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New approach for solving of linear fredholm fuzzy integral equations using sinc function

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Abstract

A numerical method is proposed to solve linear fredholm fuzzy integral equations(LFFIE). The proposed method in this paper is based on concept of the parametric form of fuzzy numbers and Sinc wavelet. By using the parametric form of fuzzy numbers linear fredholm fuzzy integral equations have been converted into a system of fredholm integral equations in the crisp form, and Sinc approach this problem reduced to solving algebraic equations. The efficiency of the proposed approach is demonstrated by numerical examples.

Keywords: Sinc function; Linear fredholm fuzzy integral equation; Fuzzy parametric form.

1 Introduction

Fuzzy set theory was initiated by Zadeh in the early 1960s. The fuzzy differential and integral equations are important part of the fuzzy analysis theory. Fixed

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point theorems for fuzzy mappings, an important tool for demonstrating existence and uniqueness of solutions to fuzzy differential and integral equations, have recently been used by various authors, see [2, 3, 5, 4, 6]. Ghanbari et al. applied the block-pulse functions for the solution of a system of LFFIE of second kind with two variable [7]. Homotopy perturbation method was used in [8]. Friedman et al. solved the integral equations by using the embedding method [9]. The interested reader can see [10, 11, 12, 13] for some presented methods to solve LFFIE.
The Sinc method is a highly efficient numerical method developed by Stenger. Maleknejad et al. solved the first kind fredholm integral equation by using the sinc function [14].
In this paper LFFIE is considered as follows:

$$y(t) = g(t) + \lambda \int_a^b K(s,t)y(s)ds \tag{1}$$

where $\lambda > 0$, $K(s,t)$ is an arbitrary given kernel function over the square $a \leq s, t \leq b$ and $g(t)$ is a given fuzzy function of $t : a \leq t \leq b$.
In the present paper, we get parametric form of LFFIE and we observe that the parametric form of the LFFIE is a system of linear fredholm integral equations in crisp case for each $0 < r < 1$. Then we apply Sinc method to solve system of linear fredholm integral equations in crisp form. In our method, the system of fredholm integral equations are transformed into algebraic equations.
The remainder of the paper is organized as follows: Section 2 is the introduction of sinc function properties and fuzzy set properties. The proposed method is discussed in Section 3. In Section 4 is presented numerical examples.

2 preliminary

In this section we will review sinc function properties and fuzzy set properties.

2.1 Sinc function properties

The Sinc function for $h > 0$ is defined on the whole real line by

- $sinc(t) = \begin{cases} \frac{\sin(\pi t)}{\pi t}, & t \neq 0, \\ 1, & t = 0. \end{cases}$
- $S(j,h)(t) = sinc(\frac{t-jh}{h}), \quad j = 0, \pm 1, \pm 2, \cdots .$
- $C(f,h)(t) = \sum_{j=-\infty}^{\infty} f(jh) sinc(\frac{t-jh}{h}),$
  Here $C(f,h)(t)$ is called the Whittaker cardinal expansion of $f(t)$ whenever this series converges.
- $C(f,h)(t) = \sum_{j=-N}^{N} f(jh) sinc(\frac{t-jh}{h}),$

These properties are derived in the infinite strip D_d of the complex plane where for $d > 0$

$$D_d = \{\zeta = \xi + i\eta \ : |\eta| < d \leq \frac{\pi}{2}\}.$$

In addition, we choose

$$h = \sqrt{\frac{\pi d}{\alpha N}}, \quad 0 < \alpha \leq 1. \tag{2}$$

To construct approximation on interval (a,b), which are used in this paper, consider the conformal map

$$\phi(t) = \ln(\frac{t-a}{b-t}). \tag{3}$$

The map ϕ carries the eye-shaped region

$$D_E = \{z = x + iy : |arg(\frac{z-a}{b-z})| < d \leq \frac{\pi}{2}\},$$

onto D_d. The basis functions on (a,b) are then given by

$$S(j,h) \circ \phi(t) = sinc(\frac{\phi(t) - jh}{h}). \tag{4}$$

Notice that these functions exhibit Kronecker delta behavior on the grid points $t_j \in (a,b)$ defined by

$$t_j = \phi^{-1}(jh) = \frac{a + be^{jh}}{1 + e^{jh}}. \tag{5}$$

The mesh size h represents the mesh size in D_d for the unform grids $\{jh\}$, $j = 0, \pm 1, \pm 2, \ldots$. The sinc grid points $t_j \in (a,b)$ in D_E will be denoted by t_j because they are real, let us also define ρ by $\rho(z) = e^{\phi(z)}$, and Γ be defined by $\Gamma = \{z \in C : z = \phi^{-1}(t), t \in \mathbb{R}\}$, we need the following theorems and lemma in [16].

Theorem 1 *If $\phi F \in L_\alpha(D)$ then for all $x \in \Gamma$*

$$|F(z) - \sum_{k=-\infty}^{\infty} F(z_k)S(k,h) \circ \phi(z)| \leq \frac{2N(F\phi^{'})}{\pi d} e^{-\pi d/h}.$$

Moreover, if $|F(z)| \leq Ce^{-\alpha|\phi(z)|}$, $z \in \Gamma$, for some positive constants C and α, and if the selection $h = \sqrt{\pi d/\alpha N} \leq 2\pi d/\ln 2$, then

$$|F(z) - \sum_{k=-N}^{N} F(z_k)S(k,h) \circ \phi(z)| \leq C_2\sqrt{N}e^{-\sqrt{\pi d\alpha N}}, z \in \Gamma,$$

where C_2 depends only on F, d and α.

Theorem 2 *Let $L_\alpha(D)$ be the set of all analytic functions, let $\frac{F}{\phi'} \in L_\alpha(D)$, let N be a positive integer and let h be selected by the formula*

$$h = (\frac{\pi d}{\alpha N})^{\frac{1}{2}},$$

then there exist positive constant c_1, independent of N, such that

$$|\int_\Gamma F(z)dz - h\sum_{k=-N}^{N} \frac{F(z_k)}{\phi'(z_k)}| \leq c_1 e^{(-\pi d\alpha N)^{\frac{1}{2}}}.$$

Lemma 1 *Let ϕ be the conformal one-to-one mapping of the simply connected domain D_E onto D_E, given by (3.1). Then*

$$\delta_{ji}^{(0)} = [S(j,h) \circ \phi(t)] \mid_{t=t_i} = \begin{cases} 1, & j = i, \\ 0, & j \neq i. \end{cases} \tag{6}$$

2.2 Fuzzy set properties

Definition 1: Let X be a universe of discourse, then a fuzzy set is defined as

$$A = \{< x, \mu_A(x) > \ \mid x \in X\}$$

where $\mu_A(x) : X \to [0,1]$ is the membership function of the fuzzy set A.
Definition 2: r-cut of fuzzy sets

$$A_r = \{x \in \mathbb{R} \mid \mu_A(x) \geq r\}$$

Definition 3: A fuzzy number u in parametric form is a pair $u = [\underline{u}(r), \overline{u}(r)]$of function $\underline{u}(r), \overline{u}(r)$, $0 \leq r \leq 1$, which satisfies the following requirements:

1.$\underline{u}(r)$ is a bounded monotonic increasing left continuous function,

2.$\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,

3.$\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

Definition 4: u is a trapezoidal fuzzy number denoted by $\widetilde{u} = (a, b, \alpha, \beta)$, in this case we will give

$$\mu_{\widetilde{u}}(x) = \begin{cases} 0 & x < a - \alpha, \\ \frac{x-a+\alpha}{\alpha} & a - \alpha \leq x \leq a, \\ 1 & a \leq x \leq b, \\ \frac{b+\beta-x}{\beta} & b \leq x \leq b + \beta, \\ 0 & b + \beta \leq x. \end{cases} \tag{7}$$

The set of all fuzzy numbers is denoted by $TF(\mathbb{R})$. For arbitrary fuzzy numbers $v = [\underline{v}, \overline{v}], u = [\underline{u}, \overline{u}]$ and $k > 0$ we define addition $(u + v)$ and multiplication by scaler k as

$$u + v = [\underline{u} + \underline{v}, \overline{u} + \overline{v}],$$

$$ku = [k\underline{u}, k\overline{u}],$$

Definition 5: For arbitrary fuzzy numbers $v = [\underline{v}, \overline{v}], u = [\underline{u}, \overline{u}]$ the quantity

$$D(u, v) = \sup_{0\ <r\ <l} \{\max[|\ \underline{u}(r) - \underline{v}(r)\ |, |\ \ \overline{u}(r) - \overline{v}(r)\ |]\}$$

is the distance between u and v.

Let $f := [a, b] \rightarrow TF(\mathbb{R})$ for each partition $p = \{t_0, t_1, \ldots, t_n\}$ of $[a, b]$ with $h = \max |t_i - t_{i-1}|$ and for arbitrary $\xi_i : t_{i-1} \leq \xi_i \leq t_i,\ 1 \leq i \leq n$ let

$$R_p = \sum_{i=1}^{n} f(\xi_i)(t_i - t_{i-1}),$$

the definite integral of $f(t)$ over $[a, b]$ is

$$\int_a^b f(t)dt = \lim R_p,\ h \rightarrow 0,$$

provided that this limit exists in the metric D. If the fuzzy function $f(t)$ is continuous in the metric D, its definite integral exists. Furthermore

$$\left(\underline{\int_a^b f(t, r)dt}\right) = \int_a^b \underline{f}(t, r)dt, \quad \left(\overline{\int_a^b f(t, r)dt}\right) = \int_a^b \overline{f}(t, r)dt,$$

Now, we introduce parametric form of a LFFIE with respect to equation (1). Parametric form of LFFIE is as follows:

$$\begin{array}{l} \underline{y}(t,r) = \underline{g}(t,r) + \lambda \int_a^b K_1(s,t,\underline{y}(s,r),\overline{y}(s,r))ds \\ \overline{y}(t,r) = \overline{g}(t,r) + \lambda \int_a^b K_2(s,t,\underline{y}(s,r),\overline{y}(s,r))ds \end{array} \tag{8}$$

where

$$K_1(s,t,\underline{y}(s,r),\overline{y}(s,r)) = \begin{cases} K(s,r)\underline{y}(s,r) & K(s,r) \geq 0, \\ K(s,r)\overline{y}(s,r) & K(s,r) \leq 0. \end{cases}$$

and

$$K_2(s,t,\underline{y}(s,r),\overline{y}(s,r)) = \begin{cases} K(s,r)\overline{y}(s,r) & K(s,r) \geq 0, \\ K(s,r)\underline{y}(s,r) & K(s,r) \leq 0. \end{cases}$$

for each $0 \leq r \leq 1$ and $a \leq s,t \leq b$. We can see that (8) is a linear system of fredholm integral equations with two variables in crips case for each $0 \leq r \leq 1$ and $a \leq s,t \leq b$.

3 The Sinc method

In this section the Sinc method is described and an algorithm is presented.
In first, $\underline{y}(t,r)$ and $\overline{y}(t,r)$ is approximated by

$$\begin{array}{l} \underline{y}(t,r) = \sum_{j=-N}^{N} \alpha_j S(k,h) \circ \phi(t). \\ \underline{y}(t,r) = \sum_{j=-N}^{N} \beta_j S(k,h) \circ \phi(t). \end{array} \tag{9}$$

Obviously by using (5), (6) in (9) we have

$$\underline{y}(t,r) = \alpha_j,$$

$$\overline{y}(t,r) = \beta_j,$$

where $j = -N \dots N$, replacing approximation defined in (9) into (8) then the system of linear fredholm integral equations in crisp case turns into $4N + 2$ algebraic equations the following form:

$$\begin{array}{l} \sum_{j=-N}^{N} \alpha_j S(j,h) \circ \phi(t) = \underline{g}(t,r) + \lambda \int_a^b K_1(s,t,\sum_{j=-N}^{N} \alpha_j S(j,h) \circ \phi(s), \sum_{j=-N}^{N} \beta_j S(j,h) \circ \phi(s))ds, \\ \sum_{j=-N}^{N} \beta_j S(j,h) \circ \phi(t) = \overline{g}(t,r) + \lambda \int_a^b K_2(s,t,\sum_{j=-N}^{N} \alpha_j S(j,h) \circ \phi(s), \sum_{j=-N}^{N} \beta_j S(j,h) \circ \phi(s))ds, \end{array} \tag{10}$$

by solving this system we can find unknowns $\{\alpha_j\}_{-N}^{N}$ and $\{\beta_j\}_{-N}^{N}$.

The following algorithm show the process of proposed method.

Algorithm of the method

Step 1: Apply the parametric form of LFFIE for (1) to obtain equation (8).

Step 2: Approximate $\underline{y}(t,r)$ and $\overline{y}(t,r)$ by (9).

Step 3: Solve system (10) and obtain $\{\alpha_j\}_{-N}^{N}$ and $\{\beta_j\}_{-N}^{N}$.

4 Numerical results

In this section two examples are presented which shows efficiency of the presented method. Choose $\alpha = 1/2$, $d = \pi/2$ which lead to $h = \pi/\sqrt{N}$. All the computations were performed with Maple.

Example 1. The following fuzzy fredholm integral equation is considered [15]

$$\underline{y} = rt + \frac{3}{26} - \frac{3}{26}r - \frac{1}{13}t^2 - \frac{1}{13}t^2 r,$$

$$\overline{y} = 2t - rt + \frac{3}{26}r + \frac{1}{13}t^2 r - \frac{3}{26} - \frac{3}{13}t^2,$$

and kernel

$$K(s,t) = \frac{s^2 + t^2 - 2}{13} \quad 0 \le s, t \le 2,$$

and $a = 0$, $b = 2$ and $\lambda = 1$. The exact solution in this case is given by

$$\underline{y}(t,r) = rt,$$

$$\overline{y}(t,r) = (2-r)t.$$

Example 2. Consider the fuzzy fredholm integral equation [9]

$$\underline{y} = sin(\frac{t}{2})(\frac{13}{15}(r^2 + r) + \frac{2}{15}(4 - r^3 - r)),$$

$$\overline{y} = sin(\frac{t}{2})(\frac{2}{15}(r^2 + r) + \frac{3}{15}(4 - r^3 - r)),$$

and kernel

$$K(s,t) = 0.1 sin(s) sin(\frac{t}{2}) \quad 0 \le s, t \le 2\pi$$

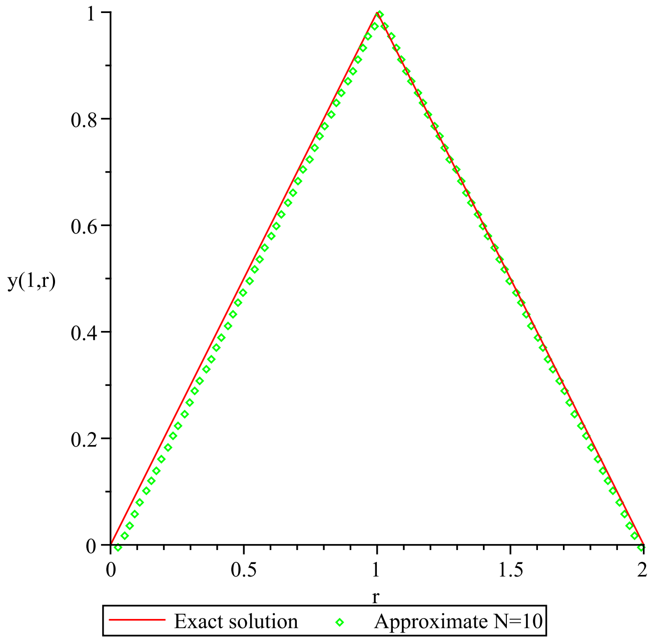

Figure 1: The exact and approximate solutions for Example 1.

and $a = 0$, $b = 2\pi$ and $\lambda = 1$. The exact solution in this case is given by

$$\underline{y}(t, r) = (r^2 + r)sin(\frac{t}{2})$$

$$\overline{y}(t, r) = (4 - r^3 - r)sin(\frac{t}{2})$$

5 Conclusion

In this article, we proposed a numerical method for solving of the linear fredholm fuzzy integral equations, using defined Sinc basis functions and the parametric form of fuzzy numbers. Comparison of the obtained results with the exact solution shows that the method is very effective and convenient. We can get much better results with increasing the N.

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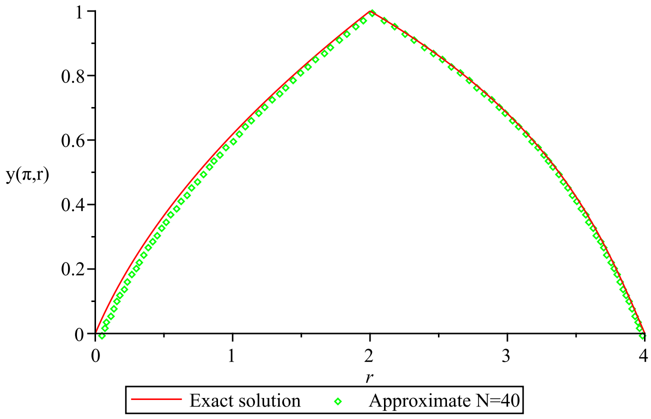

Figure 2: The exact and approximate solutions for Example 2.

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