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OPTIMIZED SOLUTION OF PRESSURE VESSEL DESIGN USING GEOMETRIC PROGRAMMING

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ABSTRACT. Geometric programming is a methodology for solving algebraic nonlinear optimization problems. It provides a powerful tool for solving nonlinear problems where nonlinear relations can be well presented by an exponential or power function. This feature is especially advantageous in situations where the optimal value of the objective function may be all that is of interest. In such cases, calculation of the optimum design vectors can be omitted. The goal of this paper is to state the problem of Pressure vessel design and after that finding a better optimized solution using geometric programme.

1. INTRODUCTION

In the real world, many applications of geometric programming are engineering design problems. One of the remarkable properties of geometric programming is that a problem with highly nonlinear constraints can be stated equivalently as one with only linear constraints. This is because there is a strong duality theorem for geometric programming problems. The dual constrains are linear and linearly constrained programs are generally easier to solve than ones with nonlinear constraints. Its attractive structural properties as well as its elegant theoretical basis have led to a number of interesting

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applications and the development of numerous useful results. The optimazation problem which is introduced in the following has been solved before by Deb and Gene [2] using Genetic Adaptive Search, by Kannan and Kramer [3] using an augmented Lagrangian Multiplier approach, and by Coello [4] using Genetic Algorithm and then by M. Mahdavi et al. [1] using an improved harmony search algorithm. In this paper, we first use the duality theorem [6] and then find the optimal solution which is better optimized than any other earlier solutions reported before. The remainder of this paper is organized as follows: The fuzzy geometric programming problem is first introduced. Next, the problem is stated. By the duality theorem, we write the dual of the problem and finally the comparison of results are shown in a table.

2. Mathematical formulation

A constrained posynomial geometric program is an optimization problem of the following form:

$$\min_x f_o(x) = \sum_{t=1}^{s_o} C_{ot} \prod_{j=1}^{n} {x_j}^{a_{otj}} \tag{2.1}$$

$$s.t. \quad f_i(x) = \sum_{t=1}^{s_i} C_{it} \prod_{j=1}^{n} {x_j}^{\gamma_{itj}} \leqslant 1, \quad i = 1, ..., m.$$

$$x_j > 0, \quad j = 1, ..., n.$$

The posynomial $f_o(x)$ containing s_o terms is the objective function, while the posynomials $f_i(x)$ for $i = 1, ..., m$ containing s_i terms represent m inequality constraints. By the definition of posynomial all the coefficients C_{it} for $i = 0, 1, ..., m$ and $t = 1, ..., s_m$ are positive. If the right hand sides of the constraints in the geometric program (2.1) are modified as

$$\min_x f_o(x) = \sum_{t=1}^{s_o} C_{ot} \prod_{j=1}^{n} {x_j}^{a_{otj}} \tag{2.2}$$

$$s.t. \quad f_i(x) = \sum_{t=1}^{s_i} C_{it} \prod_{j=1}^{n} {x_j}^{\gamma_{itj}} \leqslant b_i, \quad i = 1, ..., m.$$

$$x_j > 0, \quad j = 1, ..., n.$$

where all b_i are positive numbers. If $b_i = 1 \quad \forall i$, then this modified geometric program coincides with the original one. Otherwise, the constraints need some amendment to be consistent with model (2.1).

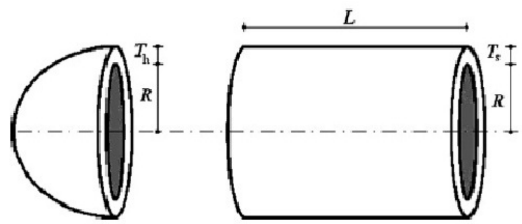

FIGURE 1. Schematic of pressure vessel

3. PRESSURE VESSEL DESIGN

A cylindrical vessel is capped at both ends by hemispherical heads as shown in Fig(1). The objective is to minimize the total cost, including the cost of material, forming and welding. There are four design variables: T_s (thickness of the shell, x_1), T_h (thickness of the head, x_2), R (inner radius, x_3) and L (length of cylindrical section of the vessel, not including the head, x_4). T_s and T_h are integer multiples of 0.0625 inch, which are the available thickness of rolled steel plates, and R and L are continuous. By using the same notation given by Coello [5], the problem is stated as follows:

$$\min \qquad f(\overrightarrow{x}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$$

$$s.t. \quad g_1(\overrightarrow{x}) = -x_1 + 0.0193x_3 \leqslant 0$$

$$g_2(\overrightarrow{x}) = -x_2 + 0.00954x_3 \leqslant 0$$

$$g_3(\overrightarrow{x}) = -\pi x_3^2x_4 - \frac{4}{3}\pi$$

$$x_3^3 + 1.296000 \leqslant 0$$

$$g_4(\overrightarrow{x}) = x_4 - 240 \leqslant 0$$

The comparisons of results are shown in Table (1).

4. Solution approach

First of all we modify the problem as an original geometric program as follow:

$$\begin{aligned}
\min \quad & f(\overrightarrow{x}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \\
& s.t. \quad g_1(\overrightarrow{x}) = \frac{1}{0.0193}x_1x_3^{-1} \geqslant 1 \\
& g_2(\overrightarrow{x}) = \frac{1}{0.00954}x_2x_3^{-1} \geqslant 1 \\
& g_3(\overrightarrow{x}) = \frac{\pi}{1296000}x_3^2x_4 + \frac{\pi}{972000}x_3^3 \geqslant 1 \\
& g_4(\overrightarrow{x}) = \frac{1}{240}x_4 \leqslant 1
\end{aligned}$$

Solution: In this problem $m = 4, N_o = 4, N_1 = 1, N_2 = 1, N_3 = 2, N_4 = 1, n = 4$. The signum functions are $\sigma_o = 1, \sigma_1 = -1, \sigma_2 = -1, \sigma_3 = -1$ and $\sigma_4 = 1$. The dual objective function can be stated as follows:

$$\max v(\lambda) = \Pi_{k=0}^{4}\Pi_{j=1}^{N_k}\left(\frac{C_{kj}}{\lambda_{kj}}\Sigma_{l=1}^{N_k}\lambda_{kl}\right)^{\sigma_k\lambda_{kj}}$$

The constraints are given by:

$$\begin{aligned}
& \Sigma_{j=1}^{N_o}\lambda_{oj} = 1 \\
& \Sigma_{k=0}^{m}\Sigma_{j=1}^{N_k}\sigma_k a_{kij}\lambda_{kj} = 0, \quad i = 1, ..., n \\
& \Sigma_{j=1}^{N_k}\lambda_{kj} \geqslant 0, \quad k = 1, ..., m.
\end{aligned}$$

where C_{kj} are the coefficients, a_{kij} are the exponents, m indicates the total number of constraints, N_0 denotes the number of terms in the objective function and N_k represents the number of terms in the kth constraint.

That is:

$\max v(\lambda) = (\frac{0.6224}{\lambda_{01}})^{\lambda_{01}}(\frac{1.7781}{\lambda_{02}})^{\lambda 02}(\frac{3.1661}{\lambda_{03}})^{\lambda 03}(\frac{19.84}{\lambda_{04}})^{\lambda_{04}}(\frac{1}{0.0193})^{-\lambda_{11}}(\frac{1}{0.00954})^{-\lambda_{21}}(\frac{1}{240})^{\lambda_{41}}(\frac{\frac{\pi}{1296000}}{\lambda_{31}}(\lambda_{31}+$

$\lambda_{32}))^{-\lambda_{31}}(\frac{\frac{\pi}{972000}}{\lambda_{32}}(\lambda_{31}+\lambda_{32}))^{-\lambda_{32}}$

$$s.t. \quad \lambda_{01}+\lambda_{02}+\lambda_{03}+\lambda_{04}=1$$

$$\lambda_{01}+2\lambda_{03}+2\lambda_{04}-\lambda_{11}=0$$

$$\lambda_{02}-\lambda_{21}=0$$

$$\lambda_{01}+2\lambda_{02}+\lambda_{04}+\lambda_{11}+\lambda_{21}-2\lambda_{31}-3\lambda_{32}=0$$

$$\lambda_{01}+\lambda_{03}-\lambda_{31}+\lambda_{41}=0$$

$$\lambda_{31}+\lambda_{32}\geqslant 0$$

$$\lambda_{11}\geqslant 0, \lambda_{21}\geqslant 0, \lambda_{41}\geqslant 0.$$

The dual problem has the desirable features of being linearly constrained.
After solving the dual problem, the optimum value of the objective function $v^* = f^* = 5807.390$ is known and the values of the design variables x_i^* are as follow:
$x_1^* = 0.7277, x_2^* = 0.3597, x_3^* = 37.70, x_4^* = 240.00.$
The results obtained using geometric programming were better optimized than any other earlier solutions which has been reported before.

Table 1

Optimal results for pressure vessel design

methods	M. Mahdavi[1]	Deb and Gene[2]	Kannan and Kramer[3]	Coello[4]	proposed method
results	5849.7617	6410.3811	7198.0428	6069.3267	5807.390

5. Conclusions

Geometric programming is a known method of solving a class of nonlinear programming problems. In particular, we have mentioned a optimization problem which was been solved by different methods and each method had a result but the results obtained using Geometric programming were better optimized.

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