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Quantified neutrosophic set (Q^tNS)-based MCDM algorithms for sustainable material selection for anti-microbial bio-fabricated textile manufacturing

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Abstract

This paper proposes a modified structure for the neutrosophic set called the Quantified Neutrosophic Set (Q^tNS) with a parameterized setting. Unlike conventional approaches, the Q^tNS provides a quantified environment for the indeterminacy by its dependence on truthness and falsity components. This innovative approach quantifies the uncertainty and improves the assessment process via expert-guided opinions, customising it according to the specific situations in real-world decision-making scenarios. Some Q^tNS operations along with useful characteristics are addressed. Furthermore, two algorithms, Q^tNSUI and Q^tNSAO, are developed for the proposed operations of union, intersection, AND, and OR based on Q^tNS. In the world of sustainable materials, biofabricated textiles are making progress. The MCDM methods based on Q^tNS are developed for material preferences in the industry of biofabricated textiles, specifically with anti-microbial properties. The study's main purpose is to develop a novel technique to quantify and reduce the predicted uncertainties in the material preference problem for antimicrobial biofabricated textile manufacturing. For eco-conscious decision-making, our work would provide an optimised environment at the industrial level, especially for ecologically conscious textile industries, for enhanced and sustainable selection with greater accuracy.

Keywords: Neutrosophic set, quantified neutrosophic set (Q^tNS), Q^tNSUI-algorithm, Q^tNSAO-algorithm, optimization, decision making, bio-fabricated textile.

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1. Introduction

Fuzzy sets [53] provided a method for dealing with uncertainty by allowing components to belong to a set with various degrees of membership. This notion has been expanded to interval-valued fuzzy sets (IVFS) [54], where items are associated with intervals, representing a variety of potential membership degrees. Intuitionistic fuzzy sets (IFSs) [4] go a step higher by including non-membership and reluctance. By connecting intervals with both membership and non-membership degrees, IVIFS [5] easily incorporates the two concepts. Smarandache's concept of neutrosophic sets adds a new level to this environment by adding indeterminacy to truth and falsity degrees. This approach gracefully integrates uncertainty,

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vagueness, and irregularity in a single framework, making it appropriate for complicated real-world scenarios. Neutrosophic sets reflect uncertainty more richly, allowing for better informed decision-making and various information analyses in a variety of domains. IV neutrosophic sets (IVNSs) expand upon the foundation of neutrosophic sets by incorporating intervals into the truth, indeterminacy, and falsity degrees. This extension provides a more flexible and nuanced representation of uncertainty, allowing for a range of possible values for each component [7, 37, 38]. MCDM is a method for taking the best option from a number of choices and circumstances. There are several complicated DM issues in the fields of material sciences, social science, economics, engineering, and environmental science that cannot be completely described by traditional mathematical approaches due to the presence of numerous sorts of uncertainties. Others employed hybrid methodologies to handle data, like the INVAR method [19] or the CODAS-COMET method [47]. Since then, certain well-known mathematical models have been developed to cope with the uncertainty that arises in such DM challenges, including fuzzy set theory [53], IFS-theory [4, 5], IVIFS-theory [5, 30], hesitant fuzzy set theory [32, 39], soft set theory [26], fuzzy soft set theory [23], and so on. Triangular fuzzy integers are employed in a fuzzy version of a simple best-worst approach in [3] is a good instance of this. An innovative approach of best-worst method for the evaluation of performance indicators is developed in [34]. The enhanced concept of structured element in neutrosophic sets was developed in [24] named as neutrosophic structured element and applied to solve MADM problems. Also, the embedded algebraic structures of soft members and soft elements are introduced by saeed et al. [9]. Several DM algorithmic strategies based on hypersoft set parameters in the fuzzy, IF, and neutrosophic set contexts have been presented in [43]. An innovative MCDM methodology implemented on plithogenic hypersoft sets in a fuzzy neutrosophic environment is presented in [1]. [57] provide some basic operations on interval-valued neutrosophic sets with the combination of hypersoft sets and their attributes.

In order to resolve the DM problem, a mathematical analysis in [33] of the neutrosophic soft set is performed by the use of generalised fuzzy TOPSIS. There are also some basics on interval-valued neutrosophic sets, along with the characteristics they possess and tangent similarity measures, in the context of hypersoft sets of single and multi-valued neutrosophic sets in the literature [36, 57]. To better deal with the decision-making process, uncertainty research often involves generalised techniques, which include Dempster-Shafer evidence theory (DSET) [51] or quantum evidence theory (QET) [50]. Similarly, other approaches include entropy-based techniques [48] and distance measures [49]. Distance and similarity measures for neutrosophic hypersoft sets have been determined in [17] with use for selecting a suitable location for systems of the management of solid waste. The challenge of supplier selection is an MCDM problem. The majority of the time, expert data is used in the process of choosing suppliers, and the opinion of experts consists of imprecise and uncertain data. Recent advancements [6, 25, 56] under the neutrosophic environment with uncertain conditions initiated a significant role at industrial level by developing various strategie in decision making problems. To solve this problem, the study [42, 45] proposed a fuzzy model and an integrated fuzzy MCDM model, including the Fuzzy Analytic Hierarchy Process (FAHP), Fuzzy Operational Competitiveness Rating (Fuzzy OCRA), and supply chain operations reference model (SCOR). A fuzzy MCDM model was built for sustainable supplier ranking and selection in the garment industry applying triple bottom line techniques, as well as sewing machine procurement for a textile workplace utilising the method of EDAS [41, 46]. Some of the basic features, like subset, empty set, absolute set, and complement with union, intersection, AND, OR operations, dealt with the possibility of hypersoft sets, and methods based on AND/OR operations are discussed and evaluated in [44]. [20] merged two possibility neutrosophic soft sets and gave the notion of AND-product and OR-product operations with a DM technique based on the AND-product operation known as the PNS-decision-making method.

Textiles are pervasive and serve an important role in human culture. Clothing microbiology and the influence and interaction of garments with human skin microbiota [35] have lately been explored. Natural antimicrobial agent coatings on textiles or fabrics date back to ancient Egypt, when spices and herbal coatings on cloths were used to produce mummy wraps. Traditionally, bamboo fibres containing an antibacterial component known as 'Bamboo-kun' were used for home structure by the Chinese. Bamboo fibres have also been studied for their natural antibacterial and antifungal activities, which are mediated by dendrocin and 2-6-dimethoxy-p-benzoquinone [22]. During the Second World War, antibiotics were widely used; at the same time, antimicrobials to prevent textile rot were in high demand. Tents, tarpaulins, and vehicle coverings have to be protected from microbial assault during heavy rain and snow, which would degrade fibres and raise the risk of infection. Several military materials were treated with antimony salts, copper, and a combination of chlorinated waxes to protect them against microbial colonisation and boost their longevity [31]. These treatments not only stiffened the fabrics but also gave them a unique odour. Initially, the side effects of these antimicrobials were not taken into account; however, more attention was paid to the negative effects of these substances on the environment and health. After the release of Rachel Carson's book Silent Spring in 1962, the notion of safer antimicrobial chemicals and fabrics was born. Ecologists, scientists, industrialists, and chemists collaborated to create eco-friendly antibiotics [10].

Antimicrobial fabrics are particularly useful in today's hospitals, environments, and areas that are prone to harmful bacteria. The garments worn by patients, healthcare staff, and physicians may contain a large number of germs that can readily be passed from one person to another. When it comes to limiting the transmission of pathogenic germs, commercial potential for antimicrobial materials abounds [27]. Antimicrobial textiles are classified according to their antimicrobial specificity, such as antibacterial, antifungal, or antiviral. Several antimicrobial fabrics may also function against bacteria, fungus, and viruses at the same time. Some compounds, known as antimicrobials, can be used to target a wide spectrum of bacteria [16]. Such fabric is widely required in common public areas such as hotels, restaurants, or trains; for example, the towel used to wipe up fluids, curtains, and carpet might be a source of infection. There are also notable unmet requirements for odour management, which is a growing study subject in this discipline. The cloth may contain many pathogenic germs that can spread from an affected individual to others.

Continuous laundering of garments is the only practical and efficient technique to reduce the microbiological load from textiles; however, this is not achievable in hospitals with continuous shifts. On the other hand, producing antimicrobial textiles is another technique to decrease the spread of microbial illness from one person to another via textile. These antimicrobial fabrics may also be advantageous for people involved in sanitary work and sewage treatment, where there is a significant danger of infection. Surface modification of textiles has been done using electrospinning, nanotechnologies, plasma treatment, polymerization, microencapsulation, and sol-gel methods to impart certain unique functional features to textiles, such as water repellency, flame retardancy, and antibacterial activity [28].

Antimicrobial winter apparel is becoming more popular since these garments are rarely laundered and are rarely exposed to sunshine. These garments are often stored for a longer period of time, which may allow bacteria to develop; thus, antimicrobial fabric may be a good alternative. Similarly, antimicrobial fabrics may be beneficial in locations where non-plastic bags are used. Food packaging made of biodegradable materials is better for the environment and does not influence food characteristics; nonetheless, antimicrobial coating in such wrappers is needed to inhibit the growth of pathogenic and food spoilage bacteria. The following industries, in particular, demand antimicrobial textiles as well as appealing colour, pattern, and design combinations.

- Clothing: caps, coats, sanitary pads, sportswear, bottoms, winter wear.
- Commercial: Rugs, covering for seats, windows, vehicles, etc.; cleaning vipes; shelter; costumes.
- Plaster, headphones, cleansers, respirators, lab coats, and protective kits.
- Sleeping arrangements, the flooring, concealment, window coverings, mop, pillows, and napkins.

Antimicrobial textiles are naturally active textiles that can kill or prevent the formation of microorganisms. [15] studies the use of different synthetic and natural antibacterial compounds in the production of antimicrobial textiles. Fabrics that are antibacterial, antifungal, or antiviral have also been discussed. Air filters, food packaging, health care, hygiene, pharmaceuticals, athletics, storage, ventilation, and water purification systems are just a few applications for antimicrobial textiles. Over the last several years, there has been an upsurge in both public awareness and commercial possibilities for antimicrobial materials. Not only are antimicrobial properties important for fashionable clothing, but so are durability, colour, patterns, and design; as a result, numerous commercial businesses are increasingly focusing on such materials.

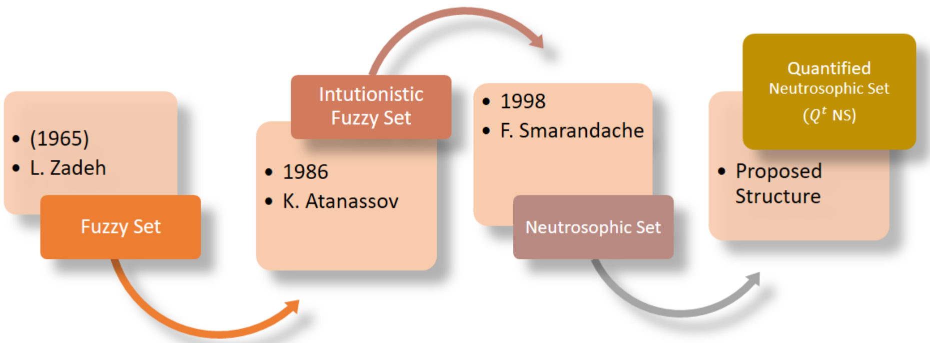

Figure 1: Illustrative diagram of existing structures.

1.1. *Main objectives*

The main objectives for this paper are as follows.

• To develop a structure that offers an advanced framework for neutrosophic sets, representing controlled and reduced uncertainty by quantifying neutrosophic components through adjustable parameters. The proposed structure provides a more nuanced and accurate depiction of the inherent uncertainties in decision-making processes. Figure 1 exhibits the fuzzy structure and its advancements.

• To introduce the adjustment mechanism in the neutrosophic set, using the parameters that provide the fine-tuned degrees of existent ship, indeterminacy, and non-existent ship. This feature is particularly valuable in real-life applications where decisions are sensitive to varying levels of uncertainty.

• To develop new structures for the operations of union, intersection, AND, and OR product between the collection of Q^tNSs based on the techniques to reduce indeterminacy.

• To establish MCDM algorithms based on union, intersection, AND, and OR products based on Q^tNSs and apply them to select sustainable materials for anti-microbial bio-fabricated textile manufacturing.

1.2. *Research gap and significant contributions*

Several studies in the textile and garment industries have used various MCDM methods using fuzzy set frameworks, as summarised in Table 1. However, there is a research gap in the application of neutrosophic MCDM approaches evolving the technique of uncertainty reduction for deciding the sustainable materials for the anti-microbial textiles manufacturing industry.

- The uniqueness of our proposed neutrosophic set structure lies in its ability to precisely control indeterminacy while making decisions about real-world problems. Our approach quantifies and controls the indeterminacy factor through its dependency on truthfulness and falsity components. We achieve this by developing parameterized modifications in the structure of the neutrosophic set.

- This innovation is vital in the context of sustainable material choices, as it enables us to navigate complex decisions with clarity and accuracy. In contrast, standard neutrosophic sets lack this specific mechanism to reduce or control indeterminacy, making our approach a valuable tool for industries in different fields, ensuring better-informed and more impactful decisions.

- We focus on anti-microbial bio-fabricated textiles because there are few resources on selecting an appropriate material for bio-fabricated textiles. According to the articles on selecting textile materials, our research provides the most thorough collection of criterion selection approaches.

- The Q^tNS MCDM approach suggested in this paper is very adaptable and accurate as it involves a controlled environment for indeterminacy and may be used in any other MCDM situations as well.

Table 1: Existing literature.

S.No.	Structure	Method	Field	References
i	Fuzzy	ANFIS	Garment & Textile	[55]
ii	Fuzzy	Fuzzy TOPSIS	Textile	[52]
iii	Fuzzy	Fuzzy TOPSIS	Garment	[18]
iv	Fuzzy	Fuzzy Inference System	Garment	[2]
v	Fuzzy	FST,AHP	Garment	[14]
vi	Fuzzy	AHP/DEA	Textile	[11]
vii	Fuzzy	DEA/PCA/VIKOR	Garment	[21]
viii	Intuitionistic Fuzzy	Topsis	Textile	[29]
ix	Fuzzy	Intuitionistic fuzzy TOPSIS	Textile	[13]
x	Q^tNS	Q^tNSUI, Q^tNSAO	Bio-fabricated Textile	**Proposed approach**

The proposed research has been organised into five sections. The Section 2 consists of important definitions, and Section 3 contains the proposed Q^tNS along with its major properties and operations. Section 4 examines the Q^tNSUI and Q^tNSAO MCDM algorithms as well as their application in the selection of bio-fabricated textile materials, whereas the conclusion and some potential future plans are discussed in Section 5.

2. Preliminaries

This section introduces important concepts that will aid in gaining an understanding of the research. Throughout this study, we will assume that $[\tilde{I}]$ stands for the closed interval of $[0,1]$, and the symbols $\succeq, \preceq, =$ of the elements α_1 and α_2 of $[\tilde{I}]$ are defined as:

$$\grave{n}_1 \succeq \grave{n}_2 \Leftrightarrow \grave{n}_1 \geqslant \grave{n}_2 \quad \text{and} \quad \grave{n}_1 \preceq \grave{n}_2 \Leftrightarrow \grave{n}_1 \leqslant \grave{n}_2.$$

Similarly, we can define $\grave{n}_1 = \grave{n}_2 \Leftrightarrow \grave{n}_1 \succeq \grave{n}_2$ as well as $\grave{n}_1 \preceq \grave{\grave{n}}_2$.

Definition 2.1 ([53, 54]). Suppose $\tilde{U}$ be a non-empty set. A fuzzy set in set $\tilde{U}$ is a function $\tilde{F} : \tilde{U} \to [\tilde{I}]$ and can be expressed as $\tilde{F} = \{< \sigma, \xi_{\tilde{F}}(\sigma) > | \sigma \in \tilde{U}\}$, where $\xi_{\tilde{F}}(\sigma)$ is referred to as the existent ship/membership value of σ in $\tilde{F}$ and $\xi_{\tilde{F}}(\sigma) \in [\tilde{I}]$. A function $\tilde{F} : \tilde{U} \to [\tilde{I}]$ is called an **IVFS** in $\tilde{U}$. The element $\tilde{F} = \left[\tilde{f}^-(\sigma), \tilde{f}^+(\sigma)\right]$ for every $\tilde{F} \in [\tilde{I}]^{\tilde{U}}$ and $\sigma \in \tilde{U}$, is the existent ship value of an element σ to the set $\tilde{F}$. The IVFS simply can be presented as $\tilde{F} = \left[\tilde{f}^-, \tilde{f}^+\right]$.

Definition 2.2 ([4]). Let $I_{\tilde{F}} =< \mu_{I_{\tilde{F}}}(\sigma), \nu_{I_{\tilde{F}}}(\sigma) >$ be an IFS of $\tilde{U}$ with the values of existent ship and non-existent ship denoted by $\mu_{I_{\tilde{F}}}(\sigma)$ and $\nu_{I_{\tilde{F}}}(\sigma)$, respectively along with the condition $0 \preceq \mu_{I_{\tilde{F}}}(\sigma) + \nu_{I_{\tilde{F}}}(\sigma) \preceq 1$.

Definition 2.3. In the crisp set $\tilde{U}$, the sort of structure classified as a neutrosophic set ([37]) is

$$\grave{N} = \{\langle \mu(\sigma), \lambda(\sigma), \nu(\sigma)\rangle \mid \sigma \in \tilde{U}\},$$

where μ, λ, and ν are the existent ship value, indeterminate value and non-existent ship value of σ w.r.t $\grave{N}$ in $\tilde{U}$ and $0 \preceq \mu(\sigma) + \lambda(\sigma) + \nu(\sigma) \preceq 3$.

Definition 2.4 ([7])**.** For two neutrosophic sets $\grave{N}_A =< \mu_A(\sigma), \lambda_A(\sigma), \nu_A(\sigma) >$ and $\grave{N}_B =< \mu_B(\sigma), \lambda_B(\sigma), \nu_B(\sigma) >$, the union $C = \grave{N}_A \cup \grave{N}_C =< \mu_C(\sigma), \lambda_C(\sigma), \nu_C(\sigma) >$ is,

$$\mu_C(\sigma) = \max(\mu_A(\sigma), \mu_B(\sigma)), \;\; \lambda_C(\sigma) = \min(\lambda_A(\sigma), \lambda_B(\sigma)), \;\; \nu_C(\sigma) = \min(\nu_A(\sigma), \nu_B(\sigma)), \;\; \text{for all} \;\; \sigma \in \tilde{U}.$$

Definition 2.5. [7] For two neutrosophic sets, the intersection $C = \grave{N}_A \cap \grave{N}_B$ is given by

$$\mu_C(\sigma) = \min(\mu_A(\sigma), \mu_B(\sigma)), \;\; \lambda_C(\sigma) = \max(\lambda_A(\sigma), \lambda_B(\sigma)), \;\; \nu_C(\sigma) = \max(\nu_A(\sigma), \nu_B(\sigma)), \;\; \text{for all} \;\; \sigma \in \tilde{U}.$$

Definition 2.6. For the family of a fuzzy set $\tilde{F}_i = \{< \sigma, \xi_{\tilde{F}_i}(\sigma) >\mid \sigma \in \tilde{U}\}$ in $\tilde{U}$, where I_n stands for the index set and $i \in I_n$, The definitions of join and meet represented by $(\vee)$ and $(\wedge)$, respectively, are as

$$\vee_{i\in I_n} \tilde{F}_i = (\vee_{i\in I_n} \xi_{\tilde{F}_i})(\sigma) = \max\{\xi_{\tilde{F}_i} \mid i \in I_n\} \quad \text{and} \quad \wedge_{i\in I_n} \tilde{F}_i = (\wedge_{i\in I_n} \xi_{\tilde{F}_i})(\sigma) = \min\{\xi_{\tilde{F}_i} \mid i \in I_n\},$$

$\forall\ \sigma \in \tilde{U}$.

3. Quantified neutrosophic set (Q^tNS)

This section presents the notion of Q^tNS, a modified structure of smarandache's neutrosophic set ([37]) with some operations and properties as follows.

Definition 3.1 (Quantified neutrosophic set)**.** A neutrosophic set $\grave{N} = \{\langle \mu_{\grave{N}}(\sigma), \lambda_{\grave{N}}(\sigma), \nu_{\grave{N}}(\sigma)\rangle \mid \sigma \in \tilde{U}\} = \langle \mu, \lambda, \nu\rangle$ in a non empty crisp set $\tilde{U}$ is called an Q^tNS represented by $\grave{N}^{qt}$, where qt represents the abbreviated form of quantification having the form

$$\grave{N}^{qt} = \{< \sigma, \mu_{\grave{N}^{qt}}(\sigma), \lambda_{\grave{N}^{qt}}(\sigma), \nu_{\grave{N}^{qt}}(\sigma) >\mid \sigma \in \tilde{U}\} = \left\langle \mu^{qt}, \lambda^{qt}, \nu^{qt} \right\rangle$$

with two independent components that are $\mu^{qt}(\sigma)$ and $\nu^{qt}(\sigma)$ along one dependent component $\lambda^{qt}(\sigma)$ $\forall$ $\sigma \in \tilde{U}$. These components of existent ship, indeterminacy, and non-existent ship are quantified through the adjustments along with the parameters α and β as

$$\mu^{qt}(\sigma) = \max\{\mu(\sigma), \alpha\}, \quad \lambda^{qt}(\sigma) = \min\{\lambda(\sigma), (\alpha+\beta)/2\}, \quad \nu^{qt}(\sigma) = \max\{\nu(\sigma), \beta\},$$

where μ^{qt} is the existent ship value function, λ^{qt} is an indeterminate value function, and ν^{qt} is a non-existent ship value function with the condition that $0 \preceq \mu^{qt}(\sigma) + \lambda^{qt}(\sigma) + \nu^{qt}(\sigma) \preceq 3$.

The dependency of indeterminate value function on existent ship and non-existent ship value functions is intertwined with the selection of parameters, as these parameters define the rules and thresholds governing the adjustment process. The chosen values of α and β shape the nature of this dependency and contribute to the overall effectiveness and applicability of our quantified neutrosophic set structure.

Remark 3.2. For this approach, the values of α, β, and $(\alpha+\beta)/2$ are always from the closed interval of $[\tilde{I}]$.

3.1. Explanatory example

Consider a decision-making scenario where we evaluate a project's success.

Parameterized quantification of existent ship value function with α

- Initial project success rate: 0.6.

- Alpha (α) at 0.8 emphasizes preserving the existent ship.
- The perimeterically adjusted existent ship becomes $\max(0.6, 0.8) = 0.8$, reflecting a positive adjustment towards a more optimistic view.

Parameterized quantification of non-existent ship value function with β

- Misleading indicators suggest a project failure rate of 0.2.
- Beta (β) at 0.3 indicates a conservative approach to non-existent ship value adjustment.
- The adjusted non-existent ship value becomes $\max(0.2, 0.3) = 0.3$, reflecting a slight adjustment with lower value of β towards a more conservative representation.

Quantified reduction of indeterminate value function:

- Now, let's consider indeterminacy. With α and β influencing the indeterminacy component, if there's indeterminacy about the project's success rate, say around 0.7, the adjustment of indeterminacy would depend on the interplay between the parameters α and β. Here, with a higher $\alpha = 0.8$ and a lower$\beta = 0.3$, the indeterminacy is reduced to 0.5, i.e., towards existent ship, emphasizing a more optimistic uncertainty range aligned with the parameterically quantified existent ship value.
- The final indeterminacy value is determined, ensuring that the overall sum of existent ship, indeterminacy, and non-existent ship components adheres to the neutrosophic set conditions.

3.1.1. *Selection of parameters* α *and* β *for* Q^tNS

The selection of α and β values are taken from the interval $[0, 1]$ and it is not a one-time decision but involves an iterative refinement process. As the dependency of indeterminacy on truthness and falsity components is a dynamic aspect of our structure, the values of α and β are adjusted iteratively based on ongoing analyses, feedback, and evolving insights. The iterative refinement acknowledges the dynamic nature of research, allowing for gradual enhancements to the model's precision, flexibility, and applicability. The parameters α and β, and the interplay of these parameters influence not only existent ship and non-existent ship adjustments but also the controlled and reduced representation of indeterminacy, allowing decision-makers to enhance the accuracy of the decision in the any real life decision problem.

3.2. *Some operations and properties on the* Q^tNS

This section will introduce some basic operations on the proposed structure of Q^tNS. Let $\grave{N}_A^{qt}$ and $\grave{N}_B^{qt}$ be two Q^tNSs as $\grave{N}_A^{qt} = \langle \mu_A^{qt}, \lambda_A^{qt}, \nu_A^{qt} \rangle$ and $\grave{N}_B^{qt} = \langle \mu_B^{qt}, \lambda_B^{qt}, \nu_B^{qt} \rangle$. Then for all $\sigma \in \tilde{U}$, there are the following relations.

(i) Containment $\grave{N}_A^{qt} \subseteq \grave{N}_B^{qt}$ shows $\grave{N}_A^{qt}$ is contained in $\grave{N}_B^{qt}$ iff $\mu_A^{qt}(\sigma) \preceq \mu_B^{qt}(\sigma)$, $\lambda_A^{qt}(\sigma) \preceq \lambda_B^{qt}(\sigma)$, and $\nu_A^{qt}(\sigma) \succeq \nu_B^{qt}(\sigma)$.

(ii) Equality $\grave{N}_A^{qt} = \grave{N}_B^{qt}$ is equal iff $\grave{N}_A^{qt} \subseteq \grave{N}_B^{qt}$ and $\grave{N}_B^{qt} \subseteq \grave{N}_A^{qt}$.

Theorem 3.3. $\grave{N}_A^{qt} \subseteq \grave{N}_B^{qt} \leftrightarrow \bar{C}(\grave{N}_B^{qt}) \subseteq \bar{C}(\grave{N}_A^{qt})$.

Proof. $\grave{N}_A^{qt} \subseteq \grave{N}_B^{qt} \Leftrightarrow \mu_A^{qt}(\sigma) \preceq \mu_B^{qt}(\sigma)$, $\lambda_A^{qt}(\sigma) \preceq \lambda_B^{qt}(\sigma)$, and $\nu_A^{qt}(\sigma) \succeq \nu_B^{qt}(\sigma) \Leftrightarrow \nu_B^{qt}(\sigma) \preceq \nu_A^{qt}(\sigma)$, $1 - \lambda_B^{qt}(\sigma) \preceq \lambda_A^{qt}(\sigma)$, and $\mu_B^{qt}(\sigma) \succeq \mu_A^{qt}(\sigma)$. □

The operational laws for two Q^tNSs $\grave{N}_A^{qt}$ and $\grave{N}_B^{qt}$ are introduced as follows.

(iii) Addition: $$\grave{N}_A^{qt} \oplus \grave{N}_B^{qt}(\sigma) = \left\{ \begin{array}{l} \langle \sigma, \mu_A^{qt}(\sigma) + \mu_B^{qt}(\sigma) - \mu_A^{qt}(\sigma)\mu_B^{qt}(\sigma), \\ \lambda_A^{qt}(\sigma)\lambda_B^{qt}(\sigma), \nu_A^{qt}(\sigma)\nu_B^{qt}(\sigma) \rangle \mid \sigma \in \tilde{U} \end{array} \right\}.$$

(iv) Multiplication: $\grave{N}^{qt}_A \otimes \grave{N}^{qt}_B(\sigma) = \left\{ \begin{array}{l} \langle \sigma, \mu^{qt}_A(\sigma)\mu^{qt}_B(\sigma), \\ \lambda^{qt}_A(\sigma) + \lambda^{qt}_B(\sigma) - \lambda^{qt}_A(\sigma)\lambda^{qt}_B(\sigma), \\ \nu^{qt}_A(\sigma) + \nu^{qt}_B(\sigma) - \nu^{qt}_A(\sigma)\nu^{qt}_B(\sigma)\rangle \mid \sigma \in \tilde{U} \end{array} \right\}.$

(v) Scaler multiplication: $\delta\grave{N}^{qt}_A = \left\langle 1-(1-\mu^{qt}_A)^\delta, (\lambda^{qt}_A)^\delta, (\nu^{qt}_A)^\delta \right\rangle$, where $\delta > 0$.

(vi) Power of $\grave{N}^{qt}_A$: $(\grave{N}^{qt}_A)^\delta = \left\langle (\mu^{qt}_A)^\delta, 1-(1-\lambda^{qt}_A)^\delta, 1-(1-\nu^{qt}_A)^\delta \right\rangle$.

(vii) Subtraction: $\grave{N}^{qt}_A \ominus \grave{N}^{qt}_B(\sigma) = \left\{ \langle \sigma, \frac{\mu^{qt}_A(\sigma)-\mu^{qt}_B(\sigma)}{1-\mu^{qt}_B(\sigma)}, \frac{\lambda^{qt}_A(\sigma)}{\lambda^{qt}_B(\sigma)}, \frac{\nu^{qt}_A(\sigma)}{\nu^{qt}_B(\sigma)}\rangle \mid \sigma \in \tilde{U} \right\}$, which is valid under the conditions $\grave{N}^{qt}_A \succeq \grave{N}^{qt}_B$, $\mu^{qt}_B(\sigma) \neq 1, \lambda^{qt}_B(\sigma) \neq 0$, and $\nu^{qt}_B(\sigma) \neq 0$.

(viii) Division $\grave{N}^{qt}_A \oslash \grave{N}^{qt}_B(\sigma) = \left\{ \langle \sigma, \frac{\mu^{qt}_A(\sigma)}{\mu^{qt}_B(\sigma)}, \frac{\lambda^{qt}_A(\sigma)-\lambda^{qt}_B(\sigma)}{1-\lambda^{qt}_B(\sigma)}, \frac{\nu^{qt}_A(\sigma)-\nu^{qt}_B(\sigma)}{1-\nu^{qt}_B(\sigma)}\rangle \mid \sigma \in \tilde{U} \right\}$, which is valid under the conditions $\grave{N}^{qt}_B \succeq \grave{N}^{qt}_A$, $\mu^{qt}_B(\sigma) \neq 0, \lambda^{qt}_B(\sigma) \neq 1$, and $\nu^{qt}_B(\sigma) \neq 1$.

(ix) Difference: the difference of two Q^{t}NSs $\grave{N}^{qt}_A$ and $\grave{N}^{qt}_B$, written as $C = \grave{N}^{qt}_A \setminus \grave{N}^{qt}_B$, whose components are related to those of $\grave{N}^{qt}_A$ and $\grave{N}^{qt}_B$ by

$$\mu^t_C(\sigma) = \min(\mu^{qt}_A(\sigma), \nu^{qt}_B(\sigma)), \quad \lambda^t_C(\sigma) = \min(\lambda^{qt}_A(\sigma), 1-\lambda^{qt}_B(\sigma)), \quad \nu^t_C(\sigma) = \max(\nu^{qt}_A(\sigma), \mu^{qt}_B(\sigma)),$$

for all $\sigma \in \tilde{U}$.

For the family of Q^{t}NSs $\grave{N}^{qt}_i = \{< \sigma, \mu^{qt}_i(\sigma), \lambda^{qt}_i(\sigma), \nu^{qt}_i(\sigma) >\mid \sigma \in \tilde{U}\} =< \mu^{qt}_i, \lambda^{qt}_i, \nu^{qt}_i >$ for $i \in I_n$, the proposed structures of $\grave{N}^{qt}_i$-union, $\grave{N}^{qt}_i$-intersection, $\grave{N}^{qt}_i$-AND product, and $\grave{N}^{qt}_i$-OR product are, respectively, defined as follows.

Definition 3.4 (Union operator). $\underset{i\in I_n}{\cup} \grave{N}^{qt}_i = \left\{ \left\langle \eta; (\vee_{i\in I_n}\mu^{qt}_i)(\eta), [\vee(\lambda^{qt}_i(\eta))]^n, (\wedge_{i\in I_n}\nu^{qt}_i)(\eta) \right\rangle \mid \eta \in \tilde{U} \right\}.$

Definition 3.5 (Intersection operator). $\underset{i\in I_n}{\cap} \grave{N}^{qt}_i = \left\{ \left\langle \eta; (\wedge_{i\in I_n}\mu^{qt}_i)(\eta), [\wedge(\lambda^{qt}_i(\eta))]^n, (\vee_{i\in I_n}\nu^{qt}_i)(\eta) \right\rangle \mid \eta \in \tilde{U} \right\}.$

Definition 3.6 (AND-product operator). $\underset{i\in I_n}{\triangle} \grave{N}^{qt}_i = \left\{ \left\langle \eta; (\wedge_{i\in I_n}\mu^{qt}_i)(\eta), [\vee(\lambda^{qt}_i(\eta))]^n, (\vee_{i\in I_n}\nu^{qt}_i)(\eta) \right\rangle \mid \eta \in \tilde{U} \right\}.$

Definition 3.7 (OR-product operator). $\underset{i\in I_n}{\triangledown} \grave{N}^{qt}_i = \left\{ \left\langle \eta; (\vee_{i\in I_n}\mu^{qt}_i)(\eta), [\wedge(\lambda^{qt}_i(\eta))]^n, (\wedge_{i\in I_n}\nu^{qt}_i)(\eta) \right\rangle \mid \eta \in \tilde{U} \right\}.$

3.2.1. Numerical explanation

Assume there are three Q^{t}NSs, as shown by the numerical representation in Table 2 and the $\grave{N}^{qt}_i$-union, $\grave{N}^{qt}_i$-intersection, $\grave{N}^{qt}_i$-AND product, and $\grave{N}^{qt}_i$-OR product of these three sets are represented by the Tables 3, 4, and 5,

$$\begin{aligned} \grave{N}^{qt}_1 &= \{< \sigma, \mu^{qt}_1(\sigma), \lambda^{qt}_1(\sigma), \nu^{qt}_1(\sigma) >\mid \sigma \in \tilde{U}\} =< \mu^{qt}_1, \lambda^{qt}_1, \nu^{qt}_1 >, \\ \grave{N}^{qt}_2 &= \{< \sigma, \mu^{qt}_2(\sigma), \lambda^{qt}_2(\sigma), \nu^{qt}_2(\sigma) >\mid \sigma \in \tilde{U}\} =< \mu^{qt}_2, \lambda^{qt}_2, \nu^{qt}_2 >, \\ \grave{N}^{qt}_3 &= \{< \sigma, \mu^{qt}_3(\sigma), \lambda^{qt}_3(\sigma), \nu^{qt}_3(\sigma) >\mid \sigma \in \tilde{U}\} =< \mu^{qt}_3, \lambda^{qt}_3, \nu^{qt}_3 > . \end{aligned}$$

Table 2: Numerical Representation of $\grave{N}^{qt}_i$ for $i = 1, 2, 3$.

$\grave{N}^{qt}_i$	$\mu^{qt}(\sigma)$	$\lambda^{qt}(\sigma)$	$\nu^{qt}(\sigma)$
$\grave{N}^{qt}_1$	0.5	0.3	0.1
$\grave{N}^{qt}_2$	0.9	0.2	1.0
$\grave{N}^{qt}_3$	0.6	0.4	0.7

Table 3: Tabular representation of $\grave{N}^{qt}_i$-union and $\grave{N}^{qt}_i$-intersection.

	$\mu^{qt}(\sigma)$	$\lambda^{qt}(\sigma)$	$\nu^{qt}(\sigma)$
$\underset{i\in I_3}{\cup} \grave{N}^{qt}_i$	0.9	0.064	0.1
$\underset{i\in I_3}{\cap} \grave{N}^{qt}_i$	0.5	0.008	1.0

Table 4: Tabular representation of $\grave{N}_i^{qt}$-AND product.

$\underset{i\in I_3}{\triangle}\grave{N}_i^{qt}$	$\mu^{qt}(\sigma)$	$\lambda^{qt}(\sigma)$	$\nu^{qt}(\sigma)$
$\tilde{c}_{11}$	0.5	0.027	0.1
$\tilde{c}_{12}$	0.5	0.027	1.0
$\tilde{c}_{13}$	0.5	0.064	0.7
$\tilde{c}_{21}$	0.5	0.027	1.0
$\tilde{c}_{22}$	0.9	0.008	1.0
$\tilde{c}_{23}$	0.6	0.064	1.0
$\tilde{c}_{31}$	0.5	0.064	0.7
$\tilde{c}_{32}$	0.6	0.064	1.0
$\tilde{c}_{33}$	0.6	0.064	0.7

Table 5: Tabular representation of $\grave{N}_i^{qt}$-OR product.

$\underset{i\in I_3}{\bigtriangledown}\grave{N}_i^{qt}$	$\mu^{qt}(\sigma)$	$\lambda^{qt}(\sigma)$	$\nu^{qt}(\sigma)$
$\tilde{c}_{11}$	0.5	0.027	0.1
$\tilde{c}_{12}$	0.9	0.008	0.1
$\tilde{c}_{13}$	0.6	0.027	0.1
$\tilde{c}_{21}$	0.9	0.008	0.1
$\tilde{c}_{22}$	0.9	0.008	1.0
$\tilde{c}_{23}$	0.9	0.008	0.7
$\tilde{c}_{31}$	0.6	0.027	0.1
$\tilde{c}_{32}$	0.9	0.008	0.7
$\tilde{c}_{33}$	0.6	0.064	0.7

3.2.2. Neutrosophic set properties for proposed operators based on Q^tNS

Some of the properties with their respective mathematical proofs for Q^tNSs under the proposed operators are as follows.

(I) Idempotent property: $\grave{N}_A^{qt}\cup\grave{N}_A^{qt}=\grave{N}_A^{qt}$ and $\grave{N}_A^{qt}\cap\grave{N}_A^{qt}=\grave{N}_A^{qt}$.

Proof. Consider an element η that belongs to both $\grave{N}_A^{qt}$ and $\grave{N}_A^{qt}$. By definition, an element in $\grave{N}_A^{qt}$ has the form: $\left\langle\eta;\mu_A^{qt}(\eta),\lambda_A^{qt}(\eta),\nu_A^{qt}(\eta)\right\rangle$. Similarly, an element in $\grave{N}_A^{qt}$ also has the same form: $\left\langle\eta;\mu_A^{qt}(\eta),\lambda_A^{qt}(\eta),\nu_A^{qt}(\eta)\right\rangle$. As both elements have the same degree of membership $\mu_A^{qt}(\eta)$, $\lambda_A^{qt}(\eta)$, and $\nu_A^{qt}(\eta)$, it follows that they are identical. Therefore, any element in the intersection of $\grave{N}_A^{qt}$ with itself is simply an element of $\grave{N}_A^{qt}$. Conversely, any element in $\grave{N}_A^{qt}$ is also in the intersection, since it satisfies the conditions of both sets. Hence, $\grave{N}_A^{qt}\cap\grave{N}_A^{qt}=\grave{N}_A^{qt}$. Similarly, this can be done for the union operation. □

(II) Commutative property: $\grave{N}_A^{qt}\cup\grave{N}_B^{qt}=\grave{N}_B^{qt}\cup\grave{N}_A^{qt}$ and $\grave{N}_A^{qt}\cap\grave{N}_B^{qt}=\grave{N}_B^{qt}\cap\grave{N}_A^{qt}$.

Proof. We want to show that $\grave{N}_A^{qt}\cup\grave{N}_B^{qt}=\grave{N}_B^{qt}\cup\grave{N}_A^{qt}$ and $\grave{N}_A^{qt}\cap\grave{N}_B^{qt}=\grave{N}_B^{qt}\cap\grave{N}_A^{qt}$. First, let's prove the union property. Consider the union of $\grave{N}_A^{qt}$ and $\grave{N}_B^{qt}$:

$$\grave{N}_A^{qt}\cup\grave{N}_B^{qt}=\left\{\left\langle\eta;(\mu_A^{qt}\vee\mu_B^{qt})(\eta),[\vee(\lambda_A^{qt}(\eta),\lambda_B^{qt}(\eta))]^2,(\nu_A^{qt}\wedge\nu_B^{qt})(\eta)\right\rangle\mid\eta\in\tilde{U}\right\}.$$

By the commutativity of the logical OR and AND operators, we have:

$$(\mu_A^{qt}\vee\mu_B^{qt})(\eta)=(\mu_B^{qt}\vee\mu_A^{qt})(\eta),\quad\vee(\lambda_A^{qt}(\eta),\lambda_B^{qt}(\eta))=\vee(\lambda_B^{qt}(\eta),\lambda_A^{qt}(\eta)),\quad(\nu_A^{qt}\wedge\nu_B^{qt})(\eta)=(\nu_B^{qt}\wedge\nu_A^{qt})(\eta).$$

Therefore, $\grave{N}_A^{qt}\cup\grave{N}_B^{qt}=\grave{N}_B^{qt}\cup\grave{N}_A^{qt}$. Next, let's prove the intersection property. Consider the intersection of $\grave{N}_A^{qt}$ and $\grave{N}_B^{qt}$:

$$\grave{N}_A^{qt}\cap\grave{N}_B^{qt}=\left\{\left\langle\eta;(\mu_A^{qt}\wedge\mu_B^{qt})(\eta),[\wedge(\lambda_A^{qt}(\eta),\lambda_B^{qt}(\eta))]^2,(\nu_A^{qt}\vee\nu_B^{qt})(\eta)\right\rangle\mid\eta\in\tilde{U}\right\}.$$

By the commutativity of the logical OR and AND operators, we have

$$(\mu_A^{qt}\wedge\mu_B^{qt})(\eta)=(\mu_B^{qt}\wedge\mu_A^{qt})(\eta),\ \wedge(\lambda_A^{qt}(\eta),\lambda_B^{qt}(\eta))=\wedge(\lambda_B^{qt}(\eta),\lambda_A^{qt}(\eta)),\ (\nu_A^{qt}\vee\nu_B^{qt})(\eta)=(\nu_B^{qt}\vee\nu_A^{qt})(\eta).$$

Therefore, $\grave{N}_A^{qt}\cap\grave{N}_B^{qt}=\grave{N}_B^{qt}\cap\grave{N}_A^{qt}$. □

(III) Identity property: $\grave{N}_A^{qt}\cup\phi=\phi$ and $\grave{N}_A^{qt}\cup U=U$.

Proof. We want to show that $\grave{N}_A^{qt} \cup \phi = \grave{N}_A^{qt}$ and $\grave{N}_A^{qt} \cup U = U$. First, let's prove $\grave{N}_A^{qt} \cup \phi = \grave{N}_A^{qt}$. Consider the union of $\grave{N}_A^{qt}$ and the empty set ϕ:

$$\grave{N}_A^{qt} \cup \phi = \left\{\left\langle \eta; (\mu_A^{qt} \vee \mu_\phi^{qt})(\eta), [\vee(\lambda_A^{qt}(\eta), \lambda_\phi^{qt}(\eta))]^2, (\nu_A^{qt} \wedge \nu_\phi^{qt})(\eta)\right\rangle \mid \eta \in \tilde{U}\right\}.$$

Since the empty set ϕ does not contain any elements, its membership, non-membership, and indeterminacy functions are all zero: $\mu_\phi^{qt}(\eta) = 0$, $\lambda_\phi^{qt}(\eta) = 0$, $\nu_\phi^{qt}(\eta) = 0$. Therefore, we have $\mu_A^{qt} \vee \mu_\phi^{qt} = \mu_A^{qt}$, $\vee(\lambda_A^{qt}(\eta), \lambda_\phi^{qt}(\eta)) = \lambda_A^{qt}(\eta)$, $\nu_A^{qt} \wedge \nu_\phi^{qt} = \nu_A^{qt}$. Hence, $\grave{N}_A^{qt} \cup \phi = \grave{N}_A^{qt}$. Next, let's prove $\grave{N}_A^{qt} \cup U = U$. Consider the union of $\grave{N}_A^{qt}$ and the universal set U:

$$\grave{N}_A^{qt} \cup U = \left\{\left\langle \eta; (\mu_A^{qt} \vee \mu_U^{qt})(\eta), [\vee(\lambda_A^{qt}(\eta), \lambda_U^{qt}(\eta))]^2, (\nu_A^{qt} \wedge \nu_U^{qt})(\eta)\right\rangle \mid \eta \in \tilde{U}\right\}.$$

Since the universal set U contains all elements, its membership, non-membership, and hesitancy functions are all one: $\mu_U^{qt}(\eta) = 1$, $\lambda_U^{qt}(\eta) = 1$, $\nu_U^{qt}(\eta) = 1$. Therefore, we have $\mu_A^{qt} \vee \mu_U^{qt} = 1$, $\vee(\lambda_A^{qt}(\eta), \lambda_U^{qt}(\eta)) = 1$, $\nu_A^{qt} \wedge \nu_U^{qt} = \nu_A^{qt}$. Hence, $\grave{N}_A^{qt} \cup U = U$. □

(IV) Null property: $\grave{N}_A^{qt} \cap \phi = \grave{N}_A^{qt}$ and $\grave{N}_A^{qt} \cap U = \grave{N}_A^{qt}$.

Proof. We want to show that $\grave{N}_A^{qt} \cap \phi = \phi$ and $\grave{N}_A^{qt} \cap U = \grave{N}_A^{qt}$. First, let's prove $\grave{N}_A^{qt} \cap \phi = \phi$. Consider the intersection of $\grave{N}_A^{qt}$ and the empty set ϕ:

$$\grave{N}_A^{qt} \cap \phi = \left\{\left\langle \eta; (\mu_A^{qt} \wedge \mu_\phi^{qt})(\eta), [\wedge(\lambda_A^{qt}(\eta), \lambda_\phi^{qt}(\eta))]^2, (\nu_A^{qt} \vee \nu_\phi^{qt})(\eta)\right\rangle \mid \eta \in \tilde{U}\right\}.$$

Since the empty set ϕ does not contain any elements, its membership, non-membership, and hesitancy functions are all zero: $\mu_\phi^{qt}(\eta) = 0$, $\lambda_\phi^{qt}(\eta) = 0$, $\nu_\phi^{qt}(\eta) = 0$. Therefore, we have:

$$\mu_A^{qt} \wedge \mu_\phi^{qt} = 0, \quad \wedge(\lambda_A^{qt}(\eta), \lambda_\phi^{qt}(\eta)) = 0, \quad \nu_A^{qt} \vee \nu_\phi^{qt} = \nu_A^{qt}.$$

Hence, $\grave{N}_A^{qt} \cap \phi = \phi$. Next, let's prove $\grave{N}_A^{qt} \cap U = \grave{N}_A^{qt}$. Consider the intersection of $\grave{N}_A^{qt}$ and the universal set U:

$$\grave{N}_A^{qt} \cap U = \left\{\left\langle \eta; (\mu_A^{qt} \wedge \mu_U^{qt})(\eta), [\wedge(\lambda_A^{qt}(\eta), \lambda_U^{qt}(\eta))]^2, (\nu_A^{qt} \vee \nu_U^{qt})(\eta)\right\rangle \mid \eta \in \tilde{U}\right\}.$$

Since the universal set U contains all elements, its membership, non-membership, and hesitancy functions are all one: $\mu_U^{qt}(\eta) = 1$, $\lambda_U^{qt}(\eta) = 1$, $\nu_U^{qt}(\eta) = 1$. Therefore, we have

$$\mu_A^{qt} \wedge \mu_U^{qt} = \mu_A^{qt}, \quad \wedge(\lambda_A^{qt}(\eta), \lambda_U^{qt}(\eta)) = \lambda_A^{qt}(\eta), \quad \nu_A^{qt} \vee \nu_U^{qt} = 1.$$

Hence, $\grave{N}_A^{qt} \cap U = \grave{N}_A^{qt}$. □

(V) Associative property: $\grave{N}_A^{qt} \cup (\grave{N}_B^{qt} \cup \grave{N}_C^{t}) = (\grave{N}_A^{qt} \cup \grave{N}_B^{qt}) \cup \grave{N}_C^{t}$ and $\grave{N}_A^{qt} \cap (\grave{N}_B^{qt} \cap \grave{N}_C^{t}) = (\grave{N}_A^{qt} \cap \grave{N}_B^{qt}) \cap \grave{N}_C^{t}$.

Proof. To prove the first equation, let's consider each side separately:

$$\begin{aligned}
&\grave{N}_A^{qt} \cup (\grave{N}_B^{qt} \cup \grave{N}_C^{t}) \\
&\quad = \left\{\left\langle \eta; (\mu_A^{qt} \vee (\mu_B^{qt} \vee \mu_C^{t}))(\eta), [\wedge(\lambda_A^{qt}(\eta), \vee(\lambda_B^{qt}(\eta), \lambda_C^{t}(\eta)))]^3, (\nu_A^{qt} \wedge (\nu_B^{qt} \wedge \nu_C^{t}))(\eta)\right\rangle \mid \eta \in \tilde{U}\right\}, \\
&(\grave{N}_A^{qt} \cup \grave{N}_B^{qt}) \cup \grave{N}_C^{t} \\
&\quad = \left\{\left\langle \eta; ((\mu_A^{qt} \vee \mu_B^{qt}) \vee \mu_C^{t})(\eta), [\wedge(\vee(\lambda_A^{qt}(\eta), \lambda_B^{qt}(\eta)), \lambda_C^{t}(\eta))]^3, ((\nu_A^{qt} \wedge \nu_B^{qt}) \wedge \nu_C^{t})(\eta)\right\rangle \mid \eta \in \tilde{U}\right\}.
\end{aligned}$$

To show that both sides are equal, we need to prove that their components are equal. We'll focus on membership functions, non-membership functions, and hesitancy functions,

$$\mu_A^{qt} \vee (\mu_B^{qt} \vee \mu_C^{t}) = (\mu_A^{qt} \vee \mu_B^{qt}) \vee \mu_C^{t},$$
$$\wedge(\lambda_A^{qt}(\eta), \vee(\lambda_B^{qt}(\eta), \lambda_C^{t}(\eta))) = \wedge(\vee(\lambda_A^{qt}(\eta), \lambda_B^{qt}(\eta)), \lambda_C^{t}(\eta)),$$
$$\nu_A^{qt} \wedge (\nu_B^{qt} \wedge \nu_C^{t}) = ((\nu_A^{qt} \wedge \nu_B^{qt}) \wedge \nu_C^{t}).$$

Therefore, both sides are equal. Similarly, we can prove the second part. □

(VI) Distributive property: $\grave{N}_A^{qt} \cup (\grave{N}_B^{qt} \cap \grave{N}_C^{t}) = (\grave{N}_A^{qt} \cup \grave{N}_B^{qt}) \cap (\grave{N}_A^{qt} \cup \grave{N}_C^{t})$ and $\grave{N}_A^{qt} \cap (\grave{N}_B^{qt} \cup \grave{N}_C^{t}) = (\grave{N}_A^{qt} \cap \grave{N}_B^{qt}) \cup (\grave{N}_A^{qt} \cap \grave{N}_C^{t})$.

Proof. To prove the first equation, let's consider each side separately:

$$\grave{N}_A^{qt} \cup (\grave{N}_B^{qt} \cap \grave{N}_C^{t}) = \{\langle \eta; (\mu_A^{qt} \vee (\mu_B^{qt} \wedge \mu_C^{t}))(\eta), [\wedge(\lambda_A^{qt}(\eta), \vee(\lambda_B^{qt}(\eta), \lambda_C^{t}(\eta)))], (\nu_A^{qt} \wedge (\nu_B^{qt} \vee \nu_C^{t}))(\eta)\rangle \mid \eta \in \tilde{U}\},$$
$$(\grave{N}_A^{qt} \cup \grave{N}_B^{qt}) \cap (\grave{N}_A^{qt} \cup \grave{N}_C^{t}) = \{\langle \eta; ((\mu_A^{qt} \vee \mu_B^{qt}) \wedge (\mu_A^{qt} \vee \mu_C^{t}))(\eta), [\wedge(\vee(\lambda_A^{qt}(\eta), \lambda_B^{qt}(\eta)), \vee(\lambda_A^{qt}(\eta), \lambda_C^{t}(\eta)))], \rangle \mid \eta \in \tilde{U}\}.$$

To show that both sides are equal, we need to prove that their components are equal. We'll focus on membership functions, non-membership functions, and indeterminacy functions,

$$\mu_A^{qt} \vee (\mu_B^{qt} \wedge \mu_C^{t}) = (\mu_A^{qt} \vee \mu_B^{qt}) \wedge (\mu_A^{qt} \vee \mu_C^{t}),$$
$$\wedge(\lambda_A^{qt}(\eta), \vee(\lambda_B^{qt}(\eta), \lambda_C^{t}(\eta))) = \wedge(\vee(\lambda_A^{qt}(\eta), \lambda_B^{qt}(\eta)), \vee(\lambda_A^{qt}(\eta), \lambda_C^{t}(\eta))),$$
$$\nu_A^{qt} \wedge (\nu_B^{qt} \vee \nu_C^{t}) = (\nu_A^{qt} \wedge \nu_B^{qt}) \vee (\nu_A^{qt} \wedge \nu_C^{t}).$$

Therefore, both sides are equal. Similarly, we can prove the second equation. □

Definition 3.8 ([12, 40]). A function $N : [\tilde{I}] \longrightarrow [\tilde{I}]$ is said to be a operator of negation with $N(0) = 1, N(1) = 0$ and is decreasing $(\sigma \leqslant \sigma_1 \longrightarrow N(\sigma) \geqslant N(\sigma_1))$. For strict negation, it should be strictly decreasing $(\sigma < \sigma_1 \to N(\sigma) > N(\sigma_1))$ and continuous. If it is also involutive, i.e., $N(N(\sigma)) = \sigma$, then strict is said to be a strong negation. Smarandache [37] defined the negation operator for the neutrosophic set structure by reversing the components of truthness and falsity.

In a similar way, now we define operators for negation and strict negation to Q^tNS as follows.

Definition 3.9. A function $N : [\tilde{I}] \times [\tilde{I}] \times [\tilde{I}] \to [\tilde{I}] \times [\tilde{I}] \times [\tilde{I}]$ is called a negation if $N(0) = 1, N(1) = 0$, and N is nonincreasing $(\sigma \leqslant \sigma_1 \to N(\sigma) \geqslant N(\sigma_1))$. A negation is called a strict negation if it is strictly decreasing $(\sigma < \sigma_1 \to N(\sigma) > N(\sigma_1))$ and continuous.

Definition 3.10. The complement of $\grave{N}_A^{qt} = \left\langle \mu_A^{qt}, \lambda_A^{qt}, \nu_A^{qt} \right\rangle$ is denoted by $\bar{C}(\grave{N}_A^{qt})$ and is defined by

$$N(\mu_A^{qt}(\sigma)) = \nu_A^{qt}(\sigma), \quad N(\lambda_A^{qt}(\sigma)) = [1 - \lambda_A^{qt}(\sigma)]^n, \quad \text{and} \quad N(\nu_A^{qt}(\sigma)) = \mu_A^{qt}(\sigma).$$

Numerical example:

$$\grave{N}^{qt} = \left\{ \begin{array}{c} < \tilde{u}_1, 0.5, 0.5, 0.8 > \\ < \tilde{u}_2, 0.3, 0.5, 0.8 > \\ < \tilde{u}_3, 0.4, 0.3, 0.8 > \end{array} \right\}, \quad \bar{C}(\grave{N}^{qt}) = \left\{ \begin{array}{c} < \tilde{u}_1, 0.8, 0.5, 0.5 > \\ < \tilde{u}_2, 0.8, 0.5, 0.3 > \\ < \tilde{u}_3, 0.8, 0.7, 0.4 > \end{array} \right\}.$$

Theorem 3.11. *For two Q^tNSs $\grave{N}_A^{qt}$ and $\grave{N}_B^{qt}$, the relations for De-Morgan's Law are as follows:*

I. $\bar{C}(\grave{N}_A^{qt} \cup \grave{N}_B^{qt}) = \bar{C}(\grave{N}_A^{qt}) \cap \bar{C}(\grave{N}_B^{qt})$;
II. $\bar{C}(\grave{N}_A^{qt} \cap \grave{N}_B^{qt}) = \bar{C}(\grave{N}_A^{qt}) \cup \bar{C}(\grave{N}_B^{qt})$.

Proof.

I.

$$
\begin{aligned}
&\bar{C}(\grave{N}_A^{qt} \cup \grave{N}_B^{qt}) \\
&= \bar{C}\left\{\left(\frac{u_j}{\left(\mu_A^{qt}(\sigma) \vee \mu_B^{qt}(\sigma), [\lambda_A^{qt}(\sigma) \vee \lambda_B^{qt}(\sigma)]^n, \nu_A^{qt}(\sigma) \wedge \nu_B^{qt}(\sigma)\right)}\right) : u_j \in U\right\} \\
&= \left\{\left(\frac{u_j}{\left(\mu_A^{qt}(\sigma) \wedge \mu_B^{qt}(\sigma), N[(\lambda_A^{qt}(\sigma) \vee \lambda_B^{qt}(\sigma))]^n, \nu_A^{qt}(\sigma) \vee \nu_B^{qt}(\sigma)\right)}\right) : u_j \in U\right\} \\
&= \left\{\left(\frac{u_j}{\left(\mu_A^{qt}(\sigma) \wedge \mu_B^{qt}(\sigma), N[(\lambda_A^{qt}(\sigma))]^n \wedge N[(\lambda_B^{qt}(\sigma))]^n, \nu_A^{qt}(\sigma) \vee \nu_B^{qt}(\sigma)\right)}\right) : u_j \in U\right\} \\
&= \left\{\left(\frac{u_j}{\left(\mu_A^{qt}(\sigma), N[(\lambda_A^{qt}(\sigma))]^n, \nu_A^{qt}(\sigma)\right)}\right) : u_j \in U\right\} \cap \left\{\left(\frac{u_j}{\left(\mu_B^{qt}(\sigma), N[(\lambda_B^{qt}(\sigma))]^n, \nu_B^{qt}(\sigma)\right)}\right) : u_j \in U\right\} \\
&= \bar{C}\left\{\left(\frac{u_j}{\left(\mu_A^{qt}(\sigma), [\lambda_A^{qt}(\sigma)]^n, \nu_A^{qt}(\sigma)\right)}\right) : u_j \in U\right\} \cap \bar{C}\left\{\left(\frac{u_j}{\left(\mu_B^{qt}(\sigma), [\lambda_B^{qt}(\sigma)]^n, \nu_B^{qt}(\sigma)\right)}\right) : u_j \in U\right\} \\
&= \bar{C}(\grave{N}_A^{qt}) \cap \bar{C}(\grave{N}_B^{qt}).
\end{aligned}
$$

II. The procedures applied to prove (II) are the same as those used to prove (I), thus we skip the proof. □

4. MCDM algorithms for biofabricated textile material selection based on Q^tNS

Biofabricated textiles, in simple terms, are fabrics and materials that are made not from traditional sources like cotton or synthetic fibres but are created using living organisms like bacteria, yeast, or fungi. These living organisms are used to produce the fibres and materials in a controlled environment, often through a process called fermentation. The result is a textile material that can be used to make clothing, accessories, and even home goods. Biofabricated textiles are gaining popularity because they are often more sustainable than traditional textiles. They can be produced with fewer resources, generate less waste, and have the potential to be biodegradable, which means they break down naturally and don't contribute to pollution.

4.1. Critiria selection problem for biofabricated textiles

For the healthcare industry, a biofabricated textile material, specifically "antimicrobial biofabricated textile" is more essential. These textiles are specially designed to inhibit the growth of harmful microorganisms, such as bacteria and viruses, on their surface. They are typically made using biofabrication methods, incorporating antimicrobial agents derived from natural sources or through biotechnological processes, and have following benefits.

Infection control: Antimicrobial textiles help reduce the risk of infections in healthcare settings, promoting patient safety.

Hygiene: They can enhance hygiene by preventing the buildup of harmful microorganisms on surfaces like hospital linens, gowns, or masks.

Sustainability: Many of the antimicrobial agents used are biodegradable and eco-friendly, aligning with healthcare sustainability goals. The materials that can be incorporated into textile production processes to create antimicrobial biofabricated textiles, each with its own unique antimicrobial properties suitable for various healthcare applications, including hospital linens, patient gowns, masks, and more, are as follows.

Safety and skin compatibility: Consider the material's safety for prolonged skin contact, especially in healthcare settings. Evaluate potential skin sensitivities, allergies, or irritations associated with the material. Textiles in healthcare are in close contact with patients and healthcare workers. Safety and comfort are critical to preventing skin issues and allergies.

Environmental impact and sustainability: Assess the environmental impact of the material, including its biodegradability, recyclables, and the resources used in its production. Consider eco-friendly alternatives. Sustainability is increasingly important in textile manufacturing. Eco-friendly materials reduce the environmental footprint and align with green practices.

Antimicrobial efficacy: The ability of the material to effectively inhibit the growth of harmful microorganisms, such as bacteria and viruses, is crucial. This includes assessing the material's spectrum of antimicrobial activity and its resistance to microbial resistance development. As antimicrobial textiles are designed to prevent infections and maintain hygiene in healthcare, high antimicrobial efficacy is paramount.

Durability and longevity of antimicrobial properties: Materials should maintain their antimicrobial properties over the expected lifespan of the textiles, even after repeated use and laundering. Assess the material's ability to withstand wear and tear. In healthcare and other applications, textiles are frequently used and laundered. Materials that lose their antimicrobial effectiveness quickly may pose a risk.

Chitosan-based textiles: Chitosan is derived from chitin, which is found in the shells of crustaceans like shrimp and crabs. Chitosan has natural antimicrobial properties and can be incorporated into textiles during production to inhibit the growth of bacteria and fungi.

Neem extract-infused textiles: Neem extracts contain strong antibacterial properties. Textiles treated with neem extracts can give long-term protection against germs and fungus, making them appropriate for a wide range of uses.

Turmeric-infused textiles: Turmeric includes curcumin, a compound with antibacterial effects. Infusing turmeric into textiles can help suppress the growth of germs and fungus, providing a natural alternative to antimicrobial materials.

Seewead based textiles: Materials produced from seaweed, such as alginate fibres, are antibacterial. These fibres can be utilised in textile manufacturing to create materials that prevent the growth of germs and fungus.

Bamboo charcoal textiles: Activated bamboo charcoal has natural antibacterial and deodorising properties. It can be integrated into textiles to create materials that help control odour and inhibit microbial growth.

Furthermore, when selecting materials for antimicrobial biofabricated textiles, some factors that determine the suitability of materials for antimicrobial biofabricated textiles in healthcare applications are:
In the textile industry, the proposed structure of a neutrosophic set with indeterminacy dependent on truthness and falsity offers a powerful approach to optimising the textile material selection process. The set of alternatives includes various textile materials available on the market, each with its own unique

composition, strength, eco-friendliness, and cost. The set of criteria encompasses essential factors that textile manufacturers consider during material selection, such as fibre composition, tensile strength, colour availability, environmental impact, and cost-effectiveness. Additionally, the set of experts involves textile specialists, sustainability experts, procurement managers, and other relevant stakeholders who bring their domain knowledge and expertise to assess and evaluate the materials and suppliers.

For the concerned approach, let $\aleph = \{\aleph_1, \aleph_2, \ldots, \aleph_m\}$ be a set of alternatives $\Im = \{\Im_1, \Im_2, \ldots, \Im_n\}$ represents a set of criteria, and $\check{E} = \{\check{e}_1, \check{e}_2, \ldots, \check{e}_K\}$ be the set of experts. Suppose each alternative $\aleph_i (i = 1, 2, \ldots, m)$ is assessed by the experts $\check{e}_k (k = 1, 2, \ldots, K)$ w.r.t the criteria $\Im_j (j = 1, 2, \ldots, n)$ using Q^tNSs. The set of alternatives is represented by $\aleph = \{\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5\}$ which respectively are chitosan, neem extract infused, turmeric infused, seewead based and bamboo charcoal Textiles with $\Im = \{\Im_1 =$ Antimicrobial Efficacy, $\Im_2 =$ Biocompatibility, $\Im_3 =$ Sustainability, $\Im_4 =$ Durability and Washability$\}$ be the set of criteria and two experts that are materials scientists and infectious disease specialists, $\check{E} = \{\check{e}_1, \check{e}_2\}$.

4.2. **Calibration process**

For the concern of sustainable material selection problem for anti microbial bio fabricated textiles manufacturing, calibration process is very crucial. Figure 2 represents the complete process of calibration regarding this problem. We will now proceed with the following algorithms:

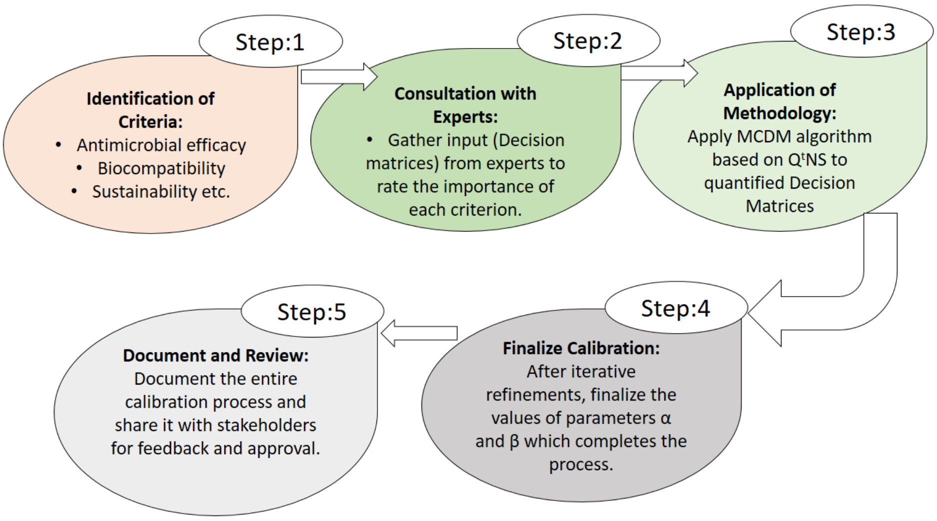

Figure 2: Pictorial representation of calibration process.

4.3. Q^tNS*UI-algorithm*

The proposed MCDM Q^tNSUI-Algothim based on Q^tNS consists the following steps.

Step 1. Construct the decision matrices $\check{R}^k_{NS} = \left(\check{r}^k_{ij}\right)_{m\times n}$ based on the assessed values of expert $\check{e}_k$ in the form of neutrosophic set $\check{r}^k_{ij}$.

Step 2. Reconstruct the decision matrices $\check{R}^k_{Q^tNS} = \left(\check{r}^k_{ij}\right)_{m\times n}$ in the form of Q^tNSs $\check{r}^k_{ij}$ based on the specific values of parameters according too each expert $\check{e}_k$ respectively.

Step 3. Calculate the aggregated decision matrix $\check{R}_{\cup} = (\check{r}_{ij})_{m\times n}$ and $\check{R}_{\cap} = (\check{r}_{ij})_{m\times n}$by using the proposed operations of union and intersection as discussed in Definitions 3.4 and 3.5, where $\check{r}_{ij} = \bigcup_{k=1,2,\ldots,K} \check{r}^k_{ij}$ or $\check{r}_{ij} = \bigcap_{k=1,2,\ldots,K} \check{r}^k_{ij}$.

Step 4. Calculate the score value of each $\check{r}_{ij}$ of each $\check{R}_{\cup}$ and $\check{R}_{\cap}$ by

$$Sc(\grave{N}_A^{qt}) = \frac{1}{3}\left[\frac{\lambda_A^{qt}}{\sqrt{(\mu_A^{qt})^2 + (\lambda_A^{qt})^2 + (\nu_A^{qt})^2}}\right],$$

where $Sc(\grave{N}_A^{qt}) \in [\tilde{I}]$.

Step 5. Calculate the preference scores of each $\aleph_i (i = 1, 2, \ldots, m)$ with $\wp_{rf}(\aleph_i) = \sum_{i=1}^{m}\sum_{j=1}^{n} \check{r}_{ij}$.

Step 6. Generate the ranking order of alternatives according to the descending order of the preference scores.

4.4. Q^{t}NS*AO-algorithm*

The proposed MCDM Q^{t}NSAO-Algothim based on Q^{t}NS consists of the following steps.

Steps 1, 2. Construct the decision matrices $\check{R}^k_{NS} = \left(\check{r}^k_{ij}\right)_{m\times n}$ based on the assessed values of expert $\sigma(k = 1, 2, \ldots, K)$ in the form of neutrosophic sets $\check{r}^k_{ij}$. Then reconstruct the decision matrices $\check{R}^k_{Q^tNS} = \left(\check{r}^k_{ij}\right)_{m\times n}$ in the form of Q^{t}NSs $\check{r}^k_{ij}$ based on the specific values of parameters according too each expert $e_k (k = 1, 2, \ldots, K)$, respectively.

Step 3. Calculate the aggregated decision matrix $\check{R}_{\triangle} = (\check{r}_{ij})_{m\times n}$ and $\check{R}_{\bigtriangledown} = (\check{r}_{ij})_{m\times n}$by using the proposed operations of AND/OR product as discussed in Definitions 3.6 and 3.7, where $\check{r}_{ij} = \underset{k=1,2,\ldots,K}{\triangle} \check{r}^k_{ij}$ or $\check{r}_{ij} = \underset{k=1,2,\ldots,K}{\bigtriangledown} \check{r}^k_{ij}$.

Step 4. Present $\triangle/\bigtriangledown$-product matrices in reduced fuzzy numerical grades matrix notation by using $\mathfrak{R}_{rfg} = |\mu^{qt} - \nu^{qt} + \lambda^{qt}|$ and mark the highest numerical grade.

Step 5. Calculate score with $Sc(\aleph_i) = \Sigma\mathfrak{R}_{hrfg}/n(\mathfrak{R}_{hrfg})$, where $\Sigma\mathfrak{R}_{hrfg}$ represents the sum of highest numerical grades in a row of $\mathfrak{I}_{ij}$ and $n(\mathfrak{R}_{hrfg})$ represents the total number of highest numerical grades in the column $\aleph_i$.

Step 6. Generate the ranking order of alternatives according to the non-increasing order of the score values and the best alternative will be of highest score.

4.5. Application of Q^{t}NS*UI-algorithm and* Q^{t}NS*AO-algorithm*

In this section, we will apply both the algorithms one by one to select the best material for bio-fabricated textiles.

4.5.1. Application of Q^{t}NS*UI-algorithm*

The algorithm is defined below.

Step 1. In accordance with the opinion of experts, the individual decision matrices $\check{R}^1_{NS}$ and $\check{R}^2_{NS}$ are constructed in neutrosophic environment, which can be seen in Tables 6 and 7.

Table 6: Decision matrix $\check{R}^1_{NS}$ provided by expert $\check{e}_1$.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_1$	$\langle 0.7, 1, 0.7\rangle$	$\langle 0.4, 0.7, 0.6\rangle$	$\langle 0.1, 0.6, 0.9\rangle$	$\langle 0.8, 0.8, 0.7\rangle$	$\langle 0.4, 0.6, 0.9\rangle$
$\mathfrak{I}_2$	$\langle 0.4, 0.2, 0.9\rangle$	$\langle 0.5, 1.0, 0.7\rangle$	$\langle 0.2, 0.5, 0.9\rangle$	$\langle 0.9, 0.9, 0.7\rangle$	$\langle 0.5, 0.9, 0.1\rangle$
$\mathfrak{I}_3$	$\langle 0.5, 0.9, 0.5\rangle$	$\langle 0.3, 0.7, 0.3\rangle$	$\langle 0.6, 0.9, 0.4\rangle$	$\langle 0.2, 0.6, 0.1\rangle$	$\langle 0.9, 0.8, 0.4\rangle$
$\mathfrak{I}_4$	$\langle 1.0, 1.0, 0.0\rangle$	$\langle 0.9, 1.0, 0.1\rangle$	$\langle 0.8, 0.5, 0.5\rangle$	$\langle 0.4, 0.9, 0.4\rangle$	$\langle 0.5, 0.5, 0.5\rangle$

Table 7: Decision matrix $\check{R}^2_{NS}$ provided by expert $\check{e}_2$.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_1$	$\langle 0.4, 0.6, 0.7\rangle$	$\langle 0.2, 0.5, 0.9\rangle$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.3, 0.4, 0.8\rangle$	$\langle 0.2, 0.3, 0.7\rangle$
$\mathfrak{I}_2$	$\langle 0.6, 0.9, 0.5\rangle$	$\langle 0.6, 0.6, 0.2\rangle$	$\langle 0.3, 0.4, 0.1\rangle$	$\langle 0.2, 0.5, 0.7\rangle$	$\langle 0.7, 0.8, 0.4\rangle$
$\mathfrak{I}_3$	$\langle 0.2, 0.2, 0.6\rangle$	$\langle 0.7, 0.8, 0.5\rangle$	$\langle 0.2, 0.4, 0.5\rangle$	$\langle 0.4, 0.6, 0.6\rangle$	$\langle 0.3, 0.9, 0.9\rangle$
$\mathfrak{I}_4$	$\langle 0.4, 0.5, 0.5\rangle$	$\langle 0.5, 0.4, 0.6\rangle$	$\langle 0.7, 0.8, 0.7\rangle$	$\langle 0.5, 0.4, 0.9\rangle$	$\langle 0.1, 0.9, 1.0\rangle$

Step 2. The individual decision matrices $\check{R}^1_{Q^tNS}$ and $\check{R}^2_{Q^tNS}$ are now constructed in the Q^tNS environment, which can be seen in Tables 8 and 9.

Table 8: Decision matrix $\check{R}^1_{NS}$ provided by expert $\check{e}_1$ with parameters $\alpha = 0.1$ and $\beta = 0.5$.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_1$	$\langle 0.7, 0.3, 0.7\rangle$	$\langle 0.4, 0.3, 0.6\rangle$	$\langle 0.1, 0.3, 0.9\rangle$	$\langle 0.8, 0.3, 0.7\rangle$	$\langle 0.4, 0.3, 0.9\rangle$
$\mathfrak{I}_2$	$\langle 0.4, 0.2, 0.9\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.2, 0.3, 0.9\rangle$	$\langle 0.9, 0.3, 0.7\rangle$	$\langle 0.5, 0.3, 0.5\rangle$
$\mathfrak{I}_3$	$\langle 0.5, 0.3, 0.5\rangle$	$\langle 0.3, 0.3, 0.5\rangle$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.2, 0.3, 0.5\rangle$	$\langle 0.9, 0.3, 0.5\rangle$
$\mathfrak{I}_4$	$\langle 1.0, 0.3, 0.5\rangle$	$\langle 0.9, 0.3, 0.5\rangle$	$\langle 0.8, 0.3, 0.5\rangle$	$\langle 0.4, 0.3, 0.5\rangle$	$\langle 0.5, 0.3, 0.5\rangle$

Table 9: Decision matrix $\check{R}^2_{NS}$ provided by expert $\check{e}_2$ with parameters $\alpha = 0.3$ and $\beta = 0.7$.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_1$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.5, 0.9\rangle$	$\langle 0.6, 0.3, 0.7\rangle$	$\langle 0.3, 0.4, 0.8\rangle$	$\langle 0.3, 0.3, 0.7\rangle$
$\mathfrak{I}_2$	$\langle 0.6, 0.5, 0.7\rangle$	$\langle 0.6, 0.5, 0.7\rangle$	$\langle 0.3, 0.4, 0.7\rangle$	$\langle 0.3, 0.5, 0.7\rangle$	$\langle 0.7, 0.5, 0.7\rangle$
$\mathfrak{I}_3$	$\langle 0.3, 0.2, 0.7\rangle$	$\langle 0.7, 0.5, 0.7\rangle$	$\langle 0.3, 0.4, 0.7\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.5, 0.9\rangle$
$\mathfrak{I}_4$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.5, 0.4, 0.7\rangle$	$\langle 0.7, 0.5, 0.7\rangle$	$\langle 0.5, 0.4, 0.9\rangle$	$\langle 0.3, 0.5, 1.0\rangle$

Step 3. The aggregated decision matrices $\check{R}_{\cup} = (\check{r}_{ij})_{4\times5}$ and $\check{R}_{\cap} = (\check{r}_{ij})_{4\times5}$ are shown in Tables 10 and 11.

Table 10: Aggregated decision matrix $\check{R}_{\cup}$ by applying the union operation.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_1$	$\langle 0.7, 0.25, 0.7\rangle$	$\langle 0.4, 0.25, 0.6\rangle$	$\langle 0.6, 0.09, 0.7\rangle$	$\langle 0.8, 0.16, 0.7\rangle$	$\langle 0.4, 0.09, 0.7\rangle$
$\mathfrak{I}_2$	$\langle 0.6, 0.25, 0.7\rangle$	$\langle 0.6, 0.25, 0.7\rangle$	$\langle 0.3, 0.16, 0.7\rangle$	$\langle 0.9, 0.25, 0.7\rangle$	$\langle 0.7, 0.25, 0.5\rangle$
$\mathfrak{I}_3$	$\langle 0.4, 0.09, 0.5\rangle$	$\langle 0.7, 0.25, 0.5\rangle$	$\langle 0.6, 0.16, 0.5\rangle$	$\langle 0.4, 0.25, 0.5\rangle$	$\langle 0.9, 0.25, 0.5\rangle$
$\mathfrak{I}_4$	$\langle 1.0, 0.25, 0.5\rangle$	$\langle 0.9, 0.16, 0.5\rangle$	$\langle 0.8, 0.25, 0.5\rangle$	$\langle 0.5, 0.16, 0.5\rangle$	$\langle 0.5, 0.25, 0.5\rangle$

Table 11: Aggregated decision matrix $\check{R}_{\cap}$ by applying the intersection operation.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_1$	$\langle 0.4, 0.09, 0.7\rangle$	$\langle 0.3, 0.09, 0.9\rangle$	$\langle 0.1, 0.09, 0.9\rangle$	$\langle 0.3, 0.09, 0.8\rangle$	$\langle 0.3, 0.09, 0.9\rangle$
$\mathfrak{I}_2$	$\langle 0.4, 0.04, 0.9\rangle$	$\langle 0.5, 0.09, 0.7\rangle$	$\langle 0.2, 0.09, 0.9\rangle$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.5, 0.09, 0.7\rangle$
$\mathfrak{I}_3$	$\langle 0.3, 0.04, 0.7\rangle$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.2, 0.09, 0.7\rangle$	$\langle 0.3, 0.09, 0.9\rangle$
$\mathfrak{I}_4$	$\langle 0.4, 0.09, 0.7\rangle$	$\langle 0.5, 0.09, 0.7\rangle$	$\langle 0.7, 0.09, 0.7\rangle$	$\langle 0.4, 0.09, 0.9\rangle$	$\langle 0.3, 0.09, 1.0\rangle$

Step 4: The matrices of the score values of the elements of $\check{R}_{\cup}$ and $\check{R}_{\cap}$, respectively, can be seen in Tables 12 and 13.

Table 12: Score values of the aggregated union decision matrix $\check{R}_{\cup}$.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\Im_1$	0.0816	0.1091	0.0323	0.0496	0.0369
$\Im_2$	0.0872	0.0872	0.0685	0.0713	0.0930
$\Im_3$	0.0420	0.0930	0.0668	0.1212	0.0786
$\Im_4$	0.0727	0.0511	0.0853	0.0735	0.1111

Table 13: Score values of the aggregated intersection decision matrix $\check{R}_{\cap}$.

Cr.	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\Im_1$	0.0369	0.0314	0.0329	0.0349	0.0314
$\Im_2$	0.0135	0.0346	0.0323	0.0391	0.0346
$\Im_3$	0.0174	0.0391	0.0391	0.0408	0.0314
$\Im_4$	0.0369	0.0346	0.0301	0.0303	0.0286

Step 5, 6. Finally, the preference score $\wp_{rf}(\aleph_i), i = 1,2,3,4,5$ of each alternative is calculated where $\wp_{rf}(\aleph_i) = \sum_{i=1}^{5}\sum_{j=1}^{4}\check{r}_{ij}$. The preference scores of alternatives in $\check{R}_{\cup}$ and $\check{R}_{\cap}$, respectively, are given below:

$$\wp_{rf}(\aleph_1) = 0.2853, \quad \wp_{rf}(\aleph_2) = 0.2529, \quad \wp_{rf}(\aleph_3) = 0.3404, \quad \wp_{rf}(\aleph_4) = 0.3156, \quad \wp_{rf}(\aleph_5) = 0.3196,$$
$$\wp_{rf}(\aleph_1) = 0.1260, \quad \wp_{rf}(\aleph_2) = 0.1047, \quad \wp_{rf}(\aleph_3) = 0.1451, \quad \wp_{rf}(\aleph_4) = 0.1397, \quad \wp_{rf}(\aleph_5) = 0.1344.$$

In accordance to the non-increasing order of preference scores, the alternatives are ranked for $\check{R}_{\cup}$ and $\check{R}_{\cap}$, respectively, as $\aleph_3 \succeq \aleph_5 \succeq \aleph_4 \succeq \aleph_1 \succeq \aleph_2$ and $\aleph_3 \succeq \aleph_4 \succeq \aleph_5 \succeq \aleph_1 \succeq \aleph_2$.

4.5.2. *Application of* Q^{t}NSAO*-algorithm*

Now, we apply Q^{t}NSAO-algorithm step by step for the MCDM approach.

Steps 1, 2. In accordance with the opinion of experts, the individual decision matrices $\check{R}^1_{NS}$ and $\check{R}^2_{NS}$ are constructed in neutrosophic environment, which can be seen in Tables 6 and 7 and decision matrices $\check{R}^1_{Q^tNS}$ and $\check{R}^2_{Q^tNS}$ constructed in the Q^{t}NS environment can be seen in Tables 8 and 9.

Step 3. The aggregated decision matrixes $\check{R}_{\triangle} = (\check{r}_{ij})_{16\times 5}$ and $\check{R}_{\bigtriangledown} = (\check{r}_{ij})_{16\times 5}$ are calculated with the help of the proposed AND/OR products are shown below.as

$\check{R}_{\triangle}$	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\Im_{11}$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.9\rangle$	$\langle 0.1, 0.09, 0.9\rangle$	$\langle 0.3, 0.16, 0.8\rangle$	$\langle 0.3, 0.09, 0.9\rangle$
$\Im_{12}$	$\langle 0.6, 0.25, 0.7\rangle$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.1, 0.16, 0.9\rangle$	$\langle 0.3, 0.25, 0.7\rangle$	$\langle 0.4, 0.25, 0.9\rangle$
$\Im_{13}$	$\langle 0.3, 0.04, 0.7\rangle$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.1, 0.16, 0.9\rangle$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.9\rangle$
$\Im_{14}$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.4, 0.16, 0.7\rangle$	$\langle 0.1, 0.25, 0.9\rangle$	$\langle 0.5, 0.16, 0.9\rangle$	$\langle 0.3, 0.25, 1.0\rangle$
$\Im_{21}$	$\langle 0.4, 0.25, 0.9\rangle$	$\langle 0.5, 0.25, 0.7\rangle$	$\langle 0.2, 0.09, 0.9\rangle$	$\langle 0.3, 0.16, 0.8\rangle$	$\langle 0.3, 0.09, 0.7\rangle$
$\Im_{22}$	$\langle 0.4, 0.25, 0.9\rangle$	$\langle 0.5, 0.25, 0.7\rangle$	$\langle 0.2, 0.16, 0.9\rangle$	$\langle 0.3, 0.25, 0.7\rangle$	$\langle 0.5, 0.25, 0.7\rangle$
$\Im_{23}$	$\langle 0.3, 0.04, 0.9\rangle$	$\langle 0.5, 0.25, 0.7\rangle$	$\langle 0.2, 0.16, 0.9\rangle$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.9\rangle$
$\Im_{24}$	$\langle 0.4, 0.25, 0.9\rangle$	$\langle 0.5, 0.09, 0.7\rangle$	$\langle 0.2, 0.25, 0.9\rangle$	$\langle 0.5, 0.16, 0.9\rangle$	$\langle 0.3, 0.25, 1.0\rangle$
$\Im_{31}$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.9\rangle$	$\langle 0.6, 0.09, 0.7\rangle$	$\langle 0.2, 0.16, 0.8\rangle$	$\langle 0.3, 0.09, 0.7\rangle$
$\Im_{32}$	$\langle 0.5, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.7\rangle$	$\langle 0.3, 0.16, 0.7\rangle$	$\langle 0.2, 0.25, 0.7\rangle$	$\langle 0.7, 0.25, 0.7\rangle$
$\Im_{33}$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.3, 0.25, 0.7\rangle$	$\langle 0.3, 0.16, 0.7\rangle$	$\langle 0.2, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.9\rangle$
$\Im_{34}$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.3, 0.16, 0.7\rangle$	$\langle 0.6, 0.25, 0.7\rangle$	$\langle 0.2, 0.16, 0.9\rangle$	$\langle 0.3, 0.25, 0.1\rangle$
$\Im_{41}$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.9\rangle$	$\langle 0.6, 0.09, 0.7\rangle$	$\langle 0.3, 0.16, 0.8\rangle$	$\langle 0.3, 0.09, 0.7\rangle$
$\Im_{42}$	$\langle 0.6, 0.25, 0.7\rangle$	$\langle 0.6, 0.25, 0.7\rangle$	$\langle 0.3, 0.16, 0.7\rangle$	$\langle 0.3, 0.25, 0.7\rangle$	$\langle 0.5, 0.25, 0.7\rangle$
$\Im_{43}$	$\langle 0.3, 0.04, 0.7\rangle$	$\langle 0.7, 0.25, 0.7\rangle$	$\langle 0.3, 0.16, 0.7\rangle$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.3, 0.25, 0.9\rangle$
$\Im_{44}$	$\langle 0.4, 0.25, 0.7\rangle$	$\langle 0.5, 0.16, 0.7\rangle$	$\langle 0.7, 0.25, 0.7\rangle$	$\langle 0.4, 0.16, 0.7\rangle$	$\langle 0.3, 0.25, 1.0\rangle$

,

$\check{R}_{\bigtriangledown}$	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_{11}$	$\langle 0.7, 0.09, 0.7\rangle$	$\langle 0.4, 0.09, 0.6\rangle$	$\langle 0.6, 0.09, 0.7\rangle$	$\langle 0.8, 0.09, 0.7\rangle$	$\langle 0.4, 0.09, 0.7\rangle$
$\mathfrak{I}_{12}$	$\langle 0.7, 0.09, 0.7\rangle$	$\langle 0.6, 0.09, 0.6\rangle$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.8, 0.09, 0.7\rangle$	$\langle 0.7, 0.09, 0.7\rangle$
$\mathfrak{I}_{13}$	$\langle 0.7, 0.04, 0.7\rangle$	$\langle 0.7, 0.09, 0.6\rangle$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.8, 0.09, 0.7\rangle$	$\langle 0.4, 0.09, 0.9\rangle$
$\mathfrak{I}_{14}$	$\langle 0.7, 0.09, 0.7\rangle$	$\langle 0.5, 0.09, 0.6\rangle$	$\langle 0.7, 0.09, 0.7\rangle$	$\langle 0.8, 0.09, 0.7\rangle$	$\langle 0.4, 0.09, 0.9\rangle$
$\mathfrak{I}_{21}$	$\langle 0.4, 0.04, 0.7\rangle$	$\langle 0.5, 0.09, 0.7\rangle$	$\langle 0.6, 0.09, 0.7\rangle$	$\langle 0.9, 0.09, 0.7\rangle$	$\langle 0.5, 0.09, 0.5\rangle$
$\mathfrak{I}_{22}$	$\langle 0.6, 0.04, 0.7\rangle$	$\langle 0.6, 0.09, 0.7\rangle$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.9, 0.09, 0.7\rangle$	$\langle 0.7, 0.09, 0.5\rangle$
$\mathfrak{I}_{23}$	$\langle 0.4, 0.04, 0.7\rangle$	$\langle 0.7, 0.09, 0.7\rangle$	$\langle 0.3, 0.09, 0.7\rangle$	$\langle 0.9, 0.09, 0.7\rangle$	$\langle 0.5, 0.09, 0.5\rangle$
$\mathfrak{I}_{24}$	$\langle 0.4, 0.04, 0.7\rangle$	$\langle 0.5, 0.09, 0.7\rangle$	$\langle 0.7, 0.09, 0.7\rangle$	$\langle 0.9, 0.09, 0.7\rangle$	$\langle 0.5, 0.09, 0.5\rangle$
$\mathfrak{I}_{31}$	$\langle 0.5, 0.09, 0.5\rangle$	$\langle 0.3, 0.09, 0.5\rangle$	$\langle 0.6, 0.09, 0.5\rangle$	$\langle 0.3, 0.09, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$
$\mathfrak{I}_{32}$	$\langle 0.6, 0.09, 0.5\rangle$	$\langle 0.6, 0.09, 0.5\rangle$	$\langle 0.6, 0.09, 0.5\rangle$	$\langle 0.3, 0.09, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$
$\mathfrak{I}_{33}$	$\langle 0.5, 0.04, 0.5\rangle$	$\langle 0.7, 0.09, 0.5\rangle$	$\langle 0.6, 0.09, 0.5\rangle$	$\langle 0.4, 0.09, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$
$\mathfrak{I}_{34}$	$\langle 0.5, 0.04, 0.5\rangle$	$\langle 0.5, 0.09, 0.5\rangle$	$\langle 0.7, 0.09, 0.5\rangle$	$\langle 0.5, 0.09, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$
$\mathfrak{I}_{41}$	$\langle 1.0, 0.09, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$	$\langle 0.8, 0.09, 0.5\rangle$	$\langle 0.4, 0.09, 0.5\rangle$	$\langle 0.5, 0.09, 0.5\rangle$
$\mathfrak{I}_{42}$	$\langle 1.0, 0.09, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$	$\langle 0.8, 0.09, 0.5\rangle$	$\langle 0.4, 0.09, 0.5\rangle$	$\langle 0.7, 0.09, 0.5\rangle$
$\mathfrak{I}_{43}$	$\langle 1.0, 0.04, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$	$\langle 0.8, 0.09, 0.5\rangle$	$\langle 0.4, 0.09, 0.5\rangle$	$\langle 0.5, 0.09, 0.5\rangle$
$\mathfrak{I}_{44}$	$\langle 1.0, 0.09, 0.5\rangle$	$\langle 0.9, 0.09, 0.5\rangle$	$\langle 0.8, 0.09, 0.5\rangle$	$\langle 0.5, 0.09, 0.5\rangle$	$\langle 0.5, 0.09, 0.5\rangle$

.

Step 4. Present $\check{R}_{\bigtriangleup}/\check{R}_{\bigtriangledown}$ in reduced fuzzy numerical grades matrix notation can be seen on the next page.

Steps 5, 6. Score values for $\check{R}_{\bigtriangleup}$ and $\check{R}_{\bigtriangledown}$, respectively, with $S(\aleph_i) = \Sigma\mathfrak{R}_{hrfg}/n(\mathfrak{R}_{hrfg})$,

$$S(\aleph_1) = 0.46, \quad S(\aleph_2) = 0.35, \quad S(\aleph_3) = 0.54, \quad S(\aleph_4) = 0.41, \quad S(\aleph_5) = 0.37,$$
$$S(\aleph_1) = 0.31, \quad S(\aleph_2) = 0.25, \quad S(\aleph_3) = 0.57, \quad S(\aleph_4) = 0.43, \quad S(\aleph_5) = 0.29,$$

$\check{R}_{\bigtriangleup}$	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_{11}$	⟨0.05⟩	⟨0.35⟩	⟨**0.71**⟩	⟨0.34⟩	⟨0.51⟩
$\mathfrak{I}_{12}$	⟨0.15⟩	⟨0.05⟩	⟨**0.64**⟩	⟨0.15⟩	⟨0.25⟩
$\mathfrak{I}_{13}$	⟨0.36⟩	⟨0.05⟩	⟨**0.64**⟩	⟨0.05⟩	⟨0.35⟩
$\mathfrak{I}_{14}$	⟨0.05⟩	⟨0.14⟩	⟨**0.55**⟩	⟨0.24⟩	⟨0.45⟩
$\mathfrak{I}_{21}$	⟨0.25⟩	⟨0.05⟩	⟨**0.61**⟩	⟨0.34⟩	⟨0.31⟩
$\mathfrak{I}_{22}$	⟨0.25⟩	⟨0.05⟩	⟨**0.54**⟩	⟨0.15⟩	⟨0.05⟩
$\mathfrak{I}_{23}$	⟨**0.56**⟩	⟨0.05⟩	⟨0.54⟩	⟨0.05⟩	⟨0.35⟩
$\mathfrak{I}_{24}$	⟨0.25⟩	⟨0.11⟩	⟨**0.45**⟩	⟨0.24⟩	⟨**0.45**⟩
$\mathfrak{I}_{31}$	⟨0.05⟩	⟨0.35⟩	⟨0.01⟩	⟨**0.44**⟩	⟨0.31⟩
$\mathfrak{I}_{32}$	⟨0.05⟩	⟨0.15⟩	⟨0.24⟩	⟨**0.25**⟩	⟨**0.25**⟩
$\mathfrak{I}_{33}$	⟨0.31⟩	⟨0.15⟩	⟨0.24⟩	⟨0.25⟩	⟨**0.35**⟩
$\mathfrak{I}_{34}$	⟨0.05⟩	⟨0.24⟩	⟨0.15⟩	⟨**0.54**⟩	⟨0.45⟩
$\mathfrak{I}_{41}$	⟨0.05⟩	⟨**0.35**⟩	⟨0.01⟩	⟨0.34⟩	⟨0.31⟩
$\mathfrak{I}_{42}$	⟨0.15⟩	⟨0.15⟩	⟨**0.24**⟩	⟨0.15⟩	⟨0.05⟩
$\mathfrak{I}_{43}$	⟨**0.36**⟩	⟨0.25⟩	⟨0.24⟩	⟨0.05⟩	⟨0.35⟩
$\mathfrak{I}_{44}$	⟨0.05⟩	⟨0.05⟩	⟨0.24⟩	⟨0.25⟩	⟨**0.45**⟩

,

$\check{R}_{\bigtriangledown}$	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$
$\mathfrak{I}_{11}$	⟨0.09⟩	⟨**0.21**⟩	⟨0.01⟩	⟨0.19⟩	⟨**0.21**⟩
$\mathfrak{I}_{12}$	⟨0.09⟩	⟨0.09⟩	⟨**0.31**⟩	⟨0.19⟩	⟨0.09⟩
$\mathfrak{I}_{13}$	⟨0.04⟩	⟨0.19⟩	⟨0.31⟩	⟨0.19⟩	⟨**0.41**⟩
$\mathfrak{I}_{14}$	⟨0.09⟩	⟨0.01⟩	⟨0.09⟩	⟨0.19⟩	⟨**0.41**⟩
$\mathfrak{I}_{21}$	⟨0.26⟩	⟨0.11⟩	⟨0.01⟩	⟨**0.29**⟩	⟨0.09⟩
$\mathfrak{I}_{22}$	⟨0.06⟩	⟨0.01⟩	⟨**0.31**⟩	⟨0.29⟩	⟨0.29⟩
$\mathfrak{I}_{23}$	⟨0.26⟩	⟨0.09⟩	⟨**0.31**⟩	⟨0.29⟩	⟨0.09⟩
$\mathfrak{I}_{24}$	⟨0.26⟩	⟨**0.29**⟩	⟨0.09⟩	⟨**0.29**⟩	⟨0.09⟩
$\mathfrak{I}_{31}$	⟨0.09⟩	⟨0.11⟩	⟨0.19⟩	⟨0.11⟩	⟨**0.49**⟩
$\mathfrak{I}_{32}$	⟨0.19⟩	⟨0.19⟩	⟨0.19⟩	⟨0.11⟩	⟨**0.49**⟩
$\mathfrak{I}_{33}$	⟨0.04⟩	⟨0.29⟩	⟨0.19⟩	⟨0.01⟩	⟨**0.49**⟩
$\mathfrak{I}_{34}$	⟨0.04⟩	⟨0.09⟩	⟨0.29⟩	⟨0.09⟩	⟨**0.49**⟩
$\mathfrak{I}_{41}$	⟨**0.59**⟩	⟨0.49⟩	⟨0.39⟩	⟨0.01⟩	⟨0.09⟩
$\mathfrak{I}_{42}$	⟨**0.59**⟩	⟨0.49⟩	⟨0.39⟩	⟨0.01⟩	⟨0.29⟩
$\mathfrak{I}_{43}$	⟨**0.54**⟩	⟨0.49⟩	⟨0.39⟩	⟨0.01⟩	⟨0.09⟩
$\mathfrak{I}_{44}$	⟨**0.59**⟩	⟨0.49⟩	⟨0.39⟩	⟨0.09⟩	⟨0.09⟩

.

We can see that the ranking order of alternatives according to the non-increasing order of their scores for $\check{R}_{\bigtriangleup}$ and $\check{R}_{\bigtriangledown}$, respectively, is $\aleph_3 \succeq \aleph_1 \succeq \aleph_4 \succeq \aleph_5 \succeq \aleph_2$ and $\aleph_3 \succeq \aleph_4 \succeq \aleph_1 \succeq \aleph_5 \succeq \aleph_2$.

4.6. Results and discussion

We can observe the ranking order of materials to be preferred for anti-microbial bio-fabricated textiles with both algorithms in Table 14.

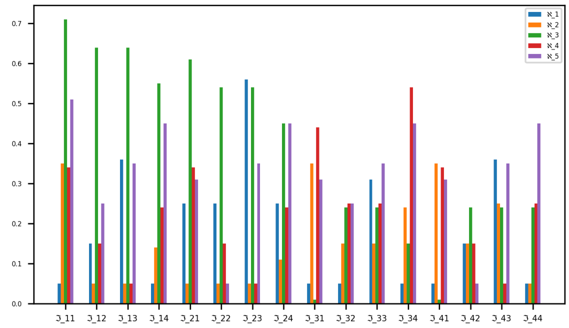

Figure 3: Fuzzy numerical grades under $\check{\mathrm{R}}_{\triangle}$.

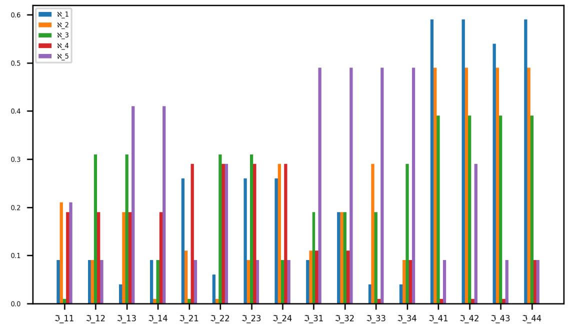

Figure 4: Fuzzy numerical grades under $\check{\mathrm{R}}_{\bigtriangledown}$.

Table 14: Comparison between score values under Q^{t}NSUI-algorithm and Q^{t}NSAO-algorithm.

Algorithms	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$	$\aleph_5$	Ranking
Q^{t}NS**U**I	0.2853	0.2529	0.3404	0.3156	0.3196	$\aleph_3 \succeq \aleph_5 \succeq \aleph_4 \succeq \aleph_1 \succeq \aleph_2$
Q^{t}NSU**I**	0.1260	0.1047	0.1451	0.1397	0.1344	$\aleph_3 \succeq \aleph_4 \succeq \aleph_5 \succeq \aleph_1 \succeq \aleph_2$
Q^{t}NS**A**O	0.46	0.35	0.54	0.41	0.37	$\aleph_3 \succeq \aleph_1 \succeq \aleph_4 \succeq \aleph_5 \succeq \aleph_2$
Q^{t}NSA**O**	0.31	0.25	0.57	0.43	0.29	$\aleph_3 \succeq \aleph_4 \succeq \aleph_1 \succeq \aleph_5 \succeq \aleph_2$

In examining the preference scores obtained through $\grave{\mathrm{N}}_i^{qt}$-union and $\grave{\mathrm{N}}_i^{qt}$-intersection operations in the Q^{t}NSUI-algorithm and the score values derived from $\grave{\mathrm{N}}_i^{qt}$-AND product and $\grave{\mathrm{N}}_i^{qt}$-OR product operations in the Q^{t}NSAO-algorithm, a clear pattern emerges in determining the suitability of materials for the manufacturing of anti-microbial bio-fabricated textiles. In the Q^{t}NSUI-algorithm, the $\grave{\mathrm{N}}_i^{qt}$-union operation captures the collective preferences for each alternative (material) across multiple criteria. Conversely, the $\grave{\mathrm{N}}_i^{qt}$-intersection operation isolates the commonality in preferences, emphasizing consensus across criteria. The outcomes of these operations reflect the intricate relationships and trade-offs between different aspects of each material. Simultaneously, the Q^{t}NSAO-algorithm employs $\grave{\mathrm{N}}_i^{qt}$-AND and $\grave{\mathrm{N}}_i^{qt}$-OR product operations to compute scores, emphasizing the conjunctive and disjunctive aspects of the neutrosophic set. These operations provide insight into the compatibility and complementarity of material characteristics, shedding light on how each alternative aligns with the desired attributes.

The consistent determination of $\aleph_3$ material as more sustainable under both algorithms underscores the robustness and adaptability of the proposed Quantified Neutrosophic Set (Q^{t}NS). The alignment of results across diverse operations and algorithms highlights the reliability of the framework in evaluating and ranking materials for anti-microbial bio-fabricated textiles. This robust and adaptable nature of Q^{t}NS is a pivotal strength, offering decision-makers a versatile tool for navigating uncertainties in material selection. The ability to produce consistent and meaningful rankings enhances confidence in the decision-making process, emphasizing the practical utility and effectiveness of the proposed framework in real-world applications. As a result, the determination of $\aleph_3$ as the most sustainable material signifies the strength and reliability of Q^{t}NS, positioning it as a valuable asset in sustainable decision-making contexts.

4.7. *Advantages of using* Q^{t}NS

The proposed Q^{t}NS offers several unique advantages compared to conventional neutrosophic sets.

Reduced representation of uncertainty: Q^{t}NS introduces a quantification mechanism that allows for a reduced representation of uncertainty. By quantifying the degree of truth, indeterminacy, and falsehood, Q^{t}NS offers a finer-grained approach to modeling uncertain information.

Flexibility and adaptability: The parameters α and β in Q^{t}NS enable flexible adjustment of the membership functions, providing users with the ability to customize the model according to specific requirements or preferences. This flexibility enhances the adaptability of Q^{t}NS to different decision-making contexts.

Robust mathematical formalism: Q^tNS incorporates rigorous mathematical formalisms and operations, ensuring robustness and reliability in its application. The defined operations and properties provide a solid foundation for the use of Q^tNS in decision support systems and other applications, offering clear guidelines for implementation and analysis.

Iterative refinement: The iterative refinement process allows for continuous improvement of the model's precision, flexibility, and applicability over time. By iteratively adjusting the parameters α and β based on ongoing analyses and feedback, Q^tNS can evolve to better meet the needs of users and address emerging challenges.

5. Conclusions

At the industrial level, the decision-making process is transparent, comprehensive, and based on both technical expertise and a thorough evaluation of criteria. As industries continue to evolve towards sustainable practices, our modified approach sets the stage for continued research and innovation in the field of neutrosophic sets and their applications to complex decision environments with controlled uncertainty. In this research, we established a parameterized quantification in the neutrosophic set, known as Q^tNS, that regulates and, probably, reduces the indeterminacy component. This innovation is vital in the context of sustainable material choices, as it enables us to navigate complex decisions with clarity and precision. In contrast, standard neutrosophic sets lack this specific mechanism to reduce or control indeterminacy, making our approach a valuable tool for industries in different fields, ensuring better-informed and more impactful decisions. Table 15 shows a comparison of the novelty of our structure to existing structures.

Table 15: Comparison of Q^tNS with some existing literature.

Model	Existent ship	Indeterminacy	Non-existent ship	Parameterized quantificationI	Controlled ndeterminacy
Fuzzy set [53]	✓	×	×	×	×
Intuitionistic FS [4]	✓	×	✓	×	×
Neutrosophic set [5]	✓	✓	✓	×	×
Rough NS [8]	✓	✓	✓	×	×
(Proposed structure)	✓	✓	✓	✓	✓

With the newly established structures of union, intersection, AND-product, and OR-product, several operations and beneficial characteristics are also presented. Furthermore, for the MCDM problem, we dealt with two algorithms, Q^tNSUI and Q^tNSAO, which provide textile companies with suitable and robust methods for successfully selecting the most suitable antimicrobial bio-fabricated textile options with controlled uncertainty for various health care industries. We focus on the bio-fabricated textiles because there are a few resources on selecting an appropriate material for bio-fabricated textiles. According to the articles on selecting textile materials, our research provides the most thorough collection of criterion selection approach. This research contributes not only to the specific context of sustainable material selection for anti-microbial bio-fabricated textiles but also to the targeted goal of indeterminacy reduction, which is crucial in real-life decision scenarios, offering decision-makers a tool to manage and control uncertainties effectively, leading to more robust and reliable decision outcomes.

In future work, more research can be conducted regarding the Q^tNS and its application in refining the decision-making process through the reduction of indeterminacy. We intend to apply our approach to more comprehensive data sets to evaluate its robustness in different scenarios of decision-making.

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