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Mathematical modeling of corrosion phenomenon in pipelines

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Abstract:

The annual cost of corrosion worldwide is over 3% of the world's GDP. There are hundreds of thousands of kilometers of pipelines in various sectors of industry, which include many uncoated pipelines in chemical manufacturing plants, interstate natural gas transmission lines, and offshore oil-and-gas production pipelines. Mathematical modeling is richly endowed with many analytic computational techniques for analyzing real life situations. This paper reviewed that the predictive models on corrosion rate for natural gas pipeline. These models were selected based on the thermodynamic properties of the fluid and the developed rate is plotted against various operating conditions.

Keywords: Mathematical models, Corrosion rate, Pipeline

1. Introduction

Mathematical modeling is the use of mathematics to: describe real-world phenomena, investigate important questions about the observed world, explain real-world phenomena, test ideas and make predictions about the real world. The real world refers to: engineering, physics, physiology, ecology, wildlife management, chemistry, economics, sports and etc.

It is an incontestable fact that every human activity involves one mathematical problem or the other; the need to use mathematical modeling is increasing released in modern times. It gives us understanding into many real life processes and the interplay between or among variable(s) quantifying such models. This process saves cost and labor that would unnecessarily have been expended. Many researchers have expressed various steps taken to model a problem. The most excellent one is summarized by Ale [1, 2]. He described the process involved in a modeling a process as follows. To have a full understand of the idea of modeling, figure 1 states the steps to be taken when modeling a problem.

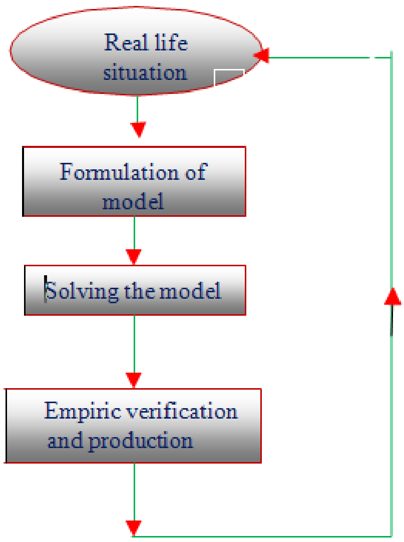

Fig 1: Idealization of a problem into a model

In real life, there is the problem whose solution is sought. This problem need to be identified, in which case, the significant features are identified and translated into mathematical entities, leading to the mathematical model. A model is nothing fanciful; it is simply the "bare bones" of the problem - what it looks like after stripping away the unimportant details. The reduced version of the original problem is what model represents. The importance of a model is related to:

- A model is more reliable than pure intuition.
- Mathematically, a model simplifies the analysis.
- A good model is economical [3]. That is, it can be labor- saving devices in more than one way.

A model used for one purpose can also be used for an entirely different purpose. Mathematical model to determine the amount of contamination arising from corrosion will have to investigate the harmful effect of the corrosion on the process of the reaction on the product quality.

2. Fundamentals of Corrosion phenomenon

Corrosion processes at the metal surface/environment interface vary from case to case and are highly complex [4-6]. However, all metallic corrosion is electrochemical and involves the operation of what is described as a "corrosion cell" (Fig. 2) [4].

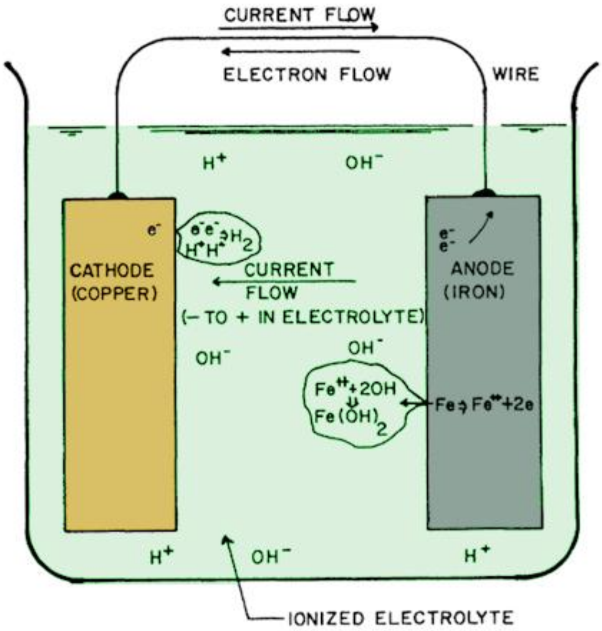

Fig. 2. Schematic diagram of a iron/copper corrosion cell

For a corrosion cell to operate, the following four conditions must be satisfied:
a-the surface of the structure must have both anodic and cathodic areas,
b-the anodes and cathodes must be immersed in a continuous ionized electrolyte,
c-the anodes and cathodes must be electrically connected through a metallic path and
d- there must be an electrical potential between the anodes and cathodes. Cathodes areas can arise in many ways:
(i) Dissimilar metals;
(ii) Corrosion products;
(iii) Inclusions in the metal, such as slag;
(iv) Less aerated areas;
(v) Areas of differential concentration;
(vi) Differential strained areas.

Consider the simplest corrosion problem in nature where iron is exposed to the atmospheric oxygen in the presence of moisture leading to formation of rust, iron (III) oxide as well as iron (III) chloride respectively. The chemical reaction can be summarized as follows:

A. $4\,Fe\,(s) + 3\,O_2\,(g) \rightarrow 2\,Fe_2O_3\,(s)$ Oxidation State of Fe is $0 \rightarrow +3$

B. $2\,Fe^{\circ}(s) + 3\,Cl^{\circ}_2\,(g) \rightarrow 2Fe{+}3Cl_3^{-1}\,(s)$

Reducing Agent Oxidizing Agent

The processes A & B illustrates the importance of redox reaction, i.e. oxidation and reduction processes. In these processes, iron is an oxidizing agent since it gains 3 electrons and chlorine is a reducing agent since it loses 1 election. From the above example and some other ones found in the literature, we note that corrosion process involves molecular and electronic charge exchanges.

3. Mathematical Models for predicting of corrosion rate

In corrosion testing, the corrosion rate is measured by the reduction in weight of a material of known area over a fixed period of time. This is expressed by the correlation (1). British standard on corrosion resistant materials showed in the table 1 [7].

$$ipy=12w/\rho At \qquad (1)$$

Table 1: Acceptable corrosion rates

	ipy	mm/y
Completely satisfactory	<0.01	0.25
use with caution	<0.03	0.75
use only for short exposure	<0.06	1.5
completely unsatisfactory	>0.06	1.5

We start our discussion with corrosion accompanied by mass - heat transfer:

3.1. Mass - Heat Transfer Model

Heat transfer in most industrial plants often accompanies corrosion; hence, we consider the following heat transfer flow in the figure (3).

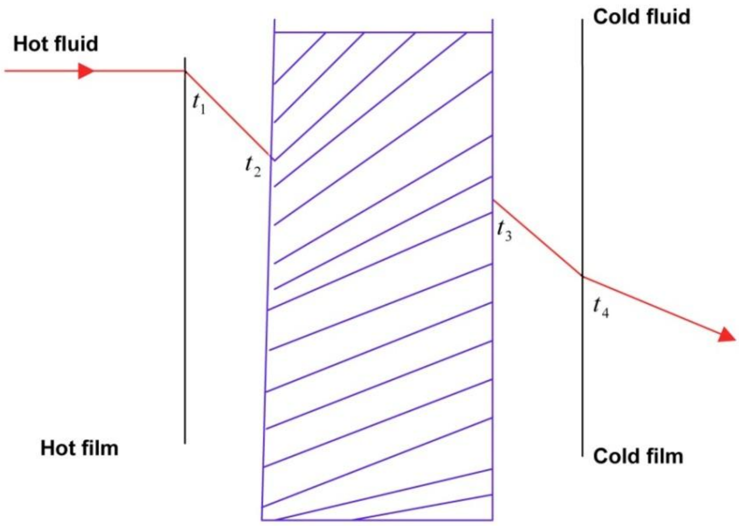

Fig 3: Variation of temperature for heat transfer across a metallic wall

The first is through a film of fluid, liquid or gas, adjacent to the metal walls of the vessel or tube. The other two resistances are:

(a) The wall of the metal, whose resistance is function of the thermal conductivity of the metal as well as its thickness,

(b) A firm that forms a thin boundary layer just at the surface of the metal on the other side before the moisture.

The basic heat transfer model is heat lost per unit area and total temperature difference [8]. Heat conducted through a material between temperature differences (T_1 - T_0) across the surface is:

$$Q = \frac{K}{t}(T_1-T_0) \quad \text{(Conductivity)} \tag{2}$$

Q is heat lost per unit area; k is thermal conductivity of the material; t is the thickness of the material. The heat lost by convection using linear model:

$$Q=h_1(T_1-T_0) \tag{3}$$

$$Q=h_2(T_2-T_1) \tag{4}$$

h_1 and h_2 are constants called convection heat transfer coefficients. Eliminating T_1 and T_2 between these three equations. We have

$$Q=[\frac{1}{h1}+\frac{a}{k}+\frac{1}{h2}]^{-1}(T_1-T_0) \tag{5}$$

In general, heat loss across the surface in fig (1), using a simple model is

$$Q=\Big[\frac{\mathbf{1}}{\boldsymbol{h1}}+\frac{\boldsymbol{2k}}{\boldsymbol{a}}+\frac{1}{\mathrm{h1}}+\frac{1}{\mathrm{h2}}\Big]^{-1}(T_1-T_0) \tag{6}$$

A more general expression for Q involving the viscosity of the fluid into the model is found in the literature and is

$$Q=[\frac{1}{\mathrm{h1}}+\frac{2\mathrm{k}}{\mathrm{a}}+\frac{1}{h1}+\frac{1}{h2}+\xi(\mu)]^{-1}(T_1-T_0) \tag{7}$$

$\xi(\mu)$ is a function depending on μ, the viscosity. To calculate the life expectancy of the plant as a result of corrosion, we use the idea of perturbation theory. Since the thickness of the surface before corrosion takes place, is Q and after corrosion, a^1 we assume that the thickness is a (Then $a = a^1 + h$). h is the part lost into the chemical reaction as a result of the corrosion and may be inform of impurities. Bedding [9] developed this methodology for laser attack on a material to be drilled. This process causes vaporization of the material and result in a moving boundary as a one dimensional hole formed. The governing heat equation for the laser - drilling equation is

$$\left.\frac{\partial^2 U}{\partial \varepsilon^2}\right| + \frac{\partial U}{\partial \varepsilon} + \frac{\partial U}{\partial T} = 0 \tag{8}$$

Where

$U(\varepsilon, T)$ is (dimensionless) temperature T and ε dimensionless is boundary position respectively. The boundary conditions are ξ is actually Z - T where Z is the momentary position of the boundary. The solution to the model is found by Bedding as

$$u(Z,T) = \frac{1}{2}\left\{erfc\left(\frac{z}{2\sqrt{T}}\right) + \exp(-Z - T)erfc\left(\frac{(2-2T)}{2\sqrt{T}}\right)\right\} \tag{9}$$

In relation to heat dissipation, Q_a

$$Q_a = \left[\frac{1}{h_1} + \frac{2k}{a} + \frac{1}{h_c} + \frac{1}{h_2} + \xi\,(\mu)\right]^{-1}[u(z,T_1) - u(z,T_2)] \tag{10}$$

Where, T_1 and T_2 are two times for heat to pass across the metallic surfaces. The quantity of heat generated in the plant can be monitor-using computer or a mathematical machines given that the temperatures $u(z,T_1)$ and $u(z,T_2)$ is known. To calculate the life expectancy of the plant as a result of corrosion, we use the idea of perturbation theory. Since the thickness of the surface before corrosion takes place, is Q and after corrosion, a^1 we assume that the thickness is a. Then

$$a = a^1 + h \tag{11}$$

h is the part lost into the chemical reaction as a result of the corrosion and may be inform of impurities. Let

$$\Gamma_a = \frac{1}{h_1} + \frac{2k}{a} + \frac{1}{h_c} + \frac{1}{h_2} + \xi\,(\mu) \tag{12}$$

$$T_a = u(t,a),\ T_b = u(t,b) \tag{13}$$

Then

$$\Gamma_a Q = u(t,a) \tag{14}$$

It follows that

Therefore

This follows that:

$$h = \frac{a}{\beta Q + \gamma} \tag{18}$$

Where

By dimensional analysis, we found that the life expectancy of the plant is related to h and the

Corrosion rate $A = \frac{12w}{tn\rho}$ by the relation

Const = C is a dimensionless constant that can be determine experimentally, hence

$$T = \frac{tAPC}{12\alpha w}(\beta\alpha + \gamma) \quad (21)$$

Acceptable working condition of the plant containing carbon and low alloy steel

Life expectancy	Time (year)
Completely satisfactory	
Use with caution	
Use only for short expensive	
Completely unsatisfactory	

For high alloy steel, brass and aluminum:

Life expectancy	Time (year)
Completely satisfactory	
Use with caution	
Use only for short expensive	
Completely unsatisfactory	

4. Investigation of the case studies

If we assume the gas flow is corrosive and turbulent, we can obtain an expression for weight loss due to corrosion. It has been shown that for a clean commercial pipe, the total annual operating cost is

$$Cf = \frac{H_p}{E} 4.13x10^{10} G^{2.84} \mu^{0.16} p^{-2} d^{-4.84} \quad (22)$$

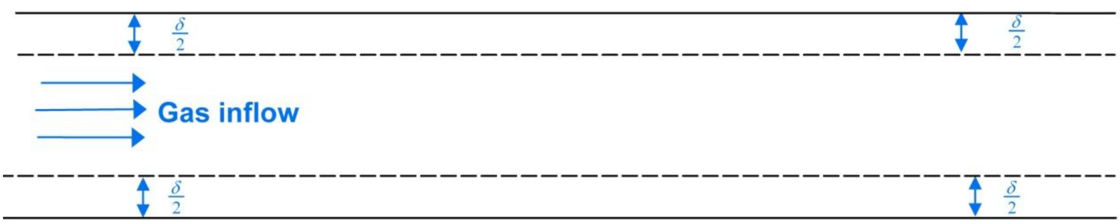

Fig 4 : Corrosion in pipeline distribution

Using the above equation and a few assumptions, life expectancy profile of various metals can be calculated when exposed to gaseous substances as [10]:

$$\frac{Md^2V}{4\pi pd^2}\left[\left(1-\frac{d}{d_0}\right)^{-4.84}\left(1-2.42\left(1-\frac{d}{d_0}\right)^{-4.84}\right)\right]LP \qquad (23)$$

Uthman [11] has been created a corrosion prediction mathematical model for risk assessment in oil and gas production and transportation facilities. This work focused on partial pressure of carbon (iv) oxide, CO_2 and the operating temperature in process equipment and transportation facility pipes as a function of corrosion rate. The model equation formulated was based on the principle of multiple linear regressions of data. The final model representing the corrosion rate of crude oil equipment was obtained

$$CR=b_0+b_1T+b_2\,P\,(CO_2) \qquad (24)$$

The model was simulated using polymath software. The correlation between the experimental and simulated resulted obtained using root mean square deviation (coefficient of determination) was 99.74% which is high, suggesting that the relationship between the predictor and response variables was linear. The variation in the model equation is 0.0066374. This low value of the variance showed that the model was accurate. Empirical models consisting of arbitrary mathematical functions of varying complexity, can have reasonable or even excellent interpolation capabilities but have to be treated with utmost caution when used to predict outside they calibration range.

Cheng et al. [12] presented a mathematical model for a buried hot crude oil pipeline during shutdown, and an unstructured grid and polar coordinate grid are respectively applied to generating grids for the soil region and the three layers in the pipe (wax layer, pipe wall, and corrosion-inhibiting coating). The governing equations are discretized using the finite volume method. The variations in temperatures of static oil and soil were investigated during pipeline shutdown in both summer and winter, in which some important parameters of the soil and crude oils of a Northeast pipeline are employed.

Cathodic protection is an important and reliable method for the corrosion control, conventionally used for prolongation of the service life of underground and underwater structures including main pipelines [13, 14]. The design of new pipelines and optimization of parameters of their protection require both carrying out experimental studies and the elaboration of mathematical models and programs for computing experiments. Bolotnov et al. [15] were proposed a mathematical model and an algorithm for solving a three-dimensional problem of electric field computation, which takes into account the potential drop in the system of extended anodes. Also, they were developed a program within the Delphi software, and the computation experiments were carried out aimed at assessing the effect of the electrical and geometrical parameters on the effectiveness of cathodic protection of main pipelines.

5. Conclusion

The aim of this paper is to introduce the reader to deterministic methods that have been developed to predict the accumulation of corrosion damage in pipelines systems. Many different mathematical models for predicting corrosion rate were used nowadays by engineers in the oil and gas industry. Some are described in the open literature, others are proprietary models. This paper reviewed that the predictive models on corrosion rate for natural gas pipeline. These models were based on the thermodynamic properties of the fluid and the developed rate is plotted against various operating conditions. It uses the mathematical modeling techniques to forecast the life expectancy of gas pipelines.

Nomenclatures

G	Flow rate, kg/s
ρ	Density, kg/m^3
μ	Viscosity, MNm^{-2}
d	Pipe diameter ,m
E	Pump efficiency per cent/100
H	Plant attainment hr/yr
d_o	Diameter of the pipe before corrosion
d	Diameter of the pipe after corrosion
w	Mass loss in time (kg)
t	Time, years
A	Surface area, m^2
CXp	Cost per unit mass, $/kg
σ_d	Design stress, N/mm^2
Q	Heat lost per unit area
k	Thermal conductivity of the material
t	The thickness of the material.
h	Convection heat transfer coefficients
L	Length of the pipe
V	Dimensionless constant ($=1.071\times10^4$)

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