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Hydrodynamic Modeling of Fluid body interaction by Fully Non-Linear Numerical Towing Tank

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Abstract

In basic design of offshore or onshore structures, prediction of surface waves due to uniform motion of the floating body is essential to achieve an optimum body shape. Whereas, in the practical hydrodynamics, using of towing tanks is common and so cost and time consuming to conduct it, so a reliable numerical tank is interesting. A wide channel with constant depth constitutes computational domain. It assumes that fluid is incompressible and non-viscous and the flow is irrotational. Therefore, Laplace's equation could describe flow field. 3D Boundary Element method based on second Green's Identity is implemented to solve Laplace's equation. Impermeable boundary condition is satisfied by Image method and Cauchy integral theorem and Poisson summation formula is used to determine Principle value integral. In this study, numerical simulation is conducted for a hemisphere and added mass and generated wave profile is presented.

Keywords: 3D Boundary Element Method, source, Green integral, Perturbation potential.

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1. Introduction

Boundary integral equation is defined in two schemes: direct and in-direct formulations in order to analyze potential flow in several hydrodynamic problems. In-direct method is based on distributing sources on a body surface, whereas its strength is calculated by Fredholm's integral equation on body. Potential flow around submerged non-lifting bodies has been widely investigated by direct method. It is based on the second Green's identity. Green function in this formulation is field's response due to a constant strength singularity presents within the uniform flow in the computational domain. Fundamental solution of a submerged source in a uniform flow is derived by Fourier transform approach. 3D Green function in finite depth was applied by Sahin and Hymann (1993, 1994). They used Cauchy Residue theorem and an adaptive quadratures set to evaluate principle value integral and determined fluid flow around axisymmetric submerged body. To approximate resistance due to generated wave pattern of a wigley hull, Insel (1995) used a modified Green function in boundary integral equation and compared results with experimental measurements.

Accuracy of BEM depends on two items. First one is definition of body geometry and another one is order of distribution velocity potential over elements. Study on these items on the numerical solutions was conducted by Kouh and Ho (1996). In their research, Non-lifting body was discretized to flat and quadratic panels with constant and linear source strength distribution over the elements.

In addition, overlapping of source point and field point is a major difficulty in boundary integral implementation. Several remedies have been found for different problems. Cao (1991) was developed a numerical approach for direct and in-direct boundary integral based on Desingularization distance for potential problems. Hwang (1998) used a subtracting and adding technique to desingularize boundary integrals for wave body interaction problem. Their approach is appropriate for open boundaries. To solve Laplace Equation in a closed domain, Grilli et al. (2001) summarize a formulation to remove singularity of direct boundary integral for curvilinear elements. Gao(2008) used curvilinear elements in desingularized in-direct method.

In this study, direct boundary integral equation is used to solve Laplace equation and isoparametric triangular element is chosen to prescribe boundary condition and geometry. Fully non-linear free-surface boundary condition is so complicated to be satisfied. By using Taylor expansion, free-surface boundary condition is satisfied in Green function and image method is used to apply impermeable bottom and walls tank. Total Velocity potential is divided in two parts. First part is uniform flow with constant velocity and second part is perturbation potential due to presentation of body in uniform flow. A new trick is presented to remove singularity of boundary integral implementation. Hydrodynamic performance of a hemisphere is considered to assess numerical procedure.

2. Mathematical Formulation

Consider a submerged or floating body in shallow water. The right hand coordinate system that is shown in Figure 1 is used. Its origin is located on undistributed free surface and z axis is directed upward. It is assumed that fluid flow is ideal, incompressible, steady and irrotational. The tank bottom is located at $z = -h$, and uniform flow with speed U streams in negative x direction. Consequently, the total steady-state velocity potential can be expressed as follow:

$$\Phi(x, y, z) = -Ux + \phi(x, y, z) \quad (1)$$

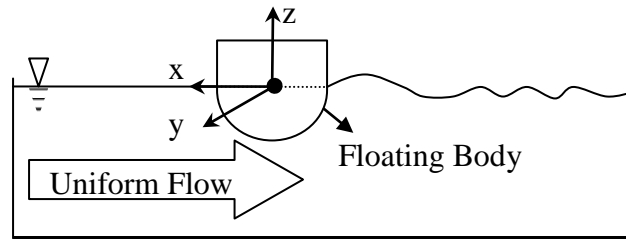


Figure 1. Definition Sketch

Indeed, the total velocity potential includes uniform flow potential and perturbation potential. Whereas the uniform flow potential satisfies the Laplace's equation, the problem can be solved in term of perturbation potential. Thus, the governing equation can be written as:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2)$$

To solve Equation 2, boundary conditions must be given which are categorized in the following:

- Bottom boundary condition

$$\frac{\partial \phi}{\partial z} = 0 \text{ on } z = -h \quad (3)$$

- Radiation condition

$$\begin{cases} \lim_{r^2 \rightarrow \infty} \phi = O(1) & \text{for } x < 0 \\ \lim_{r^2 \rightarrow \infty} \phi = 0 & \text{for } x > 0 \end{cases} \quad (4)$$

- Combined free surface boundary condition

$$\frac{\partial^2 \phi}{\partial x^2} + k_0 \frac{\partial \phi}{\partial z} = 0, \quad k_0 = \frac{g}{U^2} \text{ on } z = 0 \quad (5)$$

- Hull Boundary condition

$$\frac{\partial \Phi}{\partial n} = 0 \quad \rightarrow \quad \frac{\partial \phi}{\partial n} = Un_x \text{ on } S_b \quad (6)$$

where, S_b is body hull and n_x is x direction component of normal vector directed into fluid.

Second identity Green theorem is expressed as below:[3]

$$\iint_{S_q} \left[\phi_q \frac{\partial G_{Pq}}{\partial n_q} - G_{Pq} \frac{\partial \phi}{\partial n_q} \right] dS = \varepsilon(P)\phi(P) \quad (7)$$

where, P is field point and q is source point and S_q indicates surface integral. $\varepsilon(P)$ depends on position of field point that can be located on smoothed body's surface ($\varepsilon(P) = 2\pi$) or in control volume ($\varepsilon(P) = 4\pi$) or out of domain ($\varepsilon(P) = 0$). The modified Green function for a source point at (x_0, y_0, z_0) can be written as:

$$G = \begin{cases} -\mu \sum_{n=-\infty}^{\infty} \left[\frac{1}{r_1'} + \frac{1}{r_1''} + \frac{1}{r_2'} + \frac{1}{r_2''} \right] & , (x-x_0) > 0 \\ -\mu \sum_{n=-\infty}^{\infty} \left[\frac{1}{r_1'} + \frac{1}{r_1''} + \frac{1}{r_2'} + \frac{1}{r_2''} \right] + \frac{16\pi\mu}{W} \sum_{m=0}^{\infty} \frac{e^{-K_m h} \cosh(K_m(z_0+h))(K_0 + K_m \cos^2 \theta_m)}{K_m \cos \theta_m (1 - K_0 h \operatorname{sech}^2(K_m h) + \sin^2 \theta_m)} & , (x-x_0) < 0 \\ \frac{\cosh(K_m(z+h))}{\cosh(K_m h)} \sin(K_m(x-x_0)\cos\theta_m) \times \begin{cases} \cos(K_m y \sin \theta_m) \cos(K_m y_0 \sin \theta_m) \\ \sin(K_m y \sin \theta_m) \sin(K_m y_0 \sin \theta_m) \end{cases} & \end{cases} \quad (8)$$

where, μ is strength of source point is taken unit and [5]

$$r_1'^2 = (x - x_0)^2 + (y - y_0')^2 + (z - z_0)^2 \quad (9)$$

$$r_1''^2 = (x - x_0)^2 + (y - y_0'')^2 + (z - z_0)^2$$

$$r_2'^2 = (x - x_0)^2 + (y - y_0')^2 + (z + (2h + z_0))^2$$

$$r_2''^2 = (x - x_0)^2 + (y - y_0'')^2 + (z + (2h + z_0))^2$$

also

$$y_0' = y_0 + 2nW \quad , \quad n = -\infty \dots \infty \quad (10)$$

$$y_0'' = -y_0 + (2n+1)W \quad , \quad n = -\infty \dots \infty$$

where, W is width of numerical tank. In the image method, the tank walls as mirror image singularity point in y direction to infinity at both sides. Imaged sources' position can be described by y_0' and y_0'' in Equation 10. In Equation 8, \sin term specified to odd m and \cos term specified to even m which arises from Green function modification based on Cauchy integral theorem and Poisson summation formula. θ_m and K_m in equation 10 are derived by solving two below equations at the same time.

$$K_m - K_0 \sec^2 \theta_m \tanh(K_m h) = 0 \quad (11)$$

$$K_m \sin \theta_m = m\pi/W$$

3. Desingularization Techniques

Until now, various methods have been propounded to eliminate singular terms. In this paper, tricky approaches are proposed to obtain diagonal arrays of influence coefficient. In order to remove singularity, body's geometry surface is described by flat triangular panels ($S_j, j = 1, \dots, N$). Source points are located at center of each panel whereas; potential distribution over each element is constant. Figure 2 illustrates the location of field point and source points on the elements.

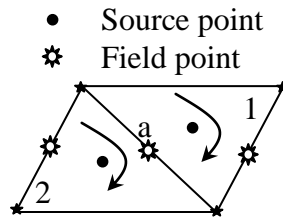


Figure 2. View of collocated source and field points

Field point (a) is positioned within the intersection edge of elements 1, 2. Field point potential can be defined by linear combination of velocity potentials of corresponding elements to each field points as below:

$$\phi_a = \frac{\frac{\phi_1}{d_{1a}} + \frac{\phi_2}{d_{2a}}}{\frac{1}{d_{1a}} + \frac{1}{d_{2a}}} \quad (12)$$

where,

$$d_{1a} = \sqrt{(x_a - x_1)^2 + (y_a - y_1)^2 + (z_a - z_1)^2} \quad (13)$$

$$d_{2a} = \sqrt{(x_a - x_2)^2 + (y_a - y_2)^2 + (z_a - z_2)^2}$$

and ϕ_1, ϕ_2 is constant potential specified to the elements. It seems that potential value over the body surface is unknown boundary value and given boundary value is potential normal flux found as Neumann boundary condition. Therefore, discretized equation 7 can be written as:

$$\sum_{j=1}^N \phi_j \iint_{S_j} \frac{\partial G_{ij}}{\partial n_j} dS_j - \varepsilon_i \frac{\frac{1}{d_{im}}}{\frac{1}{d_{im}} + \frac{1}{d_{in}}} \phi_m \delta_{im} - \varepsilon_i \frac{\frac{1}{d_{in}}}{\frac{1}{d_{im}} + \frac{1}{d_{in}}} \phi_n \delta_{in} = \sum_{j=1}^N U n_{xj} \iint_{S_j} G_{ij} dS_j, \quad i = 1, \dots, N \quad (14)$$

where, n and m are corresponding panels to field points. δ is Kronecker Delta Function. Gauss elimination method is used to solve linear algebraic system.

4. Numerical Results

A hemisphere is described by triangular flat elements is shown in Figure 3. In order to verify the present approach, surge added masses are achieved and compared with the analytical solutions for different number of mesh in table 1. Radius of hemisphere is r and uniform flow velocity is taken 1 m/s. Added mass is defined as:[6]

$$m_{ij} = \rho \iint_{S_b} \phi_j \frac{\partial \phi_i}{\partial n} dS, \quad i, j = 1, 2, \dots, 6 \quad (15)$$

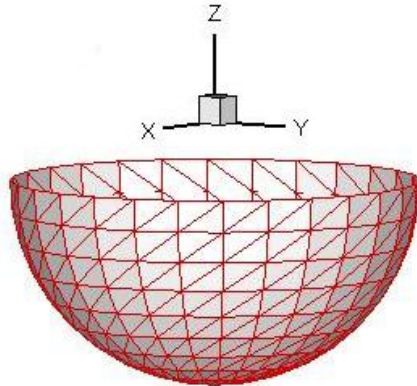


Figure 3.512 triangular elements over hemisphere

Table 1. Surge added mass of a sphere

Case	Number of Mesh	$m_{11} / \pi \rho r^3$
Case 1	256	0.356042
Case 2	528	0.346737
Case 3	756	0.335614
Case 4	1556	0.333453
Analytic	-	0.333333

Table 1 shows good agreements between numerical results and analytical results. In addition, for a tank

With 20 m depth and 30 m width, propagation wave due to uniform motion of hemisphere with different velocity is illustrated.

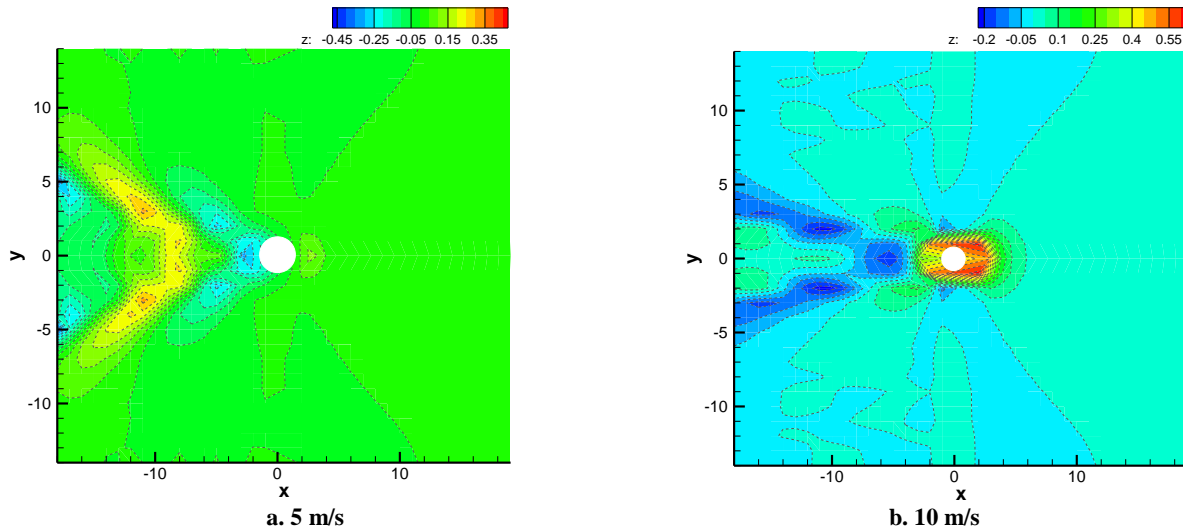


Figure 4. Wave Pattern due to uniform motion of a hemisphere

It seems that when velocity of body is increased, wave height is growing up and wave pattern is changed. This phenomenon is happened in the real world for floating marine structure. The higher wave height means higher resistance exerted by fluid to the body [2]. In the far from the body, reflected wave from the walls tank is

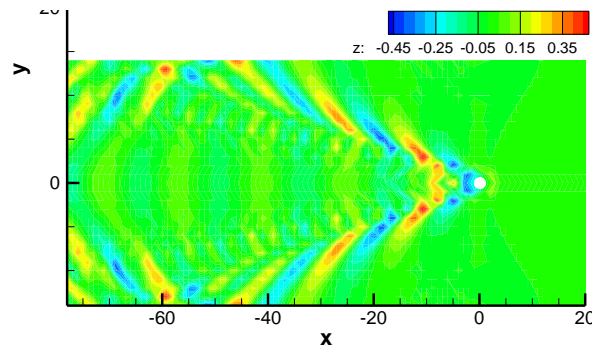


Figure 5. Reflected wave pattern from the walls tank with 20 m height and 30 m width

seen. Figure 5 shows reflected wave pattern due to the hemisphere with 5 m/s. it can be studied for different velocity.

5. Conclusion

In ocean engineering, determining resistance of floating marine structure is principal to gain the optimum hull design. Physical towing tank have been used to approximate wave making resistance for optimization process but it is so expensive and time consuming. In this paper, potential numerical towing tank based on potential theory and boundary elements methods is proposed instead of physical one. Direct boundary elements method is employed to solve boundary value problem. For BEM implementation, evaluating singular integral due to Green function is the mine difficulty. The new technique for flat triangle elements is presented for kernel surface integral. To verify the numerical results, surge added mass of a unit hemisphere is compared with analytical solution and in the following wave pattern for different velocity is illustrated to examine solver reliability. In general, presented procedure can be developed to use in applied marine hydrodynamics.

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