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Pythagorean fuzzy KU-subalgebras of KU-algebras

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Abstract

The concept of Pythagorean fuzzy sets was introduced by Yager in 2013. It is a generalization of the concepts of fuzzy sets and intuitionistic fuzzy sets. The aim of this study was to apply the concept of Pythagorean fuzzy sets to clarify in KU-algebras. The notion of Pythagorean fuzzy KU-subalgebras of KU-algebras is introduced. Then, we give some fundamental properties of Pythagorean fuzzy KU-subalgebras in KU-algebras. Finally, we investigate the relationships between the image and the preimage of Pythagorean fuzzy KU-subalgebras under a homomorphism of KU-algebras.

Keywords: KU-subalgebra, Pythagorean fuzzy set, Pythagorean fuzzy KU-subalgebra.

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1. Introduction

Two important classes of algebraic structures are BCK-algebras and BCI-algebras, which were introduced by Ise'ki [11, 12] in which the class of BCI-algebras is the general class of BCK-algebras. Afterwards, Hu and Li [9, 10] defined an algebraic structure that generalizes to BCI-algebras named BCH-algebras. Later, there were mathematicians who used the above algebraic structure to widely study various properties; for example in BCK-algebras, Hamidi [8] investigated the concept of superhyper BCK-algebras, which is a generalization of BCK-algebras. In BCI-algebras, Chaida [5] examined commutative BCI-algebras as semilattices with certain involutions in each of their sections. In BCH-algebras, Muangkarn et al. [23] used the concept of endomorphisms and bi-endomorphisms as a model to create tri-endomorphisms on BCH-algebras. For the reader requiring more details, a wider literature is available, e.g., [3, 4, 13, 14, 30].

In 2009, Prabpayak and Leerawat [24] introduced a new algebra which was called KU-algebras and studied congruences on KU-algebras. Subsequently, they discussed relationships between quotient KU-algebras and isomorphisms of KU-algebras see also [25]. Mostafa et al. [22] who applied the coding theory to KU-algebras and obtained some interesting properties. Then, the concept of a hyper structure KU-algebra was introduced and some related results were provided by Mostafa et al. [21]. Subsequently,

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Koam et al. [16] defined an extension of KU-algebras and called it an extended KU-algebra; they discussed the relations between extended KU-algebras and KU-algebras. In 2023, Manivasan and Kalidass [17] introduced BMBJ-neutrosophic sets and subalgebras as a generalization of neutrosophic sets, and examined their applications and related features to KU-algebras.

Fuzzy sets were introduced by Zadeh [32] in 1965 as a mapping from a nonempty set X to the unit interval $[0,1]$. This mapping denotes the degree of membership of each element in a set X. Then, the concepts of fuzzy KU-ideals, interval-valued fuzzy KU-ideals and anti-fuzzy KU-ideals in KU-algebras were introduced and some of their properties were investigated in a series of reports by Mostafa et al. (see [18–20]). In addition, Gulistan et al. [6, 7] presented the concepts of $(\in, \in \vee q_k)$-fuzzy KU-ideals and (α, β)-fuzzy KU-ideals of KU-algebras which are generalizations of fuzzy KU-ideals in KU-algebras. Subsequently, Senapati [26] introduced and investigated the notion of T-fuzzy KU-ideals of KU-algebras by using the t-norm T. As a generalization of fuzzy sets, Atanassov [2] made known the concept of intuitionistic fuzzy sets consisting of the degree of membership and the degree of non-membership of an element in an universe set. Senapati and Shum [28, 29] introduced the notions of intuitionistic fuzzy bi-normed KU-ideals and intuitionistic bi-normed KU-subalgebras of KU-algebras and discussed some of its properties under the homomorphism. Later, Senapati et al. [27] considered the characterizations of cubic intuitionistic KU-subalgebras and cubic intuitionistic KU-ideals of KU-algebras. Simultaneously, the notions of intuitionistic Q-fuzzy KU-ideals of KU-algebras, upper and lower-level cuts of Q-fuzzy sets and some axioms were surveyed by Alkouri et al. [1].

In 2013, Yager [31] suggested the concept of Pythagorean fuzzy sets, which is the sum of squares of the degree of membership and non-membership within the unit interval $[0,1]$. This notion generalizes the fuzzy sets and the intuitionistic fuzzy sets. The purpose of this paper is to apply the concept of Pythagorean fuzzy sets to solve problems in KU-algebras. Next, we introduce the notion of Pythagorean fuzzy KU-subsalgebras and consider some of their properties. Then, we examine the connections between the image and the preimage of Pythagorean fuzzy KU-subalgebras under a homomorphism of KU-algebras.

2. Preliminaries

Firstly, we recall some of the basis definitions and properties, which are necessary for this paper. For any nonempty set X, a mapping $\mu : X \to [0,1]$ is called a *fuzzy set* [32] of X. Let μ and λ be any two fuzzy sets of a nonempty set X. Then the fuzzy sets $\mu \cap \lambda$ and $\mu \cup \lambda$ of X are defined by $(\mu \cap \lambda)(x) = \min\{\mu(x), \lambda(x)\}$ and $(\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}$, for all $x \in X$, respectively. The *complement* of μ, denoted by μ^c, is a fuzzy set in X as defined by $\mu^c(x) = 1 - \mu(x)$, for all $x \in X$. Furthermore, the set $U(\mu, t) = \{x \in X \mid \mu(x) \geqslant t\}$ is called an *upper-level set* of μ, and the set $L(\mu, t) = \{x \in X \mid \mu(x) \leqslant t\}$ is called a *lower-level set* of μ where $t \in [0,1]$.

Definition 2.1 ([2])**.** An *intuitionistic fuzzy set* $\mathcal{A}$ on an universe set X is given by:

$$\mathcal{A} = \{\langle x, \mu_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(x)\rangle \mid x \in X\},$$

where $\mu_{\mathcal{A}} : X \to [0,1]$ and $\lambda_{\mathcal{A}} : X \to [0,1]$ are called the degree of membership and the degree of non-membership, respectively, of the element $x \in X$ in the set $\mathcal{A}$ such that $\mu_{\mathcal{A}}$ and $\lambda_{\mathcal{A}}$ satisfy the following axiom: $0 \leqslant \mu_{\mathcal{A}}(x) + \lambda_{\mathcal{A}}(x) \leqslant 1$, for all $x \in X$.

Definition 2.2 ([31])**.** A *Pythagorean fuzzy set* $\mathcal{P}$ in a nonempty set X is defined by the object:

$$\mathcal{P} = \{\langle x, \mu_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(x)\rangle \mid x \in X\},$$

where $\mu_{\mathcal{P}}(x) \in [0,1]$ denotes the degree of membership and $\lambda_{\mathcal{P}}(x) \in [0,1]$ denotes the degree of non-membership of each $x \in X$ to the set $\mathcal{P}$ with the condition that $0 \leqslant (\mu_{\mathcal{P}}(x))^2 + (\lambda_{\mathcal{P}}(x))^2 \leqslant 1$.

We observe that every intuitionistic fuzzy set on a nonempty set X is also a Pythagorean fuzzy set in a set X. For convenience, we will use the symbol $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ in place of the Pythagorean fuzzy set $\mathcal{P} = \{\langle x, \mu_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(x)\rangle \mid x \in X\}$.

Let $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ and $\mathcal{Q} = (\mu_{\mathcal{Q}}, \lambda_{\mathcal{Q}})$ be any two Pythagorean fuzzy sets on a nonempty et X. Then:

(i) $\mathcal{P} \cap \mathcal{Q} = \{\langle x, (\mu_{\mathcal{P}} \cap \mu_{\mathcal{Q}})(x), (\lambda_{\mathcal{P}} \cup \lambda_{\mathcal{Q}})(x)\rangle \mid x \in X\}$;

(ii) $\mathcal{P} \cup \mathcal{Q} = \{\langle x, (\mu_{\mathcal{P}} \cup \mu_{\mathcal{Q}})(x), (\lambda_{\mathcal{P}} \cap \lambda_{\mathcal{Q}})(x)\rangle \mid x \in X\}$.

Definition 2.3 ([24])**.** An algebra $(X, *, 0)$ of type $(2, 0)$ is called a *KU-algebra* if it satisfies the following properties: for every $x, y, z \in X$,

(i) $(x * y) * [(y * z) * (x * z)] = 0$;

(ii) $0 * x = x$;

(iii) $x * 0 = x$;

(iv) $x * y = 0 = y * x$ implies $x = y$.

In a KU-algebra $(X, *, 0)$, it obtains that $x * x = 0$, for all $x \in X$ (see, [24]). Throughout this paper, we shall use a KU-algebra X instead of the KU-algebra $(X, *, 0)$.

Definition 2.4 ([24])**.** Let T be a nonempty subset of a KU-algebra X. Then, T is said to be a *KU-subalgebra* of X if $x * y \in T$, for all $x, y \in T$.

Example 2.5. Let $X = \{0, a, b, c\}$ be a set with the multiplication $*$ on X is defined by the following table:

$*$	0	a	b	c
0	0	a	b	c
a	0	0	0	b
b	0	b	0	a
c	0	0	0	0

It turns out that $(X, *, 0)$ is a KU-algebra (see, [24]). We see that the set $A = \{0, a, b\}$ is a KU-subalgebra of X. At the same time, the set $B = \{a, b\}$ is not a KU-subalgebra of X.

3. Pythagorean fuzzy KU-subalgebras of KU-algebras

In this section, we introduce the concept of Pythagorean fuzzy KU-subalgebras in KU-algebras, and we suggest necessary and sufficient conditions to characterize the Pythagorean fuzzy KU-subalgebras in KU-algebras.

Definition 3.1. Let X be a KU-algebra. A Pythagorean fuzzy set $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ on X is called a *Pythagorean fuzzy KU-subalgebra* of X if it satisfies the following axioms:

(i) $\mu_{\mathcal{P}}(x * y) \geqslant \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\}$;

(ii) $\lambda_{\mathcal{P}}(x * y) \leqslant \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\}$,

for all $x, y \in X$.

Example 3.2. Let $X = \{0, a, b, c, d\}$ be a set with the binary operation $*$ on X defined by:

$*$	0	a	b	c
0	0	a	b	c
a	0	0	0	b
b	0	b	0	a
c	0	0	0	0

Then, $(X, *, 0)$ is a KU-algebra (see, [15]). Next, we define a Pythagorean fuzzy set $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ on X as follows:

$\mathcal{P}$	0	a	b	c
$\mu_{\mathcal{P}}$	0.8	0.4	0.8	0.4
$\lambda_{\mathcal{P}}$	0.3	0.3	0.6	0.9

By mindful calculations, we obtain that $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X.

Proposition 3.3. *Let X be a KU-algebra, and $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ be a Pythagorean fuzzy set on X. Then $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X if and only if the following conditions hold:*

(i) $\mu_{\mathcal{P}}(0) \geqslant \mu_{\mathcal{P}}(x)$ *and* $\lambda_{\mathcal{P}}(0) \leqslant \lambda_{\mathcal{P}}(x)$, *for all* $x \in X$;
(ii) $\mu_{\mathcal{P}}(x * (0 * y)) \geqslant \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\}$ *and* $\lambda_{\mathcal{P}}(x * (0 * y)) \leqslant \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\}$, *for all* $x, y \in X$.

Proof. Assume that $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. Let $x \in X$. Then, we have

$$\mu_{\mathcal{P}}(0) = \mu_{\mathcal{P}}(x * x) \geqslant \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(x)\} = \mu_{\mathcal{P}}(x)$$

and

$$\lambda_{\mathcal{P}}(0) = \lambda_{\mathcal{P}}(x * x) \leqslant \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(x)\} = \lambda_{\mathcal{P}}(x).$$

Thus (i) holds. In addition, for any $x, y \in X$, we get

$$\mu_{\mathcal{P}}(x * (0 * y)) \geqslant \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(0 * y)\} \geqslant \min\{\mu_{\mathcal{P}}(x), \min\{\mu_{\mathcal{P}}(0), \mu_{\mathcal{P}}(y)\}\} = \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\}$$

and

$$\lambda_{\mathcal{P}}(x * (0 * y)) \leqslant \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(0 * y)\} \leqslant \max\{\lambda_{\mathcal{P}}(x), \max\{\lambda_{\mathcal{P}}(0), \lambda_{\mathcal{P}}(y)\}\} = \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\}.$$

So, (ii) is obtained. Conversely, assume that (i) and (ii) are true. Let $x, y \in X$. Then, it follows

$$\mu_{\mathcal{P}}(x * y) = \mu_{\mathcal{P}}(x * (0 * y)) \geqslant \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\}$$

and

$$\lambda_{\mathcal{P}}(x * y) = \lambda_{\mathcal{P}}(x * (0 * y)) \leqslant \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\}.$$

This means that $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. □

Proposition 3.4. *The intersection of Pythagorean fuzzy KU-subalgebras of a KU-algebra X is also a Pythagorean fuzzy KU-subalgebra of X.*

Proof. Let $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ and $\mathcal{Q} = (\mu_{\mathcal{Q}}, \lambda_{\mathcal{Q}})$ be any two Pythagorean fuzzy KU-subalgebras of a KU-algebra X. Let $x, y \in X$. Then, we have

$$\begin{aligned}(\mu_{\mathcal{P}} \cap \mu_{\mathcal{Q}})(x * y) &= \min\{\mu_{\mathcal{P}}(x * y), \mu_{\mathcal{Q}}(x * y)\}\\ &\geqslant \min\{\min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\}, \min\{\mu_{\mathcal{Q}}(x), \mu_{\mathcal{Q}}(y)\}\}\\ &= \min\{\min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{Q}}(x)\}, \min\{\mu_{\mathcal{P}}(y), \mu_{\mathcal{Q}}(y)\}\}\\ &= \min\{(\mu_{\mathcal{P}} \cap \mu_{\mathcal{Q}})(x), (\mu_{\mathcal{P}} \cap \mu_{\mathcal{Q}})(y)\}\end{aligned}$$

and

$$\begin{aligned}(\lambda_{\mathcal{P}} \cup \lambda_{\mathcal{Q}})(x * y) &= \max\{\lambda_{\mathcal{P}}(x * y), \lambda_{\mathcal{Q}}(x * y)\}\\ &\leqslant \max\{\max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\}, \max\{\lambda_{\mathcal{Q}}(x), \lambda_{\mathcal{Q}}(y)\}\}\\ &= \max\{\max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{Q}}(x)\}, \max\{\lambda_{\mathcal{P}}(y), \lambda_{\mathcal{Q}}(y)\}\}\\ &= \max\{(\lambda_{\mathcal{P}} \cup \lambda_{\mathcal{Q}})(x), (\lambda_{\mathcal{P}} \cup \lambda_{\mathcal{Q}})(y)\}.\end{aligned}$$

Therefore, $\mathcal{P} \cap \mathcal{Q}$ is a Pythagorean fuzzy KU-subalgebra of X. □

On the other hand, the union of Pythagorean fuzzy KU-subalgebras of a KU-algebra X may not be a Pythagorean fuzzy KU-subalgebra of X as shown by the following example.

Example 3.5. Let $X = \{0, a, b, c, d\}$. Define the binary operation $*$ on X as follows:

$*$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	b	c	c
b	0	0	0	a	d
c	0	0	0	0	0
d	0	0	0	0	0

Then, $(X, *, 0)$ is a KU-algebra (see, [19]). The Pythagorean fuzzy sets $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ and $\mathcal{Q} = (\mu_{\mathcal{Q}}, \lambda_{\mathcal{Q}})$ on X are defined by:

$\mathcal{P}$	0	a	b	c	d
$\mu_{\mathcal{P}}$	0.9	0.8	0.7	0.4	0.2
$\lambda_{\mathcal{P}}$	0.3	0.4	0.6	0.7	0.8

and

$\mathcal{Q}$	0	a	b	c	d
$\mu_{\mathcal{Q}}$	0.8	0.3	0.5	0.3	0.7
$\lambda_{\mathcal{Q}}$	0.4	0.9	0.7	0.9	0.5

By routine calculations, we obtain $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ and $\mathcal{Q} = (\mu_{\mathcal{Q}}, \lambda_{\mathcal{Q}})$, which are Pythagorean fuzzy KU-subalgebras of X. Next, we consider the Pythagorean fuzzy set $\mathcal{P} \cup \mathcal{Q} = (\mu_{\mathcal{P}} \cup \mu_{\mathcal{Q}}, \lambda_{\mathcal{P}} \cap \lambda_{\mathcal{Q}})$ on X as the following results:

$\mathcal{P} \cup \mathcal{Q}$	0	a	b	c	d
$\mu_{\mathcal{P}} \cup \mu_{\mathcal{Q}}$	0.9	0.8	0.7	0.4	0.7
$\lambda_{\mathcal{P}} \cap \lambda_{\mathcal{Q}}$	0.3	0.4	0.6	0.7	0.5

We can see that $(\mu_{\mathcal{P}} \cup \mu_{\mathcal{Q}})(a * d) < \min\{(\mu_{\mathcal{P}} \cup \mu_{\mathcal{Q}})(a), (\mu_{\mathcal{P}} \cup \mu_{\mathcal{Q}})(d)\}$ and $(\lambda_{\mathcal{P}} \cap \lambda_{\mathcal{Q}})(a * d) > \max\{(\lambda_{\mathcal{P}} \cap \lambda_{\mathcal{Q}})(a), (\lambda_{\mathcal{P}} \cap \lambda_{\mathcal{Q}}(d))\}$. This shows that $\mathcal{P} \cup \mathcal{Q}$ is not a Pythagorean fuzzy KU-subalgebra of X.

Theorem 3.6. *Let X be a KU-algebra and $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ be a Pythagorean fuzzy set on X. Then $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X if and only if for all $s, t \in [0, 1]$, the nonempty sets $U(\mu_{\mathcal{P}}, t)$ and $L(\lambda_{\mathcal{P}}, s)$ are KU-subalgebras of X.*

Proof. Assume that $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. Let $s, t \in [0, 1]$ such that $U(\mu_{\mathcal{P}}, t) \neq \emptyset$ and $L(\lambda_{\mathcal{P}}, s) \neq \emptyset$. Let $x, y \in U(\mu_{\mathcal{P}}, t)$. Then $\mu_{\mathcal{P}}(x) \geqslant t$ and $\mu_{\mathcal{P}}(y) \geqslant t$. We obtain that $\mu_{\mathcal{P}}(x * y) \geqslant \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\} \geqslant t$. It follows that $x * y \in U(\mu_{\mathcal{P}}, t)$. Hence, $U(\mu_{\mathcal{P}}, t)$ is a KU-subalgebra of X. Similarly, we can show that $L(\lambda_{\mathcal{P}}, s)$ is also a KU-subalgebra of X. Conversely, assume that for any $s, t \in [0, 1]$, the nonempty sets $U(\mu_{\mathcal{P}}, t)$ and $L(\lambda_{\mathcal{P}}, s)$ are KU-subalgebras of X. Suppose that there exist $a, b \in X$ such that $\mu_{\mathcal{P}}(a * b) < \min\{\mu_{\mathcal{P}}(a), \mu_{\mathcal{P}}(b)\}$. Take $t = \frac{1}{2}\{\mu_{\mathcal{P}}(a * b) + \min\{\mu_{\mathcal{P}}(a), \mu_{\mathcal{P}}(b)\}\}$. It turns out that $\mu_{\mathcal{P}}(a * b) < t < \min\{\mu_{\mathcal{P}}(a), \mu_{\mathcal{P}}(b)\}$. Thus, $\mu_{\mathcal{P}}(a) > t, \mu_{\mathcal{P}}(b) > t$ and $\mu_{\mathcal{P}}(a * b) < t$. This means that $a, b \in U(\mu_{\mathcal{P}}, t)$ and $a * b \notin U(\mu_{\mathcal{P}}, t)$. So, $U(\mu_{\mathcal{P}}, t) \neq \emptyset$. By the given assumption, we have $U(\mu_{\mathcal{P}}, t)$ is a KU-subalgebra of X. Hence, $a * b \in U(\mu_{\mathcal{P}}, t)$, which is a contradiction. Therefore, $\mu_{\mathcal{P}}(x * y) \geqslant \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\}$, for all $x, y \in X$. For the case $\lambda_{\mathcal{P}}(x * y) \leqslant \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\}$, for all $x, y \in X$, we can provide a similar proof. Consequently, $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. □

Theorem 3.7. *Let X be a KU-algebra. Then $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X if and only if $\mathcal{P}^c = (\lambda^c_{\mathcal{P}}, \mu^c_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X.*

Proof. Assume that $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. Let $x, y \in X$. Then, we have

$$\lambda^c_{\mathcal{P}}(x * y) = 1 - \lambda_{\mathcal{P}}(x * y) \geqslant 1 - \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\} = \min\{1 - \lambda_{\mathcal{P}}(x), 1 - \lambda_{\mathcal{P}}(y)\} = \min\{\lambda^c_{\mathcal{P}}(x), \lambda^c_{\mathcal{P}}(x)\}$$

and

$$\mu^c_{\mathcal{P}}(x*y) = 1-\mu_{\mathcal{P}}(x*y) \leqslant 1-\min\{\mu_{\mathcal{P}}(x),\mu_{\mathcal{P}}(y)\} = \max\{1-\mu_{\mathcal{P}}(x),1-\mu_{\mathcal{P}}(y)\} = \max\{\mu^c_{\mathcal{P}}(x),\mu^c_{\mathcal{P}}(x)\}.$$

Hence, $\mathcal{P}^c = (\lambda^c_{\mathcal{P}},\mu^c_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X.

Conversely, assume that $\mathcal{P}^c = (\lambda^c_{\mathcal{P}},\mu^c_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. Let $x,y \in X$. Now, we consider

$$1-\mu_{\mathcal{P}}(x*y) = \mu^c_{\mathcal{P}}(x*y) \leqslant \max\{\mu^c_{\mathcal{P}}(x),\mu^c_{\mathcal{P}}(y)\} = \max\{1-\mu_{\mathcal{P}}(x),1-\mu_{\mathcal{P}}(y)\} = 1-\min\{\mu_{\mathcal{P}}(x),\mu_{\mathcal{P}}(y)\}$$

and

$$1-\lambda_{\mathcal{P}}(x*y) = \lambda^c_{\mathcal{P}}(x*y) \geqslant \min\{\lambda^c_{\mathcal{P}}(x),\lambda^c_{\mathcal{P}}(y)\} = \min\{1-\lambda_{\mathcal{P}}(x),1-\lambda_{\mathcal{P}}(y)\} = 1-\max\{\lambda_{\mathcal{P}}(x),\lambda_{\mathcal{P}}(y)\}.$$

It follows that $\mu_{\mathcal{P}}(x*y) \geqslant \min\{\mu_{\mathcal{P}}(x),\mu_{\mathcal{P}}(y)\}$ and $\lambda_{\mathcal{P}}(x*y) \leqslant \max\{\lambda_{\mathcal{P}}(x),\lambda_{\mathcal{P}}(y)\}$. Therefore, $\mathcal{P} = (\mu_{\mathcal{P}},\lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. □

The following theorem comes directly from Theorem 3.7.

Theorem 3.8. *Let $\mathcal{P} = (\mu_{\mathcal{P}},\lambda_{\mathcal{P}})$ be a Pythagorean fuzzy set on a KU-algebra X. Then $\mathcal{P} = (\mu_{\mathcal{P}},\lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X if only if $\mathcal{P}^\mu = (\mu_{\mathcal{P}},\mu^c_{\mathcal{P}})$ and $\mathcal{P}^\lambda = (\lambda^c_{\mathcal{P}},\lambda_{\mathcal{P}})$ are Pythagorean fuzzy KU-subalgebras of X.*

Let X be a KU-algebra and $\mathcal{P} = (\mu_{\mathcal{P}},\lambda_{\mathcal{P}})$ be any Pythagorean fuzzy set over X. We define

$$X_{\mu_{\mathcal{P}}} := \{x \in X \mid \mu_{\mathcal{P}}(x) = \mu_{\mathcal{P}}(0)\} \quad \text{and} \quad X_{\lambda_{\mathcal{P}}} := \{x \in X \mid \lambda_{\mathcal{P}}(x) = \lambda_{\mathcal{P}}(0)\}.$$

Theorem 3.9. *Let X be a KU-algebra. If $\mathcal{P} = (\mu_{\mathcal{P}},\lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X, then $X_{\mu_{\mathcal{P}}}$ and $X_{\lambda_{\mathcal{P}}}$ are KU-subalgebras of X.*

Proof. Assume that $\mathcal{P} = (\mu_{\mathcal{P}},\lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. Let $x,y \in X_{\mu_{\mathcal{P}}}$. Then, $\mu_{\mathcal{P}}(x) = \mu_{\mathcal{P}}(0) = \mu_{\mathcal{P}}(y)$. So, $\mu_{\mathcal{P}}(x*y) \geqslant \min\{\mu_{\mathcal{P}}(x),\mu_{\mathcal{P}}(y)\} = \mu_{\mathcal{P}}(0)$. Otherwise, $\mu_{\mathcal{P}}(0) \geqslant \mu_{\mathcal{P}}(x*y)$ always. It turns out that $\mu_{\mathcal{P}}(x*y) = \mu_{\mathcal{P}}(0)$, it implies $x*y \in X_{\mu_{\mathcal{P}}}$. Hence, $X_{\mu_{\mathcal{P}}}$ is a KU-subalgebra of X. Next, let $a,b \in X_{\lambda_{\mathcal{P}}}$. Then, $\lambda_{\mathcal{P}}(a) = \lambda_{\mathcal{P}}(0) = \lambda_{\mathcal{P}}(b)$. Thus, $\lambda_{\mathcal{P}}(a*b) \leqslant \max\{\lambda_{\mathcal{P}}(a),\lambda_{\mathcal{P}}(b)\} = \lambda_{\mathcal{P}}(0)$. Since $\lambda_{\mathcal{P}}(0) \leqslant \lambda_{\mathcal{P}}(a*b)$, we have $\lambda_{\mathcal{P}}(a*b) = \lambda_{\mathcal{P}}(0)$. This means that $a*b \in X_{\lambda_{\mathcal{P}}}$. Therefore, $X_{\lambda_{\mathcal{P}}}$ is a KU-subalgebra of X. □

4. Homomorphism on pythagorean fuzzy KU-subalgebras

In this section, we investigate some properties of Pythagorean fuzzy KU-subalgebras in KU-algebras under a homomorphism. A mapping $f : X \to Y$ of KU-algebras is called a *homomorphism* [25] if $f(x*y) = f(x)*f(y)$, for all $x,y \in X$. If a homomorphism f is onto, then f is called an epimorphism. We note that if $f : X \to Y$ is a homomorphism of KU-algebras, then $f(0) = 0$.

Let X and Y be any two nonempty sets, $f : X \to Y$ be a function and $\mathcal{P} = (\mu_{\mathcal{P}},\lambda_{\mathcal{P}})$ be a Pythagorean fuzzy set in X. Define the Pythagorean fuzzy set $f(\mathcal{P}) = \{\langle y,\mu_{f(\mathcal{P})}(y),\lambda_{f(\mathcal{P})}(y)\rangle \mid y \in Y\}$ in Y by,

$$\mu_{f(\mathcal{P})}(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)}, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad \lambda_{f(\mathcal{P})}(y) = \begin{cases} \inf\limits_{x\in f^{-1}(y)}, & \text{if } f^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise.} \end{cases}$$

We call $f(\mathcal{P})$ the *image of $\mathcal{P}$ under* f. On one hand, for any Pythagorean fuzzy set $\mathcal{Q} = (\mu_{\mathcal{Q}},\lambda_{\mathcal{Q}})$ over f(X), we define the *preimage of $\mathcal{Q}$ under* f denoted by

$$f^{-1}(\mathcal{Q}) = \left\{\left\langle x,\mu_{f^{-1}(\mathcal{Q})}(x),\lambda_{f^{-1}(\mathcal{Q})}(x)\right\rangle \mid x \in X\right\},$$

where $\mu_{f^{-1}(\mathcal{Q})}(x) = \mu(f(x))$ and $\lambda_{f^{-1}(\mathcal{Q})} = \lambda(f(x))$, for all $x \in X$.

Let $\mathcal{P} = (\mu_\mathcal{P}, \lambda_\mathcal{P})$ be a Pythagorean fuzzy set in a nonempty set X. We say that $\mathcal{P}$ has *sup-inf property*, if for any subset T of X, there exists $a_0 \in T$ such that $\mu_\mathcal{P}(a_0) = \sup_{t\in T} \mu_\mathcal{P}(t)$ and $\lambda_\mathcal{P}(a_0) = \inf_{t\in T} \lambda_\mathcal{P}(t)$.

Let $f : X \to Y$ be a mapping of KU-algebras and $\mathcal{P} = (\mu_\mathcal{P}, \lambda_\mathcal{P})$ be a Pythagorean fuzzy set in Y. The Pythagorean fuzzy $\mathcal{P}^f = (\mu^f_\mathcal{P}, \lambda^f_\mathcal{P})$ in X is defined by $\mu^f_\mathcal{P}(x) = \mu_\mathcal{P}(f(x))$ and $\lambda^f_\mathcal{P}(x) = \lambda_\mathcal{P}(f(x))$, for all $x \in X$.

Theorem 4.1. *Let* X *and* Y *be KU-algebras and* $f : X \to Y$ *be a homomorphism. If* $\mathcal{Q} = (\mu_\mathcal{Q}, \lambda_\mathcal{Q})$ *is a Pythagorean fuzzy KU-subalgebra of* f(X)*, then the preimage* $f^{-1}(\mathcal{Q}) = (\mu_{f^{-1}(\mathcal{Q})}, \lambda_{f^{-1}(\mathcal{Q})})$ *of* $\mathcal{Q}$ *under* f *is a Pythagorean fuzzy KU-algebra of* X*.*

Proof. Assume that $\mathcal{Q} = (\mu_\mathcal{Q}, \lambda_\mathcal{Q})$ is a Pythagorean fuzzy KU-subalgebra of f(X). Let $x, y \in X$. Then, we have

$$\mu_{f^{-1}(\mathcal{Q})}(x * y) = \mu_\mathcal{Q}(f(x * y)) = \mu_\mathcal{Q}(f(x) * f(y)) \geqslant \min\{\mu_\mathcal{Q}(f(x)), \mu_\mathcal{Q}(f(y))\} = \min\{\mu_{f^{-1}(\mathcal{Q})}(x), \mu_{f^{-1}(\mathcal{Q})}(y)\}$$

and

$$\lambda_{f^{-1}(\mathcal{Q})}(x * y) = \lambda_\mathcal{Q}(f(x * y)) = \lambda_\mathcal{Q}(f(x) * f(y)) \leqslant \max\{\lambda_\mathcal{Q}(f(x)), \lambda_\mathcal{Q}(f(y))\} = \max\{\lambda_{f^{-1}(\mathcal{Q})}(x), \lambda_{f^{-1}(\mathcal{Q})}(y)\}.$$

Hence, $f^{-1}(\mathcal{Q}) = (\mu_{f^{-1}(\mathcal{Q})}, \lambda_{f^{-1}(\mathcal{Q})})$ is a Pythagorean fuzzy KU-subalgebra of X. □

Theorem 4.2. *Let* $f : X \to Y$ *be a homomorphism of KU-algebras,* $\mathcal{P} = (\mu_\mathcal{P}, \lambda_\mathcal{P})$ *be a Pythagorean fuzzy set in* X*, and* $f(\mathcal{P}) = (\mu_{f(\mathcal{P})}, \lambda_{f(\mathcal{P})})$ *be the image of* $\mathcal{P}$ *under* f*. If* $\mathcal{P} = (\mu_\mathcal{P}, \lambda_\mathcal{P})$ *is a Pythagorean fuzzy KU-subalgebra of* X *with sup-inf property, then* $f(\mathcal{P}) = (\mu_{f(\mathcal{P})}, \lambda_{f(\mathcal{P})})$ *is a Pythagorean fuzzy KU-algebra of* f(X)*.*

Proof. Assume that $\mathcal{P} = (\mu_\mathcal{P}, \lambda_\mathcal{P})$ is a Pythagorean fuzzy KU-subalgebra of X, and it has sup-inf property. Let $x, y \in f(X)$. Then, there exist $a_0 \in f^{-1}(x)$ and $b_0 \in f^{-1}(y)$ such that

$$\mu_\mathcal{P}(a_0) = \sup_{t\in f^{-1}(x)} \mu_\mathcal{P}(t), \;\; \lambda_\mathcal{P}(a_0) = \inf_{t\in f^{-1}(x)} \lambda_\mathcal{P}(t), \;\; \mu_\mathcal{P}(b_0) = \sup_{t\in f^{-1}(y)} \mu_\mathcal{P}(t), \;\; \text{and} \;\; \lambda_\mathcal{P}(b_0) = \inf_{t\in f^{-1}(y)} \lambda_\mathcal{P}(t).$$

It turns out that

$$\begin{aligned}
\mu_{f(\mathcal{P})}(x * y) = \sup_{t\in f^{-1}(x*y)} \mu_\mathcal{P}(t) &\geqslant \mu_\mathcal{P}(a_0 * b_0) \\
&\geqslant \min\{\mu_\mathcal{P}(a_0), \mu_\mathcal{P}(b_0)\} \\
&= \min\left\{ \sup_{t\in f^{-1}(x)} \mu_\mathcal{P}(t), \sup_{t\in f^{-1}(y)} \mu_\mathcal{P}(t) \right\} = \min\{\mu_{f(\mathcal{P})}(x), \mu_{f(\mathcal{P})}(y)\}
\end{aligned}$$

and

$$\begin{aligned}
\lambda_{f(\mathcal{P})}(x * y) = \inf_{t\in f^{-1}(x*y)} \lambda_\mathcal{P}(t) &\leqslant \lambda_\mathcal{P}(a_0 * b_0) \\
&\leqslant \max\{\lambda_\mathcal{P}(a_0), \lambda_\mathcal{P}(b_0)\} \\
&= \max\left\{ \inf_{t\in f^{-1}(x)} \lambda_\mathcal{P}(t), \inf_{t\in f^{-1}(y)} \lambda_\mathcal{P}(t) \right\} = \max\{\lambda_{f(\mathcal{P})}(x), \lambda_{f(\mathcal{P})}(y)\}.
\end{aligned}$$

Therefore, $f(\mathcal{P}) = (\mu_{f(\mathcal{P})}, \lambda_{f(\mathcal{P})})$ is a Pythagorean fuzzy KU-subalgebra of f(X). □

Theorem 4.3. *Let* $f : X \to Y$ *be a homomorphism of KU-algebras, and let* $\mathcal{P} = (\mu_\mathcal{P}, \lambda_\mathcal{P})$ *be a Pythagorean fuzzy set in* Y*. If* $\mathcal{P} = (\mu_\mathcal{P}, \lambda_\mathcal{P})$ *is a Pythagorean fuzzy KU-subalgebra of* Y*, then* $\mathcal{P}^f = (\mu^f_\mathcal{P}, \lambda^f_\mathcal{P})$ *is a Pythagorean fuzzy KU-subalgebra of* X*.*

Proof. Assume that $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of Y. Let $x, y \in X$. Then, we have

$$\mu^f_{\mathcal{P}}(x * y) = \mu_{\mathcal{P}}(f(x * y)) = \mu_{\mathcal{P}}(f(x) * f(y)) \geqslant \min\{\mu_{\mathcal{P}}(f(x)), \mu_{\mathcal{P}}(f(y))\} = \min\{\mu^f_{\mathcal{P}}(x), \mu^f_{\mathcal{P}}(y)\}$$

and

$$\lambda^f_{\mathcal{P}}(x * y) = \lambda_{\mathcal{P}}(f(x * y)) = \lambda_{\mathcal{P}}(f(x) * f(y)) \leqslant \max\{\lambda_{\mathcal{P}}(f(x)), \lambda_{\mathcal{P}}(f(y))\} = \max\{\lambda^f_{\mathcal{P}}(x), \lambda^f_{\mathcal{P}}(y)\}.$$

Hence, $\mathcal{P}^f = (\mu^f_{\mathcal{P}}, \lambda^f_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. □

If f is an epimorphism, then we achieve the converse of Theorem 4.3 as shown in the following theorem.

Theorem 4.4. *Let $f : X \to Y$ be an epimorphism of KU-algebras, and let $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ be a Pythagorean fuzzy set in Y. If $\mathcal{P}^f = (\mu^f_{\mathcal{P}}, \lambda^f_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X, then $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of Y.*

Proof. Assume that $\mathcal{P}^f = (\mu^f_{\mathcal{P}}, \lambda^f_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of X. Let $x, y \in Y$. Then, there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = y$. Thus, we have

$$\begin{aligned}
\mu_{\mathcal{P}}(x * y) = \mu_{\mathcal{P}}(f(a) * f(b)) &= \mu_{\mathcal{P}}(f(a * b)) \\
&= \mu^f_{\mathcal{P}}(a * b) \\
&\geqslant \min\{\mu^f_{\mathcal{P}}(a), \mu^f_{\mathcal{P}}(b)\} \\
&= \min\{\mu_{\mathcal{P}}(f(a)), \mu_{\mathcal{P}}(f(b))\} = \min\{\mu_{\mathcal{P}}(x), \mu_{\mathcal{P}}(y)\}
\end{aligned}$$

and

$$\begin{aligned}
\lambda_{\mathcal{P}}(x * y) = \lambda_{\mathcal{P}}(f(a) * f(b)) &= \lambda_{\mathcal{P}}(f(a * b)) \\
&= \lambda^f_{\mathcal{P}}(a * b) \\
&\leqslant \max\{\lambda^f_{\mathcal{P}}(a), \lambda^f_{\mathcal{P}}(b)\} \\
&= \max\{\lambda_{\mathcal{P}}(f(a)), \lambda_{\mathcal{P}}(f(b))\} = \max\{\lambda_{\mathcal{P}}(x), \lambda_{\mathcal{P}}(y)\}.
\end{aligned}$$

Consequently, $\mathcal{P} = (\mu_{\mathcal{P}}, \lambda_{\mathcal{P}})$ is a Pythagorean fuzzy KU-subalgebra of Y. □

5. Conclusions

In this article, we applied the concept of Pythagorean fuzzy sets to study KU-algebras. The concept of Pythagorean fuzzy KU-subalgebras of KU-algebras was introduced. Then, we showed that the intersection of Pythagorean fuzzy KU-subalgebras is also a Pythagorean fuzzy KU-subalgebra, while the union of Pythagorean fuzzy KU-subalgebras need not to be a Pythagorean fuzzy KU-subalgebra as shown in Example 3.5. Finally, we indicated that the connections between the image and the preimage of Pythagorean fuzzy KU-subalgebras on a homomorphism of KU-algebras. Future studies will be able to investigate the concepts of Pythagorean fuzzy KU-ideals and Pythagorean fuzzy KU-filters of KU-algebras or other concepts in many algebraic structures can be determined by the Pythagorean fuzzy sets.

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