The Journal of
Mathematics and Computer Science

Available online at
http://www.TJMCS.com

The Journal of Mathematics and Computer Science Vol. 4 No.3 (2012) 283 - 294

A New Method for Ordering LR Fuzzy Number

S.H. Nasseri[1,*] , F.Taleshian[1], Z. Alizadeh[1] and J. Vahidi[2]

[1]Department of Mathematics, University of Mazandaran, Babolsar, IRAN

[2]Department of Mathematics, Science and Technology of Behshahr, Behshahr, IRAN

Received: February 2012, Revised: May 2012
Online Publication: July 2012

Abstract

Ranking fuzzy numbers is an important aspect of decision making in a fuzzy environment. In fuzzy decision making problems, fuzzy numbers must be ranked before an action is taken by a decision maker. This article is about ranking Fuzzy numbers and describes a ranking method for ordering LR fuzzy numbers based on the area of fuzzy numbers. This method is simple in evaluation and can rank various types of LR fuzzy numbers and also crisp numbers which are considered to be a special class of fuzzy numbers.

Keywords: fuzzy number, ranking function.

1. Introduction

Ranking fuzzy numbers are an important tool in decision making. In fuzzy decision analysis, fuzzy quantities are used to describe the performance of alternatives in modeling a real-world problem. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. It is true that fuzzy numbers are frequently partial order and cannot be compared like real numbers which can be linearly ordered. In order to rank fuzzy quantities, each fuzzy quantity is converted into a real number and compared by defining a ranking function from the set of fuzzy numbers to a set of

* Corresponding author: nasseri@umz.ac.ir

real numbers which assign a real number to each fuzzy number where a natural order exists. Ranking fuzzy numbers were first proposed by Jain [29] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [9] reviewed some of these ranking methods [3,7-10,23,28-30,40,57-59] for ranking fuzzy subsets. Chen [12] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade [24] presented the mean value of a fuzzy number. Lee and Li [34] presented a comparison of fuzzy numbers based on the probability measure of fuzzy events. Delgado et al. [21] presented a procedure for ranking fuzzy numbers. Campos and Muñoz [20] presented a subjective approach for ranking fuzzy numbers. Kim and Park [32] presented a method of ranking fuzzy numbers with index of optimism. Yuan [61] presented a criterion for evaluating fuzzy ranking methods. Heilpern [27] presented the expected value of a fuzzy number. Saade and Schwarzlander [45] presented ordering fuzzy sets over the real line. Liou and Wang [36] presented ranking fuzzy numbers with integral value. Choobineh and Li [18] presented an index for ordering fuzzy numbers. Chang and Lee [11] presented ranking of fuzzy sets based on the concept of existence. Since then several methods have been proposed by various researchers which includes ranking fuzzy numbers using area compensation, distance method, maximizing and minimizing set, decomposition principle, and signed distance [4, 17, 25, 60]. Wang and Kerre [51, 52] classified all the above ranking procedures into three classes. The first class consists of ranking procedures based on fuzzy mean and spread [3,10,18,20,25,36,57-59], and second class consists ranking procedures based on fuzzy scoring [12,28-30,32,49], whereas the third class consists of methods based on preference relations [7,8,21,23,31,41,45,61] and concluded that the ordering procedures associated with first class are relatively reasonable for the ordering of fuzzy numbers specially the ranking procedure presented by Adamo [3] which satisfies all the reasonable properties for the ordering of fuzzy quantities. The methods presented in the second class are not doing well and the methods [31, 41,45,61] which belong to class three are reasonable. Later on, ranking fuzzy numbers by preference ratio [39], left and right dominance [15], fuzzy distance measure [47], area between the centroid point and original point [19], preference weighting function expectations [37], sign distance [1], fuzzy simulation analysis method [46], an area method using radius of gyration [22], distance minimization [6], and fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers [13]. Garcia and Lamata [26] modified the index of Liou and Wang [36] for ranking fuzzy numbers. The development in ranking fuzzy numbers can also be found in [2,33-53-16-48-55-54-50-14-35-56–5]. Most of the methods presented above cannot discriminate fuzzy numbers, and some methods do not agree with human intuition, whereas some methods cannot rank crisp numbers which are a special case of fuzzy numbers. In this paper, a new method is proposed which is based on the area of fuzzy numbers. The work is organized as follows.

Section 2 introduces the basic concepts and definitions of fuzzy numbers briefly. Section 3 presents the proposed new method. In Section 4, the proposed method has been explained with examples and shows the results of comparing our method to others. Finally, the conclusions of the work are presented in Section 6.

2. Arithmetic on fuzzy numbers

In this section we review some basic definitions and notations of fuzzy set which is taken from [38, 42, 44].

Definition 2.1: Let $\boldsymbol{R}$ be the real line, then a fuzzy set $\tilde{A}$ in $\mathbb{R}$ is defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in \mathbb{R}\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set. The membership function maps each element of $\mathbb{R}$ to a membership value between 0 and 1.

Definition 2.2: The support of a fuzzy set $\tilde{A}$ is defined as fallow:

$$supp(\tilde{A}) = \{x \in \mathbb{R} | \mu_{\tilde{A}}(x) > 0\}$$

Definition 2.3: The core of a fuzzy set is the set of all points x in $\mathbb{R}$ with $\mu_{\tilde{A}}(x) = 1$.

Definition 2.4: A fuzzy set $\tilde{A}$ is called normal if its core is nonempty. In other words, ther is at least one point $x \epsilon \mathbb{R}$ with $\mu_{\tilde{A}}(x) = 1$.

Definition 2.5: The α–cut or α–level set of a fuzzy set is a crisp set defined as follows:

$$A_\alpha = \{x \in \mathbb{R} | \mu_{\tilde{A}}(x) > \alpha\}$$

Definition 2.6: A fuzzy set $\tilde{A}$on $\mathbb{R}$ is convex, if for any $x, y \epsilon \mathbb{R}$ and $\lambda \in [0,1]$, we have

$$\mu_{\tilde{A}}(\lambda x + (1 - \lambda) y) \geq min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$$

Definition 2.7: A fuzzy number $\tilde{A}$ is a fuzzy set on the real line that satisfies the condition of normality and convexity.

Definition 2.8: A fuzzy number $\tilde{A}$ on $\mathbb{R}$ is said to be LR fuzzy number, if there exist real numbers and $s, t \geq 0$ such that

$$\mu_A(x) = \begin{cases} L\left(\frac{m-x}{s}\right), x \leq m, \\ R\left(\frac{x-m}{t}\right), x \geq m. \end{cases}$$

in which $L(x)$ and $R(x)$ are continues and non decreasing functions on the real numbers line. We denote a LR fuzzy number $\tilde{A}$ by three real numbers s, t and m as $\tilde{A} = \langle m, s, t \rangle_{LR}$, whose meaning are defined in Figure 1. We also denote the set of all LR fuzzy numbers with **F**($\mathbb{R}$).

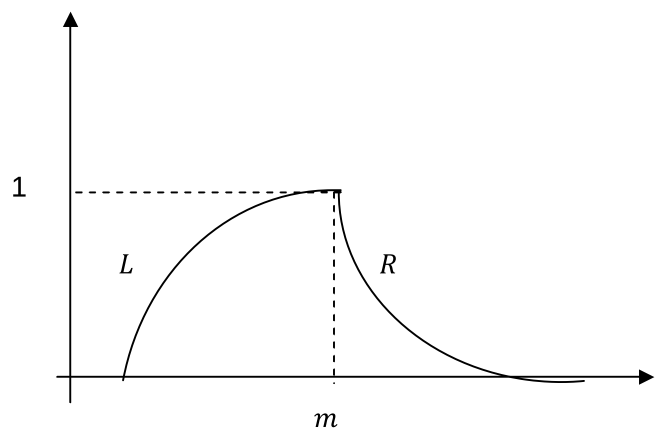

Figure 2.1

Definition 2.9: Let $\tilde{A} = \langle m_a, s_a, t_a\rangle_{LR}$ and $\tilde{B} = \langle m_b, s_b, t_b\rangle_{LR}$ be two triangular numbers and $x\epsilon\mathbb{R}$**.** Summation and multiplication of fuzzy numbers defined as [43]:

$$x\tilde{A} = \begin{cases}\langle xm_a, xs_a, xt_a\rangle_{LR}, & x \geq 0\\ \langle xm_a, -xt_a, -xs_a\rangle_{LR}, & x < 0\end{cases}$$

$$\tilde{A} + \tilde{B} = \langle m_a + m_b, s_a + s_b, t_a + t_b\rangle_{LR}$$

$$\tilde{A} - \tilde{B} = \langle m_a - m_b, s_a - t_b, t_a - s_b\rangle_{LR}$$

$\tilde{A} \leq \tilde{B}$ if and only if $m_a \leq m_b, m_a - s_a \leq m_b - s_b, m_a + t_a \leq m_b + t_b$

Definition 2.10: We let $\tilde{0} = \langle 0,0,0\rangle_{LR}$ as a zero LR o fuzzy number.

Remark 2.1: $\tilde{A} \geq \tilde{0}$ if and only if $m_a \geq 0$, $m_a - s_a \geq 0, m_a + t_a \geq 0$.

Remark 2.2: $\tilde{A} \leq \tilde{B}$ if and only if $-\tilde{A} \geq -\tilde{B}$.

3. Consstruction of a new method for ranking of fuzzy number

Here, we counstruct a new ranking system for LR fuzzy numbers which is very realistic and efficient and then introduce a new algorithm for ranking LR fuzzy numbers.

For any LR fuzzy number $\tilde{A} = \langle m_a, s_a, t_a\rangle_{LR}$, define:

$$\tilde{A}_l = m_a + \frac{1}{2}H_l, \ \tilde{A}_u = m_a + \frac{1}{2}H_u \qquad 3\text{-}1$$

where H_l and H_u are defined as fallows:

$$H_l = \frac{\int_0^1 L^{-1}(x)d\alpha}{\int_0^1 L^{-1}(x)d\alpha + \int_0^1 R^{-1}(x)d\alpha}$$

$$H_u = \frac{\int_0^1 R^{-1}(x)d\alpha}{\int_0^1 L^{-1}(x)d\alpha + \int_0^1 R^{-1}(x)d\alpha}$$

Suposs that $\tilde{A} = \langle m_a, s_a, t_a\rangle_{LR}$ and $\tilde{B} = \langle m_b, s_b, t_b\rangle_{LR}$ be two LR fuzzy numbers. Let

$$\bar{R}(\tilde{A}, \tilde{B}) = \tilde{A}_u - \tilde{B}_u, \ \underline{R}(\tilde{A}, \tilde{B}) = \tilde{A}_l - \tilde{B}_l \qquad (3\text{-}2)$$

where $\tilde{A}_l, \tilde{B}_l, \tilde{A}_u$ and $\tilde{B}_u$ are defined in (3-1).

Lemma 3-1: Assume $\tilde{A} = \langle m_a, s_a, t_a\rangle_{LR}$ and $\tilde{B} = \langle m_b, s_b, t_b\rangle_{LR}$ be two LR fuzzy numbers. Then we have:

$$\begin{aligned}\bar{R}(\tilde{A}, \tilde{B}) &= -\bar{R}(\tilde{B}, \tilde{A}) = \bar{R}(-\tilde{B}, -\tilde{A})\\ \underline{R}(\tilde{A}, \tilde{B}) &= -\underline{R}(\tilde{B}, \tilde{A}) = \underline{R}(-\tilde{B}, -\tilde{A})\end{aligned} \qquad (3\text{-}3)$$

Proof. It is straighforward from (3-2). ■

Definition 3.1: Assume $\tilde{A} = \langle m_a, s_a, t_a \rangle_{LR}$ and $\tilde{B} = \langle m_b, s_b, t_b \rangle_{LR}$ be two LR fuzzy numbers and $\underline{R}(\tilde{B}, \tilde{A}) \geq 0$. The relation $\prec$ and $\approx$ on ***F(***$\mathbb{R}$***)*** are defined as follow:

$\tilde{A} \approx \tilde{B}$ If and only if $\underline{R}(\tilde{B}, \tilde{A}) = \bar{R}(\tilde{A}, \tilde{B})$

$\tilde{A} \prec \tilde{B}$ if and only $\underline{R}(\tilde{B}, \tilde{A}) > \bar{R}(\tilde{A}, \tilde{B})$

Remark 3.1: We denot $\tilde{A} \preccurlyeq \tilde{B}$ if and only if $\tilde{A} \approx \tilde{B}$ or $\tilde{A} \prec \tilde{B}$. Than $\tilde{A} \preccurlyeq \tilde{B}$ if and only if $\underline{R}(\tilde{B}, \tilde{A}) \geq \bar{R}(\tilde{A}, \tilde{B})$. Also $\tilde{A} \prec \tilde{B}$ if and only if $\tilde{B} \succ \tilde{A}$.

Lemma 3.2: Suppose $\tilde{A} \prec \tilde{B}$. Then it can b e proved that $-\tilde{A} \succ -\tilde{B}$.

Proof. Since , we have:

$$\underline{R}(\tilde{B}, \tilde{A}) > \bar{R}(\tilde{A}, \tilde{B})$$

So by use of lemma 3-1, we have:

$$\underline{R}(-\tilde{A}, -\tilde{B}) > \bar{R}(-\tilde{B}, -\tilde{A})$$

Now from Definition 3.1 we obtained $-\tilde{B} \prec -\tilde{A}$. ■

Lemma 3.3: Assume $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ be three LR fuzzy numbers. Then the following relations are holde:

i. $\tilde{A} \approx \tilde{A}$, for every $\tilde{A}$ (reflexivity),
ii. If $\tilde{A} \approx \tilde{B}$, then $\tilde{B} \approx \tilde{A}$ (symmetry),
iii. If $\tilde{A} \approx \tilde{B}$ and $\tilde{B} \approx \tilde{C}$, then $\tilde{A} \approx \tilde{C}$ (transitivity).

Proof: First part is obvious, because

$$\tilde{A} \approx \tilde{A} \Leftrightarrow \underline{R}(\tilde{A}, \tilde{A}) = \bar{R}(\tilde{A}, \tilde{A}) \Leftrightarrow \tilde{A}_l - \tilde{A}_l = \tilde{A}_u - \tilde{A}_u$$

Now for symmetry proporty, assume that $\tilde{A} \approx \tilde{B}$, then

$$\tilde{A} \approx \tilde{B} \Leftrightarrow \underline{R}(\tilde{B}, \tilde{A}) = \bar{R}(\tilde{A}, \tilde{B}) \Leftrightarrow \tilde{B}_l - \tilde{A}_l = \tilde{A}_u - \tilde{B}_u$$

Since we can rewrite $\tilde{B}_l - \tilde{A}_l = \tilde{A}_u - \tilde{B}_u$ as a $\tilde{A}_l - \tilde{B}_l = \tilde{B}_u - \tilde{A}_u$, then

$$\tilde{A} \approx \tilde{B} \Leftrightarrow \tilde{A}_l - \tilde{B}_l = \tilde{B}_u - \tilde{A}_u \Leftrightarrow \underline{R}(\tilde{A}, \tilde{B}) = \bar{R}(\tilde{B}, \tilde{A}) \Leftrightarrow \tilde{B} \approx \tilde{A}$$

For trancivity proporty, assume that $\tilde{A} \approx \tilde{B}$ and$\tilde{B} \approx \tilde{C}$. Hence, from $\tilde{A} \approx \tilde{B}$ we have:

$$\underline{R}(\tilde{B}, \tilde{A}) = \bar{R}(\tilde{A}, \tilde{B}) \text{ } or \text{ } \tilde{B}_l - \tilde{A}_l = \tilde{A}_u - \tilde{B}_u \tag{3-4}$$

Also from $\tilde{B} \approx \tilde{C}$ we have:

$$\underline{R}(\tilde{C}, \tilde{B}) = \bar{R}(\tilde{B}, \tilde{C}) \text{ } or \text{ } \tilde{C}_l - \tilde{B}_l = \tilde{B}_u - \tilde{C}_u \tag{3-5}$$

Then (3-4) and (3-5) yeild:

$$\tilde{C}_l - \tilde{A}_l = \tilde{A}_u - \tilde{C}_u \tag{3-6}$$

Thus, $\underline{R}(\tilde{C},\tilde{A}) = \bar{R}(\tilde{A},\tilde{C})$ or equvalently we have $\tilde{A} \approx \tilde{C}$. ■

The above lemma shows that the relation is an equvalence relation on $\boldsymbol{F}(\mathbb{R})$.

We now discusse the topic of order relations and denote this subject which is necessary for future works. The reader will find it helpful to keep in mind that a partial order relation is valid (as we prove it below) by Definition 3.1 on $\boldsymbol{F}(\mathbb{R})$.

Lemma 3.4: Let $\tilde{A}, \tilde{B} \in \boldsymbol{F}(\mathbb{R})$. The relation $\preccurlyeq$ is a partial order on $\boldsymbol{F}(\mathbb{R})$.

Proof: In fact, we need to prove the below triple proporties.

i. $\tilde{A} \preccurlyeq \tilde{A}$, for every $\tilde{A}$ (reflexivity),

ii. If $\tilde{A} \preccurlyeq \tilde{B}$ and $\tilde{B} \preccurlyeq \tilde{A}$, then $\tilde{A} \approx \tilde{B}$ (symmetry),

iii. If $\tilde{A} \preccurlyeq \tilde{B}$ and $\tilde{B} \preccurlyeq \tilde{C}$, then $\tilde{A} \preccurlyeq \tilde{C}$ (transitivity).

the reflexivity proporty is valid, because

$$\tilde{A} \preccurlyeq \tilde{A} \Leftrightarrow \underline{R}(\tilde{A},\tilde{A}) \geq \bar{R}(\tilde{A},\tilde{A}) \Leftrightarrow \tilde{A}_l - \tilde{A}_l \geq \tilde{A}_u - \tilde{A}_u$$

For symmetry proporty, assume that $\tilde{A} \preccurlyeq \tilde{B}$ and $\tilde{B} \preccurlyeq \tilde{A}$, then

$$\begin{cases} \tilde{A} \preccurlyeq \tilde{B} \Leftrightarrow \underline{R}(\tilde{B},\tilde{A}) \geq \bar{R}(\tilde{A},\tilde{B}) \Leftrightarrow \tilde{B}_l - \tilde{A}_l \geq \tilde{A}_u - \tilde{B}_u \\ \tilde{B} \preccurlyeq \tilde{A} \Leftrightarrow \underline{R}(\tilde{B},\tilde{A}) \leq \bar{R}(\tilde{A},\tilde{B}) \Leftrightarrow \tilde{B}_l - \tilde{A}_l \leq \tilde{A}_u - \tilde{B}_u \end{cases} \Rightarrow \begin{cases} \tilde{A} \preccurlyeq \tilde{B} \Leftrightarrow \tilde{B}_l - \tilde{A}_l \geq \tilde{A}_u - \tilde{B}_u \\ \tilde{B} \preccurlyeq \tilde{A} \Leftrightarrow \tilde{B}_l - \tilde{A}_l \leq \tilde{A}_u - \tilde{B}_u \end{cases}$$

Now since the partial order exsist on $\mathbb{R}$, therfore it follows that $\tilde{B}_l - \tilde{A}_l = \tilde{A}_u - \tilde{B}_u$ or $\underline{R}(\tilde{B},\tilde{A}) = \bar{R}(\tilde{A},\tilde{B})$. Hence we obtained $\tilde{A} = \tilde{B}$.

Finally, for transivity proporty, assume that $\tilde{A} \preccurlyeq \tilde{B}$ and $\tilde{B} \preccurlyeq \tilde{C}$. Since $\tilde{A} \preccurlyeq \tilde{B}$ we have:

$$\underline{R}(\tilde{B},\tilde{A}) \geq \bar{R}(\tilde{A},\tilde{B}) \Rightarrow \tilde{B}_l - \tilde{A}_l \geq \tilde{A}_u - \tilde{B}_u \tag{3-7}$$

Also from $\tilde{B} \preccurlyeq \tilde{C}$ we have:

$$\underline{R}(\tilde{C},\tilde{B}) \geq \bar{R}(\tilde{B},\tilde{C}) \Rightarrow \tilde{C}_l - \tilde{B}_l \geq \tilde{B}_u - \tilde{C}_u \tag{3-8}$$

The equalityes (3-7) and (3-8) yield:

$$\tilde{C}_l - \tilde{A}_l \geq \tilde{A}_u - \tilde{C}_u \Rightarrow \underline{R}(\tilde{C},\tilde{A}) \geq \bar{R}(\tilde{A},\tilde{C}), \tag{3-9}$$

It follows that $\tilde{A} \preccurlyeq \tilde{C}$. ■

Remark 3.3: We emphasis that the relation is a linear order on $\boldsymbol{F}(\mathbb{R})$ too, beause any two elements in $\boldsymbol{F}(\mathbb{R})$ are comparable by this relation.

Lemma 3.5: If $\tilde{A} \preccurlyeq \tilde{B}$ and $\tilde{C} \preccurlyeq \tilde{D}$, then $\tilde{A} + \tilde{C} \preccurlyeq \tilde{B} + \tilde{D}$.

Proof. Since $\tilde{A} \preccurlyeq \tilde{B}$ and $\tilde{C} \preccurlyeq \tilde{D}$, the following relations are holde:

$$\underline{R}(\tilde{B},\tilde{A}) \geq \bar{R}(\tilde{A},\tilde{B}) \Longrightarrow \tilde{B}_l - \tilde{A}_l \geq \tilde{A}_u - \tilde{B}_u \tag{3-10}$$

$$\underline{R}(\tilde{D},\tilde{C}) \geq \bar{R}(\tilde{C},\tilde{D}) \Longrightarrow \tilde{D}_l - \tilde{C}_l \geq \tilde{C}_u - \tilde{D}_u \tag{3-11}$$

From (3-10) and (3-11), we obtained:

$$(\tilde{B}_l + \tilde{D}_l) - (\tilde{A}_l + \tilde{C}_l) \geq (\tilde{A}_u + \tilde{C}_u) - (\tilde{B}_u + \tilde{D}_u) \Rightarrow \underline{R}(\tilde{B} + \tilde{D}, \tilde{A} + \tilde{C}) \geq \bar{R}(\tilde{A} + \tilde{C}, \tilde{B} + \tilde{D}) \tag{3-12}$$

It follows that $\tilde{A} + \tilde{C} \preccurlyeq \tilde{B} + \tilde{D}$. ■

Algourithm 3.1: For two fuzzy numbers $\tilde{A}$ and $\tilde{B}$, assume that $\tilde{B} \geq \tilde{A}$. Compute $\underline{R}(\tilde{B},\tilde{A}) = \tilde{B}_l - \tilde{A}_l$ and $\bar{R}(\tilde{A},\tilde{B}) = \tilde{A}_u - \tilde{B}_u$ (with $\tilde{B}_l \geq \tilde{A}_l$, it is obvious that $\underline{R}(\tilde{B},\tilde{A}) \geq 0$).

Let $d = \underline{R}(\tilde{B},\tilde{A}) - \bar{R}(\tilde{A},\tilde{B})$. Then

i) If $d = 0$, then $\tilde{A} \approx \tilde{B}$,
ii) If $d > 0$, then $\tilde{A} \prec \tilde{B}$ else $\tilde{A} \succ \tilde{B}$.

4. Numerical examoles

Here we present some examples to illustrate the advantages of our method and compare our method with the others.

Example 4.1: Now for simplicity**,** consider the following trangular fuzzy numbers:

set 1: $A = (0.5, 0.1, 0.5), B = (0.7, 0.3, 0.3), C = (0.9, 0.5, 0.1)$
set 2: $A = (0.4, 0.7, 0.1, 0.2), B = (0.7, 0.4, 0.2), C = (0.7, 0.2, 0.2)$

	Fuzzy numbers	Set 1	Set 2
New method	A B C	$\underline{R}(\tilde{B},\tilde{A}) > \bar{R}(\tilde{A},\tilde{B})$ $\underline{R}(\tilde{C},\tilde{B}) > \bar{R}(\tilde{B},\tilde{C})$ $\underline{R}(\tilde{C},\tilde{A}) > \bar{R}(\tilde{A},\tilde{C})$ $A \prec B \prec C$	$\underline{R}(\tilde{B},\tilde{A}) > \bar{R}(\tilde{A},\tilde{B})$ $\underline{R}(\tilde{C},\tilde{B}) > \bar{R}(\tilde{B},\tilde{C})$ $\underline{R}(\tilde{C},\tilde{A}) > \bar{R}(\tilde{A},\tilde{C})$ $A \prec B \prec C$
Choobineh and Li	A B C	0.333 0.50 0.667 $A \prec B \prec C$	0.458 0.583 0.667 $A \prec B \prec C$
Yager	A B C	0.6 0.7 0.8 $A \prec B \prec C$	0.575 0.65 0.7 $A \prec B \prec C$
Chen	A B C	0.3375 0.50 0.667 $A \prec B \prec C$	0.4315 0.5625 0.625 $A \prec B \prec C$

Baldwin and Guild	A B C	0.3 0.33 0.44 $A \prec B \prec C$	0.27 0.27 0.37 $A \sim B \prec C$
Chu and Tsao	A B C	0.299 0.350 0.3993 $A \prec B \prec C$	0.2847 0.32478 0.35 $A \prec B \prec C$
Yao and Wu	A B C	0.6 0.7 0.8 $A \prec B \prec C$	0.575 0.65 0.7 $A \prec B \prec C$
Cheng distance	A B C	0.79 0.8602 0.9268 $A \prec B \prec C$	0.7577 0.8149 0.8602 $A \prec B \prec C$
Cheng CV uniform distribution	A B C	0.0272 0.0214 0.0225 $B \prec C \prec A$	0.0328 0.0246 0.0095 $C \prec B \prec A$
Cheng CV proportional distribution	A B C	0.0183 0.0128 0.0137 $B \prec C \prec A$	0.026 0.0146 0.0057 $C \prec B \prec A$

5. Conclusion

In this paper we proposed a method that ranks LR fuzzy numbers using a simple and naïve maner. This method ranks LR fuzzy numbers as well as triangular and trapezoidal fuzzy numbers. Also the defined ranking function, ranks crisp numbers which are special case of fuzzy numbers, whereas methods proposed by Cheng and Chu cannot rank crisp numbers as their centroid formulae are undefined for crisp numbers. This method which is simple in calculation not only gives satisfactory results to well-defined problems, but also gives a correct ranking order to problems, whereas Yager index, Fortemps and Roubens, Liou and Wang, and Chen and Lu indexes failed to discriminate fuzzy numbers, and this method also agrees with human intuition.

References

[1]: S. Abbasbandy and B. Asady, Ranking of fuzzy numbers by sign distance, Information Sciences, 176 (16) (2006) 2405–2416.

[2]: S. Abbasbandy and T. Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, Computers and Mathematics with Applications, 57(3) (2009) 413–419.

[3]: J.M. Adamo, Fuzzy decision trees, Fuzzy Sets and Systems, 4(3) (1980) 207–219.

[4]: P. Anand Raj and D. Nagesh Kumar, Ranking alternatives with fuzzy weights using maximizing set and minimizing set, Fuzzy Sets and Systems, 105(3) (1999) 365–375.

[5]: B. Asady, The revised method of ranking LR fuzzy number based on deviation degree, Expert Systems with Applications, 37(7) (2010) 5056–5060.

[6]: B. Asady and A. Zendehnam, Ranking fuzzy numbers by distance minimization, Applied Mathematical Modelling, 31(11) (2007) 2589–2598.

[7]: S.M. Baas and H. Kwakernaak, Rating and ranking of multiple-aspect alternatives using fuzzy sets, Automatica, 13(1) (1977) 47–58.

[8]: J.F. Baldwin and N.C.F. Guild, Comparison of fuzzy sets on the same decision space, Fuzzy Sets and Systems, 2(3) (1979) 213–231.

[9]: G. Bortolan and R. Degani, A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems, 15(1) (1985) 1–19.

[10]: W. Chang, Ranking of fuzzy utilities with triangular membership functions, Proceedings of International Conference on Policy Analysis and Systems, (1981) 263–272.

[11]: P.T. Chang and E.S. Lee, Ranking of fuzzy sets based on the concept of existence, Computers and Mathematics with Applications, 27(9-10) (1994) 1–21.

[12]: S.H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, 17(2) (1985) 113–129.

[13]: S.J. Chen and S.M. Chen, Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers, Applied Intelligence, 26(1) (2007) 1–11.

[14]: S.M. Chen and J.H. Chen, Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads, Expert Systems with Applications, 36(3) (2009) 6833–6842.

[15]: L.H. Chen and H.W. Lu, An approximate approach for ranking fuzzy numbers based on left and right dominance, Computers and Mathematics with Applications, 41(12) (2001) 1589–1602.

[16]: C.C. Chen and H.C. Tang, Ranking non normal p-trapezoidal fuzzy numbers with integral value, Computers and Mathematics with Applications, 56(9) (2008) 2340–2346.

[17]: C.H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems, 95(3) (1998) 307–317.

[18]: F. Choobineh and H. Li, An index for ordering fuzzy numbers, Fuzzy Sets and Systems, 54(3) (1993) 287–294.

[19]: T.C. Chu and C.T. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, Computers and Mathematics with Applications, 43 (1-2) (2002) 111–117.

[20]: L. de Campos and G.A. Muñoz, A subjective approach for ranking fuzzy numbers, Fuzzy Sets and Systems, 29(2) (1989) 145–153.

[21]: M. Delgado, J.L. Verdegay, and M.A. Vila, A procedure for ranking fuzzy numbers using fuzzy relations, Fuzzy Sets and Systems, 26(1) (1988) 49–62.

[22]: Y. Deng, Z. Zhenfu, and L. Qi, Ranking fuzzy numbers with an area method using radius of gyration, Computers and Mathematics with Applications, 51(6-7) (2006) 1127–1136.

[23]: D. Dubois and H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Information Sciences, 30(3) (1983) 183–224.

[24]: D. Dubois and H. Prade, The mean value of a fuzzy number, Fuzzy Sets and Systems, 24(3) (1987) 279–300.

[25]: P. Fortemps and M. Roubens, Ranking and defuzzification methods based on area compensation, Fuzzy Sets and Systems, 82(3) (1996) 319–330.

[26]: M.S. Garcia and M.T. Lamata, A modification of the index of liou and wang for ranking fuzzy number, International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems, 15(4) (2007) 411–424.

[27]: S. Heilpern, The expected value of a fuzzy number, Fuzzy Sets and Systems, 47(1) (1992) 81–86.

[28]: R. Jain, A procedure for multi aspect decision making using fuzzy sets, International Journal of Systems Science, 8(1) (1978) 1–7.

[29]: R. Jain, Decision making in the presence of fuzzy variables, IEEE Transactions on Systems, Man and Cybernetics, 6(10) (1976) 698–703.

[30]: E. Kerre, The use of fuzzy set theory in electrocardiological diagnostics, in Approximate Reasoning in Decision-Analysis, M. M. Gupta and E. Sanchez, Eds., pp. 277–282, North-Holland Publishing, Amsterdam, The Netherlands, 1982.

[31]: W. Kołodziejczyk, Orlovsky's concept of decision-making with fuzzy preference relation-further results, Fuzzy Sets and Systems, 19(1) (1986) 11–20.

[32]: K. Kim and K.S. Park, Ranking fuzzy numbers with index of optimism, Fuzzy Sets and Systems, 35(2) (1990) 143–150.

[33]: L.W. Lee and S.M. Chen, Fuzzy risk analysis based on fuzzy numbers with different shapes and different deviations, Expert Systems with Applications, 34(4) (2008) 2763–2771.

[34]: E.S. Lee and R.J. Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, Computers and Mathematics with Applications, 15(10) (1988) 887–896.

[35]: D.F. Li, A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems, Computers and Mathematics with Applications, 60(6) (2010) 1557–1570.

[36]: T.S. Liou and M.J. Wang, Ranking fuzzy numbers with integral value, Fuzzy Sets and Systems, 50(3) (1992) 247–255.

[37]: X.W. Liu and S.L. Han, Ranking fuzzy numbers with preference weighting function expectations, Computers and Mathematics with Applications, 49(11-12) (2005) 1731–1753.

[38]: N. Mahdavi-Amiri, S.H. Nasseri and A. Yazdani Cherati, Fuzzy primal simplex algorithm for solving fuzzy linear programming problems, Iranian Journal of Operational Research, 2 (2009) 68-84.

[39]: M. Modarres and S.S Nezhad, Ranking fuzzy numbers by preference ratio, Fuzzy Sets and Systems, 118(3) (2001) 429–436.

[40]: S. Murakami, H. Maeda, and S. Imamura, Fuzzy decision analysis on the development of centralized regional energy control system, in Proceedings of the IFAC on Fuzzy Information, Knowledge Representation and Decision Analysis, pp. 353–358, Marseille, France, 1983.

[41]: K. Nakamura, Preference relations on a set of fuzzy utilities as a basis for decision making, Fuzzy Sets and Systems, 20(2) (1986) 147–162.

[42]: S.H. Nasseri, and A. Ebrahimnejad, A fuzzy primal simplex algorithm and its application for solving flexible linear programming problems, European Journal of Industrial Engineering, 4(3) (2010) 327-389.

[43]: S.H. Nasseri, H. Attari and A. Ebrahimnejad, Revised simplex method and its application for solving fuzzy linear programming problems, European Journal of industrial Engineering, in 6(3) (2012) 259-280.

[44]: S.H. Nasseri and N. Mahdavi-Amiri, Some duality results on linear programming problems with symmetric fuzzy numbers, Fuzzy Information and Engineering, 1 (2009) 59-66.

[45]: J.J. Saade and H. Schwarzlander, Ordering fuzzy sets over the real line: an approach based on decision making under uncertainty, Fuzzy Sets and Systems, 50(3) (1992) 237–246.

[46]: H. Sun and J. Wu, A new approach for ranking fuzzy numbers based on fuzzy simulation analysis method, Applied Mathematics and Computation, 174(1) (2006) 755–767.

[47]: L. Tran and L. Duckstein, Comparison of fuzzy numbers using a fuzzy distance measure, Fuzzy Sets and Systems, 130(3) (2002) 331–341.

[48]: E. Valvis, A new linear ordering of fuzzy numbers on subsets of F (R), Fuzzy Optimization and Decision Making, 8(2) (2009) 141–163.

[49]: X. Wang, A class of approaches to ordering alternatives, M.S. thesis, Taiyuan University Technology, 1987.

[50]: Y.M. Wang, Centroid defuzzification and the maximizing set and minimizing set ranking based on alpha level sets, Computers and Industrial Engineering, 57(1) (2009) 228–236.

[51]: X. Wang and E. E. Kerre, Reasonable properties for the ordering of fuzzy quantities (I), Fuzzy Sets and Systems, 118(3) (2001) 375–385.

[52]: X. Wang and E. E. Kerre, Reasonable properties for the ordering of fuzzy quantities (II), Fuzzy Sets and Systems, 118(3) (2001) 387–405.

[53]: Y.J. Wang and H.S. Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points, Computers and Mathematics with Applications, 55(9) (2008) 2033–2042.

[54]: Z.X. Wang, Y.J. Liu, Z.P. Fan and B. Feng, Ranking LR fuzzy number based on deviation degree, Information Sciences, 179(13) (2009) 2070–2077.

[55]: Y.M. Wang and Y. Luo, Area ranking of fuzzy numbers based on positive and negative ideal points, Computers and Mathematics with Applications, 58(9) (2009) 1769–1779.

[56]: Z.X. Wang and Y.N. Mo, Ranking fuzzy numbers based on ideal solution, Fuzzy Information and Engineering, 2(1) (2010) 27–36.

[57]: R. R. Yager, A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, 24(2) (1981) 143–161.

[58]: R.R. Yager, On choosing between fuzzy subsets, Kybernetes, 9(2) (1980) 151–154.

[59]: R. R. Yager, Ranking fuzzy subsets over the unit interval, in Proceedings of the IEEE Conference on Decision and Control (CDC '78), pp. 1435–1437, Albuquerque, NM, USA, August 1978.

[60]: J.S. Yao and K. Wu, Ranking fuzzy numbers based on decomposition principle and signed distance, Fuzzy Sets and Systems, 116(2) (2000) 275–288.

[61]: Y. Yuan, Criteria for evaluating fuzzy ranking methods, Fuzzy Sets and Systems, 43(2) (1991) 139–157.

[62]: L.A. Zadeh, Fuzzy sets, Information and Control, 8(3) (1965) 338–353.