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Finite-time synchronization of fractional order neural networks via sampled data control with time delay

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Abstract

This paper investigates the problem of finite-time synchronization (FTS) of fractional-order neural networks (FONNs) with time-delay via sampled data control (SDC) scheme. To achieve FTS criteria, a sampled-data control (SDC) scheme is implemented in the slave model of FONNs. And, this investigation is based on the solution of the time-delayed NNs by using Laplace transform, Mittag-Leffler function (MLF), and the generalized Grownwall inequality. Furthermore, under the proposed SDC scheme, the FTS conditions are derived for two cases of fractional order α, such as $0 < \alpha < 1$ and $1 < \alpha < 2$. The derived conditions ensure that the slave FONNs is asymptotically synchronized with master FONNs. Finally, two numerical examples are given to show the effectiveness of derived FTS criteria, for fractional order lying between $0 < \alpha < 1$ and $1 < \alpha < 2$.

Keywords: Fractional-order derivative, neural networks, finite-time synchronization, time-delay, Mittag-leffler function.

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1. Introduction

Fractional calculus is a generalization of differential and integral operators of non-integer order that L'Hospital introduced in 1695. It is a fast-growing field of research from both theoretical and application points of view. In the last decade, fractional calculus received much attention from researchers for solving the real-world problem in the various fields like engineering, physics, economics, etc. The main advantage of fractional calculus is that it is flexible enough to utilize more degrees of freedom and incorporate infinite memory. Due to this advantage, the qualitative analysis of the dynamical system with fractional order-derivative has attracted more attention from the researchers in recent days [11, 21, 34]. As an example, the authors in [37] has been studied the stability nature of the delayed fractional order dynamical systems based on Laplace transform, MLF and Gronwall inequality. Son et al. [33] have studied a new concept in fractional differential equations called the neutrosophic problem with the help of a fuzzy function or fuzzy initial condition and also gave some examples for finding the neutroscophic differential equation solution. Recently, Arthi et al. [3] studied the fractional order dynamical system in finite-time stability using a damped time delays and their problem is to solve by using Volterra integral equation and numerical examples developed by multiterm fractional in damped concept.

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On the other hand, the analysis of dynamical NN model has received much attention among the researchers, such as Hopfield NNs, Cohengrossberg, BAM NNs, and fuzzy cellular NNs. As we know, time delay is an unavoidable factor in dynamical NNs. This factor can make whole systems perform poorly or be unstable. Various type of the time-delays are discrete delay and disturbed delay, neutral type time-delay and unbounded distributed time-delay. To investigate the effect of time-delay, many researcher have derived the necessary and sufficient conditions for various NN dynamical system in the literature [9, 10, 23, 24, 37, 49] due to its.

Recently, synchronization analysis between two dynamical systems has received much attention due various application in image processing, signal processing, combinatorial optimization [20], secure communication [26], associative memories[44], and pattern recognition [16]. As specially, the various synchronization analysis of NNs have received great attention in the literature such as adaptive synchronization [48], exponential synchronization [32], quasi and complete synchronization [43], asymptotical synchronization [40], phase synchronization [13], projective synchronization [4], and fixed-time synchronization [1]. Moreover, the synchronization problem is find the error system between master and slave systems, which is expected to reach zero within a finite time then we can say that error system is stable and this emerging concept is called FTS, and these properties have more convergence than traditional synchronization problem. Many constraints are FTS of FONNs have been modeled by memristive method based on lyapunov conditions with novel controller [41], gronwall inequality approach with quantized control [31], and uncertain method with adaptive sliding mode control with delay [18]. Among them, parameter uncertainties method [9] was discussed for fractional order fuzzy cellular NNs in FTS with delay. In addition, the authors of [28] presented the new criteria in FTS of fractional order quaternion-valued NNs using time delay. Accordingly, in [27] the authors have investigated the FTS in FONNs with help of state feedback control and discontinuous adaptive feedback control in graph theory approach.

Moreover, FONNs have been studied for various types of synchronization concepts developed over integer order NNs. As an example, the authors in [22] discussed the complete FTS conditions for fuzzy FONNs via two novel nonlinear feedback control schemes such as i) adaptive; and ii) discontinuous. In [42], new inequalities have been derived for analyzing the FTS of the delayed fractional-order quaternion-valued NNs, and the derived conditions are based on Laplace transform and MLF. Here, we would like mention that the FTS criteria have been achieved between master and salve systems by using various control schemes in the literature such as impulsive control [45], adaptive control [18], feedback and periodically intermittent control [17], sample data control [15], saturated control [38], novel hybrid controller [39], and non-fragile control [19]. These control schemes have been considered fully atmosphere hampering and logical constraints while ensuring achieved performances. For example, in switching the signal of FONNs, an adaptive control has provided better performance of neurons compared to the feedback control. Until now, different types of FONNs models has been deeply studied for the FTS problems such as BAM FONNs [43], memristive FONNs [35], memritive Bam FONNs [29], discontinuous activations-based FONNs [29], fuzzy inertial FONNs [12], fuzzy cellular FONNs [2], reaction-diffusion FONNs [25], inertial FONNs [36], fuzzy inertial Bam FONNs [50], stochastic FONNs [30], and stochastic BAM FONNs [43].

From the above discussion, we can observe that the synchronization criteria have been investigated for various types of FONN models under different control schemes (as mentioned above). On the other hand, numerous studies on various digital control systems have been conducted because of the rapid advancement of digital technology and network connections. Since the majority of digital equipment produces sampling signals, research on the management of sampled data based on these signals is now being addressed. Therefore, the sampled-data control systems have been suggested for analysing the stabilisation and synchronization problems. In particular, SDC strategies have been proposed by researchers to solve synchronization problems in chaotic systems. As a result, for example, authors in [14] discussed the SDC in exponential synchronization using switching signal with uncertain mixed delays NNs based on LMI method. In [21] authors investigated the improved synchronization in SDC extended dissipativity analysis for delayed NNs based on LMI approach. On other hand, SDC discussed in [47] synchronization of delayed inertial NNs using sampling and state quantization based Lyapunov-Krasovskii functional method. Moreover, SDC [5] have found wide applications in NNs for image secure communications of switched type based on memory approach with delay-dependent conditions.

Here, we like to mentioned that very limited investigations have been reported for the synchronization problem of delayed NNs under sampled-data control scheme. To best of authors knowledge, there is no investigation on the problem FTS for delayed FONNs under SDC in the literature.

From the above motivation, the FTS problem of FONNs with time-delay is addressed in this work. To derive FTS, SDC is implemented in the slave FONNs, then FTS criteria are derived for master and slave FONNs by using Laplace transform, generalized Gronwall's inequality, and MLF. Finally, numerical examples are validated with derived conditions to show the merit of the proposed control scheme.

Here, the main feature of this work is listed as follows.

- In this work, the FTS is analyzed for FONNs with time-delay under the SDC scheme. The corresponding FTS criteria are derived for fractional-order α as $0 < \alpha < 1$ and $1 < \alpha < 2$.
- The sampled-data-based control input is implemented in the slave FONNs. Then, the error model of FONNs is derived from slave and master FONNs.
- By using Laplace transform and MLF, the solution of error model for FONNs is derived for two cases of fractional-order such as $0 < \alpha < 1$ and $1 < \alpha < 2$. Moreover, we proved the derived solution is finite-time stable under the SDC, which means that the slave system is synchronized with master systems.
- Numerical examples and their results confirms the superiority of the proposed control input. The effectiveness of fractional-order, i.e., $0 < \alpha < 1$ and $1 < \alpha < 2$ are given in the results.

The paper is organized as follows. Section 2 demonstrates the master and slave to give a solution representation of the error system. Section 3 demonstrates a set of theorems that justifies finite-time synchronization. In Section 4, the numerical methodology is demonstrated to verify the sample data control. Section 5 ends with a conclusion.

2. Preliminaries

In this section, we introduce basic definitions, properties and lemmas for deriving the FTS conditions of the proposed FONNs.

Definition 2.1 ([7])**.** The Caputo Fractional derivative ${}^cD^\alpha$ of order $\alpha \geqslant 0$, $r-1 < \alpha < r$, $r \in N$ is defined as

$$ {}^cD^\alpha f(r) = \frac{1}{\Gamma(n-\alpha)} \int_0^r (r-s)^{n-\alpha-1} f^{(n)}(s) ds, $$

where D^α denotes the Caputo Fractional derivative.

Definition 2.2 ([27])**.** The MLFs E_α and $E_{\alpha,\beta}$ are defined by the power series

$$ E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)} \quad \text{and} \quad E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)}, $$

where $\alpha > 0, \beta > 0$, and $z \in C$.

Some basic properties of MLF are as follows [27].

Property 2.3. For any $a > 0$ and $t > 0, E_\alpha(-at^\alpha) \leqslant 1$,

(i) $E_{\alpha,\alpha}(-ar^\alpha) \leqslant \frac{1}{\Gamma(\alpha)}$;

(ii) $E_{\alpha,\alpha+\beta}(-ar^\alpha) \leqslant \frac{1}{\Gamma(\alpha+\beta)}$;

(iii) $E_{\alpha}(0) = 1, E_{\alpha,\alpha}(0) = \frac{1}{\Gamma(\alpha)}$;

(iv) $E_{\alpha,\alpha+\beta}(0) = \frac{1}{\Gamma(\alpha+\beta)}$.

Property 2.4. The Laplace transform of the MLF $E_{\alpha,\beta}(\cdot)$ is given below:

$$\mathcal{L}\left\{r^{\beta-1}E_{\alpha,\beta}\left(\pm ar^{\alpha}\right)\right\} = \frac{s^{\alpha-\beta}}{s^{\alpha} \mp a},$$

where $a \in \mathbb{C}$.

Lemma 2.5 ([6]). *Let* $u(r)$, *a(r) be non negative and locally integrable on* $0 \leqslant r < T$ *(some* $T \leqslant +\infty$ *) and let* $g(r)$ *be a non negative, non decreasing continuous function defined on* $0 \leqslant r < T, g(t) \leqslant M$, *and let* M *be a real constants,* $\alpha > 0$ *with*

$$u(r) \leqslant a(r) + g(r)\int_0^r (r-s)^{\alpha-1}u(s)ds, t \in [0,T),$$

then

$$u(r) \leqslant a(r) + \int_0^r \left[\sum_{n=1}^{\infty} \frac{(g(r)\Gamma(\alpha))^n}{\Gamma(n\alpha)}(r-s)^{n\alpha-1}a(s)\right] ds, r \in [0,T).$$

Moreover, if $a(r)$ *is a non decreasing function on* $[0,T)$, *then*

$$u(r) \leqslant a(r)E_{\alpha}\left(g(r)\Gamma(\alpha)r^{\alpha}\right),\ r \in [0,T).$$

2.1. *Model formulation*

Consider the master system of FONNs as given below:

$$D^{\alpha}y_i(r) = -a_i y_i(r) + \sum_{j=1}^{n} b_{ij}f_j(y_j(r)) + \sum_{j=1}^{n} w_{ij}f_j(y_j(r-\tau)) + I_i, \tag{2.1}$$

where $0 < \alpha < 1$, $y_i(t)$ denotes the state variables of the ith neuron, $a_{ij} > 0$ denotes the self-feedback constant, b_{ij} and w_{ij} denote the connection weight between j and i at time r, τ denotes the constant time-delay, $f_j(\cdot)$ denotes activation functions, and I_i is the external input. The initial condition of the system (2.1) in the form is

$$y_i(r) = \mu_i(r),\ r \in [-\tau, 0],\ i = 1, \dots, n.$$

Let us consider the slave system of FONNs, which is described as

$$D^{\alpha}z_i(r) = -a_i z_i(r) + \sum_{j=1}^{n} b_{ij}g_j(z_j(r)) + \sum_{j=1}^{n} w_{ij}g_i(z_j(r-\tau)) + I_i + u_i(r). \tag{2.2}$$

The initial condition of the system (2.2) is

$$z_i(r) = \rho_i(r),\ r \in [-\tau, 0],\ i = 1, \dots, n,$$

where $z_i(r)$ denotes the state variables of the i^{th} neuron, $g_j(z_j(r-\tau))$ denotes activation functions with delay, and $u_i(t)$ denote the control input.

Assumption 2.6. Let $f_i(\cdot)$ and $g_i(\cdot)$ satisfy the conditions, then for any $x, y \in \mathbb{R}$, there exist positive constants such that

$$\|f_i(x) - f_i(y)\| \leqslant l\|x-y\|, \qquad \|g_i(x) - g_i(y)\| \leqslant m\|x-y\|.$$

Defining $e_i(r)$ is the error system between master-slave system as $q_i(e_i(r)) = g_i(y_i(r)) - f_i(x_i(r))$ and given as follows:

$$D^\alpha e_i(r) = -a_i e_i(r) + \sum_{j=1}^{n} b_{ij} q_j(e_j(r)) + \sum_{j=1}^{n} w_{ij} q_j(e_j(r-\tau)) + u_i(r). \tag{2.3}$$

The initial conditions associated with (2.3) are of the form

$$e_i(r) = \mu_i(r) - \rho_i(r) = \phi_i(r), r \in [r_k, r_{k+1}].$$

Now we consider the system (2.3) in a compact form as

$$D^\alpha e(r) = -Ae(r) + Bq(e(r)) + Wq(e(r-\tau)) + u(r), \tag{2.4}$$

where $e(r) = [e_1(r)\ e_2(r) \cdots e^n(r)]^T$, $A = \text{diag}\{a_1, a_2 \cdots a_n\}$, $B = (b_{ij})_{n\times n}$, and $W = (w_{ij})_{n\times n}$ are the connection weight matrices and $u(r) = [u_1(r)\ u_2(r) \cdots u^n(r)]^T$.

To achieve the synchronization of master and slave FONNs, with help of zero order hold, we have considered the control input as sampled-data controller as

$$u(r) = Ke(r_k),\ r_k \leqslant r < r_{k+1}, \tag{2.5}$$

where K is a control gain matrix. Assume that the sampling instants satisfy $r_{k+1} - r_k = h_k \leqslant \mu$. By using the above SDC (2.5), we can rewrite the error FONNs as

$$D^\alpha e(r) = -Ae(r) + Bq(e(r)) + Wq(e(r-\tau)) + Ke(r_k). \tag{2.6}$$

To prove the error FONNs is finite-time stable, the following definition is needed.

Definition 2.7 ([35])**.** The master system (2.1) and slave system (2.2) are said to be finite-time synchronization under SDC, if the error system $\|e(r)\|$ is stable in finite-time with interval of $[r_k, r_{k+1})$, then there exists $\|e(r)\| < \epsilon$, where $e_i(r) = y_i(r) - x_i(r)$.

Next, we derive the solution of the error model for the master and slave systems by using Laplace transform and MLF properties in the following subsection.

2.2. *Solution representation*

Consider the error model of master-slave system of FONNs with delayed form

$$D^\alpha e(r) = -Ae(r) + Bq(e(r)) + Wq(e(r-\tau)) + Ke(r_k), \qquad e_0 = e(0). \tag{2.7}$$

Taking Laplace transform on both sides, we get

$$\begin{aligned}
L[D^\alpha e(r)] &= -AL[e(r)] + BL[q(e(r))] + WL[q(e(r-\tau))] + KL[e(r_k)],\\
S^\alpha L[e(r)] - S^{\alpha-1}L[e(0)] &= -AL[e(r)] + BL[q(e(r))] + WL[q(e(r-\tau))] + KL[e(r_k)],\\
(S^\alpha I - (-A))L[e(r)] &= S^{\alpha-1}L[e(0)] + BL[q(e(r))] + WL[q(e(r-\tau))] + KL[e(r_k)],\\
L[e(r)] &= \left[\frac{S^{\alpha-1}}{(S^\alpha I - (-A))}\right] L[e(0)] + \frac{BL[q(e(r))]}{(S^\alpha I - (-A))} + \frac{WL[q(e(r-\tau))]}{(S^\alpha I - (-A))} + \frac{KL[e(r_k)]}{(S^\alpha I - (-A))}.
\end{aligned}$$

Applying the inverse Laplace transform to both the sides, we have

$$L^{-1}[e(r)] = L^{-1}\left[[\frac{S^{\alpha-1}}{(S^\alpha I - (-A))}]L[e(0)]\right] + L^{-1}\left[\frac{BL[q(e(r))]}{(S^\alpha I - (-A))}\right]$$

$$+L^{-1}\left[\frac{WL[q(e(r-\tau))]}{(S^{\alpha}I-(-A))}\right]+L^{-1}\left[\frac{KL[e(r_k)]}{(S^{\alpha}I-(-A))}\right].$$

Finally, substituting the Laplace transformation of the Mittag-Leffler function in (2.7), we get the solution to the given equation

$$\begin{aligned}e(r) =&E_{\alpha}(-A(r-r_k))^{\alpha}e(r)+Kr^{\alpha}E_{\alpha,\alpha+1}(-A(r-r_k))^{\alpha}e(r_k)\\&+\int_{r_k}^{r}(r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})(Bq(e(s))+Wq(e(s-\tau)))ds.\end{aligned}$$

Based on the above solution, we derive the finite-time stability conditions are derived under the proposed SDC scheme.

3. Main results

In this section, let us derive two sufficient conditions that establish the FTS of FONNs via SDC under the two cases, such as $0<\alpha<1$ and $1<\alpha<2$.

3.1. FTS condition for $0<\alpha<1$

Theorem 3.1. *When $0<\alpha<1$ and assuming that Assumption 2.6 holds, then the FTSNNs (2.1) is synchronized with FTSNNs (2.2) in finite-time under the sample data control (2.5) and following conditions are satisfied:*

$$\|I-(A-KI)\,E_{\alpha,\alpha+1}\left(AR^{\alpha}\right)R^{\alpha}\|\,E_{\alpha}\left(mR^{\alpha}\right)\|B+W\|+\frac{m\|W\|}{\Gamma(\alpha+1)}R^{\alpha}<\frac{\in}{\delta}.$$

Proof. Initially, let $r\in[0,r_1)$ and taking Laplace and inverse Laplace transform and using MLF of (2.3) on both sides, one obtains

$$\begin{aligned}e(r)=&E_{\alpha}(-Ar)^{\alpha}e(0)+Kr^{\alpha}E_{\alpha,\alpha+1}(-Ar)^{\alpha}e(0)\\&+\int_{0}^{r}(r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})Bq(e(s))+Wq(e(s-\tau))ds.\end{aligned}$$

In general, for $r\in[r_k,r_{k+1}), K\in N$, we have

$$\begin{aligned}e(r)=&E_{\alpha}(-A(r-r_k))^{\alpha}e(r)+Kr^{\alpha}E_{\alpha,\alpha+1}(-A(r-r_k))^{\alpha}e(r_k)\\&+\int_{r_k}^{r}(r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})(Bq(e(s))+Wq(e(s-\tau)))ds.\end{aligned}$$

Let any $r\in[0,r_1)$. Then, we can obtain that

$$\begin{aligned}\|e(r)\|\leqslant&\|E_{\alpha}(-Ar)^{\alpha}+Kr^{\alpha}E_{\alpha,\alpha+1}(-Ar)^{\alpha}\|\|e(0)\|\\&+m\int_{0}^{r}(r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})\|B(e(s))+W(e(s-\tau))\|ds.\end{aligned}\tag{3.1}$$

Now consider that

$$\begin{aligned}E_{\alpha}(-Ar)^{\alpha}&=I-Ar^{\alpha}E_{\alpha,\alpha+1}(-Ar)^{\alpha},\\E_{\alpha}(-Ar)^{\alpha}+Kr^{\alpha}E_{\alpha,\alpha+1}(-Ar)^{\alpha}&=I-(A-KI)E_{\alpha,\alpha+1}(-Ar)^{\alpha}r^{\alpha}.\end{aligned}\tag{3.2}$$

Substitute (3.2) in (3.1), we can get the following inequality

$$\|e(r)\|\leqslant\|I-(A-KI)E_{\alpha,\alpha+1}(-Ar)^{\alpha}r^{\alpha}\|\|e(0)\|$$

$$+m\int_0^r (r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})\|B(e(s))+W(e(s-\tau))\|ds.$$

Using Property 2.4, we get

$$\|e(r)\| \leqslant \sup_{r\in[0,T]} \|I-(A-KI)E_{\alpha,\alpha+1}(-Ar)^{\alpha}r^{\alpha}\|\|e(0)\| + \frac{m}{\Gamma(\alpha)}\int_0^r (r-s)^{\alpha-1}\|B(e(s))+W(e(s-\tau))\|ds.$$

Moreover,

$$\|e(r)\| \leqslant \sup_{r\in[0,T]} \|I-(A-KI)E_{\alpha,\alpha+1}(-Ar)^{\alpha}r^{\alpha}\|\|e(0)\| + \frac{m}{\Gamma(\alpha)}\int_0^r (r-s)^{\alpha-1}\|B(e(s))+W(e(s)+\delta)\|ds.$$

Also,

$$\begin{aligned}\|e(r)\| \leqslant &\sup_{r\in[0,T]} \|I-(A-KI)E_{\alpha,\alpha+1}(-Ar)^{\alpha}r^{\alpha}\|\|e(0)\|\\ &+\frac{m}{\Gamma(\alpha)}\int_0^r (r-s)^{\alpha-1}\|B(e(s))+W(e(s)+\delta)\|ds + \frac{m}{\Gamma(\alpha)}\int_0^r (r-s)^{\alpha-1}\|W\delta\|ds.\end{aligned}$$

Using Lemma 2.5, the above fractional-order inequality can be written as ([46])

$$\|e(r)\| \leqslant \sup_{r\in[0,T]} \|I-(A-KI)E_{\alpha,\alpha+1}(-Ar)^{\alpha}r^{\alpha}\|\|e(0)\| + E_{\alpha}(mr^{\alpha})\|B+W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}r^{\alpha}.$$

In general, for $r\in[r_k, r_{k+1}),\ K\in N$,

$$\begin{aligned}\|e(r)\| \leqslant &\sup_{r-r_k\in[0,T]} \|I-(A-KI)E_{\alpha,\alpha+1}(-A(r-r_k))^{\alpha}(r-r_k)^{\alpha}\|\\ &+E_{\alpha}(m(r-r_k)^{\alpha})\|B+W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}(r-r_k)^{\alpha}\|e(r_k)\|.\end{aligned}$$

Thus, by recursively computing and back-substituting the above estimates of the values of $\|e(r_k)\|$ one finally obtains

$$\|e(r)\| \leqslant \left[\|I-(A-KI)E_{\alpha,\alpha+1}(-AR)^{\alpha}R^{\alpha}\| + E_{\alpha}(lR^{\alpha})\|B+W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}R^{\alpha}\right]^K \times D(a_k)\|e(r_k)\|,$$

where $a_k = r-r_k$ and $\mathcal{D}(a_k) = s_k E_{\alpha}\left(la_k^{\alpha}\right)$, with

$$s_k = \sup_{a_k\in[0,T]} \|I-(A-KI)\,E_{\alpha,\alpha+1}\left(aa_k^{\alpha}\right)a_k^{\alpha}\|.$$

Now, since $\mathcal{D}(a_k)$ is bounded for all $k\in\mathbb{N}\cup\{0\}$, $\lim_{r\to\infty}\|e(r_k)\|=0$,

$$\|e(r)\| \leqslant \lim_{k\to\infty}\left[\|I-(A-KI)\,E_{\alpha,\alpha+1}\left(-AR^{\alpha}\right)R^{\alpha}\|\,E_{\alpha}\left(mR^{\alpha}\right)\|B+W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}R^{\alpha}\right]^k.$$

Therefore,

$$\|e(r)\| \leqslant \|I-(A-KI)\,E_{\alpha,\alpha+1}\left(-AR^{\alpha}\right)T^{\alpha}\|\,E_{\alpha}\left(mR^{\alpha}\right)\|B+W\| + \frac{m\|W\|}{\Gamma(\alpha+1)}R^{\alpha}\delta.$$

From Theorem 3.1, we get

$$\|e(r)\| \leqslant \epsilon.$$

From the above inequality, we can say that the error system (2.4) is said to be stable such that the FTSNNs (2.3) is synchronized with FTSNNs (2.6) in finite-time. Then the solution $e(r)$ completes the proof. □

Remark 3.2. In summary, obtaining finite-time synchronization criteria for fractional order systems involves addressing different challenges depending on the range of the fractional order parameter α. Systems with $0 < \alpha < 1$ require dealing with slower convergence and memory effects, while systems with $1 < \alpha < 2$ pose challenges related to complex dynamics and stability considerations. Properly addressing these challenges is essential for developing effective synchronization criteria and designing controllers capable of achieving finite-time synchronization in fractional order systems.

Here, it should be mentioned that the above theorem provides the FTS criterion for fractional derivative lies between 0 and 1, i.e., $0 < \alpha < 1$. Most of research reports investigated the synchronization criteria for FONNs under consideration fractional order derivative which lies between 0 and 1. However, if we consider the second order fractional derivative, the α lies between 1 and 2. Therefore, in this work, we derive the FTS criterion for second order fractional derivative of master and slave FONNs with time-delay (2.1) and (2.2) is summarized in the following subsection.

3.2. *FTS condition for* $1 < \alpha < 2$

In this subsection, FTS criterion is derived based on Laplace and MLF and the corresponding result is summarized in the following Theorem.

Theorem 3.3. *Under Assumption 2.6 and* $1 < \alpha < 2$*, then the FTSNNs* (2.1) *will synchronize with FTSNNs* (2.2) *when the sample data control* (2.5) *satisfies the following condition:*

$$\|E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,1}(-ar)^{\alpha}\phi + kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\| + E_{\alpha}(mR)^{\alpha} + \frac{m\|w\|}{\Gamma(\alpha+1)}r^{\alpha} < \frac{\epsilon}{\delta}.$$

Proof. Let us consider the fractional-order $\alpha : 1 < \alpha < 2$, one can obtain a solution of (2.4) in the form

$$\begin{aligned} e(r) = {} & E_{\alpha,1}(-ar)^{\alpha}e_1(0) + rE_{\alpha,2}(-ar)^{\alpha}e_2(0) + kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}e(0) \\ & + \int_0^r (r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})Bq(e(s)) + Wq(e(s-\tau))ds. \end{aligned}$$

In general for $r \in [r_k, r_{k+1}), K \in N$, we have

$$\begin{aligned} e(r) = {} & E_{\alpha,1}(-a(r-r_k))^{\alpha}e_1(0) + rE_{\alpha,2}(-a(r-r_k))^{\alpha}e_2(0) + k(r-r_k)^{\alpha}E_{\alpha,\alpha+1}(a(r-r_k))^{\alpha}e(0) \\ & + \int_{r_k}^r (r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})(Bq(e(s)) + Wq(e(s-\tau)))ds. \end{aligned}$$

Let any $r \in [0, r_1)$. Then we can obtain that

$$\begin{aligned} \|e(r)\| \leqslant {} & \|E_{\alpha,1}(-ar)^{\alpha}e_1(0) + rE_{\alpha,2}(-ar)^{\alpha}e_2(0) + kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}e(0)\| \\ & + m\int_0^r (r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})\|B(e(s)) + W(e(s-\tau))\|ds. \end{aligned}$$

Moreover,

$$\begin{aligned} \|e(r)\| \leqslant {} & \|E_{\alpha,1}(-ar)^{\alpha}e_1(0) + rE_{\alpha,2}(-ar)^{\alpha}e_2(0)\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(at)^{\alpha}\|\|e(0)\| \\ & + m\int_0^r (r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})\|B(e(s)) + W(e(s-\tau))\|ds. \end{aligned}$$

Then,

$$\begin{aligned} \|e(r)\| \leqslant {} & \|E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,2}(-ar)^{\alpha}\phi\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\|\|e(0)\| \\ & + m\int_0^r (r-s)^{\alpha-1}E_{\alpha,\alpha}(-A(r-s)^{\alpha})\|B(e(s)) + W(e(s-\tau))\|ds. \end{aligned}$$

Using Lemma 2.5 and the above fractional-order Grownwall's inequality, we get ([46])

$$\begin{aligned}\|e(r)\| \leqslant & \sup_{r\epsilon[0,T]} \|\phi(E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,2}(-ar)^{\alpha})\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\|\|e(0)\| \\ & + \frac{m}{\Gamma(\alpha)}\int_0^r (r-s)^{\alpha-1}\|B(e(s)) + W(e(s)+\delta))\|ds,\end{aligned}$$

which indicates that

$$\begin{aligned}\|e(r)\| \leqslant & \sup_{r\epsilon[0,T]} \|\phi(E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,1}(-ar)^{\alpha})\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\|\|e(0)\| \\ & + \frac{m}{\Gamma(\alpha)}\int_0^r (r-s)^{\alpha-1}\|B(e(s)) + W(e(s))\|ds + \frac{m}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\|W\|\delta ds.\end{aligned}$$

Therefore,

$$\begin{aligned}\|e(r)\| \leqslant & \sup_{r\epsilon[0,T]} \|\phi(E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,2}(-ar)^{\alpha})\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\|\|e(0)\| \\ & + E_{\alpha}(mr^{\alpha})\|B + W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}r^{\alpha}.\end{aligned}$$

In general for $r \in [r_k, r_{k+1}), K \in N$, we have

$$\begin{aligned}\|e(r)\| \leqslant & \sup_{r-r_k\in[0,T]} \|\phi(E_{\alpha,1}(-a(r-r_k))^{\alpha}\phi + rE_{\alpha,2}(-a(r-r_k))^{\alpha})\| \\ & + \|k(r-r_k)^{\alpha}E_{\alpha,\alpha+1}(a(r-r_k))^{\alpha}\|\|e(r_k)\| + E_{\alpha}(m(r-r_k)^{\alpha})\|B + W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}(r-r_k)^{\alpha}.\end{aligned}$$

Thus, by recursively computing and back-substituting the above estimates of the values of $\|e(r_k)\|$ one finally obtains

$$\begin{aligned}\|e(r)\| \leqslant & \Big[\|\phi(E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,2}(-ar)^{\alpha})\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\| \\ & + E_{\alpha}(lR^{\alpha})\|B + W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}R^{\alpha}\Big]^K \times D(a_k)\|e(r_k)\|,\end{aligned}$$

where $a_k = r - r_k$ and $\mathcal{D}(a_k) = s_k E_{\alpha}\left(l a_k^{\alpha}\right)$, with

$$s_k = \sup_{a_k\in[0,T]} \|\phi(E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,1}(-ar)^{\alpha})\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\|.$$

Since $\mathcal{D}(a_k)$ is bounded for all $k \in \mathbb{N}\cup\{0\}$, $\lim_{r\to\infty}\|e(r_k)\| = 0$,

$$\|e(r)\| \leqslant \lim_{k\to\infty}\left[\|\phi(E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,2}(-ar)^{\alpha})\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\|\, E_{\alpha}(mR^{\alpha})\|B + W\| + \frac{m\|W\|\delta}{\Gamma(\alpha+1)}R^{\alpha}\right]^k.$$

Then,

$$\|e(r)\| \leqslant \|\phi(E_{\alpha,1}(-ar)^{\alpha}\phi + rE_{\alpha,2}(-ar)^{\alpha})\| + \|kr^{\alpha}E_{\alpha,\alpha+1}(ar)^{\alpha}\|\, E_{\alpha}(mR^{\alpha})\|B + W\| + \frac{m\|W\|}{\Gamma(\alpha+1)}R^{\alpha}\delta.$$

From (2.1) and (2.2), we get Theorem 3.3,

$$\|e(r)\| \leqslant \in .$$

From assumption, we get if the error system (2.4) is stable then the FTSNNs (2.1) is synchronized with FTSNNs (2.2) in finite-time. Then the solution e(r) completes the proof. □

Remark 3.4. In Theorems 3.1 and 3.3, finite-time stability conditions are derived for the error FONNs with $0 < \alpha < 1$ and $1 < \alpha < 2$, which means that the master and slave FONNs are synchronized in a finite-time. Different from the traditional LMI method, the stability condition is derived based on the solution of the error FONNs (2.6), MLF and Laplace transform.

Remark 3.5. The advantage of finite-time synchronization is particularly noteworthy, as it implies not only the convergence of system states but also the rapidity of this convergence within a finite time interval. This property is highly desirable in applications where prompt and precise synchronization is crucial, such as in secure communication protocols or real-time control systems.

Remark 3.6. Recently, the SDC scheme has received more attention among researchers for analyzing the stability and synchronization issues of integer-order systems. Here, it should be mentioned that most of the existing work is in synchronisation for integer-order NNs with time-delays based on the Lyapunov stability approach. As a result, in [4] a pinning SDC scheme has been implemented for function projective synchronization analysis of time-delay NNs with hybrid coupling. The synchronization criteria have been derived for a class of nonlinear multi-agent systems under the SDC scheme in [8]. With the help of adaptive control, master and slave exponential synchronization criteria have been derived for bounded and unbounded delayed NNs based on an analytical approach in [48]. However, in this work, the synchronization criteria are derived for master and slave FONNs with time-delay based on the Laplace transform and MLF, which is different from the traditional Lyapunov-function method.

4. Simulation results

In this section, we considered two examples to show the effectiveness of derived conditions under the SDC scheme.

Example 4.1. Consider the following FONNs with time delay as given below:

$$D^{\alpha}y_i(r) = -a_i y_i(r) + \sum_{j=1}^{2} b_{ij} f_j(y_j(r)) + \sum_{j=1}^{2} c_{ij} f_j(y_j(r-\tau)) + I_i, \tag{4.1}$$

where, $i = 1, 2,\ \tau = 1,\ a_{11} = a_{12} = 1, b_{11} = 2.1, b_{12} = -30, b_{21} = 0.1, b_{22} = -15,\ c_{11} = 0.1, c_{12} = -18, c_{21} = 0.1, c_{22} = -5,\ \alpha = 1.9,\ I_i = 0$,

$$f_i(y_i(r)) = 0.5(\|y_i + 1\| - \|y_i - 1\|),\ i = 1, 2.$$

The corresponding slave FONNs with time-delay is as given below:

$$D^{\alpha}z_i(r) = -a_i y_i(r) + \sum_{j=1}^{2} b_{ij} f_j(z_j(r)) + \sum_{j=1}^{2} w_{ij} f_i(z_j(r-\tau)) + I_i + u_i(r). \tag{4.2}$$

For simulation purpose, initial values are considered as $y_1(0) = 1.5,\ y_2(0) = 1.8,\ z_1(0) = 0.7,\ z_2(0) = 1.5$. The chaotic nature of the master FONNs (4.1) and (4.2) is given in Fig. 1. To achieve the synchronization between master FONNs (4.1) and slave FONNs (4.2), let us choose control input

$$K = \begin{bmatrix} 0.5849 & 0.1 \\ 0.2478 & 0.1 \end{bmatrix}.$$

Based on SDC gain, the master and slave FONNs are synchronized each others which is given in Fig. 2. Moreover, the error response between $y(r)$ and $z(r)$ is shown in Fig. 3. From Figs. 1-3, we can confirm that the proposed SDC scheme ensures the FTS of the master and slave models in (2.1) and (2.2).

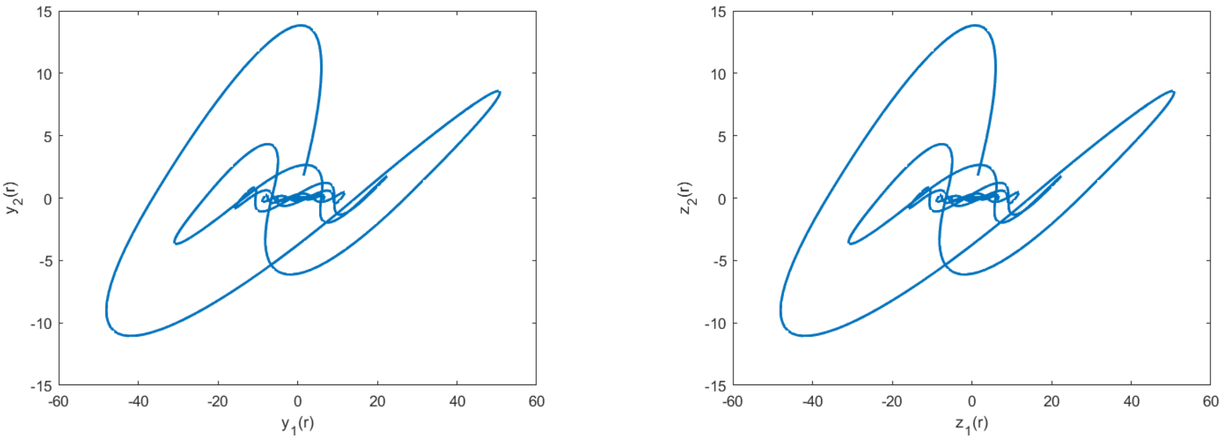

Figure 1: Chaotic trajectories of FTSNNs (4.1) and (4.2).

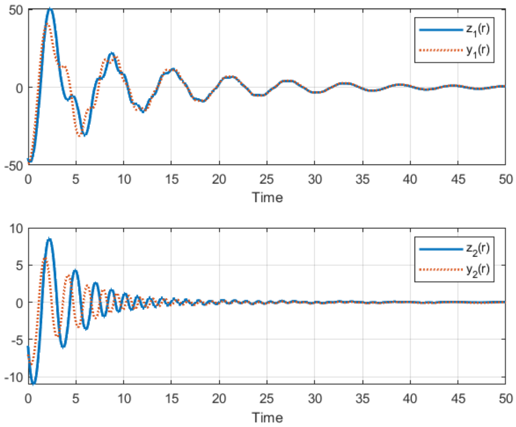

Figure 2: State trajectories of the system (4.1) and (4.2).

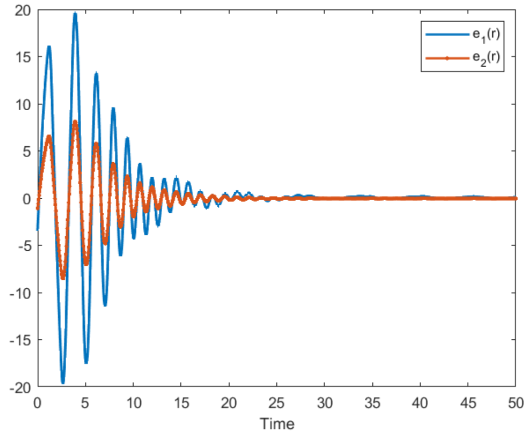

Figure 3: The FTSNNs of error system of (4.1) and (4.2).

Inference: The above figures describes the synchronization between the master and slave system implying the robustness of the network against disturbances or parameter variations, and it indicates stability of the interconnected systems. By adjusting parameters or applying control inputs to the slave system, one can achieve desired synchronization behaviors, enabling control of chaotic systems or stabilization of unstable systems. Moreover, synchronization can be employed in optimization algorithms for solving complex optimization problems.

Example 4.2. Consider the following FTSNN with time delay:

$$D^{\alpha}y_i(r) = -a_i y_i(r) + \sum_{j=1}^{3} b_{ij} f_j(x_j(r)) + \sum_{j=1}^{3} w_{ij} f_j(x_j(r-\tau)) + I_i, \tag{4.3}$$

where $i = 1,2,3$, $\tau = 1$, $a_{11} = a_{22} = a_{33} = 1, b_{11} = 0.1, b_{12} = -30, b_{13} = 0,$ $b_{21} = 0.9, b_{22} = -12, b_{23} = -15, b_{31} = 0.6, b_{32} = -5, b_{33} = 0,$ $c_{11} = 0.4, c_{12} = -18, c_{13} = 0, c_{21} = 0.5, c_{22} = -8, c_{23} = 0,$ $c_{31} = 0.1, c_{32} = -2, c_{33} = 0.5$, $\alpha = 1.9$, and $I_i = 0$,

$$f_i(y_i(r)) = (\|y_i + 1\| - \|y_i - 1\|),\ i = 1,2,3.$$

The corresponding slave FONNs with time-delay is as given below:

$$D^{\alpha} z_i(r) = -a_i z_i(r) + \sum_{j=1}^{3} b_{ij} f_j(z_j(r)) + \sum_{j=1}^{3} w_{ij} f_i(r_j(t-\tau)) + I_i + u_i(r). \tag{4.4}$$

For simulation purpose, initial values are considered as $y_1(0) = 1.5$, $y_2(0) = 1.8$, $y_3(0) = 1.6$, $z_1(0) = 0.7$, $z_2(0) = 1.5$, $z_3 = 1.4$. The chaotic nature of the master FONNs (4.3) and (4.4) is given in Fig. 4. To achieve the synchronization between master FONNs (4.3) and slave FONNs (4.4), let us choose control input

$$K = \begin{bmatrix} 0.5 & 0.4 & 0 \\ 0.2 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

Based on SDC gain, the master and slave FONNs are synchronized each others which is given in Fig. 5. Moreover, the error response between $y(r)$ and $z(r)$ is shown in Fig. 6. From Figs. 4-6, we can confirm that the proposed SDC scheme ensures the FTS of the master and slave models in (2.1) and (2.2).

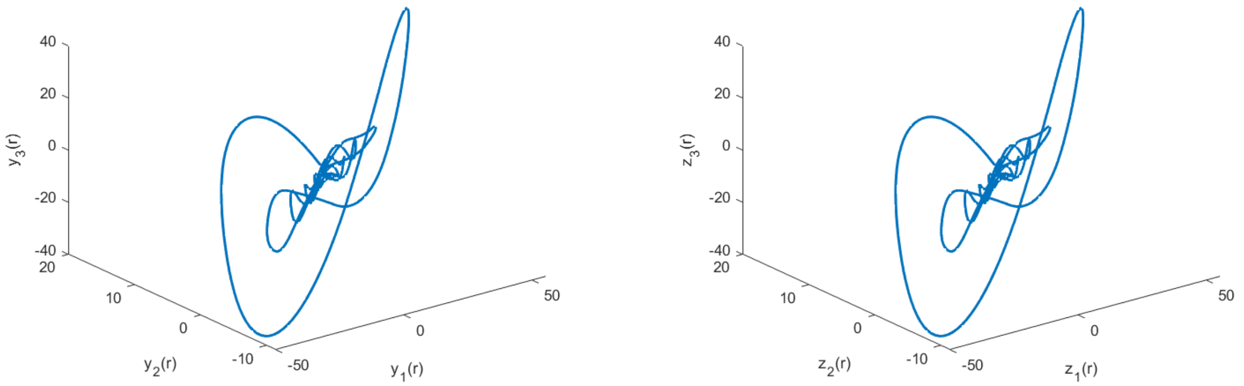

Figure 4: Chaotic trajectories of FTSNNs (4.3) and (4.4).

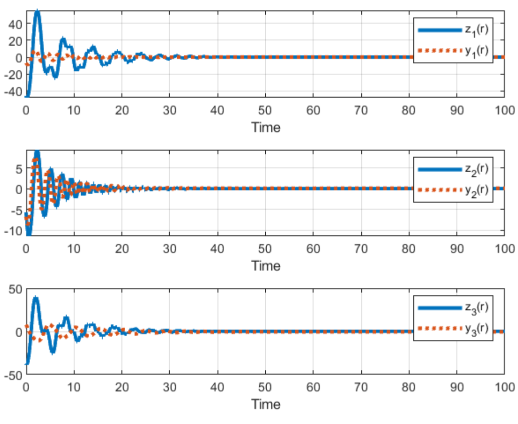

Figure 5: State trajectories of the system (4.3) and (4.4).

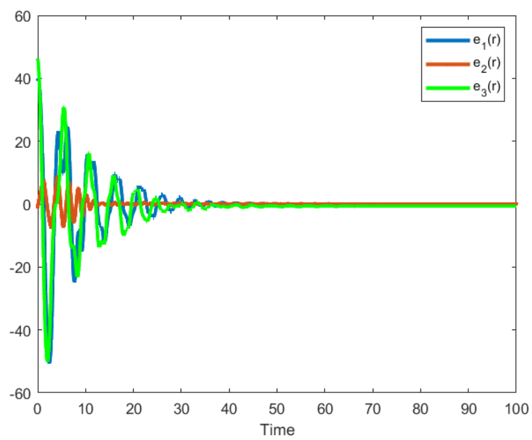

Figure 6: The FTSNNs of error system of (4.3) and (4.4).

5. Conclusion

In this work, the FTS problem of FONNs with time delays is addressed. To achieve the synchronization criteria, a sampled-data-based control scheme is implemented in the slave FONNs. Then, the finite-time stability conditions of the error model are derived based on the Laplace transform, the MLF, and the generalized Grown-wall's inequality. The derived conditions can ensure the synchronization between master and slave FONNs with time-delay under SDC scheme. Moreover, we have derived the FTS conditions for two fractional order cases, such as $0 < \alpha < 1$ and $1 < \alpha < 2$. Finally, two numerical examples are given to show the superiority of the proposed FTS conditions over SDC. Recently, the complex network has received much attention among the researchers due to various applications. Future research directions may include exploring extensions of the proposed synchronization problem to address more complex network topologies, nonlinear dynamics, and uncertain environments.

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