The Journal of
Mathematics and Computer Science

Available online at
http://www.TJMCS.com

The Journal of Mathematics and Computer Science Vol .3 No.4 (2011) 390 - 395

Decision-Making for a Single Item EOQ Model with Demand-Dependent Unit Cost and Dynamic Setup Cost

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Received: September 2011, Revised: November 2011
Online Publication: December 2011

Abstract

A Single item EOQ model is modeled using crisp arithmetic approach in decision making process with demand unit cost and dynamic setup cost varies with the quantity produced/Purchased. This paper considers the modification of objective function and storage area in the presence of estimated parameters. The model is developed for the problem by employing NLP modeling approaches over an infinite planning horizon. It incorporates all concepts of crisp arithmetic approach, the quantity ordered, the demand per unit and compares with other model that of the crisp would optimal ordering policy of the problem over an infinite time horizon is also suggested. Investigation of the properties of an optimal solution allows developing an algorithm for obtaining solution through LINGO 13.0 version whose validity is illustrated through an example problem. Sensitivity analysis of the optimal solution is also studied with respect to changes in different parameter values and to draw managerial insights.

Keywords: Single item, EOQ, Unit cost, Dynamic setup cost

Introduction

Since its formulation in 1915, the square root formula for the economic order quantity (EOQ) was used in the inventory literature for a pretty long time. Ever since its introduction in the second decade of the past century, the EOQ model has been the subject of extensive investigations and extensions by academicians. Although the EOQ formula has been widely used and accepted by many industries, some practitioners have questioned its practical application.

For several years, classical EOQ problems with different variations were solved by many researchers and had be separated in reference books and survey papers. [2], [3], [10], [11] and [12]. Recently, for a single product with demand related to unit price [1] has solved the EOQ model. Their treatments are fully analytical and much computational efforts were needed there to get the optimal solution.

During the Second World War, this OR mathematics was used in a wider sense to solve the complex executive strategic and tactical problems of military teams. Since then the subject has been enlarged in importance in the field of Economics, Management Sciences, Public Administration, Behavioral Science, Social Work Commerce Engineering and different branches of Mathematics etc. But various Paradigmatic changes in science and mathematics concern the concept of fixed setup cost. In Science, this change has been manifested by a gradual transition from the traditional view, which insists that static setup cost is undesirable and should be avoided by all possible means. According to the traditional view, science should strive for static setup cost in all its manifestations; hence it is regarded as unscientific. According to the modern view, dynamic setup cost is considered essential to real market; it is not any an unavoidable plague but has; in fact, a great utility. But to tackle dynamic setup cost [4] gives significant contributions in this direction which have been applied in many fields including production related areas.

But [4] and [5] have considered the space constraint with the objective goal in fuzzy environment and attacked the fuzzy optimization problem directly using either fuzzy non-linear or fuzzy geometric programming techniques. Two sophisticated models were developed by [6] and [7] to deal with reliability constraint with deterministic demand, [7] considered the unit cost of production is inversely related to both process reliability and demand but [6] assumed the unit cost of production is inversely related to process reliability and directly related to demand. [9] studied an inventory model with two component demand allowing price discounts for perishable items to get maximum profit of an fuzzy model. [8] extended this work and considers entropic order quantity with two component demand allowing discounts for deteriorating items.

The purpose of this paper is to investigate the effect of the approximation made by using the average cost when determining the optimal values of the policy variables. This paper focuses exclusively on the inventory holding cost with demand dependent unit cost, dynamic setup cost and storage constraint for demand dependent unit cost in crisp decision space. A policy iteration algorithm is designed for non linear programming (NLP) with the help of LINGO 13.0 versions software. Numerical experiment is carried out to analyze the magnitude of the approximation error. This model has encouraged researchers to look for a better model to optimize total costs.

Table-1 Summary of the Related Research

Author	Model	Type of Model	Demand	Setup Cost	Holding Cost	Unit Cost is a function of	Constraint	Sensitivity Study
Tripathy et al. (2009)	Crisp	NLP	Constant	Constant	$\frac{H\lambda q^2}{2r^2}$	Reliability and demand	Reliability	Yes
Tripathy et al. (2011)	Crisp	NLP	Constant	Constant	$\frac{Hq^2}{2\lambda r^2}$	Reliability and demand	Reliability	Yes

Roy et al. (1997)	Crisp	NLP	Constant	Variable	$\frac{1}{2}C_1 q$	Demand	Storage	No
Present Paper (2011)	Crisp	NLP	Constant	Variable	$\frac{1}{2\times 100}C_1 K D^{-\beta} q$	Demand	Storage	Yes

In this paper a single item EOQ model is developed where unit price varies inversely with demand and setup cost increases with the increase of production. The model is illustrated with numerical example and with the variation in tolerance limits for both shortage area and total expenditure. A sensitivity analysis is presented. The numerical results for crisp model are compared. The remainder of the paper is organized as follows. The major assumptions used in the above research articles are summarized in Table-1. The remainder of this paper is organized as follows. In section 2, assumptions and notations are provided for the development of the model and the mathematical formulation is developed. In section 3, the numerical example is presented to illustrate the development of the model. The sensitivity analysis is carried out in section 4 to observe the changes in the optimal solution. Finally section 5 deals with the summary and the concluding remarks.

2. Mathematical Model

A single item inventory model with demand dependent unit price and variable setup cost under storage constraint is formulated as

$$\text{Min } C(D,q) = C_{03}\ q^{\nu-1} D + KD^{1-\beta} + \frac{1}{2\times 100} C_1\, KDq^{-\beta}$$

S.t. $Aq \le B$

$\forall$ $D, q > 0$ (1)

Where, C = average total cost,

q = number of order quantity,

D = demand per unit time,

C_1 = holding cost per item per unit time.

C_3 = Setup cost = C03 q^{ν}, (C_{03} (> 0) and ν (0< ν < 1) are constants),

P = Unit production cost = $KD^{-\beta}$ (K (> 0) and β (> 1) are constants. A and B are non negative real numbers. Here lead time is zero, no back order is permitted and replenishment rate is infinite.

For this crisp NLP model the solution is obtained by through LINGO software with 13.0 versions.

3. Numerical Example

For a particular EOQ problem, let C_{03} = Rs. 200, K = 100, C_1 = Rs. 100, ν = 0.5, β = 1.5, A = 10 units, B = 50 units and C_0 = Rs. 2000. For these values the optimal value of productions batch quantity q*, optimal demand rate D*, minimum average total cost C* (D*, q*) and Aq* obtained by NLP are given in Table 2.

After 23 iterations Table-2 reveals the optimal replenishment policy for single item with demand dependent unit cost and dynamic setup cost. In this table the optimal numerical results of [4] are also compared with the results of present model. The optimum replenishment quantity q^* and Aq^* for both the models are equal but the optimum quantity demand D^* is 9.31 and 9.21 for comparing model, hence -1.06% less from present model. The minimum total average cost $C^*(D^*, q^*)$ is 49.60 and 54.43 for comparing model, hence 9.73% more from the present model. It permits better use of present model as compared to other related model. The results are justified and agree with the present model. It indicates the consistency of the crisp space of EOQ model from other comparing model [4].

Table-2 Optimal Values for the Proposed Inventory Model

Model	Solution Method	Iteration	Quantity q^*	Demand D^*	Total Average Cost $C^*(D^*, q^*)$	Aq^*
Crisp Model	NLP	23	5	9.308755	49.60392	50
Crisp Model Roy et al. (1997)	NLP	-	5	9.21	54.43	50
% Change	-	-	0	-1.060883007	9.729231077	0

4. Sensitivity Analysis

Now the effect of changes in the system parameters on the optimal values of q, D, C (D, q) and Aq when only one parameter changes and others remain unchanged the computational results are described in Table 3. As a result

- q^*,$D^*$and $C^*(D^*, q^*)$ are moderately sensitive to the parameter 'A' but Aq^* is insensitive to 'A'.
- D^* and $C^*(D^*, q^*)$ are moderately sensitive to the parameter 'B' and q^* but Aq^* is highly sensitive to 'B'.
- q^*, $D^*$and $C^*(D^*, q^*)$ but Aq^* is insensitive to 'C_1'.
- q^* and A$q^*$are insensitive to the parameter 'C_{03}' but $D^*$and $C^*(D^*, q^*)$ are moderately sensitive to 'C_{03}'.
- q^* and A$q^*$are insensitive to the parameter 'K' but $D^*$and $C^*(D^*, q^*)$ are moderately sensitive to 'K'.
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Table-3 Sensitivity Analysis of the Parameters A, B, C_1, C_{03} and K

Parameter	Value	Iteration	q^*	D^*	$C^*(D^*, q^*)$	% Change in $C^*(D^*, q^*)$	Aq^*
A	20	20	2.5	7.359624	55.60518	12.09835836	50
	25	24	2.0	6.825753	57.69413	16.30961827	50
	30	22	1.666667	6.419030	59.46095	19.87147387	50.00001
	40	22	1.25	5.826853	62.36260	25.72111236	50
	50	21	1.0	5.405995	64.71282	30.45908469	50
	100	22	0.5	4.284911	72.60460	46.36867409	50
B	100	21	10	11.79983	44.28378	-10.72524107	100
	150	26	15	13.57518	41.46137	-16.41513413	150
	200	24	20	15.00722	39.58056	-20.20679011	200
	250	29	25	16.23052	38.18858	-23.01297962	250
	300	24	30	17.31098	37.09340	-25.22082932	300

	500	24	50	20.78599	34.21978	-31.01396019	500
C_1	3	24	5	9.357136	49.69161	0.1767803835	50
	4	23	5	9.404909	49.77861	0.3521697479	50
	5	24	5	9.452095	49.86497	0.5262688917	50
	6	29	5	9.498713	49.95068	0.6990576551	50
	7	33	5	9.544783	50.03577	0.8705965174	50
	10	47	5	9.679871	50.28742	1.3779152940	50
C_{03}	5	24	5	8.035475	53.46459	7.7829937630	50
	6	24	5	7.126884	56.84471	14.597213280	50
	7	26	5	6.440270	59.87185	20.699835820	50
	8	23	5	5.899914	62.62672	26.253570280	50
	9	24	5	5.461596	65.16403	31.368710380	50
	10	26	5	5.097610	67.52276	36.123838600	50
K	110	24	5	9.913036	52.8465	6.5369430480	50
	120	23	5	10.49924	55.99212	12.878417670	50
	130	24	5	11.06936	59.05145	19.045934270	50
	140	22	5	11.62503	62.03323	25.057112420	50
	150	24	5	12.16760	64.94479	30.926729180	50
	200	26	5	14.71925	78.63761	58.531039480	50

5. Conclusion

Inventory modelers have so far considered auto type of setup cost that is fixed or constant. This is rarely seen to occur in the real market. In the opinion of the author, an alternative (and perhaps more realistic) approach is to consider the setup cost as a function quantity produced / purchased may represent the tractable decision making procedure in crisp environment. In this paper the real life inventory model for single item is solved by NLP technique in crisp decision space. Some sensitivity analyses on the tolerance limits have been presented. The results of the crisp model are compounded with those of other crisp model which reveals that the present model obtains better result than the other crisp model. This method is quite general and can be extended to other similar inventory models including the ones with shortages and deteriorate items.

The current work can be extended in order to incorporate the allocation of other constraint and the consideration of the multi-item problem. A further issue that is worth exploring is that of changing demand. Finally, few additional aspects that it is intended to take into account in the near future are imposing promotion and pricing through a new optimization model and stochastically of the quality of the quality of the products.

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