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Control and adaptive modified function projective synchronization of a new chaotic system

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Abstract

In this paper, the feedback control technique is used to suppress the chaos of a New chaotic system. Under some conditions on the parameters of the system, the controlled system is stable. Those conditions are based on the Routh-Hurwitz criterion. In addition, the adaptive modified function projective synchronization of two New chaotic systems is satisfied. To prove the asymptotic stability of solutions for the error system, we used the Lyapunov theorem of stability. Numerical experiment results are presented to display the impact of the proposed schemes.

Keywords: Chaotic system, linear feedback control, adaptive modified function projective synchronization.

2020 MSC: 37N35, 34D06, 34H10.

1. Introduction

Chaos is a phenomenon in some nonlinear dynamical systems. Chaotic behavior is sensitive to its initial conditions. In the last decades, control and synchronization of chaos have attracted the attention of many researchers because of their applications in engineering, electronic circuits, and secure communications [25].

Controlling chaos involves adding an input control to try stabilizing the equilibrium point. In the recent years, several control methods have been presented for chaos control such as linear and nonlinear feedback control, active control, and adaptive control [1, 3, 12–15, 22].

Many authors have studied various chaos synchronization techniques such as anti-synchronization, complete synchronization, function projective synchronization (FPS), and modified projective synchronization (MPS) [2, 4–10, 16, 18–21, 23, 26]. Later, a new kind of synchronization technique called modified function projective synchronization is presented [11, 24, 27, 28].

Our object in this paper is to control the new chaotic system by linear feedback control and to study the adaptive modified function projective synchronization (AMFPS) for two new chaotic systems.

The paper is organized as follows. Section 2 shows the description of a new chaotic system. We explain the feedback control of the new chaotic system in Section 3. Furthermore, Section 4 illustrates the (AMFPS) of two new chaotic systems. Section 5 is devoted to the numerical experiment results to illustrate the impact and effectiveness of the proposed schemes. The last section includes the conclusion.

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2. System description

The New chaotic system [17] is expressed by

$$\begin{cases} \dot{x} = -\dfrac{ab}{a+b}x - yz + c, \\ \dot{y} = ay + xz, \\ \dot{z} = bz + xy, \end{cases} \tag{2.1}$$

where the parameters a, b, and c are real constant numbers. In a wide parameter range, the system will be chaotic. For example, when $a = -10,\ b = -4$, and $c = 18.1$, the chaotic attractor is shown in Fig. 1. Furthermore, when $a = -10,\ b = -4,\ c = 0$, the chaotic attractor is shown in Fig. 2.

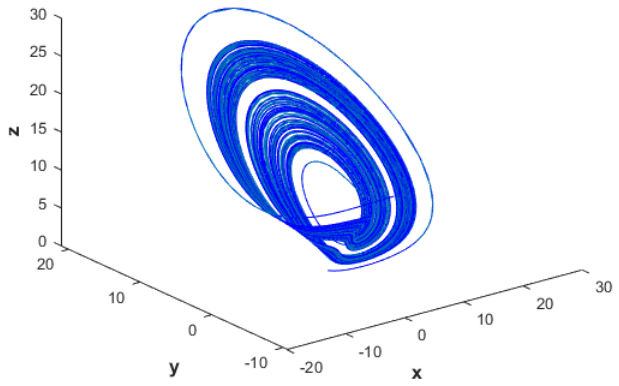

Figure 1: The new chaotic system at $a = -10$, $b = -4$, $c = 18.1$.

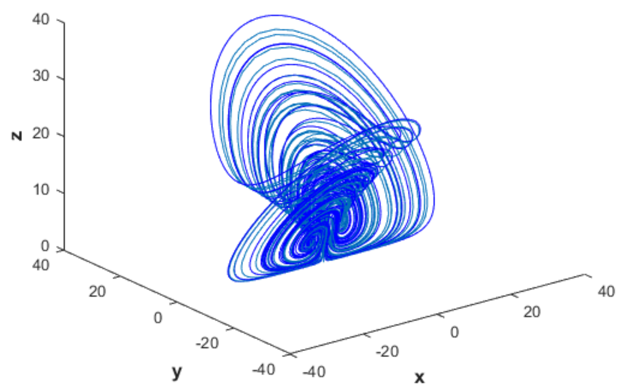

Figure 2: The new chaotic system at $a = -10$, $b = -4$, $c = 0$.

2.1. *The dissipation*

The divergence of system (2.1) is found as follows:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -\frac{ab}{a+b} + a + b = \frac{-78}{7} < 0,$$

hence, system (2.1) is dissipative.

2.2. *Equilibrium points*

There are only three equilibrium points in the system when $a = -10,\ b = -4$, and $c = 18.1$, which are

$$P_1 = (-6.335,\ 0,\ 0),\ P_{2,3} = \left(2\sqrt{10},\ \pm\sqrt{\frac{80}{7} + 3.62\sqrt{10}},\ \pm\frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10}}\right).$$

2.3. *Lyapunov exponents and its dimension*

The Lyapunov exponents of system (2.1) is found to be $\lambda_1 = 0.253223,\ \lambda_2 = 0,\ \lambda_3 = -11.3944$. If the initial value is taken as $(1, 1, 1)$, then the Lyapunov dimension is $D_L = 2.0221$.

3. Controlling a new chaotic system

To control the new chaotic system to the unstable equilibrium points, we use feedback control to guide the chaotic trajectory to the unstable equilibrium points.

3.1. First

We guide the chaotic trajectory $(x(t), y(t), z(t))$ of the system (2.1) to $P_1 = (-6.335, 0, 0)$ by using the linear feedback control method. Let system (2.1) be controlled on the form:

$$\begin{cases} \dot{x} = -\dfrac{ab}{a+b}x - yz + c - k_{11}(x - x_1), \\ \dot{y} = ay + xz - k_{12}(y - y_1), \\ \dot{z} = bz + xy - k_{13}(z - z_1). \end{cases} \tag{3.1}$$

The previous system has one equilibrium point P_1. We linearize the system (3.1) about P_1. Therefore, the linearized system is written as:

$$\begin{cases} \dot{X} = (-\dfrac{ab}{a+b} - k_{11})X - z_1 Y - y_1 Z, \\ \dot{Y} = z_1 X + (a - k_{12})Y + x_1 Z, \\ \dot{Z} = y_1 X + x_1 Y + (b - k_{13})Z, \end{cases}$$

where $P_1 = (x_1, y_1, z_1) = (-6.335, 0, 0)$, then

$$\begin{cases} \dot{X} = (-\dfrac{ab}{a+b} - k_{11})X, \\ \dot{Y} = (a - k_{12})Y - 6.335Z, \\ \dot{Z} = -6.335Y + (b - k_{13})Z. \end{cases} \tag{3.2}$$

The system (3.2) has a zero solution $(0, 0, 0)$, which is equivalent to the equilibrium point (x_1, y_1, z_1) for the controlled system (3.1).

Lemma 3.1. *If the conditions on the gained matrix are selected as* $k_{11} > -\dfrac{ab}{a+b}$, $k_{12} \geqslant \dfrac{1}{2}$, *and* $k_{13} = 0$, *then the zero solution of the linearized system* (3.2) *is asymptotically stable.*

Proof. This proof depends on the real part of eigenvalues. The Jacobian matrix of system (3.2) is shown as:

$$J = \begin{pmatrix} -\dfrac{ab}{a+b} - k_{11} & 0 & 0 \\ 0 & a - k_{12} & x_1 \\ 0 & x_1 & b \end{pmatrix},$$

then we get the characteristic equation on the form

$$(-\frac{ab}{a+b} - k_{11} - \lambda)[(a - k_{12} - \lambda)(b - \lambda) - {x_1}^2] = 0,$$
$$(-\frac{ab}{a+b} - k_{11} - \lambda)[\lambda^2 + (k_{12} - a - b)\lambda + (ab - bk_{12} - {x_1}^2)] = 0,$$

then we get $\lambda_1 = -\dfrac{ab}{a+b} - k_{11}$ and

$$\lambda_{2,3} = -\frac{(k_{12} - a - b) \pm \sqrt{(k_{12} - a - b)^2 - 4(ab - bk_{12} - {x_1}^2)}}{2}.$$

It satisfies that $\lambda_1 < 0$ and $\lambda_{2,3} < 0$, therefore the zero solution of (3.2) is asymptotically stable since the eigenvalues have negative real parts. □

3.2. Second

We guide the chaotic trajectory $(x(t), y(t), z(t))$ of the system (2.1) to P_2 by using the linear feedback control method, where

$$P_2 = (x_2, y_2, z_2) = \left(2\sqrt{10},\ \sqrt{\frac{80}{7} + 3.62\sqrt{10}},\ \frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10})} \right).$$

Let system (2.1) be controlled on the form:

$$\begin{cases} \dot{x} = -\dfrac{ab}{a+b}x - yz + c - k_{21}(x - x_2), \\ \dot{y} = ay + xz - k_{22}(y - y_2), \\ \dot{z} = bz + xy - k_{23}(z - z_2). \end{cases} \tag{3.3}$$

The previous system has one equilibrium point P_2. We linearize the system (3.3) about P_1. Therefore, the linearized system is written as:

$$\begin{cases} \dot{X} = -(\dfrac{ab}{a+b} + k_{21})X - z_2 Y - y_2 Z, \\ \dot{Y} = z_2 X + (a - k_{22})Y + x_2 Z, \\ \dot{Z} = y_2 X + x_2 Y + (b - k_{23})Z. \end{cases}$$

Let $(x_2, y_2, z_2) = (\alpha, \beta, \gamma)$, where $\alpha = 2\sqrt{10}$, $\beta = \sqrt{\frac{80}{7} + 3.62\sqrt{10}}$, and $\gamma = \frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10}}$, we get

$$\begin{cases} \dot{X} = -(\dfrac{ab}{a+b} + k_{21})X - \gamma Y - \beta Z, \\ \dot{Y} = \gamma X + (a - k_{22})Y + \alpha Z, \\ \dot{Z} = \beta X + \alpha Y + (b - k_{23})Z. \end{cases} \tag{3.4}$$

Lemma 3.2. *If the conditions on the gained matrix are selected as $k_{21} = k_{22} = 0$ and $k_{23} > 4.3$, then the zero solution of the linearized system (3.4) is asymptotically stable.*

Proof. This proof depends on the condition of the Routh-Hurwitz criterion. The Jacobian matrix of system (3.4) is shown as:

$$J = \begin{pmatrix} w & -\gamma & -\beta \\ \gamma & a & \alpha \\ \beta & \alpha & b - k_{23} \end{pmatrix}.$$

Let $w = -\dfrac{ab}{a+b}$, then we get the characteristic equation on the form

$$\begin{aligned} &\lambda^3 + (k_{23} - a - b - w)\lambda^2 + \{(b - k_{23})(a + w) + aw - \alpha^2 + \beta^2 + \gamma^2\}\lambda \\ &\quad + \{(aw + \gamma^2)(k_{23} - b) + \alpha^2 w + 2\alpha\beta\gamma - a\beta^2\} = 0, \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} a_1 &= k_{23} - a - b - w, \\ a_2 &= (b - k_{23})(a + w) + aw - \alpha^2 + \beta^2 + \gamma^2, \\ a_3 &= (aw + \gamma^2)(k_{23} - b) + \alpha^2 w + 2\alpha\beta\gamma - a\beta^2. \end{aligned}$$

The Routh-Hurwitz condition $a_1 > 0$, $a_1 a_2 > a_3$, and $a_3 > 0$ is achieved. This leads that the zero solution of (3.4) is asymptotically stable since the eigenvalues have negative real parts. □

3.3. *Third*

We guide the chaotic trajectory $(x(t), y(t), z(t))$ of the system (2.1) to P_3 by using the linear feedback control method, where

$$P_3 = (x_3, y_3, z_3) = \left(2\sqrt{10},\ -\sqrt{\frac{80}{7} + 3.62\sqrt{10}},\ -\frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10}}\right).$$

Let system (2.1) be controlled on the form:

$$\begin{cases} \dot{x} = -\dfrac{ab}{a+b}x - yz + c - k_{31}(x - x_3), \\ \dot{y} = ay + xz - k_{32}(y - y_3), \\ \dot{z} = bz + xy - k_{33}(z - z_3). \end{cases} \tag{3.6}$$

The previous system has one equilibrium point P_3. We linearize the system (3.1) about P_3. Therefore, the linearized system is written as

$$\begin{cases} \dot{X} = -(\dfrac{ab}{a+b} + k_{31})X - z_3 Y - y_3 Z, \\ \dot{Y} = z_3 X + (a - k_{32})Y + x_3 Z, \\ \dot{Z} = y_3 X + x_3 Y + (b - k_{33})Z. \end{cases}$$

Let $(x_3, y_3, z_3) = (\alpha^*, \beta^*, \gamma^*)$, where $\alpha^* = 2\sqrt{10}$, $\beta^* = -\sqrt{\frac{80}{7} + 3.62\sqrt{10}}$, and $\gamma^* = -\frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10}}$, we get

$$\begin{cases} \dot{X} = -(\dfrac{ab}{a+b} + k_{31})X - \gamma^* Y - \beta^* Z, \\ \dot{Y} = \gamma^* X + (a - k_{32})Y + \alpha^* Z, \\ \dot{Z} = \beta^* X + \alpha^* Y + (b - k_{33})Z. \end{cases} \tag{3.7}$$

Lemma 3.3. *If the conditions on the gained matrix are selected as $k_{31} = k_{32} = 0$ and $k_{33} > 4.3$, then the zero solution of the linearized system (3.7) is asymptotically stable.*

Proof. This proof depends on the condition of the Routh-Hurwitz criterion. The Jacobian matrix of system (3.7) is shown as

$$J = \begin{pmatrix} w & -\gamma^* & -\beta^* \\ \gamma^* & a & \alpha^* \\ \beta^* & \alpha^* & b - k_{33} \end{pmatrix}.$$

Let $w = -\dfrac{ab}{a+b}$, then we get the characteristic equation on the form

$$\begin{aligned} &\lambda^3 + (k_{33} - a - b - w)\lambda^2 + \{(b - k_{33})(a + w) + aw + \gamma^{*2} - \alpha^{*2} + \beta^{*2}\}\lambda \\ &\quad + \{(aw + \gamma^{*2})(k_{33} - b) + \alpha^{*2}w + 2\alpha^*\beta^*\gamma^* - a\beta^{*2}\} = 0, \end{aligned}$$

where

$$\begin{aligned} a_1 &= k_{33} - a - b - w, \\ a_2 &= (b - k_{33})(a + w) + aw + \gamma^{*2} - \alpha^{*2} + \beta^{*2}, \\ a_3 &= (aw + \gamma^{*2})(k_{33} - b) + \alpha^{*2}w + 2\alpha^*\beta^*\gamma^* - a\beta^{*2}. \end{aligned}$$

The Routh-Hurwitz condition $a_1 > 0$, $a_1 a_2 > a_3$, and $a_3 > 0$ is achieved. This leads that the zero solution of (3.7) is asymptotically stable since the eigenvalues have negative real parts. □

4. Adaptive modified function projective synchronization of a new chaotic system

When synchronization is applied between chaotic systems, there will be a master (drive) and a slave (response) system.

The master system can be written as:

$$\dot{X} = H(X), \tag{4.1}$$

and the slave system is defined as:

$$\dot{Y} = D(Y) + U(t, X, Y), \tag{4.2}$$

where $X, Y \in \mathbb{R}^n$ are the state vectors of the (4.1) and (4.2), $H, D : \mathbb{R}^n \to \mathbb{R}^n$ are differentiable vector functions, and $U(t, X, Y)$ is a control function which will be selected later.

Definition 1. *The master and slave systems are called to be synchronized in the manner of a modified projective synchronization function, if there exists a scaling function matrix such that:*

$$\lim_{t\to+\infty} \|e(t)\| = \lim_{t\to+\infty} \|Y - \Lambda(t)X\| = 0,$$

where $\Lambda(t) = \mathrm{diag}\{\alpha_1(t),\ \alpha_2(t), \ldots, \alpha_n(t)\}$ *such that* $\alpha_i(t)$ *are continuous differentiable functions and* $\alpha_i(t) \neq 0$ *for all* t.

We will study the AMFPS of the new chaotic system (2.1). Then, the master and slave systems are given as follows, respectively,

$$\begin{cases} \dot{x}_1 = -\dfrac{ab}{a+b}x_1 - y_1 z_1 + c, \\ \dot{y}_1 = a y_1 + x_1 z_1, \\ \dot{z}_1 = b z_1 + x_1 y_1, \end{cases} \tag{4.3}$$

and the new chaotic system as the slave system is given by:

$$\begin{cases} \dot{x}_2 = -\dfrac{ab}{a+b}x_2 - y_2 z_2 + c + u_1, \\ \dot{y}_2 = a y_2 + x_2 z_2 + u_2, \\ \dot{z}_2 = b z_2 + x_2 y_2 + u_3, \end{cases} \tag{4.4}$$

where u_1, u_2, and u_3 are the nonlinear controllers. The two systems (4.3) and (4.4) can be synchronized in the sense of MFPS if

$$\begin{cases} \lim_{t\to+\infty} \|x_2 - (\alpha_{11}x_1 + \alpha_{12})x_1\| = 0, \\ \lim_{t\to+\infty} \|y_2 - (\alpha_{21}y_1 + \alpha_{22})y_1\| = 0, \\ \lim_{t\to+\infty} \|z_2 - (\alpha_{31}z_1 + \alpha_{32})z_1\| = 0, \end{cases}$$

whereas $e_1 = x_2 - (\alpha_{11}x_1 + \alpha_{12})x_1,\ e_2 = y_2 - (\alpha_{21}y_1 + \alpha_{22})y_1$, and $e_3 = z_2 - (\alpha_{31}z_1 + \alpha_{32})z_1$ are state error vectors. Then, the error dynamical system between (4.3) and (4.4) is defined by

$$\begin{cases} \dot{e}_1 = -\dfrac{ab}{a+b}e_1 - y_2 z_2 + c + \dfrac{ab}{a+b}\alpha_{11}x_1^2 + 2\alpha_{11}x_1 y_1 z_1 - 2\alpha_{11}x_1 c + \alpha_{12}y_1 z_1 - \alpha_{12}c + u_1, \\ \dot{e}_2 = a e_2 + x_2 z_2 - a\alpha_{21}y_1^2 - 2\alpha_{21}x_1 y_1 z_1 - \alpha_{22}x_1 z_1 + u_2, \\ \dot{e}_3 = b e_3 + x_2 y_2 - b\alpha_{31}z_1^2 - 2\alpha_{31}x_1 y_1 z_1 - \alpha_{32}x_1 y_1 + u_3. \end{cases} \tag{4.5}$$

The aim is to the stabilizing error variables $e_i (i = 1, 2, 3)$ of (4.5) by defining a control law $u_j (j = 1, 2, 3)$. Therefore, we proposed the control law as below:

$$\begin{cases} u_1 = y_2 z_2 - c - \dfrac{ab}{a+b}\alpha_{11}x_1^2 - 2\alpha_{11}x_1 y_1 z_1 + 2\alpha_{11}x_1 c - \alpha_{12}y_1 z_1 + \alpha_{12}c + \dfrac{2ab}{a+b}e_1, \\ u_2 = -x_2 z_2 + a\alpha_{21}y_1^2 + 2\alpha_{21}x_1 y_1 z_1 + \alpha_{22}x_1 z_1, \\ u_3 = -x_2 y_2 + b\alpha_{31}z_1^2 + 2\alpha_{31}x_1 y_1 z_1 + \alpha_{32}x_1 y_1. \end{cases} \tag{4.6}$$

Hence, the following theorem was deduced.

Theorem 4.1. *AMFPS will be achieved between system (4.3) and system (4.4) under control law (4.6), where the* $\alpha_{ij}(i = 1, 2, 3, j = 1, 2)$ *are given nonzero scalars.*

Proof. Define a Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2). \tag{4.7}$$

The derivative of the Lyapunov function (4.7) is

$$\begin{aligned}\frac{dV}{dt} &= (e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3)\\ &= e_1\left(-\frac{ab}{a+b}e_1 - y_2z_2 + c + \frac{ab}{a+b}\alpha_{11}x_1^2 + 2\alpha_{11}x_1y_1z_1 - 2\alpha_{11}x_1c + \alpha_{12}y_1z_1 - \alpha_{12}c + u_1\right)\\ &\quad + e_2\left(ae_2 + x_2z_2 - a\alpha_{21}y_1^2 - 2\alpha_{21}x_1y_1z_1 - \alpha_{22}x_1z_1 + u_2\right)\\ &\quad + e_3\left(be_3 + x_2y_2 - b\alpha_{31}z_1^2 - 2\alpha_{31}x_1y_1z_1 - \alpha_{32}x_1y_1 + u_3\right).\end{aligned} \tag{4.8}$$

By substituting the control input (4.6) into (4.8), it leads to

$$\begin{aligned}\frac{dV}{dt} &= e_1\Big(-\frac{ab}{a+b}e_1 - y_2z_2 + c + \frac{ab}{a+b}\alpha_{11}x_1^2 + 2\alpha_{11}x_1y_1z_1 - 2\alpha_{11}x_1c + \alpha_{12}y_1z_1 - \alpha_{12}c + y_2z_2 - c\\ &\quad - \frac{ab}{a+b}\alpha_{11}x_1^2 - 2\alpha_{11}x_1y_1z_1 + 2\alpha_{11}x_1c - \alpha_{12}y_1z_1 + \alpha_{12}c + \frac{2ab}{a+b}e_1\Big) + e_2\Big(ae_2 + x_2z_2 - a\alpha_{21}y_1^2\\ &\quad - 2\alpha_{21}x_1y_1z_1 - \alpha_{22}x_1z_1 - x_2z_2 + a\alpha_{21}y_1^2 + 2\alpha_{21}x_1y_1z_1 + \alpha_{22}x_1z_1\Big)\\ &\quad + e_3\Big(be_3 + x_2y_2 - b\alpha_{31}z_1^2 - 2\alpha_{31}x_1y_1z_1 - \alpha_{32}x_1y_1 - x_2y_2 + b\alpha_{31}z_1^2 + 2\alpha_{31}x_1y_1z_1 + \alpha_{32}x_1y_1\Big).\end{aligned}$$

Therefore, we get

$$\frac{dV}{dt} = \frac{ab}{a+b}e_1^2 + ae_2^2 + be_3^2.$$

Then, we have

$$\frac{dV}{dt} = \frac{-20}{7}e_1^2 - 10e_2^2 - 4e_3^2 = -\{\frac{20}{7}e_1^2 + 10e_2^2 + 4e_3^2\}.$$

Since $\frac{dV}{dt} < 0$, therefore the AMFPS for two new chaotic systems is satisfied. □

5. Numerical results

A numerical experiment is presented in this section to display the effectiveness of the proposed method, where $a = -10$, $b = -4$, and $c = 18.1$. Figure 3 illustrates the trajectory of the controlled system that heads to $P_1 = (-6.335, 0, 0)$. Figure 4 illustrates the trajectory of the controlled system that goes to $P_2 = \left(2\sqrt{10}, \sqrt{\frac{80}{7} + 3.62\sqrt{10}}, \frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10}}\right)$. Figure 5 illustrates the trajectory of the controlled system that goes to $P_3 = \left(2\sqrt{10}, -\sqrt{\frac{80}{7} + 3.62\sqrt{10}}, -\frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10}}\right)$.

In all simulations, the initial conditions of the master system and slave system are selected as follows: $x_1(0) = 1$, $y_1(0) = 1$, $z_1(0) = 1$, and $x_2(0) = 3$, $y_2(0) = 7$, $z_2(0) = 2$. We take the scaling functions as: $\alpha_1 = 2x_1 + 3$, $\alpha_2 = y_1 + 2$, $\alpha_3 = 3z_1 + 4$. Figure 6 illustrates the AMFPS for two new chaotic systems. When we simplify the scaling factor as $\alpha_1 = 2$, $\alpha_2 = 3$, $\alpha_3 = -4$. Figure 7 displays the MPS for two new chaotic systems. Moreover, in Figure 8 the anti-synchronization for two new chaotic systems appears when the scaling factors are given by $\alpha_i = -1(i = 1, 2, 3)$. Finally, the complete synchronization for two new chaotic systems is shown in Figure 9, when the scaling factors are taken as $\alpha_i = 1(i = 1, 2, 3)$.

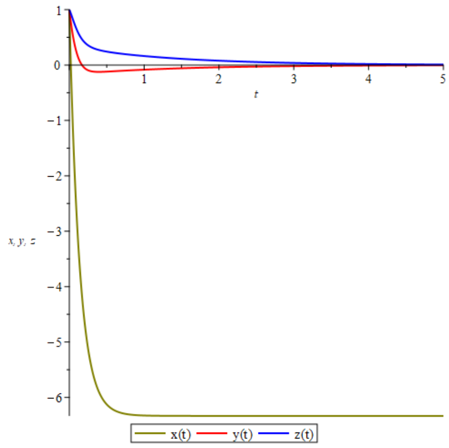

Figure 3: The trajectories of system (3.1) tend to P_1.

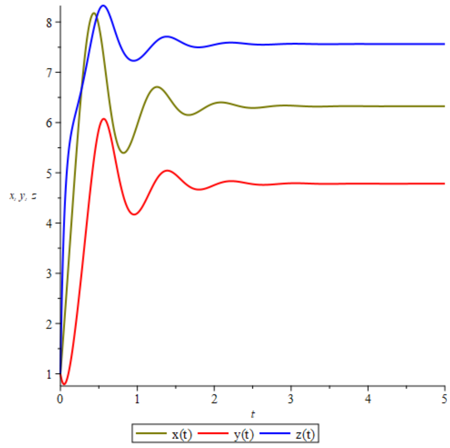

Figure 4: The trajectories of system (3.3) tend to P_2.

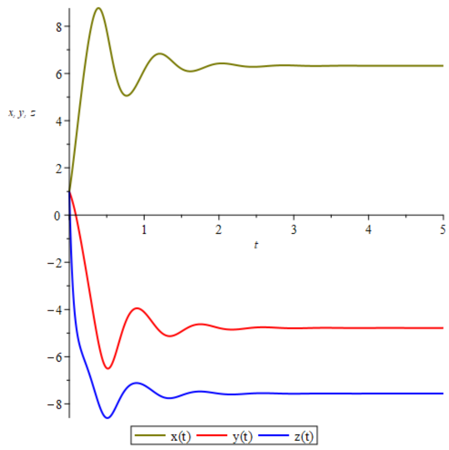

Figure 5: The trajectories of system (3.6) tend to P_3.

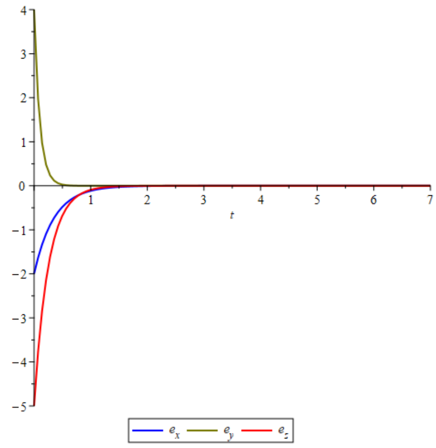

Figure 6: The error vectors e_x, e_y and e_z head to zero for achieving AMFPS.

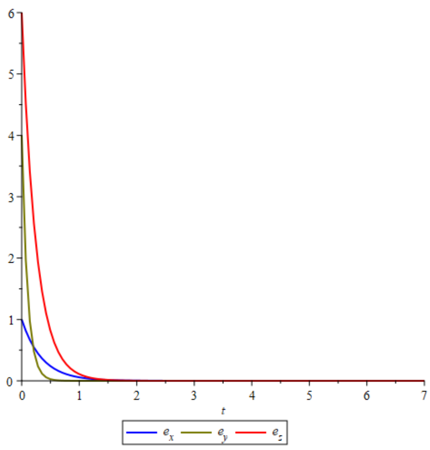

Figure 7: The error vectors e_x, e_y, and e_z head to zero for achieving MPS.

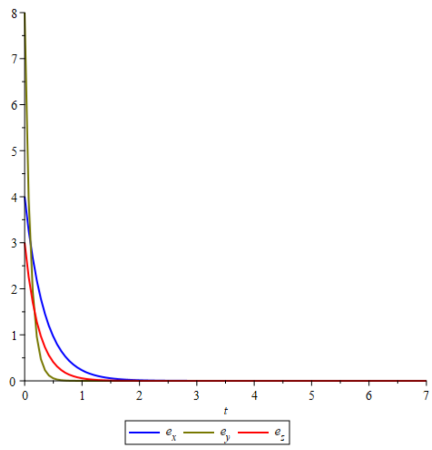

Figure 8: The error vectors e_x, e_y, and e_z head to zero for achieving anti synchronization.

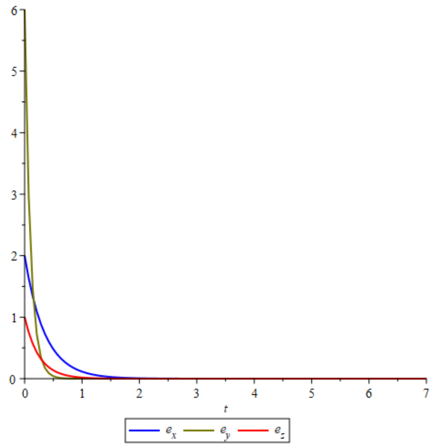

Figure 9: The error vectors e_x, e_y, and e_z head to zero for achieving complete synchronization.

6. Conclusions

In this paper, we have presented the new chaotic system and some of its characteristics. The linear feedback control scheme used for unstable equilibrium points of the new chaotic system succeeded in achieving asymptotic stability. By the Routh-Hurwitz criterion, some conditions on the parameters of the control system were determined. Furthermore, the AMFPS for two new chaotic systems is satisfied. The results of the numerical experiment showed the effectiveness of the selected methods.

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