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Fuzzy set theory applied to IUP-algebras

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Abstract

In this article, we apply fuzzy set theory to IUP-algebras, introducing four new concepts: fuzzy IUP-subalgebras, fuzzy IUP-filters, fuzzy IUP-ideals, and fuzzy strong IUP-ideals, and examining their properties and relationships. We also found a relationship between characteristic functions and the four concepts of fuzzy sets. In addition, the concepts of prime subsets and prime fuzzy sets were also applied. The notions of upper t-(strong) level subsets and lower t-(strong) level subsets of a fuzzy set are introduced in IUP-algebras.

Keywords: IUP-algebra, fuzzy IUP-subalgebra, fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal, upper t-(strong) level subset, lower t-(strong) level subset.

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1. Introduction

Imai and Iséki [11] first commenced the study of BCK-algebras in 1966. In that same year, Iséki [12] introduced another class of algebras, called BCI-algebras, which are generalizations of BCK-algebras. Both types of abstract algebras have been actively studied by a large number of academics.

The concept of fuzzy sets was first considered by Zadeh [25] in 1965. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. After Zadeh introduced the concept of fuzzy sets, many mathematicians applied it to various algebraic systems, such as Baik and Kim [3] introduced the notion of fuzzy k-ideals of semirings. Zhan [26] introduced the concept of T-fuzzy left h-ideals in hemirings. Jun and Song [13] studied fuzzy implicative ideals in BCK-algebras. Kavikumar and Khamis [15] introduced the notions of fuzzy ideals and fuzzy quasi-ideals in ternary semirings. Dymek and Walendziak [7] studied fuzzy filters of BE-algebras in 2013. In 2014, Krishnaswamy and Anitha [17] introduced the notions of fuzzy prime ideals and fuzzy m-systems in ternary semirings. Rao and Venkateswarlu [20] introduced the concept of an anti-fuzzy prime ideal, an

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anti-fuzzy semi-prime ideal, and an anti-fuzzy ideal extension in a Γ-semiring. Bhargavi and Eswarlal [4] introduced and studied the concept of fuzzy Γ-semirings. Somjanta et al. [23] applied fuzzy set theory to UP-algebras. Guntasow et al. [8] introduced the concept of fuzzy translations of a fuzzy set in UP-algebras. Rao [18, 19] introduced the notions of fuzzy left bi-quasi ideals, fuzzy right bi-quasi ideals, and fuzzy bi-quasi ideals of semirings in 2018 and introduced the notions of fuzzy ideals, fuzzy prime ideals, and fuzzy filters of ordered Γ-semirings in 2019. Sowmiya and Jeyalakshmi [24] introduced the notion of fuzzy Z-ideals of Z-algebras. Kalaiarasi and Manimozhi [14] applied fuzzy set theory to KM-algebras. Ahna et al. [1] introduced the notions of $(2,3)$-fuzzy subalgebras and closed $(2,3)$-fuzzy subalgebras in BCK/BCI-algebras. Rittichuai et al. [21] introduced the concept of fuzzy almost subsemirings of semirings. Rizal et al. [6] introduced the fuzzification of dual B-algebras. Saeid et al. [22] introduced the notion of fuzzy quasi-interior ideals of semirings.

In 2017, Iampan [9] introduced the concept of UP-algebras, and in 2022, Iampan et al. [10] introduced a new algebraic structure called an IUP-algebra, which is independent of each other. Then, they talked about IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals of IUP-algebras and looked into what they were and how they related to each other. In addition, they also discussed the concept of homomorphisms between IUP-algebras and studied the direct and inverse images of four special subsets. Our review of the study of fuzzy sets in various algebraic systems inspired us to study them in IUP-algebras. We talk about fuzzy IUP-subalgebras, fuzzy IUP-filters, fuzzy IUP-ideals, and fuzzy strong IUP-ideals of IUP-algebras and look into their properties.

2. Preliminaries

First of all, we start with the definitions and examples of IUP-algebras as well as other relevant definitions for the study in this paper, as follows.

Definition 2.1 ([10])**.** An algebra $X = (X; \cdot, 0)$ of type $(2, 0)$ is called an IUP-algebra, where X is a non-empty set, $\cdot$ is a binary operation on X, and 0 is the constant of X if it satisfies the following axioms:

$$(\forall x \in X)(0 \cdot x = x), \tag{IUP-1}$$
$$(\forall x \in X)(x \cdot x = 0), \tag{IUP-2}$$
$$(\forall x, y, z \in X)((x \cdot y) \cdot (x \cdot z) = y \cdot z). \tag{IUP-3}$$

For convenience, we refer to X as an IUP-algebra $X = (X; \cdot, 0)$ until otherwise specified.

Example 2.2. Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with the Cayley table as follows:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	4	0	3	1	5	2
2	2	5	0	4	3	1
3	5	4	1	0	2	3
4	1	3	5	2	0	4
5	3	2	4	5	1	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra.

For further study and examples of IUP-algebras, see [5, 10].

In X, the following assertions are valid (see [10]):

$$(\forall x, y \in X)((x \cdot 0) \cdot (x \cdot y) = y),$$
$$(\forall x \in X)((x \cdot 0) \cdot (x \cdot 0) = 0),$$
$$(\forall x, y \in X)((x \cdot y) \cdot 0 = y \cdot x),$$

$$
\begin{aligned}
&(\forall x \in X)((x \cdot 0) \cdot 0 = x), \\
&(\forall x, y \in X)(x \cdot ((x \cdot 0) \cdot y) = y), \\
&(\forall x, y \in X)(((x \cdot 0) \cdot y) \cdot x = y \cdot 0), \\
&(\forall x, y, z \in X)(x \cdot y = x \cdot z \Leftrightarrow y = z), \\
&(\forall x, y \in X)(x \cdot y = 0 \Leftrightarrow x = y), \\
&(\forall x \in X)(x \cdot 0 = 0 \Leftrightarrow x = 0), \\
&(\forall x, y, z \in X)(y \cdot x = z \cdot x \Leftrightarrow y = z), \\
&(\forall x, y \in X)(x \cdot y = y \Rightarrow x = 0), \\
&(\forall x, y, z \in X)((x \cdot y) \cdot 0 = (z \cdot y) \cdot (z \cdot x)), \\
&(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (z \cdot x) \cdot (z \cdot y) = 0), \\
&(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (x \cdot z) \cdot (y \cdot z) = 0), \\
&\text{the right and the left cancellation laws hold.}
\end{aligned} \tag{2.1}
$$

Definition 2.3 ([10])**.** A non-empty subset S of X is called

(i) an *IUP-subalgebra* of X if it satisfies the following condition:

$$(\forall x, y \in S)(x \cdot y \in S),$$

(ii) an *IUP-filter* of X if it satisfies the following conditions:

$$
\begin{aligned}
&\text{the constant } 0 \text{ of } X \text{ is in } S, \\
&(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S),
\end{aligned} \tag{2.2}
$$

(iii) an *IUP-ideal* of X if it satisfies the condition (2.2) and the following condition:

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S),$$

(iv) a *strong IUP-ideal* of X if it satisfies the following condition:

$$(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S).$$

From axiom (IUP-2), we have the following remark.

Remark 2.4. Every IUP-subalgebra of X satisfies (2.2).

According to [10], we know that the concept of IUP-filters is a generalization of IUP-ideals and IUP-subalgebras, and IUP-ideals and IUP-subalgebras are generalizations of strong IUP-ideals. In X, we have strong IUP-ideals and X coincide. We get the diagram of the special subsets of IUP-algebras, which is shown in Figure 1.

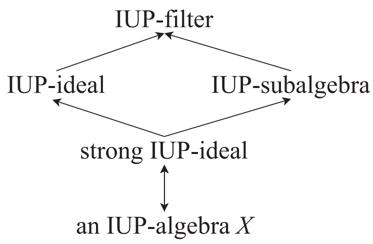

Figure 1: Special subsets of IUP-algebras.

3. Main results

This section presents all the results from this study. We will begin by reviewing the definition of a fuzzy set. After that, four new concepts, namely fuzzy IUP-subalgebras, fuzzy IUP-filters, fuzzy IUP-ideals, and fuzzy strong IUP-ideals, will be introduced in Definition 3.3 and examples will be given.

Definition 3.1 ([25])**.** A *fuzzy set* in a non-empty set X (or a fuzzy subset of X) is an arbitrary function $f : X \to [0,1]$, where $[0,1]$ is the unit segment of the real line. If $A \subseteq X$, the characteristic function f_A of X is a function of X into $\{0,1\}$ defined as follows:

$$f_A(x) = \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{otherwise.} \end{cases}$$

By the definition of characteristic function, f_A is a function of X into $\{0,1\} \subset [0,1]$. Then f_A is a fuzzy set in X.

Definition 3.2. Let f be a fuzzy set in a non-empty set X. The fuzzy set $\overline{f}$ defined by $\overline{f}(x) = 1 - f(x)$ for all $x \in X$ is called the complement of f in X.

Definition 3.3. A fuzzy set f in X is called

(i) a *fuzzy IUP-subalgebra* of X if it satisfies the following condition:

$$(\forall x, y \in X)(f(x \cdot y) \geqslant \min\{f(x), f(y)\}), \tag{3.1}$$

(ii) a *fuzzy IUP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(f(0) \geqslant f(x)), \tag{3.2}$$
$$(\forall x, y \in X)(f(y) \geqslant \min\{f(x \cdot y), f(x)\}), \tag{3.3}$$

(iii) a *fuzzy IUP-ideal* of X if it satisfies the condition (3.2) and the following condition:

$$(\forall x, y, z \in X)(f(x \cdot z) \geqslant \min\{f(x \cdot (y \cdot z)), f(y)\}), \tag{3.4}$$

(iv) a *fuzzy strong IUP-ideal* of X if it satisfies the following condition:

$$(\forall x, y \in X)(f(x \cdot y) \geqslant f(y)). \tag{3.5}$$

Example 3.4. Let $X = \{0,1,2,3,4,5\}$ with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	5	4	3	2
2	4	5	0	1	2	3
3	3	2	1	0	5	4
4	2	3	4	5	0	1
5	5	4	3	2	1	0

Then X is an IUP-algebra. We define a fuzzy set f in X as follows:

$$f = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Then f is a fuzzy IUP-subalgebra of X. Since $f(3 \cdot 0) = f(3) = 0 \ngeqslant 1 = f(0)$, we have f is not a fuzzy strong IUP-ideal of X.

Example 3.5. Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	5	0	4	2	3	1
2	3	4	0	1	5	2
3	2	3	5	0	1	4
4	4	2	1	5	0	3
5	1	5	3	4	2	0

Then X is an IUP-algebra. We define a fuzzy set f in X as follows:

$$f = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Then f is a fuzzy IUP-ideal of X. Since $f(5 \cdot 0) = f(1) = 0 \ngeq 1 = f(0)$, we have f is not a fuzzy strong IUP-ideal of X.

Example 3.6. Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	4	5	2	3
2	5	4	0	1	3	2
3	3	2	1	0	5	4
4	4	5	3	2	0	1
5	2	3	5	4	1	0

Then X is an IUP-algebra. We define a fuzzy set f in X as follows:

$$f = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0.3 & 0.3 & 0.3 & 0.4 & 0.3 \end{pmatrix}.$$

Then f is a fuzzy IUP-subalgebra of X. Since $f(1 \cdot 5) = f(3) = 0.3 \ngeq 0.4 = \min\{f(1 \cdot (4 \cdot 5)), f(4)\} = \min\{f(0), f(4)\}$, we have f is not a fuzzy IUP-ideal of X.

Example 3.7. Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	2	0	1	4	5	3
2	1	2	0	5	3	4
3	3	4	5	0	1	2
4	4	5	3	2	0	1
5	5	3	4	1	2	0

Then X is an IUP-algebra. We define a fuzzy set f in X as follows:

$$f = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Then f is a fuzzy IUP-filter of X. Since $f(4 \cdot 5) = f(1) = 0 \ngeq 1 = \min\{f(3), f(3)\} = \min\{f(4 \cdot (3 \cdot 5)), f(3)\}$, we have f is not a fuzzy IUP-ideal of X.

Lemma 3.8. *Let A be a non-empty subset of X. Then the constant 0 of X is in A if and only if f_A satisfies (3.2).*

Proof. If $0 \in A$, then $f_A(0) = 1$. Thus, $f_A(0) = 1 \geqslant f_A(x)$ for all $x \in X$, that is, it satisfies (3.2).

Conversely, assume that f_A satisfies (3.2). Then $f_A(0) \geqslant f_A(x)$ for all $x \in X$. Since A is a non-empty subset of X, we let $a \in A$. Then $f_A(0) \geqslant f_A(a) = 1$, so $f_A(0) = 1$. Hence, $0 \in A$. □

Theorem 3.9. *Every fuzzy IUP-subalgebra of* X *is a fuzzy IUP-filter of* X.

Proof. Assume that f is a fuzzy IUP-subalgebra of X. By (IUP-2) and (3.1), we have $f(0) = f(x \cdot x) \geqslant \min\{f(x), f(x)\} = f(x)$ for all $x \in X$. Let $x, y \in X$. Then

$$\begin{aligned} f(y) &= f(0 \cdot y) \text{ (by (IUP-1))} \\ &= f((x \cdot 0) \cdot (x \cdot y)) \text{ (by (IUP-3))} \\ &\geqslant \min\{f(x \cdot 0), f(x \cdot y)\} \text{ (by (3.1))} \\ &\geqslant \min\{\min\{f(x), f(0)\}, f(x \cdot y)\} \text{ (by (3.1))} \\ &= \min\{f(x \cdot y), f(x)\} \text{ (by (3.2)).} \end{aligned}$$

Hence, f is a fuzzy IUP-filter of X. □

Theorem 3.10. *Every fuzzy IUP-ideal of* X *is a fuzzy IUP-filter of* X.

Proof. Assume that f is a fuzzy IUP-ideal of X. Then $f(0) \geqslant f(x)$ for all $x \in X$. Let $x, y \in X$. Then

$$f(y) = f(0 \cdot y) \text{ (by (IUP-1))} \geqslant \min\{f(0 \cdot (x \cdot y)), f(x)\} \text{ (by (3.4))} = \min\{f(x \cdot y), f(x)\} \text{ (by (IUP-1)).}$$

Hence, f is a fuzzy IUP-filter of X. □

Theorem 3.11. *Every fuzzy strong IUP-ideal of* X *is a fuzzy IUP-ideal of* X.

Proof. Assume that f is a fuzzy strong IUP-ideal of X. By (IUP-2) and (3.5), we have $f(0) = f(x \cdot x) \geqslant f(x)$ for all $x \in X$. Let $x, y, z \in X$. Then

$$\begin{aligned} f(x \cdot z) &\geqslant \min\{f(x \cdot z), f(y)\} \\ &\geqslant \min\{f(z), f(y)\} \text{ (by (3.5))} \\ &= \min\{f(0 \cdot z), f(y)\} \text{ (by (IUP-1))} \\ &= \min\{f((y \cdot 0) \cdot (y \cdot z)), f(y)\} \text{ (by (IUP-3))} \\ &\geqslant \min\{f(y \cdot z), f(y)\} \text{ (by (3.5))} \\ &= \min\{f(0 \cdot (y \cdot z)), f(y)\} \text{ (by (IUP-1))} \\ &= \min\{f(x \cdot 0) \cdot (x \cdot (y \cdot z)), f(y)\} \text{ (by (IUP-3))} \\ &\geqslant \min\{f(x \cdot (y \cdot z)), f(y)\} \text{ (by (3.5)).} \end{aligned}$$

Hence, f is a fuzzy IUP-ideal of X. □

Theorem 3.12. *Every fuzzy strong IUP-ideal of* X *is a fuzzy IUP-subalgebra of* X.

Proof. Assume that f is a fuzzy strong IUP-ideal of X. Let $x, y \in X$. Then

$$f(x \cdot y) \geqslant \min\{f(x), f(x \cdot y)\} \geqslant \min\{f(x), f(y)\} \text{ (by (3.5)).}$$

Hence, f is a fuzzy IUP-subalgebra of X. □

Theorem 3.13. *Fuzzy strong IUP-ideals and constant fuzzy sets of* X *coincide.*

Proof. Assume that f is a fuzzy strong IUP-ideal of X. Then f satisfies (3.2), that is, $f(0) \geqslant f(x)$ for all $x \in X$. Let $x \in X$. Then

$$f(x) = f((x \cdot 0) \cdot 0) \text{ (by (2.1))} \geqslant f(0) \text{ (by (3.5))}.$$

Thus, $f(x) = f(0)$ for all $x \in X$, that is, f is constant of X. Clearly, every constant fuzzy set of X is a fuzzy strong IUP-ideal of X. Hence, fuzzy strong IUP-ideals and constant fuzzy sets of X coincide. □

Theorem 3.14. *A non-empty subset A of X is an IUP-subalgebra of X if and only if the characteristic function f_A is a fuzzy IUP-subalgebra of X.*

Proof. Assume that A is an IUP-subalgebra of X. Let $x, y \in X$.

Case 1: Suppose $x, y \in A$. Then $f_A(x) = 1$ and $f_A(y) = 1$. Since A is an IUP-subalgebra of X, we have $x \cdot y \in A$. Thus, $f_A(x \cdot y) = 1 \geqslant 1 = \min\{1, 1\} = \min\{f_A(x), f_A(y)\}$.

Case 2: Suppose $x \notin A$ or $y \notin A$. Then $f_A(x) = 0$ or $f_A(y) = 0$. Thus, $f_A(x \cdot y) \geqslant 0 = \min\{f_A(x), f_A(y)\}$. Hence, f_A is a fuzzy IUP-subalgebra of X.

Conversely, assume that f_A is a fuzzy IUP-subalgebra of X. Let $x, y \in A$. Then $f_A(x) = 1$ and $f_A(y) = 1$. By (3.1), we have $f_A(x \cdot y) \geqslant \min\{f_A(x), f_A(y)\} = \min\{1, 1\} = 1$. Thus, $f_A(x \cdot y) = 1$, that is, $x \cdot y \in A$. Hence, A is an IUP-subalgebra of X. □

Theorem 3.15. *A non-empty subset A of X is an IUP-ideal of X if and only if the characteristic function f_A is a fuzzy IUP-ideal of X.*

Proof. Assume that A is an IUP-ideal of X. Since $0 \in A$, it follows from Lemma 3.8 that f_A satisfies (3.2). Next, let $x, y, z \in X$.

Case 1: Suppose $x \cdot (y \cdot z) \in A$ and $y \in A$. Then $x \cdot z \in A$. Thus, $f_A(x \cdot z) = 1$. Hence, $f_A(x \cdot z) = 1 \geqslant \min\{f_A(x \cdot (y \cdot z)), f_A(y)\}$.

Case 2: Suppose $x \cdot (y \cdot z) \notin A$ or $y \notin A$. Then $f_A(x \cdot (y \cdot z)) = 0$ or $f_A(y) = 0$. Thus, $\min\{f_A(x \cdot (y \cdot z)), f_A(y)\} = 0$. Hence, $f_A(x \cdot z) \geqslant 0 = \min\{f_A(x \cdot (y \cdot z)), f_A(y)\}$.

Therefore, f_A is a fuzzy IUP-ideal of X.

Conversely, assume that f_A is a fuzzy IUP-ideal of X. Since f_A satisfies (3.2), it follows from Lemma 3.8 that $0 \in A$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in A$ and $y \in A$. Assume that $x \cdot z \notin A$. By (3.4), we have $0 = f_A(x \cdot z) \geqslant \min\{f_A(x \cdot (y \cdot z)), f_A(y)\}$. Thus, $\min\{f_A(x \cdot (y \cdot z)), f_A(y)\} = 0$. It means that $f_A(x \cdot (y \cdot z)) = 0$ or $f_A(y) = 0$. Thus, $x \cdot (y \cdot z) \notin A$ or $y \notin A$, a contradiction. Hence, $x \cdot z \in A$, so A is an IUP-ideal of X. □

Theorem 3.16. *A non-empty subset A of X is an IUP-filter of X if and only if the characteristic function f_A is a fuzzy IUP-filter of X.*

Proof. Assume that A is an IUP-filter of X. Since $0 \in A$, it follows from Lemma 3.8 that f_A satisfies (3.2). Next, let $x, y \in X$.

Case 1: Suppose $y \in A$. Then $f_A(y) = 1$. Thus, $f_A(y) = 1 \geqslant \min\{f_A(x \cdot y), f_A(x)\}$.

Case 2: Suppose $y \notin A$. Then $f_A(y) = 0$. Since A is an IUP-filter of X, we have $x \notin A$ or $x \cdot y \notin A$. Thus, $f_A(x) = 0$ or $f_A(x \cdot y) = 0$. Hence, $f_A(y) = 0 \geqslant 0 = \min\{f_A(x \cdot y), f_A(x)\}$.

Therefore, f_A is a fuzzy IUP-filter of X.

Conversely, assume that f_A is a fuzzy IUP-filter of X. Since f_A satisfies (3.2), it follows from Lemma 3.8 that $0 \in A$. Next, let $x, y \in X$ be such that $x \cdot y \in A$ and $x \in A$. Then $f_A(x \cdot y) = 1$ and $f_A(x) = 1$. Assume that $y \notin A$. By (3.3), we have $0 = f_A(y) \geqslant \min\{f_A(x \cdot y), f_A(x)\} = \min\{1, 1\} = 1$, a contradiction. Hence, $y \in A$, so A is an IUP-filter of X. □

Theorem 3.17. *A non-empty subset A of X is a strong IUP-ideal of X if and only if the characteristic function f_A is a fuzzy strong IUP-ideal of X.*

Proof. It is straightforward by Theorem 3.13. □

Example 3.18 ([10])**.** Let $\mathbb{R}^*$ be the set of all nonzero real numbers. Then $(\mathbb{R}^*; \cdot, 1)$ is an IUP-algebra, where $\cdot$ is the binary operation on $\mathbb{R}^*$ defined by $x \cdot y = \frac{y}{x}$ for all $x, y \in \mathbb{R}^*$. Let $S = \{x \in \mathbb{R}^* \mid x \geqslant 1\}$. Then S is an IUP-ideal and an IUP-filter of $\mathbb{R}^*$ but it is not an IUP-subalgebra of $\mathbb{R}^*$. From Theorems 3.14, 3.15, and 3.16, we have the characteristic function f_S is a fuzzy IUP-ideal and a fuzzy IUP-filter of $\mathbb{R}^*$ but it is not a fuzzy IUP-subalgebra of $\mathbb{R}^*$.

Definition 3.19. An IUP-subalgebra (resp., IUP-filter, IUP-ideal, strong IUP-ideal) A of X is called a *prime IUP-subalgebra* (resp., IUP-filter, IUP-ideal, strong IUP-ideal) of X if it is a prime subset of X, that is,

$$(\forall x, y \in X)(x \cdot y \in A \Rightarrow x \in A \text{ or } y \in A).$$

Definition 3.20. A fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) f of X is called a *prime fuzzy IUP-subalgebra* (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X if it is a prime fuzzy set in X, that is,

$$(\forall x, y \in X)(f(x \cdot y) \leqslant \max\{f(x), f(y)\}).$$

Theorem 3.21. *A non-empty subset A of X is a prime subset of X if and only if the characteristic function f_A is a prime fuzzy set in X.*

Proof. Assume that A is a prime subset of X. Let $x, y \in X$.

Case 1: Suppose $x \cdot y \in A$. Since A is a prime subset of X, we have $x \in A$ or $y \in A$. Then $f_A(x) = 1$ or $f_A(y) = 1$. Thus, $\max\{f_A(x), f_A(y)\} = 1$, so $f_A(x \cdot y) \leqslant 1 = \max\{f_A(x), f_A(y)\}$.

Case 2: Suppose $x \cdot y \notin A$. Then $f_A(x \cdot y) = 0 \leqslant \max\{f_A(x), f_A(y)$.

Hence, f_A is a prime fuzzy set in X.

Conversely, assume that f_A is a prime fuzzy set in X. Let $x, y \in X$ be such that $x \cdot y \in A$. Then $f_A(x \cdot y) = 1$. Since f_A is a prime fuzzy set in X, we have $1 = f_A(x \cdot y) \leqslant \max\{f_A(x), f_A(y)\}$ and so $\max\{f_A(x), f_A(y)\} = 1$. Hence, $f_A(x) = 1$ or $f_A(y) = 1$, that is, $x \in A$ or $y \in A$. Hence, A is a prime subset of X. □

Theorem 3.22. *A non-empty subset A of X is a prime IUP-subalgebra of X if and only if the characteristic function f_A is a prime fuzzy IUP-subalgebra of X.*

Proof. It is straightforward by Theorems 3.14 and 3.21. □

Theorem 3.23. *A non-empty subset A of X is a prime IUP-ideal of X if and only if the characteristic function f_A is a prime fuzzy IUP-ideal of X.*

Proof. It is straightforward by Theorems 3.15 and 3.21. □

Theorem 3.24. *A non-empty subset A of X is a prime IUP-filter of X if and only if the characteristic function f_A is a prime fuzzy IUP-filter of X.*

Proof. It is straightforward by Theorems 3.16 and 3.21. □

Theorem 3.25. *A non-empty subset A of X is a prime strong IUP-ideal of X if and only if the characteristic function f_A is a prime fuzzy strong IUP-ideal of X.*

Proof. It is straightforward by Theorems 3.17 and 3.21. □

Definition 3.26 ([16]). Let f be a fuzzy set in a non-empty set X. For any $t \in [0,1]$, the sets

$$U(f;t) = \{x \in X \mid f(x) \geqslant t\} \text{ and } U^{+}(f;t) = \{x \in X \mid f(x) > t\}$$

are called an *upper* t*-level subset* and an *upper* t*-strong level subset* of f, respectively, and

$$L(f;t) = \{x \in X \mid f(x) \leqslant t\} \text{ and } L^{-}(f;t) = \{x \in X \mid f(x) < t\}$$

are called a *lower* t*-level subset* and a *lower* t*-strong level subset* of f, respectively.

Theorem 3.27. *A fuzzy set f in X is a fuzzy IUP-subalgebra of X if and only if for all $t \in [0,1], U(f;t)$ is an IUP-subalgebra of X if it is non-empty.*

Proof. Assume that f is a fuzzy IUP-subalgebra of X. Let $t \in [0,1]$ be such that $U(f;t) \neq \emptyset$. Let $x, y \in U(f;t)$. Then $f(x) \geqslant t$ and $f(y) \geqslant t$, so t is a lower bound of $\{f(x), f(y)\}$. Since f is a fuzzy IUP-subalgebra of X, we have $f(x \cdot y) \geqslant \min\{f(x), f(y)\} \geqslant t$. Thus, $x \cdot y \in U(f;t)$. Hence, $U(f;t)$ is an IUP-subalgebra of X.

Conversely, assume that for all $t \in [0,1], U(f;t)$ is an IUP-subalgebra of X if it is non-empty. Let $x, y \in X$. Choose $t = \min\{f(x), f(y)\}$. Then $f(x) \geqslant t$ and $f(y) \geqslant t$. Thus, $x, y \in U(f;t) \neq \emptyset$. By the assumption, we have $U(f;t)$ is an IUP-subalgebra of X. So $x \cdot y \in U(f;t)$. Hence, $f(x \cdot y) \geqslant t = \min\{f(x), f(y)\}$. Therefore, f is a fuzzy IUP-subalgebra of X. □

Theorem 3.28. *A fuzzy set f in X is a fuzzy IUP-ideal of X if and only if for all $t \in [0,1], U(f;t)$ is an IUP-ideal of X if it is non-empty.*

Proof. Assume that f is a fuzzy IUP-ideal of X. Let $t \in [0,1]$ be such that $U(f;t) \neq \emptyset$. Let $a \in U(f;t)$. Then $f(a) \geqslant t$. Since f is a fuzzy IUP-ideal of X, we have $f(0) \geqslant f(a) \geqslant t$. Thus, $0 \in U(f;t)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(f;t)$ and $y \in U(f;t)$. Then $f(x \cdot (y \cdot z)) \geqslant t$ and $f(y) \geqslant t$. Thus, t is a lower bound of $\{f(x \cdot (y \cdot z)), f(y)\}$. Since f is fuzzy IUP-ideal of X, we have $f(x \cdot z) \geqslant \min\{f(x \cdot (y \cdot z)), f(y)\} \geqslant t$. So $x \cdot z \in U(f;t)$. Hence, $U(f;t)$ is an IUP-ideal of X.

Conversely, assume that for all $t \in [0,1], U(f;t)$ is an IUP-ideal of X if it is non-empty. Let $x \in X$. Choose $t = f(x)$. Then $f(x) \geqslant t$. Thus, $x \in U(f;t) \neq \emptyset$. By the assumption, we have $U(f;t)$ is an IUP-ideal of X. So $0 \in U(f;t)$. Hence, $f(0) \geqslant t = f(x)$. Next, let $x, y, z \in X$. Choose $t' = \min\{f(x \cdot (y \cdot z)), f(y)\}$. Then $f(x \cdot (y \cdot z)) \geqslant t'$ and $f(y) \geqslant t'$. Thus, $x \cdot (y \cdot z), y \in U(f;t') \neq \emptyset$. By the assumption, we have $U(f;t')$ is an IUP-ideal of X. So $x \cdot z \in U(f;t')$. Hence, $f(x \cdot z) \geqslant t' = \min\{f(x \cdot (y \cdot z)), f(y)\}$. Therefore, f is a fuzzy IUP-ideal of X. □

Theorem 3.29. *A fuzzy set f in X is a fuzzy IUP-filter of X if and only if for all $t \in [0,1], U(f;t)$ is an IUP-filter of X if it is non-empty.*

Proof. Assume that f is a fuzzy IUP-filter of X. Let $t \in [0,1]$ be such that $U(f;t) \neq \emptyset$. Let $a \in U(f;t)$. Then $f(a) \geqslant t$. Since f is a fuzzy IUP-filter of X, we have $f(0) \geqslant f(a) \geqslant t$. Thus, $0 \in U(f;t)$. Next, let $x, y \in X$ be such that $x \cdot y \in U(f;t)$ and $x \in U(f;t)$. Then $f(x \cdot y) \geqslant t$ and $f(x) \geqslant t$. Thus, t is a lower bound of $\{f(x \cdot y), f(x)\}$. Since f is a fuzzy IUP-filter of X, we have $f(y) \geqslant \min\{f(x \cdot y), f(x)\} \geqslant t$. So $y \in U(f;t)$. Hence, $U(f;t)$ is an IUP-filter of X.

Conversely, assume that for all $t \in [0,1], U(f;t)$ is an IUP-filter of X if it is non-empty. Let $x \in X$. Choose $t = f(x)$. Then $f(x) \geqslant t$. Thus, $x \in U(f;t) \neq \emptyset$. By the assumption, we have $U(f;t)$ is an IUP-filter of X. So $0 \in U(f;t)$. Hence, $f(0) \geqslant t = f(x)$. Next, let $x, y \in X$. Choose $t' = \min\{f(x \cdot y), f(x)\}$. Then $f(x \cdot y) \geqslant t'$ and $f(x) \geqslant t'$. Thus, $x \cdot y, x \in U(f;t') \neq \emptyset$. By the assumption, we have $U(f;t')$ is an IUP-filter of X. So $y \in U(f;t')$. Hence, $f(y) \geqslant t' = \min\{f(x \cdot y), f(x)\}$. Therefore, f is a fuzzy IUP-filter of X. □

Theorem 3.30. *A fuzzy set f in X is a fuzzy strong IUP-ideal of X if and only if for all $t \in [0,1], U(f;t)$ is a strong IUP-ideal of X if it is non-empty.*

Proof. It is straightforward by Theorem 3.13. □

Theorem 3.31. *A fuzzy set* f *in* X *is a prime fuzzy set of* X *if and only if for all* $t \in [0,1], U(f;t)$ *is a prime subset of* X *if it is non-empty.*

Proof. Assume that f is a prime fuzzy set in X. Let $t \in [0,1]$ be such that $U(f;t) \neq \emptyset$. Let $x, y \in X$ be such that $x \cdot y \in U(f;t)$. Assume that $x \notin U(f;t)$ and $y \notin U(f;t)$. Then $f(x) < t$ and $f(y) < t$. Thus, t is an upper bound of $\{f(x), f(y)\}$. Since f is a prime fuzzy set in X, we have $f(x \cdot y) \leqslant \max\{f(x), f(y)\} < t$. So $x \cdot y \notin U(f;t)$, a contradiction. Hence, $x \in U(f;t)$ or $y \in U(f;t)$. Therefore, $U(f;t)$ is a prime subset of X.

Conversely, assume that for all $t \in [0,1], U(f;t)$ is a prime subset of X if it is non-empty. Let $x, y \in X$. Choose $t = f(x \cdot y)$. Then $f(x \cdot y) \geqslant t$. Thus, $x \cdot y \in U(f;t) \neq \emptyset$. By the assumption, we have $U(f;t)$ is a prime subset of X. So $x \in U(f;t)$ or $y \in U(f;t)$. Hence, $t \leqslant f(x)$ or $t \leqslant f(y)$, so $f(x \cdot y) = t \leqslant \max\{f(x), f(y)\}$. Therefore, f is a prime fuzzy set in X. □

Theorem 3.32. *A fuzzy set* f *in* X *is a prime fuzzy IUP-subalgebra of* X *if and only if for all* $t \in [0,1], U(f;t)$ *is a prime IUP-subalgebra of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.27 and 3.31. □

Theorem 3.33. *A fuzzy set* f *in* X *is a prime fuzzy IUP-ideal of* X *if and only if for all* $t \in [0,1], U(f;t)$ *is a prime IUP-ideal of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.28 and 3.31. □

Theorem 3.34. *A fuzzy set* f *in* X *is a prime fuzzy IUP-filter of* X *if and only if for all* $t \in [0,1], U(f;t)$ *is a prime IUP-filter of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.29 and 3.31. □

Theorem 3.35. *A fuzzy set* f *in* X *is a prime fuzzy strong IUP-ideal of* X *if and only if for all* $t \in [0,1], U(f;t)$ *is a prime strong IUP-ideal of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.30 and 3.31. □

Theorem 3.36. *A fuzzy set* f *in* X *is a fuzzy IUP-subalgebra of* X *if and only if for all* $t \in [0,1], U^+(f;t)$ *is an IUP-subalgebra of* X *if it is non-empty.*

Proof. Assume that f is a fuzzy IUP-subalgebra of X. Let $t \in [0,1]$ be such that $U^+(f;t) \neq \emptyset$. Let $x, y \in U^+(f;t)$. Then $f(x) > t$ and $f(y) > t$. Thus, t is a lower bound of $\{f(x), f(y)\}$. Since f is a fuzzy IUP-subalgebra of X, we have $f(x \cdot y) \geqslant \min\{f(x), f(y)\} > t$. So $x \cdot y \in U^+(f;t)$. Hence, $U^+(f;t)$ is an IUP-subalgebra of X.

Conversely, assume that for all $t \in [0,1], U^+(f;t)$ is an IUP-subalgebra of X if it is non-empty. Let $x, y \in X$. Assume that $f(x \cdot y) < \min\{f(x), f(y)\}$. Choose $t = f(x \cdot y)$. Then $f(x) > t$ and $f(y) > t$. Thus, $x, y \in U^+(f;t) \neq \emptyset$. By the assumption, we have $U^+(f;t)$ is an IUP-subalgebra of X. Thus, $x \cdot y \in U^+(f;t)$. So $f(x \cdot y) > t = f(x \cdot y)$, a contradiction. Hence, $f(x \cdot y) \geqslant \min\{f(x), f(y)\}$ for all $x, y \in X$. Therefore, f is a fuzzy IUP-subalgebra of X. □

Theorem 3.37. *A fuzzy set* f *in* X *is a fuzzy IUP-ideal of* X *if and only if for all* $t \in [0,1], U^+(f;t)$ *is an IUP-ideal of* X *if it is non-empty.*

Proof. Assume that f is a fuzzy IUP-ideal of X. Let $t \in [0,1]$ be such that $U^+(f;t) \neq \emptyset$ and let $a \in U^+(f;t)$. Then $f(a) > t$. Since f is a fuzzy IUP-ideal of X, we have $f(0) \geqslant f(a) > t$. Thus, $0 \in U^+(f;t)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U^+(f;t)$ and $y \in U^+(f;t)$. Then $f(x \cdot (y \cdot z)) > t$ and $f(y) > t$. Thus, t is a lower bound of $\{f(x \cdot (y \cdot z)), f(y)\}$. Since f is a fuzzy IUP-ideal of X, we have $f(x \cdot z) \geqslant \min\{f(x \cdot (y \cdot z)), f(y)\} > t$. So $x \cdot z \in U^+(f;t)$. Hence, $U^+(f;t)$ is an IUP-ideal of X.

Conversely, assume that for all $t \in [0,1], U^+(f;t)$ is an IUP-ideal of X if it is non-empty. Let $x \in X$. Assume $f(0) < f(x)$. Then $x \in U^+(f;f(0)) \neq \emptyset$. Since $U^+(f;f(0))$ is an IUP-ideal of X, we have

$0 \in U^+(f; f(0))$. Thus, $f(0) > f(0)$, a contradiction. Hence, $f(0) \geqslant f(x)$ for all $x \in X$. Let $x, y, z \in X$. Assume that $f(x \cdot z) < \min\{f(x \cdot (y \cdot z)), f(y)\}$. Then $f(x \cdot z) < f(x \cdot (y \cdot z))$ and $f(x \cdot z) < f(y)$. Thus, $x \cdot (y \cdot z), y \in U^+(f; f(x \cdot z)) \neq \emptyset$. By the assumption, we have $U^+(f; f(x \cdot z))$ is an IUP-ideal of X. Thus, $x \cdot z \in U^+(f; f(x \cdot z))$. Thus, $f(x \cdot z) > f(x \cdot z)$, a contradiction. Hence, $f(x \cdot z) \geqslant \min\{f(x \cdot (y \cdot z)), f(y)\}$ for all $x, y, z \in X$. Therefore, f is a fuzzy IUP-ideal of X. □

Theorem 3.38. *A fuzzy set* f *in* X *is a fuzzy IUP-filter of* X *if and only if for all* $t \in [0, 1], U^+(f; t)$ *is an IUP-filter of* X *if it is non-empty.*

Proof. Assume that f is a fuzzy IUP-filter of X. Let $t \in [0, 1]$ be such that $U^+(f; t) \neq \emptyset$ and let $a \in U^+(f; t)$. Then $f(a) > t$. Since f is a fuzzy IUP-filter of X, we have $f(0) \geqslant f(a) > t$. Thus, $0 \in U^+(f; t)$. Let $x, y \in X$ be such that $x \cdot y \in U^+(f; t)$ and $x \in U^+(f; t)$. Then $f(x \cdot y) > t$ and $f(x) > t$. Thus, t is a lower bound of $\{f(x \cdot y), f(x)\}$. Since f is a fuzzy IUP-filter of X, we have $f(y) \geqslant \min\{f(x \cdot y), f(x)\} > t$. So $y \in U^+(f; t)$. Hence, $U^+(f; t)$ is an IUP-filter of X.

Conversely, assume that for all $t \in [0, 1], U^+(f; t)$ is an IUP-filter of X if it is non-empty. Let $x \in X$. Assume $f(0) < f(x)$. Then $x \in U^+(f; f(0)) \neq \emptyset$. Since $U^+(f; f(0))$ is an IUP-filter of X, we have $0 \in U^+(f; f(0))$. Thus, $f(0) > f(0)$, a contradiction. Hence, $f(0) \geqslant f(x)$ for all $x \in X$. Let $x, y \in X$. Assume that $f(y) < \min\{f(x \cdot y), f(x)\}$. Then $f(y) < f(x \cdot y)$ and $f(y) < f(x)$. Thus, $x \cdot y, x \in U^+(f; f(y)) \neq \emptyset$. By the assumption, we have $U^+(f; f(y))$ is an IUP-filter of X. Thus, $y \in U^+(f; f(y))$. Thus, $f(y) > f(y)$, a contradiction. Hence, $f(y) \geqslant \min\{f(x \cdot y), f(x)\}$ for all $x, y \in X$. Therefore, f is a fuzzy IUP-filter of X. □

Theorem 3.39. *A fuzzy set* f *in* X *is a fuzzy strong IUP-ideal of* X *if and only if for all* $t \in [0, 1], U^+(f; t)$ *is a strong IUP-ideal of* X *if it is non-empty.*

Proof. It is straightforward by Theorem 3.13. □

Theorem 3.40. *A fuzzy set* f *in* X *is a prime fuzzy set of* X *if and only if for all* $t \in [0, 1], U^+(f; t)$ *is a prime subset of* X *if it is non-empty.*

Proof. Assume that f is a prime fuzzy set in X. Let $t \in [0, 1]$ be such that $U^+(f; t) \neq \emptyset$. Let $x, y \in X$ be such that $x \cdot y \in U^+(f; t)$. Assume that $x \notin U^+(f; t)$ and $y \notin U^+(f; t)$. Then $f(x) \leqslant t$ and $f(y) \leqslant t$. Thus, t is an upper bound of $\{f(x), f(y)\}$. Since f is a prime fuzzy set in X, we have $f(x \cdot y) \leqslant \max\{f(x), f(y)\} \leqslant t$. So $x \cdot y \notin U^+(f; t)$, a contradiction. Hence, $x \in U^+(f; t)$ or $y \in U^+(f; t)$. Therefore, $U^+(f; t)$ is a prime subset of X.

Conversely, assume that for all $t \in [0, 1], U^+(f; t)$ is a prime subset of X if it is non-empty. Let $x, y \in X$. Assume that $f(x \cdot y) > \max\{f(x), f(y)\}$. Choose $t = \max\{f(x), f(y)\}$. Then $f(x \cdot y) > t$. Thus, $x \cdot y \in U^+(f; t) \neq \emptyset$. By the assumption, we have $U^+(f; t)$ is a prime subset of X. So $x \in U^+(f; t)$ or $y \in U^+(f; t)$. Thus, $f(x) > t = \max\{f(x), f(y)\}$ or $f(y) > t = \max\{f(x), f(y)\}$, a contradiction. Hence, $f(x \cdot y) \leqslant \max\{f(x), f(y)\}$ for all $x, y \in X$. Therefore, f is a prime fuzzy set in X. □

Theorem 3.41. *A fuzzy set* f *in* X *is a prime fuzzy IUP-subalgebra of* X *if and only if for all* $t \in [0, 1], U^+(f; t)$ *is a prime IUP-subalgebra of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.36 and 3.40. □

Theorem 3.42. *A fuzzy set* f *in* X *is a prime fuzzy IUP-ideal of* X *if and only if for all* $t \in [0, 1], U^+(f; t)$ *is a prime IUP-ideal of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.37 and 3.40. □

Theorem 3.43. *A fuzzy set* f *in* X *is a prime fuzzy IUP-filter of* X *if and only if for all* $t \in [0, 1], U^+(f; t)$ *is a prime IUP-filter of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.38 and 3.40. □

Theorem 3.44. *A fuzzy set* f *in* X *is a prime fuzzy strong IUP-ideal of* X *if and only if for all* $t \in [0,1], U^+(f;t)$ *is a prime strong IUP-ideal of* X *if it is non-empty.*

Proof. It is straightforward by Theorems 3.39 and 3.40. □

Theorem 3.45. *The complement* $\overline{f}$ *of a fuzzy set* f *in* X *is a fuzzy IUP-subalgebra of* X *if and only if for all* $t \in [0,1], L(f;t)$ *is an IUP-subalgebra of* X *if it is non-empty.*

Proof. Assume that $\overline{f}$ is a fuzzy IUP-subalgebra of X. Let $t \in [0,1]$ be such that $L(f;t) \neq \emptyset$. Let $x, y \in L(f;t)$. Then $f(x) \leqslant t$ and $f(y) \leqslant t$. Thus, t is an upper bound of $\{f(x), f(y)\}$. Since $\overline{f}$ is a fuzzy IUP-subalgebra of X, we have $\overline{f}(x \cdot y) \geqslant \min\{\overline{f}(x), \overline{f}(y)\}$. Thus, $1 - f(x \cdot y) \geqslant \min\{1 - f(x), 1 - f(y)\} = 1 - \max\{f(x), f(y)\}$, that is, $f(x \cdot y) \leqslant \max\{f(x), f(y)\} \leqslant t$. Thus, $x \cdot y \in L(f;t)$. Therefore, $L(f;t)$ is an IUP-subalgebra of X.

Conversely, assume that for all $t \in [0,1], L(f;t)$ is an IUP-subalgebra of X if it is non-empty. Let $x, y \in X$. Choose $t = \max\{f(x), f(y)\}$. Then $f(x) \leqslant t$ and $f(y) \leqslant t$. Thus, $x, y \in L(f;t) \neq \emptyset$. By the assumption, we have $L(f;t)$ is an IUP-subalgebra of X and thus $x \cdot y \in L(f;t)$. So $f(x \cdot y) \leqslant t = \max\{f(x), f(y)\}$. Thus, $\overline{f}(x \cdot y) = 1 - f(x \cdot y) \geqslant 1 - \max\{f(x), f(y)\} = \min\{1 - f(x), 1 - f(y)\} = \min\{\overline{f}(x), \overline{f}(y)\}$. Therefore, $\overline{f}$ is a fuzzy IUP-subalgebra of X. □

Theorem 3.46. *The complement* $\overline{f}$ *of a fuzzy set* f *in* X *is a fuzzy IUP-ideal of* X *if and only if for all* $t \in [0,1], L(f;t)$ *is an IUP-ideal of* X *if it is non-empty.*

Proof. Assume that $\overline{f}$ is a fuzzy IUP-ideal of X. Let $t \in [0,1]$ be such that $L(f;t) \neq \emptyset$ and let $a \in L(f;t)$. Then $f(a) \leqslant t$. Since $\overline{f}$ is a fuzzy IUP-ideal of X, we have $\overline{f}(0) \geqslant \overline{f}(a)$. Thus, $1 - f(0) \geqslant 1 - f(a)$, that is, $f(0) \leqslant f(a) \leqslant t$. Hence, $0 \in L(f;t)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L(f;t)$ and $y \in L(f;t)$. Then $f(x \cdot (y \cdot z) \leqslant t$ and $f(y) \leqslant t$. Thus, t is an upper bound of $\{f(x \cdot (y \cdot z)), f(y)\}$. Since $\overline{f}$ is a fuzzy IUP-ideal of X, we have $\overline{f}(x \cdot z) \geqslant \min\{\overline{f}(x \cdot (y \cdot z)), \overline{f}(y)\}$. Thus, $1 - f(x \cdot z) \geqslant \min\{1 - f(x \cdot (y \cdot z)), 1 - f(y)\} = 1 - \max\{f(x \cdot (y \cdot z)), f(y)\}$, that is, $f(x \cdot z) \leqslant \max\{f(x \cdot (y \cdot z)), f(y)\} \leqslant t$. Thus, $x \cdot z \in L(f;t)$. Therefore, $L(f;t)$ is an IUP-ideal of X.

Conversely, assume that for all $t \in [0,1], L(f;t)$ is an IUP-ideal of X if it is non-empty. Let $x \in X$. Choose $t = f(x)$. Then $f(x) \leqslant t$. Thus, $x \in L(f;t) \neq \emptyset$. By the assumption, we have $L(f;t)$ is an IUP-ideal of X. Thus, $0 \in L(f;t)$. So $f(0) \leqslant t = f(x)$. Tuus $\overline{f}(0) = 1 - f(0) \geqslant 1 - f(x) = \overline{f}(x)$. Let $x, y, z \in X$. Choose $t' = \max\{f(x \cdot (y \cdot z)), f(y)\}$. Then $f(x \cdot (y \cdot z)) \leqslant t'$ and $f(y) \leqslant t'$. Thus, $x \cdot (y \cdot z), y \in L(f;t') \neq \emptyset$. By the assumption, we have $L(f;t')$ is an IUP-ideal of X. Thus, $x \cdot z \in L(f;t')$. So $f(x \cdot z) \leqslant t' = \max\{f(x \cdot (y \cdot z)), f(y)\}$. Thus, $\overline{f}(x \cdot z) = 1 - f(x \cdot z) \geqslant 1 - \max\{f(x \cdot (y \cdot z)), f(y)\} = \min\{1 - f(x \cdot (y \cdot z)), 1 - f(y)\} = \min\{\overline{f}(x \cdot (y \cdot z)), \overline{f}(y)\}$. Therefore, $\overline{f}$ is a fuzzy IUP-ideal of X. □

Theorem 3.47. *The complement* $\overline{f}$ *of a fuzzy set* f *in* X *is a fuzzy IUP-filter of* X *if and only if for all* $t \in [0,1], L(f;t)$ *is an IUP-filter of* X *if it is non-empty.*

Proof. Assume that $\overline{f}$ is a fuzzy IUP-filter of X. Let $t \in [0,1]$ be such that $L(f;t) \neq \emptyset$ and let $a \in L(f;t)$. Then $f(a) \leqslant t$. Since $\overline{f}$ is a fuzzy IUP-filter of X, we have $\overline{f}(0) \geqslant \overline{f}(a)$. Thus, $1 - f(0) \geqslant 1 - f(a)$, that is, $f(0) \leqslant f(a) \leqslant t$. Hence, $0 \in L(f;t)$. Let $x, y \in X$ be such that $x \cdot y \in L(f;t)$ and $x \in L(f;t)$. Then $f(x \cdot y) \leqslant t$ and $f(x) \leqslant t$. Thus, t is an upper bound of $\{f(x \cdot y), f(x)\}$. Since $\overline{f}$ is a fuzzy IUP-filter of X, we have $\overline{f}(y) \geqslant \min\{\overline{f}(x \cdot y), \overline{f}(x)\}$. Thus, $1 - f(y) \geqslant \min\{1 - f(x \cdot y), 1 - f(x)\} = 1 - \max\{f(x \cdot y), f(x)\}$, that is, $f(y) \leqslant \max\{f(x \cdot y), f(x)\} \leqslant t$ and thus $y \in L(f;t)$. Therefore, $L(f;t)$ is an IUP-filter of X.

Conversely, assume that for all $t \in [0,1], L(f;t)$ is an IUP-filter of X if it is non-empty. Let $x \in X$. Choose $t = f(x)$. Then $f(x) \leqslant t$. Thus, $x \in L(f;t) \neq \emptyset$. By the assumption, we have $L(f;t)$ is an IUP-filter of X. Thus, $0 \in L(f;t)$. So $f(0) \leqslant t = f(x)$. Thus, $\overline{f}(0) = 1 - f(0) \geqslant 1 - f(x) = \overline{f}(x)$. Let $x, y \in X$. Choose $t' = \max\{f(x \cdot y), f(x)\}$. Then $f(x \cdot y) \leqslant t'$ and $f(x) \leqslant t'$. Thus, $x \cdot y, x \in L(f;t') \neq \emptyset$. By the assumption, we have $L(f;t')$ is an IUP-filter of X. Thus, $y \in L(f;t')$. So $f(y) \leqslant t' = \max\{f(x \cdot y), f(x)\}$. Thus, $\overline{f}(y) = 1 - f(y) \geqslant 1 - \max\{f(x \cdot y), f(x)\} = \min\{1 - f(x \cdot y), 1 - f(x)\} = \min\{\overline{f}(x \cdot y), \overline{f}(x)\}$. Therefore, $\overline{f}$ is a fuzzy IUP-filter of X. □

Theorem 3.48. *The complement $\overline{f}$ of a fuzzy set f in X is a fuzzy strong IUP-ideal of X if and only if for all $t \in [0,1], L(f;t)$ is a strong IUP-ideal of X if it is non-empty.*

Proof. It is straightforward by Theorem 3.13. □

Theorem 3.49. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy set in X if and only if for all $t \in [0,1], L(f;t)$ is a prime subset of X if it is non-empty.*

Proof. Assume that $\overline{f}$ is a prime fuzzy set in X. Let $t \in [0,1]$ be such that $L(f;t) \neq \emptyset$. Let $x, y \in X$ be such that $x \cdot y \in L(f;t)$. Assume that $x \notin L(f;t)$ and $y \notin L(f;t)$. Then $f(x) > t$ and $f(y) > t$. Thus, t is a lower bound of $\{f(x), f(y)\}$. Since $\overline{f}$ is a prime fuzzy set in X, we have $\overline{f}(x \cdot y) \leqslant \max\{\overline{f}(x), \overline{f}(y)\}$. Thus, $1 - f(x \cdot y) \leqslant \max\{1 - f(x), 1 - f(y)\} = 1 - \min\{f(x), f(y)\}$, that is, $f(x \cdot y) \geqslant \min\{f(x), f(y)\} > t$. Thus, $x \cdot y \notin L(f;t)$, a contradiction. Hence, $x \in L(f;t)$ or $y \in L(f;t)$. Therefore, $L(f;t)$ is a prime subset of X.

Conversely, assume that for all $t \in [0,1], L(f;t)$ is a prime subset of X if it is non-empty. Let $x, y \in X$. Choose $t = f(x \cdot y)$. Then $f(x \cdot y) \leqslant t$. Thus, $x \cdot y \in L(f;t) \neq \emptyset$. By the assumption, we have $L(f;t)$ is a prime subset of X. So $x \in L(f;t)$ or $y \in L(f;t)$. Thus, $t \geqslant f(x)$ or $t \geqslant f(y)$, so $f(x \cdot y) = t \geqslant \min\{f(x), f(y)\}$. Thus, $\overline{f}(x \cdot y) = 1 - f(x \cdot y) \leqslant 1 - \min\{f(x), f(y)\} = \max\{1 - f(x), 1 - f(y)\} = \max\{\overline{f}(x), \overline{f}(y)\}$. Therefore, $\overline{f}$ is a prime fuzzy set in X. □

Theorem 3.50. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy IUP-subalgebra of X if and only if for all $t \in [0,1], L(f;t)$ is a prime IUP-subalgebra of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.45 and 3.49. □

Theorem 3.51. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy IUP-ideal of X if and only if for all $t \in [0,1], L(f;t)$ is a prime IUP-ideal of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.46 and 3.49. □

Theorem 3.52. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy IUP-filter of X if and only if for all $t \in [0,1], L(f;t)$ is a prime IUP-filter of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.47 and 3.49. □

Theorem 3.53. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy strong IUP-ideal of X if and only if for all $t \in [0,1], L(f;t)$ is a prime strong IUP-ideal of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.48 and 3.49 □

Theorem 3.54. *The complement $\overline{f}$ of a fuzzy set f in X is a fuzzy IUP-subalgebra of X if and only if for all $t \in [0,1], L^{-}(f;t)$ is an IUP-subalgebra of X if it is non-empty.*

Proof. Assume that $\overline{f}$ is a fuzzy IUP-subalgebra of X. Let $t \in [0,1]$ be such that $L^{-}(f;t) \neq \emptyset$. Let $x, y \in L^{-}(f;t)$. Then $f(x) < t$ and $f(y) < t$. Thus, t is an upper bound of $\{f(x), f(y)\}$. Since $\overline{f}$ is a fuzzy IUP-subalgebra of X, we have $\overline{f}(x \cdot y) \geqslant \min\{\overline{f}(x), \overline{f}(y)\}$. Thus, $1 - f(x \cdot y) \geqslant \min\{1 - f(x), 1 - f(y)\} = 1 - \max\{f(x), f(y)\}$, that is, $f(x \cdot y) \leqslant \max\{f(x), f(y)\} < t$. So $x \cdot y \in L^{-}(f;t)$. Therefore, $L^{-}(f;t)$ is an IUP-subalgebra of X.

Conversely, assume that for all $t \in [0,1], L^{-}(f;t)$ is an IUP-subalgebra of X if it is non-empty. Let $x, y \in X$. Assume that $\overline{f}(x \cdot y) < \min\{\overline{f}(x), \overline{f}(y)\}$. Then $1 - f(x \cdot y) < \min\{1 - f(x), 1 - f(y)\} = 1 - \max\{f(x), f(y)\}$, that is, $f(x \cdot y) > \max\{f(x), f(y)\}$. Choose $t = f(x \cdot y)$. Then $f(x) < t$ and $f(y) < t$. Thus, $x, y \in L^{-}(f;t) \neq \emptyset$. By the assumption, we have $L^{-}(f;t)$ is an IUP-subalgebra of X. Thus, $x \cdot y \in L^{-}(f;t)$. So $f(x \cdot y) < t = f(x \cdot y)$, a contradiction. Hence, $\overline{f}(x \cdot y) \geqslant \min\{\overline{f}(x), \overline{f}(y)\}$ for all $x, y \in X$. Therefor, $\overline{f}$ is a fuzzy IUP-subalgebra of X. □

Theorem 3.55. *The complement $\overline{f}$ of a fuzzy set f in X is a fuzzy IUP-ideal of X if and only if for all $t \in [0,1], L^{-}(f;t)$ is an IUP-ideal of X if it is non-empty.*

Proof. Assume that $\overline{f}$ is a fuzzy IUP-ideal of X. Let $t \in [0,1]$ be such that $L^-(f;t) \neq \emptyset$ and let $a \in L^-(f;t)$. Then $f(a) < t$. Since $\overline{f}$ is a fuzzy IUP-ideal of X, we have $\overline{f}(0) \geqslant \overline{f}(a)$. Thus, $1 - f(0) \geqslant 1 - f(a)$, that is, $f(0) \leqslant f(a) < t$. Hence, $0 \in L^-(f;t)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L^-(f;t)$ and $y \in L^-(f;t)$. Then $f(x \cdot (y \cdot z)) < t$ and $f(y) < t$. Thus, t is an upper bound of $\{f(x \cdot (y \cdot z)), f(y)\}$. Since $\overline{f}$ is a fuzzy IUP-ideal of X, we have $\overline{f}(x \cdot z) \geqslant \min\{\overline{f}(x \cdot (y \cdot z)), \overline{f}(y)\}$. Thus, $1 - f(x \cdot z) \geqslant \min\{1 - f(x \cdot (y \cdot z)), 1 - f(y)\} = 1 - \max\{f(x \cdot (y \cdot z)), f(y)\}$, that is, $f(x \cdot z) \leqslant \max\{f(x \cdot (y \cdot z)), f(y)\} < t$. Thus, $x \cdot z \in L^-(f;t)$. Therefore, $L^-(f;t)$ is an IUP-ideal of X.

Conversely, assume that for all $t \in [0,1], L^-(f;t)$ is an IUP-ideal of X if it is non-empty. Let $x \in X$. Assume that $\overline{f}(0) < \overline{f}(x)$. Then $1 - f(0) < 1 - f(x)$, that is, $f(0) > f(x)$. Choose $t = f(0)$. Then $f(x) < t$. Thus, $x \in L^-(f;t) \neq \emptyset$. By the assumption, we have $L^-(f;t)$ is an IUP-ideal of X. Thus, $0 \in L^-(f;t)$. So $f(0) < t = f(0)$, a contradiction. Hence, $\overline{f}(0) \geqslant \overline{f}(x)$ for all $x \in X$. Let $x, y, z \in X$. Assume that $\overline{f}(x \cdot z) < \min\{\overline{f}(x \cdot (y \cdot z)), \overline{f}(y)\}$. Then $1 - f(x \cdot z) < \min\{1 - f(x \cdot (y \cdot z)), 1 - f(y)\} = 1 - \max\{f(x \cdot (y \cdot z)), f(y)\}$, that is, $f(x \cdot z) > \max\{f(x \cdot (y \cdot z)), f(y)\}$. Choose $t' = f(x \cdot z)$. Then $f(x \cdot (y \cdot z)) < t'$ and $f(y) < t'$. Thus, $x \cdot (y \cdot z), y \in L^-(f;t') \neq \emptyset$. By the assumption, we have $L^-(f;t')$ is an IUP-ideal of X. Thus, $x \cdot z \in L^-(f;t')$. So $f(x \cdot z) < t' = f(x \cdot z)$, a contradiction. Hence, $\overline{f}(x \cdot z) \geqslant \min\{\overline{f}(x \cdot (y \cdot z)), \overline{f}(y)\}$ for all $x, y, z \in X$. Therefore, $\overline{f}$ is a fuzzy IUP-ideal of X. □

Theorem 3.56. *The complement* $\overline{f}$ *of a fuzzy set* f *in* X *is a fuzzy IUP-filter of* X *if and only if for all* $t \in [0,1], L^-(f;t)$ *is an IUP-filter of* X *if it is non-empty.*

Proof. Assume that $\overline{f}$ is a fuzzy IUP-filter of X. Let $t \in [0,1]$ be such that $L^-(f;t) \neq \emptyset$ and let $a \in L^-(f;t)$. Then $f(a) < t$. Since $\overline{f}$ is a fuzzy IUP-filter of X, we have $\overline{f}(0) \geqslant \overline{f}(a)$. Thus, $1 - f(0) \geqslant 1 - f(a)$, that is, $f(0) \leqslant f(a) < t$. Hence, $0 \in L^-(f;t)$. Let $x, y \in X$ be such that $x \cdot y \in L^-(f;t)$ and $x \in L^-(f;t)$. Then $f(x \cdot y) < t$ and $f(x) < t$. Thus, t is an upper bound of $\{f(x \cdot y), f(x)\}$. Since $\overline{f}$ is a fuzzy IUP-filter of X, we have $\overline{f}(y) \geqslant \min\{\overline{f}(x \cdot y), \overline{f}(x)\}$. Thus, $1 - f(y) \geqslant \min\{1 - f(x \cdot y), 1 - f(x)\} = 1 - \max\{f(x \cdot y), f(x)\}$, that is, $f(y) \leqslant \max\{f(x \cdot y), f(x)\} < t$. Thus, $y \in L^-(f;t)$. Therefore, $L^-(f;t)$ is an IUP-filter of X.

Conversely, assume that for all $t \in [0,1], L^-(f;t)$ is an IUP-filter of X if it is non-empty. Let $x \in X$. Assume that $\overline{f}(0) < \overline{f}(x)$. Then $1 - f(0) < 1 - f(x)$, that is, $f(0) > f(x)$. Choose $t = f(0)$. Then $f(x) < t$. Thus, $x \in L^-(f;t) \neq \emptyset$. By the assumption, we have $L^-(f;t)$ is an IUP-filter of X. Thus, $0 \in L^-(f;t)$. So $f(0) < t = f(0)$, a contradiction. Hence, $\overline{f}(0) \geqslant \overline{f}(x)$ for all $x \in X$. Let $x, y \in X$. Assume that $\overline{f}(y) < \min\{\overline{f}(x \cdot y), \overline{f}(x)\}$. Then $1 - f(y) < \min\{1 - f(x \cdot y), 1 - f(x)\} = 1 - \max\{f(x \cdot y), f(x)\}$, that is, $f(y) > \max\{f(x \cdot y), f(x)\}$. Choose $t' = f(y)$. Then $f(x \cdot y) < t'$ and $f(x) < t'$. Thus, $x \cdot y, x \in L^-(f;t') \neq \emptyset$. By the assumption, we have $L^-(f;t')$ is an IUP-filter of X. Thus, $y \in L^-(f;t')$. So $f(y) < t' = f(y)$, a contradiction. Hence, $\overline{f}(y) \geqslant \min\{\overline{f}(x \cdot y), \overline{f}(x)\}$ for all $x, y \in X$. Therefor, $\overline{f}$ is a fuzzy IUP-filter of X. □

Theorem 3.57. *The complement* $\overline{f}$ *of a fuzzy set* f *in* X *is a fuzzy strong IUP-ideal of* X *if and only if for all* $t \in [0,1], L^-(f;t)$ *is a strong IUP-ideal of* X *if it is non-empty.*

Proof. It is straightforward by Theorem 3.13. □

Theorem 3.58. *The complement* $\overline{f}$ *of a fuzzy set* f *in* X *is a prime fuzzy set in* X *if and only if for all* $t \in [0,1], L^-(f;t)$ *is a prime subset of* X *if it is non-empty.*

Proof. Assume that $\overline{f}$ is a prime fuzzy set in X. Let $t \in [0,1]$ be such that $L^-(f;t) \neq \emptyset$. Let $x, y \in X$ be such that $x \cdot y \in L^-(f;t)$. Assume that $x \notin L^-(f;t)$ and $y \notin L^-(f;t)$. Then $f(x) \geqslant t$ and $f(y) \geqslant t$. Thus, t is a lower bound of $\{f(x), f(y)\}$. Since $\overline{f}$ is a prime fuzzy set in X, we have $\overline{f}(x \cdot y) \leqslant \max\{\overline{f}(x), \overline{f}(y)\}$. Thus, $1 - f(x \cdot y) \leqslant \max\{1 - f(x), 1 - f(y)\} = 1 - \min\{f(x), f(y)\}$, that is, $f(x \cdot y) \geqslant \min\{f(x), f(y)\} \geqslant t$. Thus, $x \cdot y \notin L^-(f;t)$, a contradiction. Hence, $x \in L^-(f;t)$ or $y \in L^-(f;t)$. Therefore, $L^-(f;t)$ is a prime subset of X.

Conversely, assume that for all $t \in [0,1], L^-(f;t)$ is a prime subset of X if it is non-empty. Let $x, y \in X$. Assume that $\overline{f}(x \cdot y) > \max\{\overline{f}(x), \overline{f}(y)\}$. Then $1 - f(x \cdot y) > \max\{1 - f(x), 1 - f(y)\} = 1 - \min\{f(x), f(y)\}$, that is, $f(x \cdot y) < \min\{f(x), f(y)\}$. Choose $t = \min\{f(x), f(y)\}$. Then $f(x \cdot y) < t$. Thus, $x \cdot y \in L^-(f;t) \neq \emptyset$.

By the assumption, we have $L^-(f;t)$ is a prime subset of X. Thus, $x \in L^-(f;t)$ or $y \in L^-(f;t)$. So $f(x) < t = \min\{f(x), f(y)\}$ or $f(y) < t = \min\{f(x), f(y)\}$, a contradiction. Hence, $\overline{f}(x \cdot y) \leqslant \max\{\overline{f}(x), \overline{f}(y)\}$ for all $x, y \in X$. Therefore, $\overline{f}$ is a prime fuzzy set in X. □

Theorem 3.59. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy IUP-subalgebra of X if and only if for all $t \in [0,1]$, $L^-(f;t)$ is a prime IUP-subalgebra of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.54 and 3.58. □

Theorem 3.60. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy IUP-ideal of X if and only if for all $t \in [0,1]$, $L^-(f;t)$ is a prime IUP-ideal of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.55 and 3.58. □

Theorem 3.61. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy IUP-filter of X if and only if for all $t \in [0,1]$, $L^-(f;t)$ is a prime IUP-filter of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.56 and 3.58. □

Theorem 3.62. *The complement $\overline{f}$ of a fuzzy set f in X is a prime fuzzy strong IUP-ideal of X if and only if for all $t \in [0,1]$, $L^-(f;t)$ is a prime strong IUP-ideal of X if it is non-empty.*

Proof. It is straightforward by Theorems 3.57 and 3.58. □

4. Conclusions and future work

In this paper, we have introduced the concepts of fuzzy IUP-subalgebras, fuzzy IUP-filters, fuzzy IUP-ideals, and fuzzy strong IUP-ideals of IUP-algebras and investigated important properties. Our research found that these four concepts are also related to characteristic functions and level sets.

In the near future, our research team will also study the concept of intuitionistic fuzzy sets as defined by Atanasov [2].

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