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A remark on positive solution for a class of (p,q)-Laplacian nonlinear system with sign-changing weight and combined nonlinear effects

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Abstract

In this article, we study the existence of positive solution for a class of (p,q)- Laplacian system

$$\begin{cases} -\Delta_p u = \lambda\, a(x)\, f(u)\, h(v), & x \in \Omega, \\ -\Delta_q v = \lambda\, b(x)\, g(u)\, k(v), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z = div\,(|\nabla z|^{p-2}\nabla z)$, $p > 1$, $\lambda > 0$ is a parameter and Ω is a bounded domain in $R^N(N > 1)$ with smooth boundary $\partial\Omega$. Here

$a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe negative near the boundary and f, g, k, h are C^1 nondecreasing functions such that $f, g, h, k : [0, \infty) \to [0, \infty)$; $f(s)$, $k(s)$, $h(s)$, $g(s) > 0$; $s > 0$ and

$$\lim_{x\to\infty} \frac{h\Big(A\,(g(x))^{\frac{1}{q-1}}\Big)\,(f(x))^{p-1}}{x^{p-1}} = 0,$$

for every $A > 0$.

We discuss the existence of positive solution when h, k, f, g, $a(x)$ and $b(x)$ satisfy certain additional conditions. We use the method of sub-super solutions to establish our results.

Keywords: (p, q)- Laplacian system; Sign-changing weight.
AMS Subject Classification: 35J60, 35J66, 35J92.

1 Introduction

In this paper we consider the existence of positive solution for the nonlinear system

$$\begin{cases} -\Delta_p u = \lambda\, a(x)\, f(u)\, h(v), & x \in \Omega, \\ -\Delta_q v = \lambda\, b(x)\, g(u)\, k(v), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z = div\,(|\nabla z|^{p-2}\nabla z)$, $p > 1$, $\lambda > 0$ is a parameter, Ω is a bounded domain in $R^N (N > 1)$ with smooth boundary $\partial\Omega$, $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe negative near the boundary and $k, h, f, g : [0, \infty) \to [0, \infty)$ are C^1 nondecreasing functions such that $h(s)$, $k(s)$, $f(s)$, $g(s) > 0$ for $s > 0$.

Problems involving the p-Laplace operator arise in some physical models like the flow of non-Newtonian fluids: pseudo-plastic fluids correspond to $p \in (1, 2)$ while dilatant fluids correspond to $p > 2$. The case $p = 2$ expresses Newtonian fluids [1]. On the other hand, quasilinear elliptic systems like (1) has an extensive practical background. It can be used to describe the multiplicate chemical reaction catalyzed by the catalyst grains under constant or variant temperature, it can be used in the theory of quasiregular and quasiconformal mappings in Riemannian manifolds with boundary (see [2, 3]) and can be a simple model of tubular chemical reaction, more naturally, it can be a correspondence of the stable station of dynamical system determined by the reaction-diffusion system, see Ladde and Lakshmikantham et al. [4]. More naturally, it can be the populations of two competing species [5]. So, the study of positive solutions of elliptic systems has more practical meanings.

For the single-equation, namely equation of the form

$$\begin{cases} -\Delta_p u = \lambda\, a(x)\, f(u), & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases}$$

with sign-changing weight function has been studied by several authors (see [6, 7]). See [8, 9] where the authors discussed the system (1) when $p = q = 2$, $f \equiv 1$, $k \equiv 1$, $a \equiv 1$, $b \equiv 1$, h, g are increasing and $h, g \geq 0$. In [10], the authors extended the study of [8], to the case when no sign conditions on $h(0)$ or $g(0)$ were required, and in [11] they extend this study to the case when $p = q > 1$. Here we focus on further extending the study in [11] to the system (1). In fact, we study the existence of positive solution to the system (1) with sign-changing weight functions $a(x)$ and $b(x)$. Due to this weight functions, the extensions are challenging and nontrivial. Our approach is based on the method of sub-super solutions, see [12, 13]. Several methods have been used to treat quasilinear equations and systems. In the scalar case, weak solutions can be obtained through variational methods which provide critical points of the corresponding energy functional, an approach which is also fruitful in the case of potential systems i.e, the nonlinearities on the right hand side are the gradient of a C^1-functional [14]. However, due to the loss of the variational structure, the treatment of nonvariational systems like (1) is more complicated and is based mostly on topological methods [15]. We refer to [16], [17], [18], [19] for additional results on elliptic problems involving the p-Laplacian.

To precisely state our existence result we consider the eigenvalue problem

$$\begin{cases} -\Delta_r \phi = \lambda\, |\phi|^{r-2}\phi, & x \in \Omega, \\ \phi = 0, & x \in \partial\Omega. \end{cases} \tag{2}$$

Let $\phi_{1,r}$ be the eigenfunction corresponding to the first eigenvalue $\lambda_{1,r}$ of (2) such that $\phi_{1,r}(x) > 0$ in Ω, and $||\phi_{1,r}||_\infty = 1$ for $r = p, q$. It can be shown that $\frac{\partial \phi_{1,r}}{\partial n} < 0$ on $\partial\Omega$ for $r = p, q$. Here n is the outward normal. This result is well known and hence, depending on Ω, there exist positive constants m, δ, σ such that

$$\lambda_{1,r}\, \phi_{1,r}^r - |\nabla \phi_{1,r}|^r \;\leq\; -m, \qquad x \in \bar{\Omega}_\delta, \tag{3}$$

$$\phi_{1,r} \geq \sigma, \qquad x \in \Omega_0 = \Omega \setminus \bar{\Omega}_\delta, \tag{4}$$

with $\bar{\Omega}_\delta = \{x \in \Omega \mid d(x, \partial\Omega) \leq \delta\}$. We will also consider the unique solution $\zeta_r(x) \in W_0^{1,r}(\Omega)$ of the boundary value problem

$$\begin{cases} -\Delta_r \zeta_r(x) = 1, & x \in \Omega, \\ \zeta_r(x) = 0, & x \in \partial\Omega, \end{cases}$$

to discuss our existence result. It is known that $\zeta_r(x) > 0$ in Ω and $\frac{\partial \zeta_r(x)}{\partial n} < 0$ on $\partial\Omega$.

Here we assume that the weight functions $a(x)$ and $b(x)$ take negative values in $\bar{\Omega}_\delta$, but require $a(x)$ and $b(x)$ be strictly positive in $\Omega - \bar{\Omega}_\delta$. To be precise we assume that there exist positive constants a_0, a_1, b_0 and b_1 Such that $a(x) \geq -a_0$, $b(x) \geq -b_0$ on $\bar{\Omega}_\delta$ and $a(x) \geq a_1$, $b(x) \geq b_1$ on $\Omega - \bar{\Omega}_\delta$.

2 Existence results

In this section, we shall establish our existence result via the method of sub and supersolutions. A pair of nonnegative functions (ψ_1, ψ_2), (z_1, z_2) are called a subsolution and supersolution of (1) if they satisfy $(\psi_1, \psi_2) = (0, 0) = (z_1, z_2)$ on $\partial\Omega$ and

$$\int_\Omega |\nabla \psi_1|^{p-2} |\nabla \psi_1| \cdot \nabla w \, dx \leq \lambda \int_\Omega a(x) \, f(\psi_1) \, h(\psi_2) \, w dx,$$

$$\int_\Omega |\nabla \psi_2|^{q-2} |\nabla \psi_2| \cdot \nabla w \, dx \leq \lambda \int_\Omega b(x) \, g(\psi_2) \, k(\psi_1) \, w dx,$$

$$\int_\Omega |\nabla z_1|^{p-2} |\nabla z_1| \cdot \nabla w \, dx \geq \lambda \int_\Omega a(x) \, f(z_1) \, h(z_2) \, w dx,$$

$$\int_\Omega |\nabla z_2|^{q-2} |\nabla z_2| \cdot \nabla w \, dx \geq \lambda \int_\Omega b(x) \, g(z_2) \, k(z_1) \, w \, dx,$$

for all $w \in W = \{w \in C_0^\infty(\Omega) | w \geq 0 \in \Omega\}$. Then the following result holds:

Lemma 2.1. (See [12]) Suppose there exist sub and super- solutions (ψ_1, ψ_2) and (z_1, z_2) respectively of (1) such that $(\psi_1, \psi_2) \leq (z_1, z_2)$. Then (1) has a solution (u, v) such that $(u, v) \in [(\psi_1, \psi_2), (z_1, z_2)]$.

We make the following assumptions:

- **(H1)** $f, g, h, k : [0, \infty) \to [0, \infty)$ are C^1 nondecreasing functions such that $f(s)$, $h(s)$, $k(s)$, $g(s) > 0$, for $s > 0$, $\lim_{x\to\infty} k(x) < \infty$ and $\lim_{x\to\infty} g(x) = \infty$.
- **(H2)** For all $A > 0$,

$$\lim_{x\to\infty} \frac{h\Big(A\,(g(x))^{\frac{1}{q-1}}\Big)\,(f(x))^{p-1}}{x^{p-1}} = 0.$$

(H3) Suppose that there exists $\epsilon > 0$ such that:

$$\frac{\lambda_{1,p}\, a_0}{m}\, f(\epsilon^{\frac{1}{p-1}})\, h(\epsilon^{\frac{1}{q-1}}) < \min\Big\{b_1\, g\Big((\tfrac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, k\Big((\tfrac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big), \\ a_1\, f\Big((\tfrac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, h\Big((\tfrac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big)\Big\},$$

$$\frac{\lambda_{1,q}\, b_0}{m}\, g(\epsilon^{\frac{1}{p-1}})\, k(\epsilon^{\frac{1}{q-1}}) < \min\Big\{b_1\, g\Big((\tfrac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, k\Big((\tfrac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big), \\ a_1\, f\Big((\tfrac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, h\Big((\tfrac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big)\Big\},$$

and

$$\max\Big\{\frac{\epsilon\,\lambda_{1,q}}{b_1\, g\Big((\frac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, k\Big((\frac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big)}, \\ \frac{\epsilon\,\lambda_{1,p}}{a_1\, f\Big((\frac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, h\Big((\frac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big)}\Big\} \leq \frac{1}{||b||_\infty\, k\Big(g(c)^{\frac{1}{q-1}}\, ||\zeta_q||_\infty\Big)},$$

where c is positive constant that will be fixed later. Now we are ready to state our existence result.

Theorem 2.2. Let $(H1) - (H3)$ hold. Then there exists a positive solution of (1) for every $\lambda \in [\lambda_*(\epsilon), \lambda^*(\epsilon)]$, where

$$\lambda^* = \min\Big\{\frac{\epsilon\, m}{a_0\, f(\epsilon^{\frac{1}{q-1}})\, h(\epsilon^{\frac{1}{q-1}})}, \frac{m\,\epsilon}{b_0\, g(\epsilon^{\frac{1}{p-1}})\, k(\epsilon^{\frac{1}{q-1}})}, \frac{1}{||b||_\infty\, k(g(c)^{\frac{1}{q-1}})\, ||\zeta_q||_\infty}\Big\},$$

$$\lambda_* = \max\Big\{\frac{\epsilon\,\lambda_{1,q}}{b_1\, g\Big((\frac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, k\Big((\frac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big)}, \frac{\epsilon\,\lambda_{1,p}}{a_1\, f\Big((\frac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\, h\Big((\frac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big)}\Big\}.$$

Remark 2.3. Note that $(H3)$ implies $\lambda_* < \lambda^*$.

Proof of Theorem 2.2. We shall verify that

$$(\psi_1, \psi_2) = \Big((\frac{p-1}{p})\,\epsilon^{\frac{1}{p-1}}\,\phi_{1,p}^{\frac{p}{p-1}}\,,\,(\frac{q-1}{q})\,\epsilon^{\frac{1}{q-1}}\,\phi_{1,q}^{\frac{q}{q-1}}\Big),$$

is a sub-solution of (1). Let $w \in W$. Then a calculation shows that

$$\begin{aligned} \int_\Omega |\nabla\psi_1|^{p-2}\,\nabla\psi_1\,\nabla w\,dx &= \epsilon\int_\Omega \phi_{1,p}\,|\nabla\phi_{1,p}|^{p-2}\,\nabla\phi_{1,p}\,\nabla w\,dx \\ &= \epsilon\Big\{\int_\Omega |\nabla\phi_{1,p}|^{p-2}\,\nabla\phi_{1,p}\,\nabla(\phi_{1,p}w)\,dx - \int_\Omega |\nabla\phi_{1,p}|^p\,w\,dx\Big\} \\ &= \epsilon\Big\{\int_\Omega [\lambda_{1,p}\,\phi_{1,p}^p - |\nabla\phi_{1,p}|^p]w\,dx\Big\}. \end{aligned}$$

Similarly

$$\int_\Omega |\nabla\psi_2|^{q-2}\nabla\psi_2\nabla w\,dx = \epsilon\Big\{\int_\Omega [\lambda_{1,q}\,\phi_{1,q}^q - |\nabla\phi_{1,q}|^q]w\,dx\Big\}.$$

First we consider the case when $x \in \bar{\Omega}_\delta$. We have $|\nabla\phi_r|^r - \lambda_1\,\phi_r^r \geq m$ on $\bar{\Omega}_\delta$ for $r = p, q$, and since $\lambda \leq \lambda^*$, then $\lambda \leq \frac{m\,\epsilon}{a_0\,f(\epsilon^{\frac{1}{p-1}})\,h(\epsilon^{\frac{1}{q-1}})}$. Hence

$$\begin{aligned} \epsilon\,(\lambda_{1,p}\,\phi_{1,p}^p - |\nabla\phi_{1,p}|^p) &\leq -m\,\epsilon \\ &\leq -\lambda\,a_0\,f(\epsilon^{\frac{1}{p-1}})\,h(\epsilon^{\frac{1}{q-1}}) \\ &\leq -\lambda\,a_0\,f\Big((\frac{p-1}{p})\,\epsilon^{\frac{1}{p-1}}\,\phi_{1,p}^{\frac{p}{p-1}}\Big)\,h\Big((\frac{q-1}{q})\,\epsilon^{\frac{1}{q-1}}\,\phi_{1,q}^{\frac{q}{q-1}}\Big) \\ &\leq \lambda\,a(x)\,f(\psi_1)\,h(\psi_2). \end{aligned}$$

A similar argument shows that

$$\epsilon\,(\lambda_{1,q}\,\phi_{1,q}^q - |\nabla\phi_{1,q}|^q) \leq \lambda\,b(x)\,g(\psi_1)\,k(\psi_2)$$

when $x \in \bar{\Omega}_\delta$.

On the other hand, on $\Omega \setminus \bar{\Omega}_\delta$, we note that $\phi_{1,r} \geq \sigma$ for $r = p, q$. Also $a(x) \geq a_1$, $b(x) \geq b_1$ and since $\lambda \geq \lambda_*$, we have $\lambda \geq \frac{\epsilon\,\lambda_{1,p}}{a_1\,f\Big((\frac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\,h\Big((\frac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big)}$. Hence

$$\begin{aligned} \epsilon\,(\lambda_{1,p}\,\phi_{1,p}^p - |\nabla\phi_{1,p}|^p) &\leq \epsilon\,\lambda_{1,p} \\ &\leq \lambda\,a_1\,f\Big((\frac{p-1}{p})\epsilon^{\frac{1}{p-1}}\,\sigma^{\frac{p}{p-1}}\Big)\,h\Big((\frac{q-1}{q})\epsilon^{\frac{1}{q-1}}\,\sigma^{\frac{q}{q-1}}\Big) \\ &\leq \lambda\,a(x)\,f(\psi_1)\,h(\psi_2). \end{aligned}$$

A similar argument shows that

$$-\epsilon\,(\lambda_{1,q}\,\phi_{1,q}^q - |\nabla\phi_{1,q}|^q) \leq \lambda\,b(x)\,g(\psi_1)\,k(\psi_2).$$

Hence

$$\int_\Omega |\nabla\psi_1|^{p-2}\,|\nabla\psi_1|\cdot\,\nabla w\,dx \leq \lambda\int_\Omega a(x)\,f(\psi_1)\,h(\psi_2)\,wdx,$$

$$\int_\Omega |\nabla\psi_2|^{q-2}\,|\nabla\psi_2|\cdot\,\nabla w\,dx \le \lambda\int_\Omega b(x)\,g(\psi_1)\,k(\psi_2)\,wdx,$$

i.e., (ψ_1, ψ_2) is a sub-solution of (1).

Now, we will prove there exists a c large enough so that

$$(z_1, z_2) = \Big(\frac{c}{||\zeta_p(x)||_\infty}\,\zeta_p(x)\,,\; g(c)^{\frac{1}{q-1}}\,\zeta_q(x)\Big),$$

is a super-solution of (1). A calculation shows that:

$$\begin{aligned}\int_\Omega |\nabla z_1|^{p-2}\,\nabla z_1\,\nabla w\,dx &= (\frac{c}{||\zeta_p(x)||_\infty})^{p-1}\int_\Omega |\nabla\zeta_p|^{p-2}\,\nabla\zeta_p\,\nabla w\,dx\\ &= (\frac{c}{||\zeta_p(x)||_\infty})^{p-1}\int_\Omega w\,dx.\end{aligned}$$

By $(H2)$ we can choose c large enough so that

$$c^{p-1}\left(\lambda\,||a(x)||_\infty\,||\zeta_p(x)||_\infty^{p-1}\right)^{-1} \ge h\Big(||\zeta_q(x)||_\infty\,g(c)^{\frac{1}{q-1}}\Big)\,f(c)^{p-1}.$$

Hence

$$\begin{aligned}(\frac{c}{||\zeta_p(x)||_\infty})^{p-1} &\ge \lambda\,||a(x)||_\infty\,h\Big(||\zeta_q(x)||_\infty\,g(c)^{\frac{1}{q-1}}\Big)\,f\Big(c\,\frac{\zeta}{||\zeta_{p(x)}||_\infty}\Big)\\ &\ge \lambda\,a(x)\,h\Big(\zeta_q(x)\,g(c)\Big)\\ &= \lambda\,a(x)\,h(z_2)\,f(z_1).\end{aligned}$$

Next, since $\lambda \le \lambda^*$ we have $\lambda \le \frac{1}{||b||_\infty\,k\left(g(c)^{\frac{1}{q-1}}\,||\zeta_q||_\infty\right)}$. Hence

$$\begin{aligned}\int_\Omega |\nabla z_2|^{q-2}\,\nabla z_2\,\nabla w\,dx &= \int_\Omega g(c)\,w\,dx\\ &\ge \int_\Omega g\Big(\frac{c}{||\zeta_p(x)||_\infty}\,\zeta_p(x)\Big)\,w\,dx\\ &\ge \lambda\,||b||_\infty\,k\Big(g(c)^{\frac{1}{q-1}}\,||\zeta_q||_\infty\Big)\int_\Omega g\Big(\frac{c}{||\zeta_p(x)||_\infty}\,\zeta_p(x)\Big)\,wdx\\ &\ge \lambda\int_\Omega b(x)\,g(z_1)\,k(z_2)\,w\,dx.\end{aligned}$$

i.e. (z_1, z_2) is a super-solution of (1) with $z_i \ge \psi_i$ for c large, $i = 1, 2$. (Note $|\nabla\zeta_r| \ne 0; \partial\Omega$ for $r = p, q$). Thus, there exists a positive solution (u, v) of (1) such that $(\psi, \psi) \le (u, v) \le (z_1, z_2)$. This completes the proof of Theorem 2.2. $\square$

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