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Research of a nonlinear dynamic system describing the mathematical model of the bipolar world

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Abstract

Over the past few decades, mathematical modeling of social processes, such as information warfare, language globalization, ethnic assimilation, political conflicts, the process of state territorial stability, etc. has been of particular interest. Currently, the ongoing military events related to the Russian invasion of Ukraine are directly Russia's desire to change the natural course of geopolitical influence distribution on the world policy of the two main powers USA and China, where the their economic components in world GDP (Gross Domestic Product) are respectively 28.78% and 18.53%. For comparison, Russia's contribution to world GDP is 2.54%.

This paper proposes new nonlinear mathematical models describing both a bipolar (USA, China) system of real influence on world politics. Mathematical models are described by two- dimensional nonlinear dynamic systems with variable coefficients and corresponding initial conditions characterizing the current state of influence of the world's main actors. The models consider the conditions for rationing solutions, which imply a complete redistribution of world influence in case of a bipolar world between the United States and China, and the contribution of other countries is considered insignificant. In the mathematical model of the bipolar arrangement of the world, exact analytical solutions are obtained in quadratures, showing the dynamics of changes in the influence of these two powers on world politics, i.e., changes in their relative contribution to the redistribution of world influence. At some values of the variable coefficients of the mathematical model, accurate periodic solutions of the dynamic system were found, describing the process of alternating dominance of the political weight of the United States and China.

Keywords: Bipolar political model, mathematical modelling, nonlinear dynamic systems, some particular exact solutions.

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1. Introduction

The study of a number of social processes, such as information warfare, assimilation of languages (people), globalization, settlement of political conflicts, secession of regions, territorial integrity of states, etc. is of great interest. From our point of view, the only scientific approach to an adequate quantitative

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and qualitative description of these problems is the mathematical modeling of processes, i.e., the creation of mathematical models describing these current problems [2–9].

In [2] nonlinear continuous mathematical model of linguistic globalization is considered. Two categories of the world's population are considered: a category that hinders and a category conducive to the dominant position of the English language. With a positive demographic factor of the population, which prevents globalization and the negative demographic factor of the population contributing to globalization, it is shown that the dynamic system describing this process allows for the existence of two topologically not equivalent phase portraits (a stable node, a limit cycle). Under certain restrictions on the parameters of the model, the theorem on the absence of periodic trajectories of the dynamical system is proved and an asymptotically stable equilibrium position (limit cycle) is found. Thus, it is established that complete linguistic globalization is impossible if the demographic factor of the category of the world population contributing to the dominance of the English language is non-positive. Full linguistic globalization is possible only if the demographic factor is positive for the category of the world's population, which contributes to the dominance of the English language and a certain restriction on the parameters of the model associated with the coefficient of assimilation.

In a very brief period of time, three or four decades, mathematical political science has covered a great deal of territory [13]. Mathematical models first became popular in political science in the 1950s, when inferential statistics came into common usage. The 1950s and 1960s are referred as the "behavioral era", because most effort was focused on the detection of empirical patterns in voting behavior and public opinion data. A considerable infrastructure has developed to administer and make available national surveys on a regular basis [13].

The study of complex political phenomena such as parties polarization calls for mathematical models of political systems. These questions, mainly with the use of statistical mathematical models, are considered in the works [1, 10–12].

As you know, the term "bipolar" is associated with the Cold War, when two clear poles stood out in the structure of the international system (the United States of America and the Soviet Union). Each of them had a whole group of satellite states, politically, economically, and ideologically dependent on the these superpowers. The confrontation between the two blocks determined the nature of the system. The bipolar system assumes a certain distribution of power between the poles, but this is not a system of balance of power. It can be hard and soft. With a rigid bipolar system, almost every country is a member of one of two blocs (for example, at the initial stage of the Cold War, until the mid-1950s), and with a soft bipolar system, a significant number of states do not belong to any bloc and are represented separately in the international system (the international system from the second half of the 50-s until the end of the Cold War).

After the collapse of the USSR, and the weakening of the political weight of Russia at the world level, China comes to the fore of political action, which consistently and systematically strengthens its political influence at the world level. Given the military, economic, technological capabilities, and demographic resources of countries and associations, the top two world political players (actors) could be distinguished: the United States and China.

Power structures (generals, KGB, military industrialists, etc.), despite the fact that they were well aware of the economic and technological capabilities of Russia, due to the presence of nuclear and, in general, military potential, could not accept to fall the country out of the top two world players. Apparently, Russian invasion of Ukraine on February 24, 2022 year is directly related to desire of Russia to change the natural course of development of geopolitical influence on the world policy of the two main powers and associations (USA, China).

The political weight at the global level of a country or a union of countries is determined by at least four components:

- Military force (military power, armament, presence of nuclear weapons, availability of biological weapons, modern military equipment, military technology, army, history of successful military operations, etc.);

- Economic opportunities (GDP, energy estimation, sales markets, etc.);
- Technological capabilities (modern digital technology), as well as technology for conducting successful information (disinformation) warfare;
- Demographic resources and mentality of citizens to strengthen the political weight of the country (hybrid wars, soft occupation).

Taking into account the certain experience we have accumulated in mathematical and computer modeling of social processes (information warfare; management models; political elections; assimilation of peoples (languages); language globalization; political conflicts) and the excessive relevance of a quantitative and qualitative description of global political processes taking place against the backdrop of the war between Russia and Ukraine, we propose an innovative method of describing the distribution of political influences (weight) on the world policy of the main players, which involves the creation of adequate new mathematical models and the study of relevant nonlinear dynamic systems.

2. Mathematical model of the bipolar world system of equations

Qualitative analysis of the dynamics of the military (military power, weapons, the presence of nuclear and biological weapons, modern military equipment, military technology, army, history of successful military operations, etc.), economic (economic development rates, new product markets, global GDP growth, etc.), and technological (conducting information wars, propaganda at the scientific level, etc.) capabilities of countries, as well as the mentality and inclination of citizens to increase political weight (influence) of the country on world politics, in the political model of the bipolar world unambiguously leads to a model of the United States and China.

The dynamic system of equations describing the bipolar arrangement of the world, taking into account the two main superpowers (USA, China), has the following form

$$\begin{cases} \frac{dI_1(t)}{dt} = \alpha_1(t)I_1(t) + \beta_1(t)I_1(t)I_2(t), \\ \frac{dI_2(t)}{dt} = \alpha_2(t)I_2(t) + \beta_2(t)I_1(t)I_2(t), \end{cases} \tag{2.1}$$

$$I_1(0) = I_{10},\ I_2(0) = I_{20},,\ I_1, I_2 \in C^1[0;T],\ \alpha_1(t),\ \alpha_2(t),\ \beta_1(t),\ \beta_2(t) \in C[0;T], \tag{2.2}$$

where

- $I_1(t)$ is a function describing the political weight of the United States in world politics at time t;
- $I_2(t)$ is a function describing the political weight of China in world politics at time t;
- $\alpha_1(t), \alpha_2(t)$ are changes in the relative speed of the United States influence and China, respectively, on world politics at time t, respectively, due to the dynamics of exclusively domestic resources (military, economic, technological capabilities, mentality, and inclination of citizens aimed at increasing the political weight of the country on world politics);
- $\beta_1(t), \beta_2(t)$ are the coefficients of political weight, respectively, of the United States and China on world politics at time t, due to their confrontation, expansion or narrowing of their political influence (spheres of influence on third countries, struggle for new sales markets, etc.);
- $[0;T]$ is the mathematical model review period.

The solution normalization condition has the form

$$I_1(t) + I_2(t) = 1,\ \forall t \in C[0;T]. \tag{2.3}$$

In the phase plane of solutions $(O, I_1(t), I_2(t))$, (2.3) is a segment, the hypotenuse of an isosceles right triangle with unit legs. The non-triviality and adequacy of the mathematical model implies finding solutions within a segment (interval, open set) imposing certain restrictions on the coefficients of the dynamic system (2.1),

$$\begin{cases} \alpha_1(t) < 0, \\ \beta_1(t) > 0, \\ \alpha_2(t) > 0, \\ \beta_2(t) < 0, \end{cases} \quad t \in [0; T], \tag{2.4}$$

or

$$\begin{cases} \alpha_1(t) < 0, \\ \beta_1(t) > 0, \\ \alpha_2(t) > 0, \\ \beta_2(t) < 0, \end{cases} \quad t \in [0; t_*], \qquad \begin{cases} \alpha_1(t) > 0, \\ \beta_1(t) < 0, \\ \alpha_2(t) < 0, \\ \beta_2(t) > 0, \end{cases} \quad t \in [t_*; T].$$

From (2.1)-(2.3) you can get the Cauchy problem

$$\begin{cases} \dfrac{dI_1(t)}{dt} = (\alpha_1(t) + \beta_1(t))I_1(t) - \beta_1(t)I_1^2(t), \\ I_1(0) = I_{10}, \end{cases}$$

the exact solution of which has the form

$$I_1(t) = \frac{I_{10} \exp\left[\int\limits_0^t (\alpha_1(\tau) + \beta_1(\tau))d\tau\right]}{1 + I_{10} \int\limits_0^t \exp\left[\int\limits_0^\tau (\alpha_1(s) + \beta_1(s))ds\right] \beta_1(\tau)d\tau}, \tag{2.5}$$

and correspondingly the problem

$$\begin{cases} \dfrac{dI_2(t)}{dt} = (\alpha_2(t) + \beta_2(t))I_1(t) - \beta_2(t)I_2^2(t), \\ I_2(0) = I_{20}, \end{cases}$$

the exact solution of which will be recorded in the following form

$$I_2(t) = \frac{I_{20} \exp\left[\int\limits_0^t (\alpha_2(\tau) + \beta_2(\tau))d\tau\right]}{1 + I_{20} \int\limits_0^t \exp\left[\int\limits_0^\tau (\alpha_2(s) + \beta_2(s))ds\right] \beta_2(\tau)d\tau}. \tag{2.6}$$

In this case, the condition of solutions normalization (2.3) leads to the following limitation

$$\frac{I_{10} \exp\left[\int\limits_0^t (\alpha_1(\tau) + \beta_1(\tau))d\tau\right]}{1 + I_{10} \int_0^t \exp\left[\int\limits_0^\tau (\alpha_1(s) + \beta_1(s))ds\right] \beta_1(\tau)d\tau} + + \frac{I_{20} \exp\left[\int\limits_0^t (\alpha_2(\tau) + \beta_2(\tau))d\tau\right]}{1 + I_{20} \int_0^t \exp\left[\int\limits_0^\tau (\alpha_2(s) + \beta_2(s))ds\right] \beta_2(\tau)d\tau} = 1. \tag{2.7}$$

Naturally, the variable coefficients of the system of equations (2.1), except for conditions (2.4) must satisfy the condition (2.7).

3. Some particular exact solutions

Let's consider some exact solutions of the dynamic system (2.1)-(2.2).

1. If the variable coefficients of the dynamic system (2.1) are

$$\alpha_1(t) = -a\left(I_{20} + \frac{\sin\frac{4\pi t}{T}}{k}\right) < 0, \qquad \alpha_2(t) = a\left(I_{10} - \frac{\sin\frac{4\pi t}{T}}{k}\right) > 0, \quad a > 0,\ k > 0,$$

$$\beta_1(t) = a - \frac{\frac{4\pi\cos\frac{4\pi t}{T}}{Tk}}{\left(I_{10} - \frac{\sin\frac{4\pi t}{T}}{k}\right)\left(I_{20} + \frac{\sin\frac{4\pi t}{T}}{k}\right)} > 0, \quad \beta_2(t) = -a + \frac{\frac{4\pi\cos\frac{4\pi t}{T}}{Tk}}{\left(I_{10} - \frac{\sin\frac{4\pi t}{T}}{k}\right)\left(I_{20} + \frac{\sin\frac{4\pi t}{T}}{k}\right)} < 0,$$

$$\beta_1(t) = -\beta_2(t),$$

then the exact solution of the Cauchy problem (2.1)-(2.2) has the form

$$I_1(t) = I_{10} - \frac{\sin\frac{4\pi t}{T}}{k}, \quad I_2(t) = I_{20} + \frac{\sin\frac{4\pi t}{T}}{k}, \tag{3.1}$$

and $a > 0$, $k > 0$ must be selected the way that

$$k > \frac{1}{I_{10}}, \quad k > \frac{1}{I_{20}}, \quad a > \frac{\frac{4\pi}{Tk}}{(I_{10} - \frac{1}{k})(I_{20} - \frac{1}{k})}.$$

2. If the variable coefficients of the dynamic system (2.1) are

$$\alpha_1(t) = -a\left(I_{20} - \frac{\sin\frac{4\pi t}{T}}{k}\right) < 0, \qquad \alpha_2(t) = a\left(I_{10} + \frac{\sin\frac{4\pi t}{T}}{k}\right) > 0, \quad a > 0,\ k > 0,$$

$$\beta_1(t) = a + \frac{\frac{4\pi\cos\frac{4\pi t}{T}}{Tk}}{\left(I_{10} + \frac{\sin\frac{4\pi t}{T}}{k}\right)\left(I_{20} - \frac{\sin\frac{4\pi t}{T}}{k}\right)} > 0, \quad \beta_2(t) = -a - \frac{\frac{4\pi\cos\frac{4\pi t}{T}}{Tk}}{\left(I_{10} + \frac{\sin\frac{4\pi t}{T}}{k}\right)\left(I_{20} - \frac{\sin\frac{4\pi t}{T}}{k}\right)} < 0,$$

$$\beta_1(t) = -\beta_2(t),$$

then the exact solution of the Cauchy problem (2.1)-(2.2) has the form

$$I_1(t) = I_{10} + \frac{\sin\frac{4\pi t}{T}}{k}, \quad I_2(t) = I_{20} - \frac{\sin\frac{4\pi t}{T}}{k}, \tag{3.2}$$

and $a > 0$, $k > 0$ must be selected the way that

$$k > \frac{1}{I_{10}}, \quad k > \frac{1}{I_{20}}, \quad a > \frac{\frac{4\pi}{Tk}}{\left(I_{10} - \frac{1}{k}\right)\left(I_{20} - \frac{1}{k}\right)}.$$

3. If the variable coefficients of the dynamic system (2.1) are

$$\alpha_2(t) = -\alpha_1(t) > 0, \qquad \alpha_1(t) = \frac{(1-\alpha_1)\frac{4\pi}{Tk}\cos\frac{4\pi t}{T}}{\alpha_1 I_{10} - I_{20} - (1+\alpha_1)\frac{\sin\frac{4\pi t}{T}}{k}} < 0,$$

$$\alpha_1 > 0, \quad k > 0, \qquad 0 > \beta_2(t) = -\alpha_1\beta_1(t),$$

$$\beta_3(t) = \frac{4\pi\cos\frac{4\pi t}{T}}{Tk}\left(I_{10} - I_{20} - 2\frac{\sin\frac{4\pi t}{T}}{k}\right), \qquad \beta_4(t) = \left(I_{10} - \frac{\sin\frac{4\pi t}{T}}{k}\right)\left(I_{20} + \frac{\sin\frac{4\pi t}{T}}{k}\right),$$

$$\beta_5(t) = \alpha_1 I_{10} - I_{20} - (1+\alpha_1)\frac{\sin\frac{4\pi t}{T}}{k}, \qquad \beta_1(t) = -\frac{\beta_3(t)}{\beta_4(t)\beta_5(t)} > 0,$$

$$\alpha_1(0) = \frac{(1-\alpha_1)\frac{4\pi}{Tk}}{\alpha_1 I_{10} - I_{20}} < 0, \qquad \beta_1(0) = -\frac{\frac{4\pi}{Tk}(I_{10} - I_{20})}{I_{10}I_{20}(\alpha_1 I_{10} - I_{20})} > 0,$$

$$\alpha_1 I_{10} - I_{20} < 0,\ 1-\alpha_1 > 0,\ 0 < \alpha_1 < 1,\ k > \frac{1}{I_{20}},$$

then the exact solution of the Cauchy problem (2.1)-(2.2) also has the form (3.1).

4. If the variable coefficients of the dynamic system (2.1) are

$$\alpha_2(t) = -\alpha_1(t) > 0, \qquad \alpha_1(t) = \frac{(-1+\alpha_1)\frac{4\pi}{Tk}\cos\frac{4\pi t}{T}}{\alpha_1 I_{10} - I_{20} + (1+\alpha_1)\frac{\sin\frac{4\pi t}{T}}{k}} < 0,$$

$$\alpha_1 > 0, \quad k > 0, \qquad 0 > \beta_2(t) = -\alpha_1\beta_1(t),$$

$$\beta_6(t) = \frac{4\pi\cos\frac{4\pi t}{T}}{Tk}\left(I_{10} - I_{20} + 2\frac{\sin\frac{4\pi t}{T}}{k}\right), \qquad \beta_7(t) = \left(I_{10} + \frac{\sin\frac{4\pi t}{T}}{k}\right)\left(I_{20} - \frac{\sin\frac{4\pi t}{T}}{k}\right),$$

$$\beta_8(t) = \alpha_1 I_{10} - I_{20} + (1+\alpha_1)\frac{\sin\frac{4\pi t}{T}}{k}, \qquad \beta_1(t) = \frac{\beta_6(t)}{\beta_7(t)\beta_8(t)} > 0,$$

$$\alpha_1(0) = \frac{(-1+\alpha_1)\frac{4\pi}{Tk}}{\alpha_1 I_{10} - I_{20}} < 0, \qquad \beta_1(0) = \frac{\frac{4\pi}{Tk}(I_{10} - I_{20})}{I_{10}I_{20}(\alpha_1 I_{10} - I_{20})} > 0,$$

$$\alpha_1 I_{10} - I_{20} > 0,\ 1-\alpha_1 > 0,\ 0 < \alpha_1 < 1,\ k > \frac{1}{I_{20}},$$

then the exact solution of the Cauchy problem (2.1)-(2.2) also has the form (3.2).

4. Conclusions

In conclusion, we note that in the general case of the mathematical model variable coefficients in quadratures, an exact solution was found as (2.5) and (2.6) of the dynamic system (2.1)-(2.2). At some values of the variable coefficients of the model, accurate periodic solutions of the dynamic system were found, describing the process of alternating dominance of the United States and China's political weight. In reality, whether or not China can bypass the political weight of the United States in the future depends on many factors, including constant economic growth, increased military power, and the involvement of allied countries in its political orbit.

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