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Generalized function projective synchronization of identical and nonidentical chaotic systems

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Abstract

In this paper, the generalized function projective synchronization of two identical Lü-Chen-Cheng four-scroll chaotic systems is satisfied. Also, we studied the generalized function projective synchronization between two nonidentical chaotic systems, Lü-Chen-Cheng four-scroll chaotic system and new chaotic system, with known parameters. To prove the solutions of the error system that are asymptotic stable, we based on the Lyapunov theorem of stability. The proposed schemes were evaluated through numerical experiments to showcase their effectiveness and impact.

Keywords: Chaotic system, generalized function projective synchronization, nonidentical chaotic systems.

2020 MSC: 37N35, 34D06, 34H10.

1. Introduction

The phenomenon of chaos synchronization in nonlinear science has garnered significant interest among scientists, engineers, and researchers due to its potential applications in a variety of fields such as biology, physics, economics, and secure communications [22]. Following the groundbreaking research by Pecora and Carroll [19], which introduced an effective method for synchronizing chaotic systems with different initial conditions, many types of synchronization schemes have been discovered and studied [6–8, 12, 20]. Projective synchronization (PS) [1, 2, 4, 13, 15, 17, 21, 23, 25] has emerged as a particularly promising area of research in recent years due to its ability to achieve rapid synchronization by proportional features. Many methods have been derived from it, such as modified projective synchronization (MPS) and function projective synchronization (FPS) [3, 5, 9–11, 14, 18, 27, 28]. Recently, a novel form of synchronization method known as generalized function projective synchronization (GFPS) was introduced [24, 26].

This article showcases a GFPS scheme between two identical LLü-Chen-Cheng four-scroll chaotic systems and between two nonidentical chaotic systems Lü-Chen-Cheng four-scroll and new chaotic system with certain parameters.

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The remainder of this essay is structured as follows. The fundamental dynamic features of Lü-Chen-Cheng four-scroll chaotic system and new chaotic system are described in Section 2. The GFPS scheme definition is given in Section 3. In Section 4, GFPS of two identical Lü-Chen-Cheng four-scroll chaotic systems is presented. GFPS between the Lü-Chen-Cheng four-scroll and new chaotic systems is investigated in Section 5. Section 6 displays numerical examples to validate the effectiveness of the technique chosen. Lastly, Section 7 provides the conclusion.

2. Description of systems

In this section, we demonstrate several basic dynamic properties of some chaotic systems.

2.1. Lü-Chen-Cheng four-scroll chaotic system

The model of Lü-Chen-Cheng four-scroll chaotic system [22] is expressed by the following derivative equations:

$$\begin{cases} \dot{s} = \mathrm{d}s - \nu w, \\ \dot{\nu} = -\mathrm{h}\nu + sw + \mathrm{m}, \\ \dot{w} = -\mathrm{l}w + s\nu, \end{cases} \tag{2.1}$$

where $\mathrm{d}, \mathrm{h}, \mathrm{l}$, and m are parameters of the system that are positive constants. At the values $\mathrm{d} = \frac{20}{7}$, $\mathrm{h} = 10$, $\mathrm{l} = 4$, and $\mathrm{m} = 5$, the system has chaotic behavior, which is displayed in Figures 1, 2, and 3.

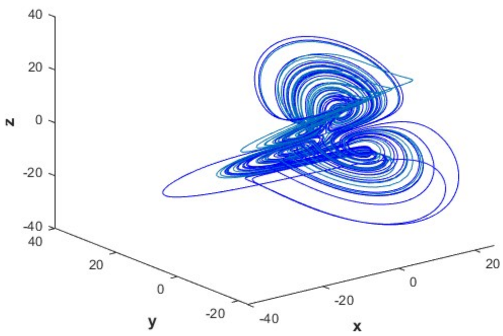

Figure 1: Lü-Chen-Cheng four-scroll chaotic system in XYZ dimensions.

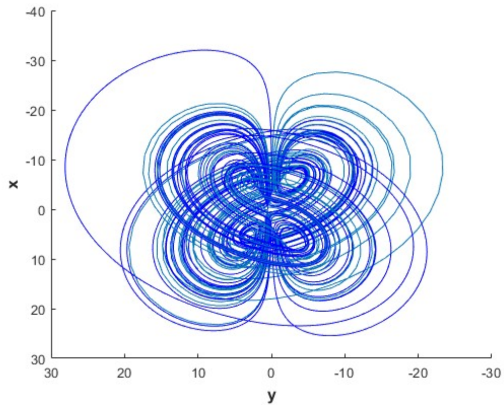

Figure 2: Lü-Chen-Cheng four-scroll chaotic system in YX dimensions.

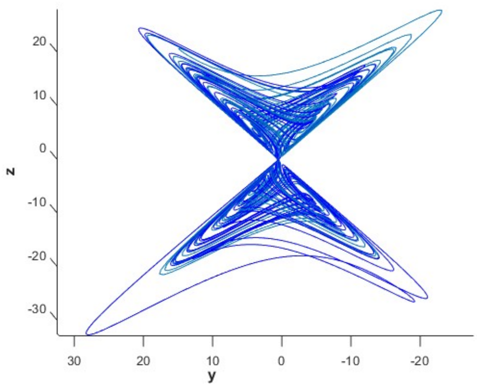

Figure 3: Lü-Chen-Cheng four-scroll chaotic system in YZ dimensions.

2.1.1. *The dissipation*

The divergence of system (2.1) can be obtained as:

$$\nabla.F = \frac{\partial \dot{s}}{\partial s} + \frac{\partial \dot{v}}{\partial v} + \frac{\partial \dot{w}}{\partial w} = d - h - l = -11.1428071 < 0.$$

Hence, the (2.1) is the dissipative system.

2.1.2. *Equilibrium points*

By using the usual method for finding equilibrium points by setting the right side of the system's equations equal to zero and then solving them, consequently the system has four equilibrium points, which are:

$$P_1 = \left(5.838143559,\ \sqrt{\frac{80}{7}},\ 4.934131869\right), \qquad P_2 = \left(-5.838143559,\ \sqrt{\frac{80}{7}},\ -4.934131869\right),$$
$$P_3 = \left(-6.776140478,\ -\sqrt{\frac{80}{7}},\ 5.726883956\right), \quad P_4 = \left(6.776140478,\ -\sqrt{\frac{80}{7}},\ -5.726883956\right).$$

2.2. *New chaotic system*

The new chaotic system [16] is expressed by:

$$\begin{cases} \dot{s} = -\dfrac{ab}{a+b}s - vw + c, \\ \dot{v} = av + sw, \\ \dot{z} = bw + sv, \end{cases} \tag{2.2}$$

where the parameters a, b, and c are real constant numbers. In a wide parameter range, the system will be chaotic. For example, when $a = -10,\ b = -4$, and $c = 18.1$, the chaotic attractor is shown in Figure 4. Furthermore, when $a = -10,\ b = -4,\ c = 0$, the chaotic attractor is shown in Figure 5.

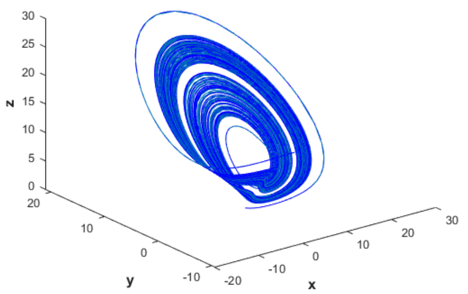

Figure 4: The new chaotic system at $a = -10,\ b = -4,\ c = 18.1$.

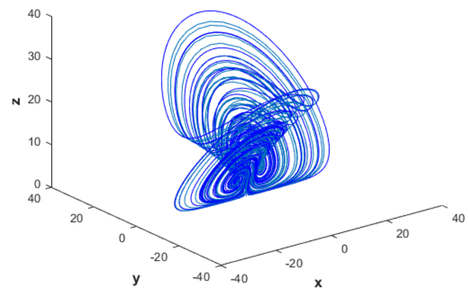

Figure 5: The new chaotic system at $a = -10,\ b = -4,\ c = 0$.

2.2.1. *The dissipation*

The divergence of system (2.2) is found as follows:

$$\nabla F = \frac{\partial \dot{s}}{\partial s} + \frac{\partial \dot{v}}{\partial v} + \frac{\partial \dot{w}}{\partial w} = -\frac{ab}{a+b} + a + b = \frac{-78}{7} < 0.$$

Hence, system (2.2) is dissipative.

2.2.2. Equilibrium points

The system (2.2) has three equilibrium points when $a = -10,\ b = -4$, and $c = 18.1$, which are

$$P_1 = (-6.335,\ 0,\ 0),\ P_{2,3} = \left(2\sqrt{10},\ \pm\sqrt{\frac{80}{7} + 3.62\sqrt{10}},\ \pm\frac{1}{2}\sqrt{\frac{800}{7} + 36.2\sqrt{10}}\right).$$

3. Definition of generalized function projective synchronization scheme

In the synchronization process, there is a drive system and a response system, which can be written in the following formula, respectively,

$$\dot{S} = F(S), \tag{3.1}$$

$$\dot{V} = G(V) + U(t, S, V), \tag{3.2}$$

where $S = (s_1, s_2, \dots, s_n)^T, V = (v_1, v_2, \dots, v_n)^T \in \mathbb{R}^n$ are the state vectors of the systems (3.1) and (3.2), $F, G : \mathbb{R}^n \to \mathbb{R}^n$ are differentiable vector functions, $U(t, S, V)$ is a controller function which will be designed later.

Definition 1. *The concept of generalized function projective synchronization (GFPS) for the drive system* (3.1) *and the response system* (3.2) *involves the existence of a scaling function matrix such that*

$$\lim_{t\to+\infty} \|e(t)\| = \lim_{t\to+\infty} \|V - \Lambda(s)S\| = 0,$$

where $e(t) = (e_s,\ e_v, e_w)$ *are called the error vectors,* $\Lambda(S) = \mathrm{diag}\{h_1(S),\ h_2(S), \dots, h_n(S)\}$ *such that* $h_i(S)(i = 1, 2, \dots, n)$ *are continuous differentiable functions and* $h_i(S) \neq 0$ *for all* t, $\|.\|$ *represents a vector norm induced by the matrix norm.*

4. GFPS of two identical Lü-Chen-Cheng four-scroll chaotic dynamical systems

This study will involve examining the GFPS of two identical Lü-Chen-Cheng four-scroll chaotic system (2.1) with predefined parameters and selected controller functions for the GFPS of the drive and response systems. The goal is to devise a controller that allows the response system to mimic and achieve the behavior of the drive system.

For GFPS of Lü-Chen-Cheng four-scroll chaotic system (2.1), the derive and response systems are defined as follows, respectively,

$$\begin{cases} \dot{s}_1 = ds_1 - v_1 w_1, \\ \dot{v}_1 = -hv_1 + s_1 w_1 + m, \\ \dot{w}_1 = -lw_1 + s_1 v_1, \end{cases} \tag{4.1}$$

and Lü-Chen-Cheng four-scroll chaotic system as a response system can be written as

$$\begin{cases} \dot{s}_2 = ds_2 - v_2 w_2 + u_1, \\ \dot{v}_2 = -hv_2 + s_2 w_2 + m + u_2, \\ \dot{w}_2 = -lw_2 + s_2 v_2 + u_3, \end{cases} \tag{4.2}$$

where u_1, u_2, and u_3 are the nonlinear controller functions. Based on the GFPS scheme outlined previously, we can select the scaling function matrix without loss of generality as: $\Lambda(S) = \mathrm{diag}\{h_{11}s_1 + h_{12},\ h_{21}v_1 + h_{22},\ h_{31}w_1 + h_{32}\}$, where $h_{ij}(i = 1, 2, 3,\ j = 1, 2)$ are constant numbers.

Therefore, GFPS between the two systems (4.1) and (4.2) occurs if the following is achieved:

$$\lim_{t\to+\infty} \|e_s\| = \lim_{t\to+\infty} \|s_2 - (h_{11}s_1 + h_{12})s_1\| = 0,$$

$$\lim_{t\to+\infty}\|e_v\| = \lim_{t\to+\infty}\|v_2-(h_{21}v_1+h_{22})v_1\| = 0,$$
$$\lim_{t\to+\infty}\|e_w\| = \lim_{t\to+\infty}\|w_2-(h_{31}w_1+h_{32})w_1\| = 0.$$

Then the error dynamical system is given by:

$$\begin{cases} \dot{e}_s = de_s - v_2w_2 - dh_{11}s_1^2 + 2h_{11}s_1v_1w_1 + h_{12}v_1w_1 + u_1, \\ \dot{e}_v = -he_v + s_2w_2 + m + hh_{21}v_1^2 - 2h_{21}s_1v_1w_1 - 2h_{21}v_1m - h_{22}s_1w_1 - h_{22}m + u_2, \\ \dot{e}_w = -le_w + s_2v_2 + lh_{31}w_1^2 - 2h_{31}s_1v_1w_1 - h_{32}s_1v_1 + u_3. \end{cases} \tag{4.3}$$

The aim is to determine a control law that will stabilize the error variables of the system (4.3). To achieve this, we can take the control law as:

$$\begin{cases} u_1 = v_2w_2 + dh_{11}s_1^2 - 2h_{11}s_1v_1w_1 - h_{12}v_1w_1 - 9s_2 + 9h_{11}s_1^2 + 9h_{12}s_1, \\ u_2 = -s_2w_2 - m - hh_{21}v_1^2 + 2h_{21}s_1v_1w_1 + 2h_{21}v_1m + h_{22}s_1w_1 + h_{22}m, \\ u_3 = -s_2v_2 - lh_{31}w_1^2 + 2h_{31}s_1v_1w_1 + h_{32}s_1v_1. \end{cases} \tag{4.4}$$

Theorem 4.1. *GFPS will be achieved between system (4.1) and system (4.2) under controler functions (4.4), where the $h_{ij}\,(i=1,2,3, j=1,2)$ are given nonzero scalars.*

Proof. Define a Lyapunov function:

$$L = \frac{1}{2}(e_s^2 + e_v^2 + e_w^2). \tag{4.5}$$

The derivative of (4.5) with respect to time is:

$$\begin{aligned} \frac{dL}{dt} &= (e_s\dot{e}_s + e_v\dot{e}_v + e_w\dot{e}_w) \\ &= e_s\left(de_s - v_2w_2 - dh_{11}s_1^2 + 2h_{11}s_1v_1w_1 + h_{12}v_1w_1 + u_1\right) \\ &\quad + e_v\left(-he_v + s_2w_2 + m + hh_{21}v_1^2 - 2h_{21}s_1v_1w_1 - 2h_{21}v_1m - h_{22}s_1w_1 - h_{22}m + u_2\right) \\ &\quad + e_w\left(-le_w + s_2v_2 + lh_{31}w_1^2 - 2h_{31}s_1v_1w_1 - h_{32}s_1v_1 + u_3\right). \end{aligned}$$

Substituting the controller functions (4.4) into (4.3), we get:

$$\begin{aligned} \frac{dL}{dt} = e_s\Big(&de_s - v_2w_2 - dh_{11}s_1^2 + 2h_{11}s_1v_1w_1 + h_{12}v_1w_1 + v_2w_2 + dh_{11}s_1^2 - 2h_{11}s_1v_1w_1 - h_{12}v_1w_1 \\ &- 9s_2 + 9h_{11}s_1^2 + 9h_{12}s_1\Big) + e_v\Big(-he_v + s_2w_2 + m + hh_{21}v_1^2 - 2h_{21}s_1v_1w_1 - 2h_{21}v_1m \\ &- h_{22}s_1w_1 - h_{22}m + -s_2w_2 - m - hh_{21}v_1^2 + 2h_{21}s_1v_1w_1 + 2h_{21}v_1m + h_{22}s_1w_1 + h_{22}m\Big) \\ &+ e_w\Big(-le_w + s_2v_2 + lh_{31}w_1^2 - 2h_{31}s_1v_1w_1 - h_{32}s_1v_1 - s_2v_2 - lh_{31}w_1^2 + 2h_{31}s_1v_1w_1 + h_{32}s_1v_1\Big). \end{aligned}$$

Therefore, we have:

$$\frac{dL}{dt} = (d-9)e_s^2 - he_v^2 - le_w^2 = -\Big(\frac{43}{7}e_s^2 + he_v^2 + le_w^2\Big). \tag{4.6}$$

Hence, we can write the previous equation (4.6) as:

$$\frac{dL}{dt} = -e^T De,$$

where

$$D = \begin{bmatrix} \frac{43}{7} & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & l \end{bmatrix} = \begin{bmatrix} \frac{43}{7} & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \quad e = \begin{bmatrix} e_s \\ e_v \\ e_w \end{bmatrix}.$$

Since $\frac{dL}{dt}$ is a negative definite, the GFPS is attained for two Lü-Chen-Cheng four-scroll chaotic systems. □

5. GFPS between Lü-Chen-Cheng four-scroll system and new chaotic system

This study will involve examining the GFPS of two nonidentical systems, Lü-Chen-Cheng four-scroll chaotic system (2.1), and new chaotic system (2.2) with predefined parameters and selected controller functions for the GFPS of the drive and response systems. The goal is to devise a controller that allows the response system to mimic and achieve the behavior of the drive system.

For GFPS between Lü-Chen-Cheng four-scroll chaotic system (2.1) and new chaotic system (2.2), the derive and response systems are defined as follows, respectively,

$$\begin{cases} \dot{s}_1 = ds_1 - v_1 w_1, \\ \dot{v}_1 = -hv_1 + s_1 w_1 + m, \\ \dot{w}_1 = -lw_1 + s_1 v_1, \end{cases} \tag{5.1}$$

and new chaotic system as a response system can be written as:

$$\begin{cases} \dot{s}_2 = -\dfrac{ab}{a+b} s_2 - v_2 w_2 + c + u_1, \\ \dot{v}_2 = av_2 + s_2 w_2 + u_2, \\ \dot{w}_2 = bw_2 + s_2 v_2 + u_3, \end{cases} \tag{5.2}$$

where u_1, u_2, and u_3 are the nonlinear controller functions. Based on the GFPS scheme outlined previously, we can select the scaling function matrix without loss of generality as: $\Lambda(S) = \text{diag}\{h_{11}s_1 + h_{12},\ h_{21}v_1 + h_{22},\ h_{31}w_1 + h_{32}\}$, where $h_{ij}\,(i = 1, 2, 3,\ j = 1, 2)$ are constant numbers.

Therefore, GFPS between the two systems (5.1) and (5.2) occurs if the following is achieved:

$$\lim_{t\to+\infty} \|e_s\| = \lim_{t\to+\infty} \|s_2 - (h_{11}s_1 + h_{12})s_1\| = 0,$$
$$\lim_{t\to+\infty} \|e_v\| = \lim_{t\to+\infty} \|v_2 - (h_{21}v_1 + h_{22})v_1\| = 0,$$
$$\lim_{t\to+\infty} \|e_w\| = \lim_{t\to+\infty} \|w_2 - (h_{31}w_1 + h_{32})w_1\| = 0.$$

Then the error dynamical system is given by:

$$\begin{cases} \dot{e}_s = -\dfrac{ab}{a+b} s_2 - v_2 w_2 + c - 2dh_{11}s_1^2 + 2h_{11}s_1 v_1 w_1 - dh_{12}s_1 + h_{12}v_1 w_1 + u_1, \\ \dot{e}_v = av_2 + s_2 w_2 + 2hh_{21}v_1^2 - 2h_{21}s_1 v_1 w_1 - 2mh_{21}v_1 + hh_{22}v_1 - h_{22}s_1 w_1 - h_{22}m + u_2, \\ \dot{e}_w = bw_2 + s_2 v_2 + 2h_{31}lw_1^2 - 2h_{31}s_1 v_1 w_1 + h_{32}lw_1 - h_{32}s_1 v_1 + u_3. \end{cases} \tag{5.3}$$

The aim is to determine a control law that will stabilize the error variables of the system (5.3). To achieve this, we can take the control law as:

$$\begin{cases} u_1 = \dfrac{ab}{a+b} s_2 + v_2 w_2 - c + dh_{11}s_1^2 - 2h_{11}s_1 v_1 w_1 - h_{12}v_1 w_1 + ds_2 - 8s_2 + 8h_{11}s_1^2 + 8h_{12}s_1, \\ u_2 = -av_2 - s_2 w_2 - hh_{21}v_1^2 + 2h_{21}s_1 v_1 w_1 + 2mh_{21}v_1 + h_{22}s_1 w_1 + h_{22}m - hv_2, \\ u_3 = -bw_2 - s_2 v_2 - h_{31}lw_1^2 + 2h_{31}s_1 v_1 w_1 + h_{32}s_1 v_1 - lw_2. \end{cases} \tag{5.4}$$

Theorem 5.1. *GFPS will be achieved between system (5.1) and system (5.2) under controler functions (5.4), where the $h_{ij}\,(i = 1, 2, 3, j = 1, 2)$ are given nonzero scalars.*

Proof. Define a Lyapunov function:

$$L^* = \frac{1}{2}(e_s^2 + e_v^2 + e_w^2). \tag{5.5}$$

The derivative of (5.5) with respect to time is:

$$\begin{aligned}\frac{dL^*}{dt} &= (e_s\dot{e}_s + e_v\dot{e}_v + e_w\dot{e}_w)\\ &= e_s\left(-\frac{ab}{a+b}s_2 - v_2w_2 + c - 2dh_{11}s_1^2 + 2h_{11}s_1v_1w_1 - dh_{12}s_1 + h_{12}v_1w_1 + u_1\right)\\ &\quad + e_v\left(av_2 + s_2w_2 + 2hh_{21}v_1^2 - 2h_{21}s_1v_1w_1 - 2mh_{21}v_1 + hh_{22}v_1 - h_{22}s_1w_1 - h_{22}m + u_2\right)\\ &\quad + e_w\left(bw_2 + s_2v_2 + 2h_{31}lw_1^2 - 2h_{31}s_1v_1w_1 + h_{32}lw_1 - h_{32}s_1v_1 + u_3\right).\end{aligned}$$

Substituting the controller functions (5.4) into (5.3), we get:

$$\begin{aligned}\frac{dL^*}{dt} = e_s\Big(&-\frac{ab}{a+b}s_2 - v_2w_2 + c - 2dh_{11}s_1^2 + 2h_{11}s_1v_1w_1 - dh_{12}s_1 + h_{12}v_1w_1 - \frac{ab}{a+b}s_2 + v_2w_2 - c\\ &+ dh_{11}s_1^2 - 2h_{11}s_1v_1w_1 - h_{12}v_1w_1 + ds_2 - 8s_2 + 8h_{11}s_1^2 + 8h_{12}s_1\Big) + e_v\Big(av_2 + s_2w_2 + 2hh_{21}v_1^2\\ &- 2h_{21}s_1v_1w_1 - 2mh_{21}v_1 + hh_{22}v_1 - h_{22}s_1w_1 - h_{22}m - av_2 - s_2w_2 - hh_{21}v_1^2 + 2h_{21}s_1v_1w_1\\ &+ 2mh_{21}v_1 + h_{22}s_1w_1 + h_{22}m - hv_2\Big) + e_w\Big(bw_2 + s_2v_2 + 2h_{31}lw_1^2 - 2h_{31}s_1v_1w_1 + h_{32}lw_1\\ &- h_{32}s_1v_1 - bw_2 - s_2v_2 - h_{31}lw_1^2 + 2h_{31}s_1v_1w_1 + h_{32}s_1v_1 - lw_2\Big).\end{aligned}$$

Therefore, we have

$$\frac{dL^*}{dt} = (d-8)e_s^2 - he_v^2 - le_w^2 = -\Big(\frac{36}{7}e_s^2 + he_v^2 + le_w^2\Big). \tag{5.6}$$

Hence, we can write the previous equation (5.6) as

$$\frac{dL^*}{dt} = -e^T D^* e,$$

where

$$D^* = \begin{bmatrix}\frac{36}{7} & 0 & 0\\ 0 & h & 0\\ 0 & 0 & l\end{bmatrix} = \begin{bmatrix}\frac{36}{7} & 0 & 0\\ 0 & 10 & 0\\ 0 & 0 & 4\end{bmatrix}, \quad e = \begin{bmatrix}e_s\\ e_v\\ e_w\end{bmatrix}.$$

Since $\frac{dL^*}{dt}$ is a negative definite and the error vectors go to zero with time, GFPS is achieved between Lü-Chen-Cheng four-scroll chaotic system (2.1) and the new chaotic system (2.2). □

6. Numerical simulations

6.1. Synchronization of two identical Lü-Chen-Cheng four-scroll chaotic dynamical systems

The outcomes of the numerical simulations in this section are intended to validate the analytical results obtained in the previous Section 5. We take the initial conditions as: $s_1(0) = 22,\ v_1(0) = 10,\ w_1(0) = 1$, and $s_2(0) = 7,\ v_2(0) = 5,\ w_2(0) = 2$ in all processes.

Figure 6 shows that the GFPS for two identical Lü-Chen-Cheng four-scroll chaotic systems when the scaling functions are given by $h_1 = \frac{1}{22}s - \frac{1}{2},\ h_2 = 1,\ h_3 = w - 2$. Furthermore, the MPS for two identical Lü-Chen-Cheng four-scroll chaotic systems is shown in Figure 7, when the scaling factors are taken as: $h_1 = 0.5,\ h_2 = 1,\ h_3 = -1$. When we simplify the scaling factors as $h_1 = h_2 = h_3 = 3$, Figure 8 displays the PS for two identical Lü-Chen-Cheng four-scroll chaotic systems. Moreover, if we choose the scaling factors as $h_1 = h_2 = h_3 = 1$, Figure 9 illustrates the complete synchronization for two identical Lü-Chen-Cheng four-scroll chaotic systems. Finally, the anti synchronization for two identical Lü-Chen-Cheng four-scroll chaotic systems is displayed in Figure 10, when the scaling factors are taken as $h_1 = h_2 = h_3 = -1$.

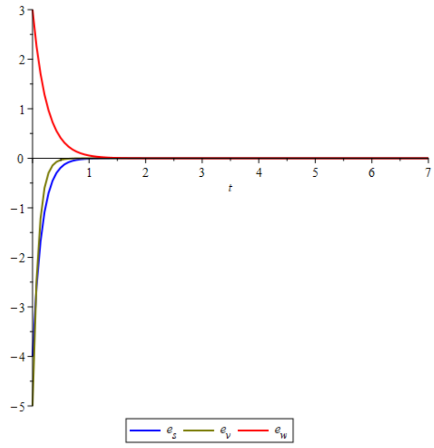

Figure 6: The error vectors e_s, e_v, and e_w converge to zero to attain GFPS between two identical Lü-Chen-Cheng four-scroll chaotic systems.

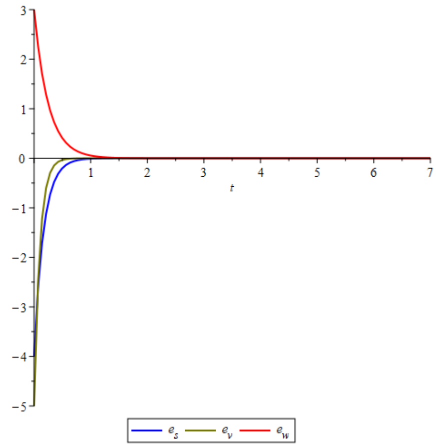

Figure 7: The error vectors e_s, e_v, and e_w converge to zero to attain MPS between two identical Lü-Chen-Cheng four-scroll chaotic systems.

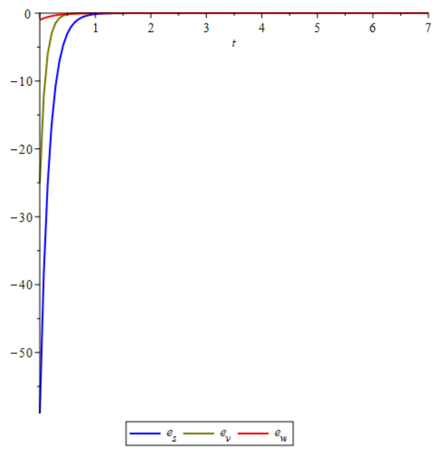

Figure 8: The error vectors e_s, e_v, and e_w converge to zero to attain PS between two identical Lü-Chen-Cheng four-scroll chaotic systems.

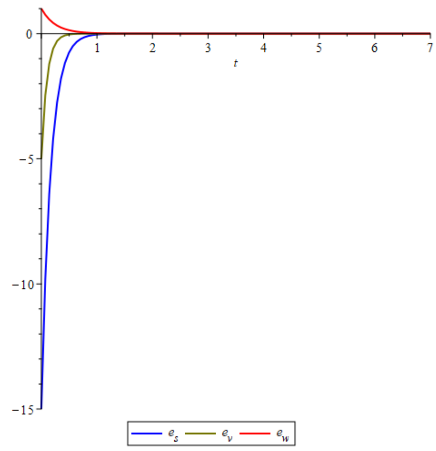

Figure 9: The error vectors e_s, e_v, and e_w converge to zero to attain complete synchronization between two identical Lü-Chen-Cheng four-scroll chaotic systems.

6.2. *Synchronization between Lü-Chen-Cheng four-scroll system and new chaotic system*

The outcomes of the numerical simulations in this section are intended to validate the analytical results obtained in the previous Section 5. We take the initial conditions as: $s_1(0) = 17,\ v_1(0) = 9,\ w_1(0) = 1$, and $s_2(0) = 7,\ v_2(0) = 6,\ w_2(0) = 2$ in all processes.

Figure 11 shows the GFPS between Lü-Chen-Cheng four-scroll chaotic system and new chaotic system when the scaling functions are given by $h_1 = -1,\ h_2 = 0.02v + 2,\ h_3 = 3w - 4$. Furthermore, the FPS between Lü-Chen-Cheng four-scroll chaotic system and new chaotic system is shown in Figure 12, when the scaling factors are taken as: $h_1 = 0.05s - 0.75,\ h_2 = 0.05v - 0.75,\ h_3 = 0.05w - 0.75$.

When we simplify the scaling factors as $h_1 = 2,\ h_2 = -1.5,\ h_3 = -5$, Figure 13 displays the MPS between Lü-Chen-Cheng four-scroll chaotic system and new chaotic system. Moreover, if we choose the scaling factors as $h_1 = h_2 = h_3 = 1$, Figure 14 illustrates the complete synchronization between Lü-Chen-Cheng four-scroll chaotic system and new chaotic system. Finally, the anti synchronization between Lü-Chen-Cheng four-scroll chaotic system and new chaotic system is displayed in Figure 15, when the scaling factors are taken as $h_1 = h_2 = h_3 = -1$.

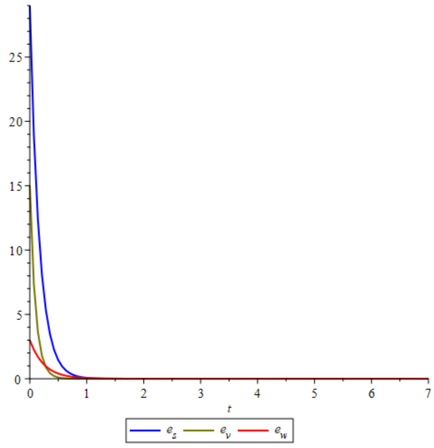

Figure 10: The error vectors e_s, e_v, and e_w converge to zero to attain anti synchronization between two identical Lü-Chen-Cheng four-scroll chaotic systems.

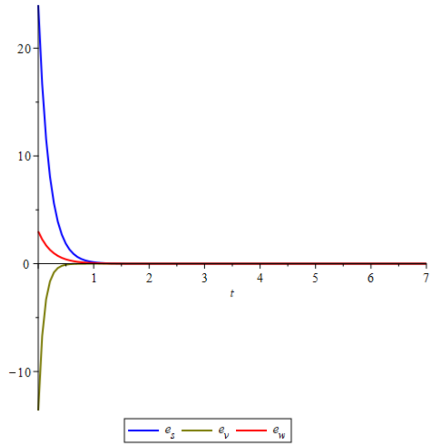

Figure 11: The error vectors e_s, e_v, and e_w converge to zero to attain GMPS between two nonidentical chaotic systems.

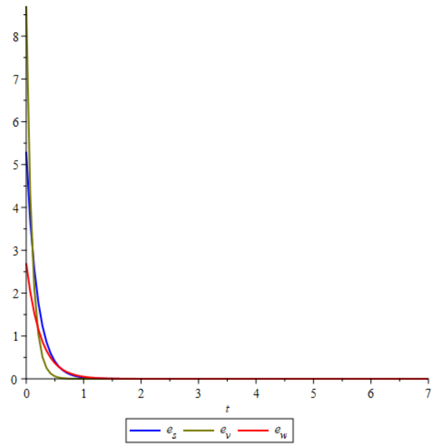

Figure 12: The error vectors e_s, e_v, and e_w converge to zero to attain FPS between two nonidentical chaotic systems.

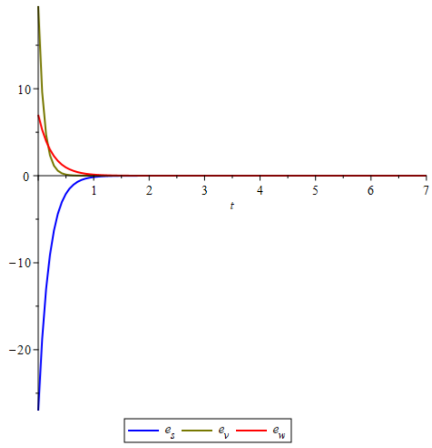

Figure 13: The error vectors e_s, e_v, and e_w converge to zero to attain MPS between two nonidentical chaotic systems.

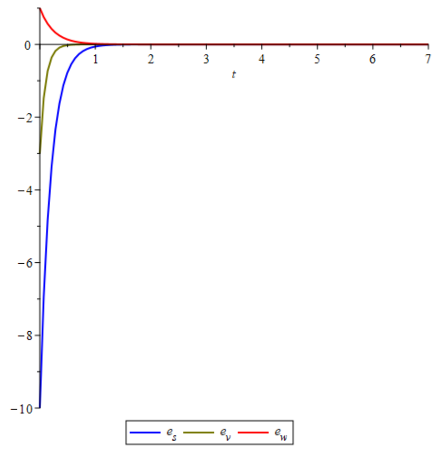

Figure 14: The error vectors e_s, e_v, and e_w converge to zero to attain complete synchronization between two nonidentical chaotic systems.

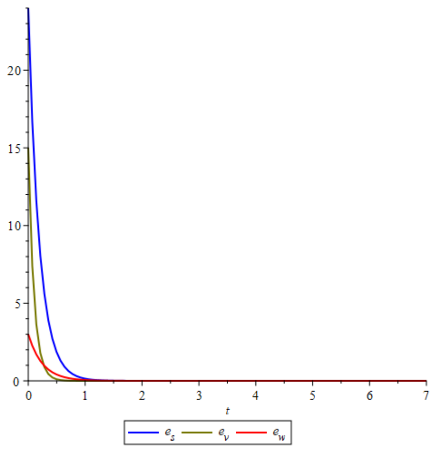

Figure 15: The error vectors e_s, e_v, and e_w converge to zero to attain anti synchronization between two nonidentical chaotic systems.

7. Conclusion

This study explores the concept of generalized function projective synchronization in two identical Lü-Chen-Cheng four-scroll chaotic systems, as well as between Lü-Chen-Cheng four-scroll and new chaotic systems with specific parameters. Through the use of adaptive control techniques and Lyapunov stability theory, nonlinear function controllers are derived to ensure the stability of the error dynamics between the driving and responding systems. Numerical simulations are conducted to validate the theoretical findings and showcase the efficacy of the proposed synchronization approach. Ultimately, this technique is deemed applicable to a wide range of chaotic systems.

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