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Robust Control of Inverted Pendulum Using Fuzzy Sliding Mode Control (FSMC)

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Abstract

Due to the vast application of inverted pendulum in robust, one of the most important Problems today is robotics and its control. In this paper by using fuzzy sliding control, we have tried to control the inverted pendulum angle by nonlinear equations. No oscillation in response in seen by using this method and the time for achieving a desirable response is appropriate. The results of this simulation have as well as its comparison with the classical controller has been mentioned at the conclusion.

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1. Introduction

There are variety methods for Inverted Pendulum control that are presented since now. The presented methods for Inverted Pendulum control are divided generally in three groups. Classic methods such as PID and PI controllers [1,2]. Modern methods (adaptation-optimum...) [3, 4, 5] . Artificial methods such as neural networks and fuzzy and Genetic Algorithm and PSO [6, 7, 8] theory are the presented methods for Inverted Pendulum angle control.

The design method in linear control comprise based on main application the wide span ' of frequency, linear controller has a weak application, because it can't compensate the nonlinear system effect completely.

2. Modeling an Inverted Pendulum

The cart with an inverted pendulum, shown below, is "bumped" with an impulse force, F. Determine the dynamic equations of motion for the system, and linearize about the pendulum's angle, theta = 0 (in other words, assume that pendulum does not move more than a few degrees away from the vertical, chosen to be at an angle of 0). Find a controller to satisfy all of the design requirements given below. For this example, let's assume that.
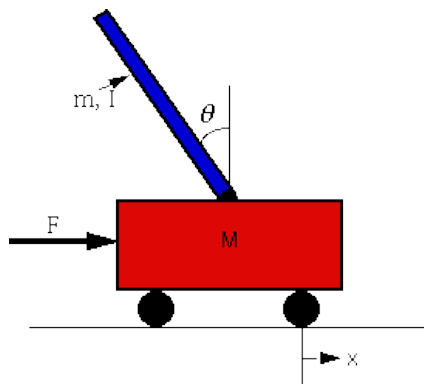

Figure 1.The structure of an Inverted Pendulum

Table 1physical parameters of Inverted Pendulum

M	mass of the cart	0.5 kg
m	mass of the pendulum	0.2 kg
b	friction of the cart	0.1 N/m/sec
l	length to pendulum center of mass	0.3 m
I	inertia of the pendulum	0.006 kg*m^2
F	force applied to the cart	
x	cart position coordinate	
theta	pendulum angle from vertical	

This system is tricky to model in Simulink because of the physical constraint (the pin joint) between the cart and pendulum which reduces the degrees of freedom in the system. Both the cart and the pendulum have one degree of freedom (X and theta, respectively). We will then model Newton's equation for these two degrees of freedom.

$$\frac{d^2x}{dt^2} = \frac{1}{M} \Sigma_{cart} \ F_x = \frac{1}{M}\left(F - N - b\frac{dx}{dt}\right) \tag{1}$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{I} \sum_{pend} \tau = \frac{1}{I}(NL \cos(\theta) + PL \sin(\theta)) \tag{2}$$

It is necessary, however, to include the interaction forces N and P between the cart and the pendulum in order to model the dynamics. The inclusion of these forces requires modeling the x and y dynamics of the pendulum in addition to its theta dynamics. Generally, we would like to exploit the modeling power of Simulink and let the simulation take care of the algebra. Therefore, we will model the additional x and y equations for the pendulum.

$$m\frac{d^2x_p}{dt^2} = \Sigma_{pend} \ F_x = N \tag{3}$$

$$\Rightarrow N = m\frac{d^2x_p}{dt^2} \tag{4}$$

$$m\frac{d^2y_p}{dt^2} = P - mg \tag{5}$$

$$\Rightarrow P = m\left(\frac{d^2y_p}{dt^2} + g\right) \tag{6}$$

However, xp and yp are exact functions of theta. Therefore, we can represent their derivatives in terms of the derivatives of theta.

$$x_p = x - L \sin(\theta) \tag{7}$$

$$\frac{dx_p}{dt} = \frac{dx}{dt} - L \cos(\theta)\frac{d^2\theta}{dt^2} \tag{8}$$

$$\frac{d^2x_p}{dt^2} = \frac{d^2x}{dt^2} + L \sin(\theta)\left(\frac{d\theta}{dt}\right) - L \cos(\theta)\frac{d^2\theta}{dt^2} \tag{9}$$

$$y_p = L \cos(\theta) \tag{10}$$

$$\frac{dy_p}{dt} = -L \sin(\theta)\frac{d\theta}{dt} \tag{11}$$

$$\frac{d^2y_p}{dt^2} = -L \cos(\theta)\left(\frac{d\theta}{dt}\right)^2 - L \sin(\theta)\frac{d^2\theta}{dt^2} \tag{12}$$

These expressions can then be substituted into the expressions for N and P. Rather than continuing with algebra here, we will simply represent these equations in Simulink. Simulink can work directly with nonlinear equations.
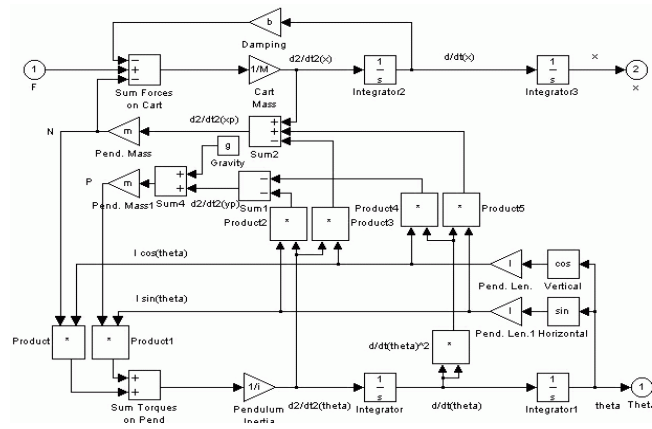

Figure 2.The block diagram of an Inverted Pendulum

3. Sliding Mode Controller

Nonlinear system control that its model isn't clear carefully works with two methods:

(1).Robust control methods.

(2).Adaptive control methods.

In control view, uncertainly in modeling is divided in two main kinds:

(1).Non certainly in existent Para meters in model

(2).Estimating the lower step for system and being UN modeled dynamics in the estimating model.

Sliding control is one of the designed modes for robust control that make access to system desired application estimating system in model.

The major idea of this method is the controlling of nonlinear first grade system is easier than n grade system control in spite of uncertainly.

But this function maybe cause the control law with more energy that is not practicable implement station.

Sliding mode is really compromise between modeling and suitable operation with inaccurate design.

We consider the nonlinear system model in this rule:

$$X^n = f(x) + b(x)u \tag{13}$$

That F(x) is nonlinear function, its high boundary characterized as X function. B(x) is a continuous function that its high and low boundaries characterized by X function.

The good of finding X is in this way that in g(x) F(x) function we can follow the desirable mode in spite of uncertainly.

$$\widetilde{X} = X - X_d = \left[\tilde{X}, \widetilde{X'}, \ldots, \tilde{X}^{n-1}\right] \tag{14}$$

In ideal state

$$\tilde{X} = 0 \tag{15}$$

Sliding surface equation defines as below:

$$s = e' + \alpha_1 e + \alpha_2 \int e \, dt \tag{16}$$

Because of the signals of control that gain with this designing method has limited energy, it is necessary to:

$$X_d(0) = X(0) \tag{17}$$

In other word:

$$S(X,t) \equiv 0 \tag{18}$$

$$\frac{1}{2}\frac{dS^2}{dt} \leq -\eta|S| \tag{19}$$

In designing, the control low on S(t) continuously is noticed in cause we should concentrate to carelessness in model in sliding surface and reduced the chattering effect.

We can write the system's dynamics when in some situation they are in sliding state.

$$S'=0 \tag{20}$$

The gained control signals for this system are as below:
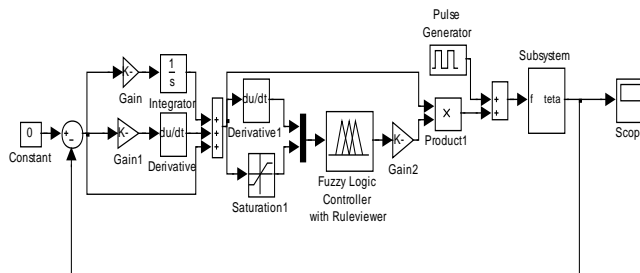
$$U = k_1 \times out_{fuzzy} \times S \tag{21}$$

Figure 3.simulink block diagram of FSMC

Fuzzy controls are designed based on created sliding surface and sliding surface changes.
All of the fuzzy rules collection came in Table II

Table2. Fuzzy Rule

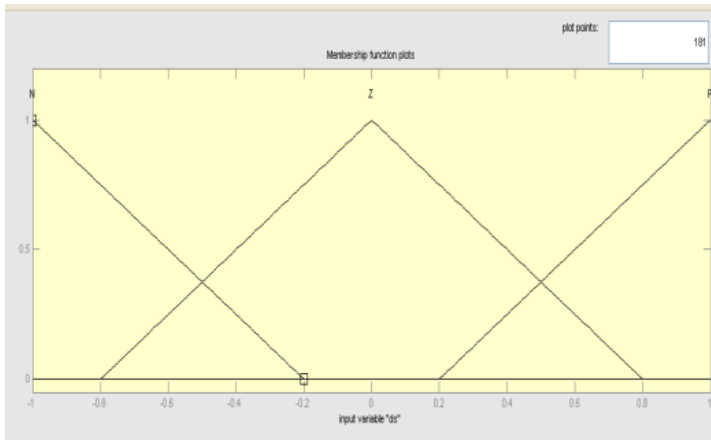| dS / S	NB	NS	ZE	PS	PB
N	B	B	M	S	B
Z	B	M	S	M	B
P	B	S	M	B	B

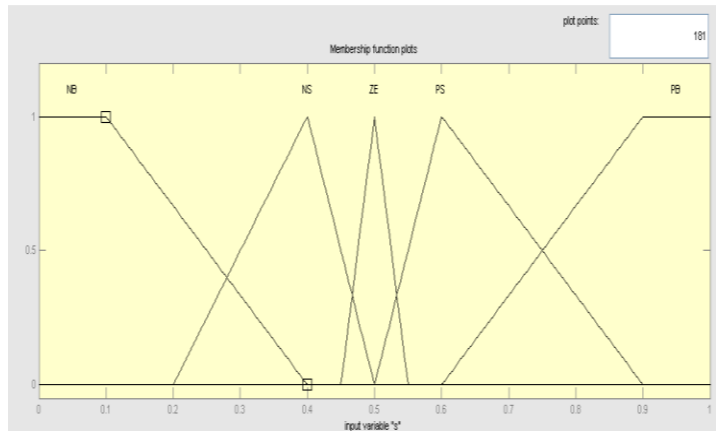Figure 4.Membership functions for (s) normalized inputs

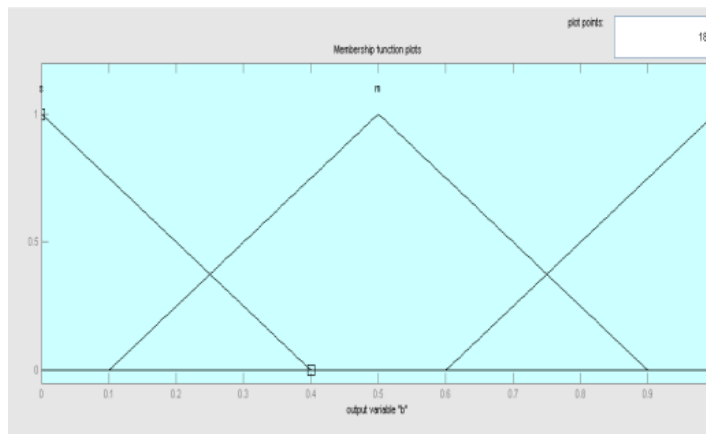Figure 5.Membership functions for (ds/dt) normalized inputs

Figure 6.Membership functions for (out fuzzy) normalized outputs

3. Conclusion

In this paper, a robust control system with the fuzzy sliding mode controller and the additional compensator is presented for an Inverted Pendulum position control. According to the simulation results, the FSMC controllers can provide the properties of insensitivity and robustness to uncertainties and external disturbances, and response of the Inverted Pendulum for FSMC controllers against uncertainties

and external disturbance is the same Fuzzy sliding mode controller gives a better response to system than the fuzzy and classical PID controllers.
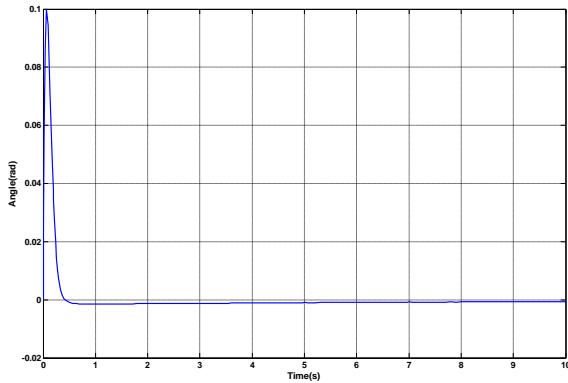

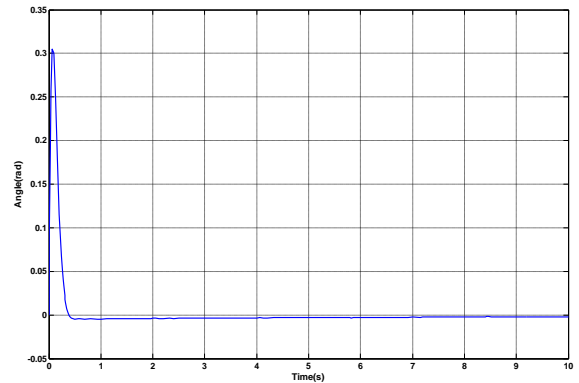**Figure 7.Inverted pendulum rod angle for initial 0.1 radians**

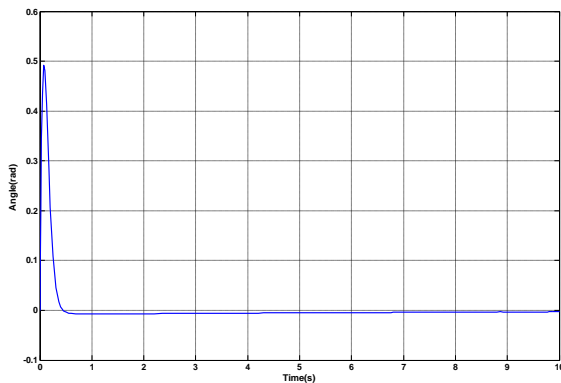**Figure 8.Inverted pendulum rod angle for initial 0.3 radians**

Figure9.Inverted pendulum rod angle for initial 0.5 radians
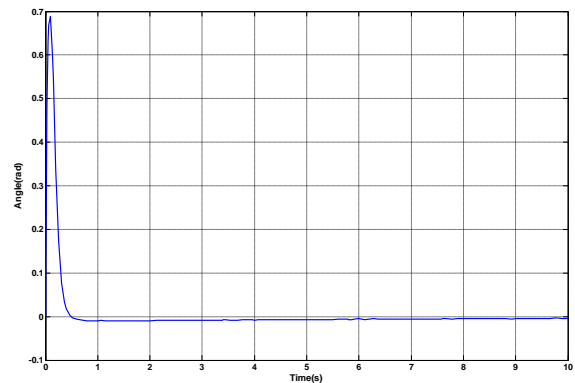

Figure10.Inverted pendulum rod angle for initial 0.7 radians

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