is the simplest UMS. Recently, the study on the control of 2-DOF UMS is a hot issue in the field of control engineering, [1, 2, 19]. Furuta pendulum is a well-known example of 2-DOF UMS. The physical structure of Furuta pendulum is shown in Fig. 1. It consists of a horizontal rotating arm and an inverted pendulum. The arm is driven by an actuator and moves in a horizontal plane. The pendulum connects to the arm by a passive joint and can freely move in vertical plane. The commonly discussed control task for Furuta pendulum is to keep the pendulum stabilized in the upright position while the horizontal arm does not rotate. But the control task is not easy to achieve because one DOF of Furuta pendulum lies in an uncertain configuration and a second non-holonomic constraint is possessed by this system [12]. In addition, this mechanical system is a complicated nonlinear system that is not feedback linearizable [8]. All of these bring the stabilization control of Furuta pendulum to be a challenging task.

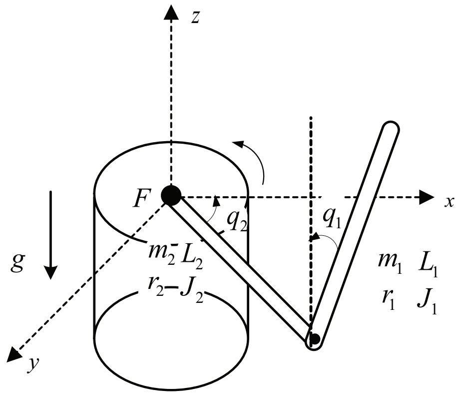

Figure 1: Model of Furuta pendulum.

In order to solve the stabilization control problem for Furuta pendulum, many control strategies have been presented in the past three decades. In [11], Olfati-Saber firstly changed Furuta pendulum into a cascade nonlinear system, and then used a backstepping method to semiglobally stabilize the nonlinear system. It enables the semiglobal stabilization of Furuta pendulum to be achieved. In [6], the local stabilization of Furuta pendulum around the unstable vertical equilibrium was realized by a Lyapunov-function-based control method. To achieve the stabilization of Furuta pendulum in the whole motion space, a common used control strategy is firstly to divide the motion space into two subspaces: swing-up area and balancing area, and then to design swing-up controller and balancing controller for each subspace, respectively. The switch from swing-up controller to balancing controller makes the stabilizing control objective of Furuta pendulum be achieved. Many control methods based on this strategy have been presented, [3–5, 17]. Although the switch strategy is effective for stabilizing Furuta pendulum sometimes, the stability of switching from swing-up area to balancing area is not guaranteed. This may cause the controller frequently to switch. It is harmful for the running of control system safely. To solve this problem, the attempts of using a single controller to achieve the stabilization of Furuta pendulum have been made, for example an energy shaping method in [10], a nonlinear sliding-mode method in [7], and a coupled sliding-mode method in [13].

As discussed above, many control methods of stabilizing Furuta pendulum have been presented. However, all these methods require the measurements of both angle position and velocity information to design the stabilizing controller. Since the velocity information generally contains noises that may affect the performance of control system, and since the installation of tachometer to measure the velocity information increases the cost of the control system, it is of great meaning to stabilize Furuta pendulum using the angle position measurements only. This is the main motivation of this study. In this paper, we present a new method of globally stabilizing Furuta pendulum. This method only uses the measurements of angle position to design the stabilizing controller. The design procedure of this method has three successive steps:

(1) Give the state space equation of Furuta pendulum by appropriately choosing state variables, which is an affine nonlinear system.

(2) Linearize the affine nonlinear system around the origin.

(3) Considering the nonlinear higher order term to be system's fictitious disturbance enables us to use an idea of equivalent input disturbance to design the stabilizing controller for the nonlinear system.

The validity of the proposed theoretical results is demonstrated by a numerical example.

2. Mathematical model of Furuta pendulum

In the model of Furuta pendulum shown in Fig. 1, the meanings of physical parameters are:

m_1 : mass of the inverted pendulum;
L_1 : length of the inverted pendulum;
r_1 : distance from the passive joint to the center of mass (COM) of the pendulum;
J_1 : moment of inertia around the COM of the pendulum;
m_2 : mass of the horizontal arm;
L_2 : length of the horizontal arm;
F : torque applied to the horizontal arm;
r_2 : distance from the active joint to the center of mass (COM) of the arm;
J_2 : moment of inertia around the COM of the arm;
g : gravitational acceleration;
q_1 : angle of the pendulum relative to the axis z;
q_2 : angle of the arm relative to the axis x.

We assume that there are no friction on the joints in this paper. Let $\boldsymbol{q} = [q_1,\ q_2]^\top$, $\dot{\boldsymbol{q}} = d\boldsymbol{q}/dt$. Then, it is not difficult to get the kinetic energy $K(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and the potential energy $P(\boldsymbol{q})$ of Furuta pendulum as

$$K(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^\top \begin{bmatrix} \alpha_1 & -\alpha_2 \cos q_1 \\ -\alpha_2 \cos q_1 & \alpha_3 + \alpha_1 \sin^2 q_1 \end{bmatrix} \dot{\boldsymbol{q}}, \quad P(\boldsymbol{q}) = \alpha_4 \cos q_1, \tag{2.1}$$

where

$$\alpha_1 = J_1 + m_1 r_1^2, \quad \alpha_2 = m_1 r_1 L_2, \quad \alpha_3 = J_2 + m_2 r_2^2 + m_1 L_2^2, \quad \alpha_4 = m_1 g r_1.$$

We take $L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = K(\boldsymbol{q}, \dot{\boldsymbol{q}}) - P(\boldsymbol{q})$ to be the Lagrangian of Furuta pendulum. The Euler-Lagrange equations give the mathematical model of this mechanical system as

$$\begin{cases} \dfrac{d}{dt}\left[\dfrac{\partial L(\boldsymbol{q}, \dot{\boldsymbol{q}})}{\partial \dot{q}_1}\right] - \dfrac{\partial L(\boldsymbol{q}, \dot{\boldsymbol{q}})}{\partial q_1} = 0, \\ \dfrac{d}{dt}\left[\dfrac{\partial L(\boldsymbol{q}, \dot{\boldsymbol{q}})}{\partial \dot{q}_2}\right] - \dfrac{\partial L(\boldsymbol{q}, \dot{\boldsymbol{q}})}{\partial q_2} = F. \end{cases} \tag{2.2}$$

From (2.1), we easily get

$$\begin{cases} \dfrac{\partial L(\boldsymbol{q},\dot{\boldsymbol{q}})}{\partial \dot{q}_1} = \alpha_1\dot{q}_1 - \alpha_2\cos q_1\dot{q}_2, \\ \dfrac{\partial L(\boldsymbol{q},\dot{\boldsymbol{q}})}{\partial \dot{q}_2} = (\alpha_3 + \alpha_1\sin^2 q_1)\dot{q}_2 - \alpha_2\cos q_1\dot{q}_1, \\ \dfrac{\partial L(\boldsymbol{q},\dot{\boldsymbol{q}})}{\partial q_1} = \dfrac{1}{2}\alpha_1\dot{q}_2^2\sin 2q_1 + \alpha_2\sin q_1\dot{q}_1\dot{q}_2 + \alpha_4\sin q_1, \\ \dfrac{\partial L(\boldsymbol{q},\dot{\boldsymbol{q}})}{\partial q_2} = 0. \end{cases} \tag{2.3}$$

Combining (2.2) and (2.3) gives the model of Furuta pendulum in the following standard form

$$\begin{bmatrix} \alpha_1 & -\alpha_2\cos q_1 \\ -\alpha_2\cos q_1 & \alpha_3+\alpha_1\sin^2 q_1 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} H_1(\boldsymbol{q},\dot{\boldsymbol{q}}) \\ H_2(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} + \begin{bmatrix} G_1(q_1) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix}, \tag{2.4}$$

where

$$H_1(\boldsymbol{q},\dot{\boldsymbol{q}}) = -\frac{1}{2}\alpha_1\dot{q}_2^2\sin 2q_1,\quad H_2(\boldsymbol{q},\dot{\boldsymbol{q}}) = \alpha_2\dot{q}_1^2\sin q_1 + \alpha_1\dot{q}_1\dot{q}_2\sin 2q_1,\quad G_1(q_1) = -\alpha_4\sin q_1.$$

The following variables are chosen to be the state variables of (2.4)

$$x_1 = q_1,\quad x_2 = \alpha_1\dot{q}_1 - \alpha_2\cos q_1\dot{q}_2,\quad x_3 = q_2,\quad x_4 = \dot{q}_2. \tag{2.5}$$

Since $x_2 = \partial L/\partial\dot{q}_1$, the equation (2.2) gives $\dot{x}_2 = \partial L/\partial q_1$. Let $\boldsymbol{x} = [x_1,\ x_2,\ x_3,\ x_4]^\top$. From (2.5), we get the state space equation of (2.4) in $\boldsymbol{x}-$state as

$$\begin{cases} \dot{x}_1 = \dfrac{x_2 + \alpha_2 x_4\cos x_1}{\alpha_1}, \\ \dot{x}_2 = \alpha_4\sin x_1 + \varphi(x_1,x_2,x_4), \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = \tau, \end{cases} \tag{2.6}$$

where

$$\varphi(x_1,x_2,x_4) = \frac{\alpha_1 x_4^2\sin 2x_1}{2} + \frac{\alpha_2 x_4(x_2+\alpha_2 x_4\cos x_1)\sin x_1}{\alpha_1},$$

and $\tau = \ddot{q}_2$ is considered to be the control input of the system (2.6). From (2.4), it is not difficult to get the relationship between control inputs τ and F as

$$F = \frac{\alpha_1(\alpha_3+\alpha_1\sin^2 q_1) - \alpha_2^2\cos^2 q_1}{\alpha_1}\tau + H_2(\boldsymbol{q},\dot{\boldsymbol{q}}) + \frac{\alpha_2\cos q_1}{\alpha_1}\left[H_1(\boldsymbol{q},\dot{\boldsymbol{q}}) + G_1(q_1)\right]. \tag{2.7}$$

The main task of this paper is to design a controller F such that $\boldsymbol{q}\to\boldsymbol{0}$ and $\dot{\boldsymbol{q}}\to\boldsymbol{0}$. From (2.5), we get

$$q_1 = x_1,\quad q_2 = x_3,\quad \dot{q}_1 = \frac{x_2+\alpha_2 x_4\cos x_1}{\alpha_1},\quad \dot{q}_2 = x_4. \tag{2.8}$$

Combining (2.5) and (2.8) yields that $\boldsymbol{q} = \dot{\boldsymbol{q}} = \boldsymbol{0}$ is equivalent to $\boldsymbol{x} = \boldsymbol{0}$. As a result, if we can design a controller τ for (2.6) that makes $\boldsymbol{x}\to\boldsymbol{0}$, then the controller F obtained from τ and (2.7) can make $\boldsymbol{q}\to\boldsymbol{0}$ and $\dot{\boldsymbol{q}}\to\boldsymbol{0}$. Thus, the stabilization control problem for Furuta pendulum changes into the problem of designing a stabilizing controller τ for (2.6). We detailedly explain how to design such controller τ below.

3. Design of stabilizing controller

Note that $\boldsymbol{x} = \mathbf{0}$ is an open-loop equilibrium point of the system (2.6). Linearizing (2.6) around $\boldsymbol{x} = \mathbf{0}$ gives

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\tau + \boldsymbol{\omega(t)}, \tag{3.1}$$

where

$$A = \begin{bmatrix} 0 & \frac{1}{\alpha_1} & 0 & \frac{\alpha_2}{\alpha_1} \\ \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

and $\boldsymbol{\omega(t)}$ is the high-order nonlinear term. This paper assumes that only the measurement of angle position $\boldsymbol{q}$ is available. From (2.5), it is reasonable to take the following output for (3.1) as

$$\boldsymbol{y} = C\boldsymbol{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}. \tag{3.2}$$

A simple calculation gives

$$\Lambda_C = \left[B,\ AB,\ A^2B,\ A^3B\right] = \begin{bmatrix} 0 & \frac{\alpha_2}{\alpha_1} & 0 & \frac{\alpha_2\alpha_4}{\alpha_1^2} \\ 0 & 0 & \frac{\alpha_2\alpha_4}{\alpha_1} & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\Lambda_O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{\alpha_1} & 0 & \frac{\alpha_2}{\alpha_1} \\ 0 & 0 & 0 & 1 \\ \frac{\alpha_4}{\alpha_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha_4}{\alpha_1^2} & 0 & \frac{\alpha_2\alpha_4}{\alpha_1^2} \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Assume that $\alpha_1 \neq \alpha_4$ is satisfied. So, it is easy to get

$$\text{rank}(\Lambda_C) = \text{rank}(\Lambda_O) = 4.$$

This means that (A, B) is controllable and (C, A) is observable.

If $\boldsymbol{\omega}(t)$ is considered to be a fictitious disturbance of (3.1), then there exists an equivalent input disturbance, $\omega_e(t)$, on the control input channel such that $\boldsymbol{\omega}(t)$ and $\omega_e(t)$ play a same influence on the output of (3.1), [15]. Thus, the system (3.1) with (3.2) is equivalent to

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + B[\tau + \omega_e(t)], \\ \boldsymbol{y} = C\boldsymbol{x}, \end{cases} \tag{3.3}$$

if we just consider the response of system's output. For the system (3.3), a state observer is designed to be

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}} + B\tau_f + L[\boldsymbol{y} - C\hat{\boldsymbol{x}}], \tag{3.4}$$

where $\tau_f = \tau + \boldsymbol{\omega}_e(t)$ and L is the observer gain. Since (C, A) is observable, $(A^\top, C^\top)$ is controllable. So, the Riccati equation

$$PA^\top + AP - PC^\top R^{-1}CP + \rho Q = 0, \tag{3.5}$$

has a positive definite solution $P = P^\top > 0$ for the weight matrices $Q > 0$, $R > 0$ and a positive scalar $\rho > 0$. It is easy to verify that the matrix $A^\top - C^\top R^{-1}CP$ is stable. Then, $\left(A^\top - C^\top R^{-1}CP\right)^\top = A - PC^\top R^{-1}C$ is also a stable matrix. Therefore, if we design the observer gain L to be

$$L = PC^\top R^{-1},$$

then the matrix $A - LC$ is stable. This guarantees the stability of the state observer (3.4).

On the other hand, we design the feedback controller to be

$$\tau_f = -R_f^{-1}B^T S\ \hat{\boldsymbol{x}}, \tag{3.6}$$

where $R_f > 0$ is a given weight matrix and S is the solution of Riccati equation

$$SA + A^\top S - SBR_f^{-1}B^\top S = -Q_f, \tag{3.7}$$

with the weight matrix $Q_f > 0$. Since the observer (3.4) is stable and $A - BR_f^{-1}B^T S$ is a stable matrix, the controller (3.6) asymptotically stabilizes the system (3.3) at the origin.

Note that $\tau_f = \tau + \omega_e(t)$ and τ_f has been designed in (3.6). In order to get the expression of the controller τ, it needs to obtain the expression of $\omega_e(t)$. We use the observer (3.4) to get an estimation for $\omega_e(t)$ now. Let $\Delta\boldsymbol{x} = \boldsymbol{x} - \hat{\boldsymbol{x}}$. Substituting $\boldsymbol{x} = \Delta\boldsymbol{x} + \hat{\boldsymbol{x}}$ into (3.3) yields

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}} + B\left[\tau + \hat{\omega}_e(t)\right], \tag{3.8}$$

where

$$\hat{\omega}_e(t) = \omega_e(t) - \Delta\omega, \quad \Delta\dot{\boldsymbol{x}} = A\Delta\boldsymbol{x} + B\Delta\omega. \tag{3.9}$$

Combining (3.4) and (3.8) gives

$$B\left[\tau + \hat{\omega}_e(t)\right] = B\tau_f + L[\boldsymbol{y} - C\hat{\boldsymbol{x}}]. \tag{3.10}$$

From (3.10), we get

$$\hat{\omega}_e(t) = B^\top L[\boldsymbol{y} - C\hat{\boldsymbol{x}}] - \tau + \tau_f.$$

Note that the expressions of (3.3) and (3.8) are analogous and $\hat{\boldsymbol{x}}$ is the observer state of $\boldsymbol{x}$. It is necessary to take $\hat{\omega}_e(t)$ as the estimation of $\omega_e(t)$. In order to guarantee the accuracy of estimation, we use the following low-pass filter to select the frequency band for estimation

$$F(s) = \frac{1}{Ts + 1}, \tag{3.11}$$

where $T > 0$ is a constant. Let the filtered estimation be $\tilde{\omega}_e(t)$. Since the state observer (3.4) is stable, we obtain that $\Delta \boldsymbol{x} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ approaches to zero as $t \to \infty$. It follows from (3.9) that $\tilde{\omega}_e(t)$ asymptotically converges to $\omega_e(t)$.

Remark 3.1. Since the controller τ_f designed in (3.6) asymptotically stabilizes all state variables of (3.3) at zero, the output of (3.3) converges to zero driven by this controller. From the definition of equivalent input disturbance, we get that the controller $\tau = \tau_f - \tilde{\omega}_e(t)$ makes $\boldsymbol{y}$ in (3.2) converge to zero, i.e., $x_1 \to 0$ and $x_3 \to 0$. It follows from the first and the third equations of (2.6) that $x_2 \to 0$ and $x_4 \to 0$. Thus, the stabilization control objective of Furuta pendulum is realized.

Remark 3.2. From (2.7), we note that the measurement of the velocity $\dot{\boldsymbol{q}}$ is needed for the expression of the controller F. But we do not measure $\dot{\boldsymbol{q}}$ in this paper. To solve this problem, the state variables of observer (3.4) is used to construct a substitution variable for $\dot{\boldsymbol{q}}$ due to the fact that the observer (3.4) is stable. Based on (2.8), we give the expression of substitution variable as

$$\dot{q}_1 = \frac{\hat{x}_2 + \alpha_2 \hat{x}_4 \cos x_1}{\alpha_1}, \quad \dot{q}_2 = \hat{x}_4.$$

Table 1: Mechanical parameters of Furuta pendulum.

m_1 [kg]	m_2 [kg]	L_1 [m]	L_2 [m]
0.098	0.080	0.215	0.150
r_1 [m]	r_2 [m]	J_1 [kg $\cdot$ m^2]	J_2 [kg $\cdot$ m^2]
0.148	0.100	2.619×10^{-3}	3.127×10^{-2}

4. Numerical example

In this section, a numerical example is presented in order to demonstrate the effectiveness of our proposed theoretical results.

The physical parameters of Furuta pendulum used in simulations are shown in Table 1. These parameters come from a Furuta pendulum device presented in [7]. In addition, the control design parameters in (3.5), (3.7), and (3.11) were chosen to be

$$Q = I_4, \quad R = 50I_2, \quad \rho = 10^6, \quad Q_f = 1000I_4, \quad R_f = 1, \quad T = 0.1,$$

where I_n is an $n \times n$ identity matrix. By using the MATLAB functions of `lqr`, we get

$$L = \begin{bmatrix} 418.4331 & 258.3406 & 0.0017 & 0.5591 \\ 0.0017 & -0.5591 & 259.1970 & 258.1983 \end{bmatrix}^T.$$

We chose the initial condition of Furuta pendulum to be

$$[q_1,\ q_2,\ \dot{q}_1,\ \dot{q}_2]^\top = [0,\ \pi,\ 1,\ 0]^\top. \tag{4.1}$$

The simulation results of $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and F with (4.1) are shown in Fig. 2. The results show that Furuta pendulum is quickly stabilized at the origin. The stabilization time is less than 10s and the maximal value of input is less than 15 Nm. These demonstrate the validity of our proposed control strategy.

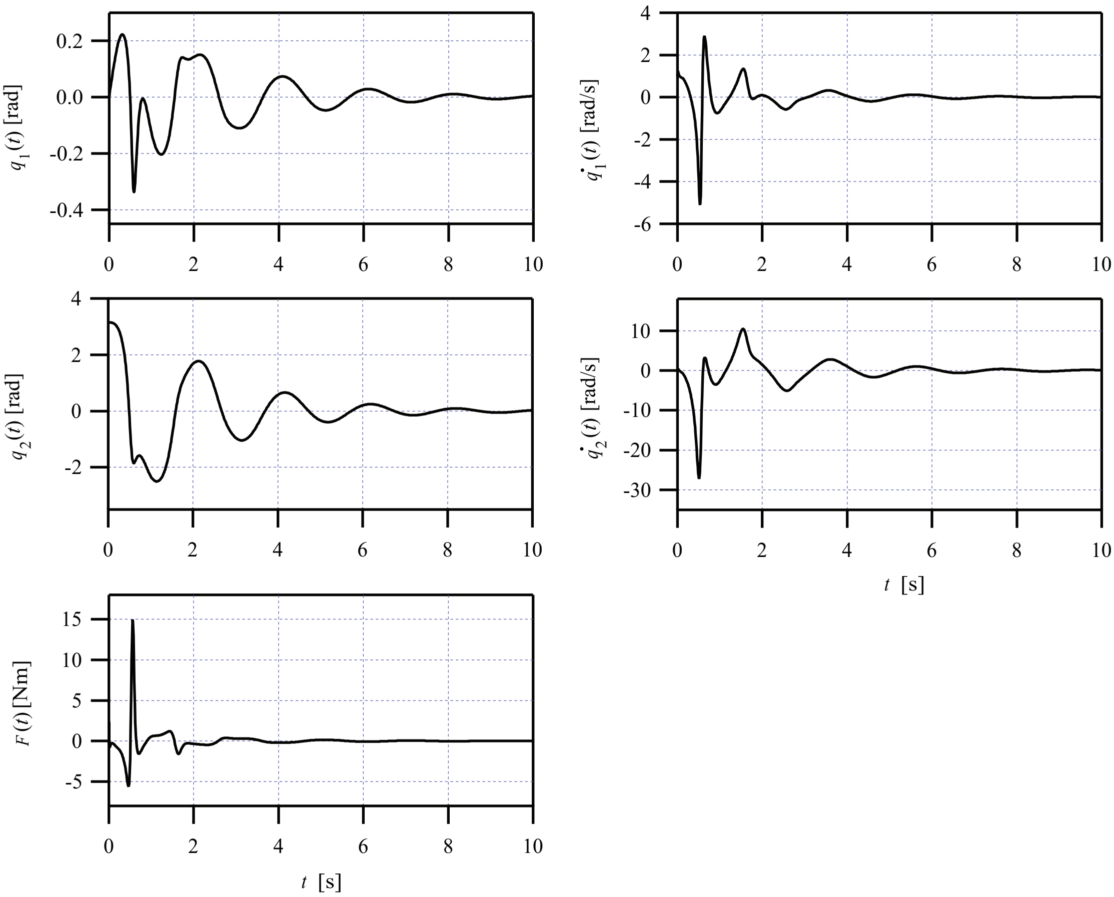

Figure 2: Time responses of $q_i(t)$, $\dot{q}_i(t)$ $(i = 1, 2)$ and $F(t)$.

5. Conclusions

This paper develops a new control method of globally stabilizing underactuated Furuta pendulum. This new method only requires the angle position measurements to design the stabilizing controller. The procedure of designing controller consists of three successive steps. First, the state space model of Furuta pendulum is given, which is an affine nonlinear system. And then, the nonlinear system is linearized around the origin and the nonlinear higher order term is taken to be the fictitious disturbance of system. After that, we use an idea of equivalent input disturbance to design a global stabilizing controller for the nonlinear system. The simulation results demonstrate the effectiveness of our proposed control strategy.

Acknowledgment

This work is supported in part by National Natural Science Foundation of China under Grant Nos. 61304023 and 11526106, by the Scientific Research Foundation for Doctor of Linyi University under Nos. LYDX2014BS002 and LYDX2016BS056, and by Applied Mathematics Enhancement Program of Linyi University. The authors declare that there is no conflict of interests regarding the publication of this article.

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