The Journal of
Mathematics and Computer Science

Available online at
http://www.TJMCS.com

The Journal of Mathematics and Computer Science Vol. 4 No.3 (2012) 301 - 309

On the anisotropic Wiener-Hopf operator, connected with Helmholtze-Sohrodinger equation

S.A. Hosseini Matikolai[1], H. Jafari[2], Roshanak Lotfikar[3]

1- Department of Mathematics, Yerevan State University, Yerevan, Armenia
seyedm895@yahoo.com
2- Department of Mathematics, University of Mazandaran, Babolsar, Iran
jafari@umz.ac.ir
3- Islamic Azad University Ilam branch
rlotfikar@yahoo.com

Received: February 2012, Revised: May 2012
Online Publication: July 2012

Abstract
In this article, solvability of one the anisotropic Helmholtz-Shrodinger equation with the boundary conditions of the first and second type is investigated in the upper and lower half –space, ($x_5>0$, $x_5<0$), in 5 dimensions. Solvability of these boundary problems reduces to solvability of Rieman-Hilbert boundary problem, in general necessary and sufficient conditions for the correctness of the problem in the Sobolev space are presented as well as explicit formulas for a factorization of the Fourier symbol matrix of the one-medium problem. The solvability analysis is based on the factorization problem of some matrix-function.[1]

Keywords: Helmholtz-Shrodinger equation, Factorization of matrix-function, Boundary value problem, Wiener- Hopf equation.

1. Introduction
Was a certain class of diffraction problems leading investigated to simultaneous 2×2 systems of Wiener-Hopf equations. First the classical Wiener- Hopf technique was represented by Noble [1].

[1] 2000 Mathematics subject classifications: 47A68,47A70

This type of problems studied by A. J. Summerfeld for the wave diffraction on the interface of two media [2, 3].

Various physical problems in diffraction theory lead us to study modification of the Sommerfeld half–plane governed by two proper elliptic partial differential equation is complementary R^3 half-space $\Omega^{\pm}$ and allow different boundary or transmission conditions on two half-planes, which together from the common boundary of $\Omega^{\pm}$ [3].

These problems were investigated in the isotropic case [3, 4], and studied the problem of finding a function "u" in a suitable space with satisfies [3].

We investigated solvability of the boundary value problem coordinated with the anisotropic Helmholtz- Shrodinger equation:

$$\begin{cases} \Delta u + \left(k_+^2 + 2\beta_+^2 \sec h^2 (\beta_+ y)\right) u = 0, \ \ in \ \ \Omega^+, \\ \Delta u + \left(k_-^2 + 2\beta_-^2 \sec h^2 (\beta_- y)\right) u = 0, \ \ in \ \ \Omega^-, \end{cases} \tag{1}$$

In the Sobolev spaces [5]. Also it was investigated the case of anisotropic Helmholtz-Shrodinger equation (1), where k_+=k_-=k, and the solution of the boundary value problem gained and then prove solvability of this [6]. Also it was surveyed the Fredholmz property of Wiener-Hopf operator for anisotropic boundary value problem for the Helmholtz-Shrodinger equation with the boundary conditions of the first and second type on the line y=0 [7].

In this paper we investigate solvability of the boundary value problem coordinated with the anisotropic Helmholtz-Shrodinger equation, in the Sobolev spaces in 5 dimensions. Further we prove that solvability of the boundary value problem is equivalent to solvability of the some Riemann-Hilbert problem in 5 dimensions.

Convention: As a rule, upper or lower indices ± are related to the half-spaces $\Omega^{\pm}$ except for some standard notation $R_{\pm}$ and $H^{\pm\frac{1}{2}}$

2. Investigate solvability of anisotropic boundary value problem

Consider the following anistropic Helmholtz-Shrodinger equation

$$\begin{cases} \Delta u + \left(k_+^2 + 2\beta_+^2 \sec h^2 (\beta_+ x_5)\right) u = 0, \ \ in \ \ \Omega^+, \\ \Delta u + \left(k_-^2 + 2\beta_-^2 \sec h^2 (\beta_- x_5)\right) u = 0, \ \ in \ \ \Omega^-, \end{cases} \tag{2}$$

Let $\Omega^{\pm} = \{(x_1, x_2, \ldots, x_5) \in R^5 : x_5 \lessgtr 0\}$ and $\sum^{\pm} = \{(x_1, x_2, x_3, x_4, 0) \in R^4 \ : \ x_4 \gtrless 0\}$, where $J_m(k_{\pm}) > 0$ and let $H^{\frac{1}{2}}(\Omega^{\pm})$ and $H^{-\frac{1}{2}}(\Omega^{\pm})$ are the corresponding Sobolev spaces (see [3]).

Now we suppose the boundary conditions:

$$\begin{cases} \begin{cases} a_0u(x_1,x_2,x_3,x_4,+0)+b_0u(x_1,x_2,x_3,x_4,-0)=h_0(x_1,x_2,x_3,x_4) \\ a_1\dfrac{\partial u(x_1,x_2,x_3,x_4,+0)}{\partial x_5}+b_1\dfrac{\partial u(x_1,x_2,x_3,x_4,-0)}{\partial x_5}=h_1(x_1,x_2,x_3,x_4) \qquad in \qquad \sum^+ \end{cases} \\ \begin{cases} c_0u(x_1,x_2,x_3,x_4,+0)+d_0u(x_1,x_2,x_3,x_4,-0)=P_0(x_1,x_2,x_3,x_4) \\ c_1\dfrac{\partial u(x_1,x_2,x_3,x_4,+0)}{\partial x_5}+d_1\dfrac{\partial u(x_1,x_2,x_3,x_4,-0)}{\partial x_5}=P_1(x_1,x_2,x_3,x_4) \qquad in \qquad \sum^+ \end{cases} \end{cases}$$

Where $h_0 \in H^{\frac{1}{2}}\left(\sum^+\right), h_1 \in H^{-\frac{1}{2}}\left(\sum^+\right), p_0 \in H^{\frac{1}{2}}\left(\sum^-\right), p_1 \in H^{-\frac{1}{2}}\left(\sum^-\right)$ and $a_0, b_0, c_0, d_0, a_1, b_1, c_1, d_1$, are complex constants. For finding the solution of the boundary value problem (2) in the $L^2(R^5)$, apply Fourier integral transform to the solution $u \in L^2(R^5)$ over the variables x_1, x_2, x_3, x_4, one derives the following system of ordinary differential equations:

$$\begin{cases} \dfrac{d^2\hat{u}}{dx_5^2}+\left(\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+2\beta_+^2\sec h^2(\beta_+x_5)\right) \quad \hat{u}=0, \qquad for \qquad x_5>0 \\ \dfrac{d^2\hat{u}}{dx_5^2}+\left(\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+2\beta_-^2\sec h^2(\beta_-x_5)\right) \quad \hat{u}=0, \qquad for \qquad x_5<0. \end{cases} \tag{4}$$

Then $\hat{u} \in L^2(R^5)$, we denote $\gamma \pm (\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\sqrt{\sum_{j=1}^{4}\lambda_j^2-k_\pm^2}=i\kappa_\pm(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$.

If follows that the general solutions of the system of ordinary differential equations (4) in the $L^2(R^5)$ -space has the following form:

$$\hat{u}(\lambda_1,\lambda_2,\lambda_3,\lambda_{4,}x_5)=\begin{cases} a(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\dfrac{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_+tanh(\beta_+x_5)}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_{4,})}\left(e^{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)x_5}\right), \ x_5>0 \\ b(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\dfrac{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-tanh(\beta_-x_5)}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left(e^{-i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)x_5}\right), \ x_5<0 \end{cases} \tag{5}$$

Let $\chi_\pm(x_5)=1/2(\operatorname{sgn} x_5)$, and

$$\begin{cases} \hat{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4,x_5)=\dfrac{\chi_+(x_5)}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty}u(x_1,x_2,\ldots,x_5)e^{i\sum_{K=1}^{4}\lambda_kx_k}dx_1dx_2\ldots dx_5 \\ \hat{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4,x_5)=\dfrac{\chi_-(x_5)}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\ldots\int_{-\infty}^{+\infty}u(x_1,x_2,\ldots,x_5)e^{i\sum_{K=1}^{4}\lambda_kx_k}dx_1dx_2\ldots dx_5 \end{cases} \tag{6}$$

Then from equation (5) it follows that

$$\hat{u}(\lambda_1,\lambda_2,\lambda_3,\lambda_4,x_5)=\hat{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4,x_5)+\hat{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4,x_5) \tag{7}$$

We introduce the following notations:

$$\begin{cases} u_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\dfrac{1}{(2\pi)^2}\int_{-\infty}^{0}\int_{-\infty}^{0}\ldots\int_{-\infty}^{0}(a_0u(x_1,x_2,x_3,x_4,+0) \\ +b_0u(x_1,x_2,x_3,x_4,-0)-h_0(x_1,x_2,x_3,x_4))e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4 \\ w_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\dfrac{1}{(2\pi)^2}\int_{-\infty}^{0}\int_{-\infty}^{0}\ldots\int_{-\infty}^{0}(a_1\dfrac{\partial u(x_1,x_2,x_3,x_4,+0)}{\partial x_5}+b_1\dfrac{\partial u(x_1,x_2,x_3,x_4,-0)}{\partial x_5} \\ -h_1(x_1,x_2,x_3,x_4))e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4. \end{cases} \tag{8}$$

Similarly

$$\begin{cases} u_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\dfrac{1}{(2\pi)^2}\int_{0}^{+\infty}\int_{0}^{+\infty}\ldots\int_{0}^{+\infty}(c_0u(x_1,x_2,x_3,x_4,+0) \\ +d_0u(x_1,x_2,x_3,x_4,-0)-p_0(x_1,x_2,x_3,x_4))e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4, \\ w_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\dfrac{1}{(2\pi)^2}\int_{0}^{+\infty}\int_{0}^{+\infty}\ldots\int_{0}^{+\infty}(c_1\dfrac{\partial u(x_1,x_2,x_3,x_4,+0)}{\partial x_5}+d_1\dfrac{\partial u(x_1,x_2,x_3,x_4,-0)}{\partial x_5} \\ -p_1(x_1,x_2,x_3,x_4))e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4. \end{cases} \tag{9}$$

So

$$\frac{\partial\hat{u}(x_1,x_2,x_3,x_4,x_5)}{\partial x_5}= \begin{cases} a(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\left[i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_+\tanh(\beta_+x_5)-\dfrac{\beta_+{}^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\cosh^2(\beta_+x_5)}\right] \\ \left(e^{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)x_5}\right),\ x_5>0 \\ -b(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\left[i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-\tanh(\beta_-x_5)+\dfrac{\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\cosh^2(\beta_-x_5)}\right] \\ \left(e^{-i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)x_5}\right),\ x_5<0. \end{cases} \tag{10}$$

Using boundary conditions (3) and taking into account equations (5), (10) one derives:

$$\begin{cases} a_0a(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+b_0b(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=u_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\hat{h}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \\ \dfrac{-a_1\left[\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2\right]a(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}+\dfrac{b_1\left[\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2\right]b(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \\ =w_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\hat{h}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \end{cases} \tag{11}$$

Where

$$\begin{cases} \hat{h}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \dfrac{1}{(2\pi)^2}\int_{-\infty}^{0}\int_{-\infty}^{0}\dots\int_{-\infty}^{0} h_0(x_1,x_2,x_3,x_4)e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4 \\ \hat{h}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \dfrac{1}{(2\pi)^2}\int_{-\infty}^{0}\int_{-\infty}^{0}\dots\int_{-\infty}^{0} h_1(x_1,x_2,x_3,x_4)e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4 \end{cases}$$

Assume that the determinant $\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$, of system (11), is not zero, i.e.

$$\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = a_0b_1\frac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} + a_1b_0\frac{\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}$$
$$= a_0b_1\frac{\gamma_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_-^2}{\gamma_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} + a_1b_0\frac{\gamma_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_+^2}{\gamma_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \neq 0 \qquad (12)$$

In view of equation (11):

$$\begin{cases} a(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \dfrac{1}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left\{b_1\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left(u_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\hat{h}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right)\right\} \\ -\dfrac{1}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left\{b_0\left(w_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\hat{h}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right)\right\} \\ b(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \dfrac{1}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left\{a_1\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left(u_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\hat{h}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right)\right\} \\ +\dfrac{1}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left\{a_0\left(w_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\hat{h}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right)\right\} \end{cases} \qquad (13)$$

Then taking into account that

$$\begin{cases} u_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = c_0a(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+d_0b(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\hat{p}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \\ w_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \dfrac{-c_1\left[\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2\right]a(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \\ +\dfrac{d_1\left[\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2\right]b(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} - \hat{p}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \end{cases} \qquad (14)$$

Where

$$\begin{cases} \hat{p}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \dfrac{1}{(2\pi)^2}\int_{0}^{+\infty}\int_{0}^{+\infty}\dots\int_{0}^{+\infty} \hat{p}_0(x_1,x_2,x_3,x_4)e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4 \\ \hat{p}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = \dfrac{1}{(2\pi)^2}\int_{-\infty}^{0}\int_{0}^{+\infty}\dots\int_{-\infty}^{0} \hat{p}_1(x_1,x_2,x_3,x_4)e^{i\sum_{k=1}^{4}\lambda_k x_k}dx_1dx_2dx_3dx_4 \end{cases}$$

Which derives the following boundary problem of Riemann-Hilbert with respect to

$$\vec{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\begin{pmatrix} u_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \\ w_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\end{pmatrix},\quad \vec{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\begin{pmatrix} u_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \\ w_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\end{pmatrix}, \qquad (15)$$

$\vec{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$ and $\vec{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$ are analytical functions respectively in the vector notations this problem takes the following form

$$\vec{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=L(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\vec{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\vec{m}(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \qquad (16)$$

Where the matrix function $L(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$ is:

$$L(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\frac{1}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\begin{pmatrix} A_{11}(\lambda_1,\lambda_2,\lambda_3,\lambda_4), & A_{12}(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \\ A_{21}(\lambda_1,\lambda_2,\lambda_3,\lambda_4), & A_{22}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\end{pmatrix} \qquad (17)$$

With

$$\begin{cases} A_{11}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=a_1d_0\dfrac{\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}+b_1c_0\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \\ =a_1d_0\dfrac{\gamma_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_+^2}{\gamma_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}+b_1c_0\dfrac{\gamma_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_-^2}{\gamma_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \\ A_{12}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=a_0d_0-b_0c_o \\ A_{21}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=(a_1d_1-b_1c_1)\dfrac{\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\times\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \\ =(a_1d_1-b_1c_1)\dfrac{\gamma_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_+^2}{\gamma_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\times\dfrac{\gamma_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_-^2}{\gamma_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \\ A_{22}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=b_0c_1\dfrac{\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}+a_0d_1\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \\ =b_0c_1\dfrac{\gamma_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_+^2}{\gamma_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}+a_0d_1\dfrac{\gamma_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\beta_-^2}{\gamma_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)} \end{cases}$$

The coordinates of the vector-function $\overrightarrow{m}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$

$$\overrightarrow{m}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\begin{pmatrix} m_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \\ m_2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\end{pmatrix} \qquad (18)$$

Have the following form

$$\begin{cases} m_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\dfrac{\hat{h}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left\{a_1 d_0\dfrac{\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}+b_1 c_0\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\right\} \\ +\dfrac{a_0 d_0-b_0 c_0}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\hat{h}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\hat{p}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \\ m_2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\dfrac{\hat{h}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\left\{b_0 c_1\dfrac{\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}+a_0 d_1\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\right\} \\ +\dfrac{a_1 d_1-b_1 c_1}{\Delta(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\hat{h}_0(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\dfrac{\kappa_+^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_+^2}{i\kappa_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}\times\dfrac{\kappa_-^2(\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\beta_-^2}{i\kappa_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4)}-\hat{p}_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \end{cases} \quad (19)$$

For find the function $u\in L^2(R^5)$ for the anisotropic boundary problem *($\mathcal{AP}$)* in space R^5, with respect to vector-functions $\left\{\overrightarrow{u_+}(\lambda_1,\lambda_2,\lambda_3,\lambda_4), \overrightarrow{u_-}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right\}$, one derives the following Riemann-Hilbert boundary problem (16), satisfies in $\hat{H}^{\frac{1}{2}}\left(\sum^+\right)\times\hat{H}^{-\frac{1}{2}}\left(\sum^-\right)$.

Therefore vector-functions $\left\{\overrightarrow{u_+}(\lambda_1,\lambda_2,\lambda_3,\lambda_4), \overrightarrow{u_-}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right\}$, with notice to (16), we take $\hat{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4,0)=a(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$, and $\hat{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4,0)=b(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$, then taking into account that

$$\begin{cases} u^+(x_1,x_2,x_3,x_4,x_5)=F^{-1}_{(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\to(x_1,x_2,x_3,x_4)}\hat{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4,x_5) \\ =F^{-1}_{(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\to(x_1,x_2,x_3,x_4)}\left\{e^{-x_5\gamma_+(\xi_1,\zeta_2,\xi_3,\xi_4)}\hat{u}_+(\lambda_1,\lambda_2,\lambda_3,\lambda_4,0)\chi_+(x_5)\right\}\in H^1(\Omega^+) \\ u^-(x_1,x_2,x_3,x_4,x_5)=F^{-1}_{(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\to(x_1,x_2,x_3,x_4)}\hat{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4,x_5) \\ =F^{-1}_{(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\to(x_1,x_2,x_3,x_4)}\left\{e^{-x_5\gamma_+(\xi_1,\zeta_2,\xi_3,\xi_4)}\hat{u}_-(\lambda_1,\lambda_2,\lambda_3,\lambda_4,0)\chi_-(x_5)\right\}\in H^1(\Omega^-) \end{cases} \quad (20)$$

And function

$$u(x_1,x_2,x_3,x_4,x_5)=u^+(x_1,x_2,x_3,x_4,x_5)+u^-(x_1,x_2,x_3,x_4,x_5) \quad (21)$$

Is the Solution anisotropic boundary problem ($\mathcal{AP}$)

In this article we have this result:

Theorem: If the function $u\in L^2(R^5)$ is the solution of the boundary problem (2), then the pair of vector –functions $\left\{\overrightarrow{u_+}(\lambda_1,\lambda_2,\lambda_3,\lambda_4), \overrightarrow{u_-}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right\}$ is a solution of the boundary problem of Riemann-Hilbert (16). Vice-versa, if applies inverse Fourier transform which is associated with vector-functions $\left\{\overrightarrow{u_+}(\lambda_1,\lambda_2,\lambda_3,\lambda_4), \overrightarrow{u_-}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\right\}$, the function

$$u\left(x_1,x_2,x_3,x_4,x_5\right) = F^{-1}_{\left(\lambda_1,\lambda_2,\lambda_3,\lambda_4\right)\to\left(x_1,x_2,x_3,x_4\right)}\left\{\overrightarrow{u_+}\left(\lambda_1,\lambda_2,\lambda_3,\lambda_4\right)+\overrightarrow{u_-}\left(\lambda_1,\lambda_2,\lambda_3,\lambda_4\right)\right\}$$

$$= u^+\left(x_1,x_2,x_3,x_4,x_5\right) + u^-\left(x_1,x_2,x_3,x_4,x_5\right) = F^{-1}_{\left(\lambda_1,\lambda_2,\lambda_3,\lambda_4\right)\to\left(x_1,x_2,x_3,x_4\right)}$$

$$\left\{e^{-x_5\gamma_+(\xi_1,\xi_2,\xi_3,\xi_4)}\widehat{u_+}\left(\lambda_1,\lambda_2,\lambda_3,\lambda_4,0\right)\chi_+(x_5) + e^{-x_5\gamma_+(\xi_1,\xi_2,\xi_3,\xi_4)}\widehat{u_-}\left(\lambda_1,\lambda_2,\lambda_3,\lambda_4,0\right)\chi_-(x_5)\right\}.$$

By the case of relations (8), (9), the solutions of the boundary problem (2), will be derived.▲

The case of $\beta_- = \beta_+ = 0$ *was studied in the papers* [2,3].

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