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Investigation of the Dynamic Behavior of Periodic Systems with Newton Harmonic Balance Method

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Abstract
In this paper, Newton Harmonic Balancing Method (NHBM) is applied to scrutinize free vibration analysis of the nonlinear oscillatory systems. This method is combined by the Harmonic Balance and Newton's methods. Two classical cases are used to illustrate the applicable of NHBM and results compared by other analytical methods and ODE solver built in MATLAB. The results of the NHBM are shown that the solution quickly convergent and does not need to complicated calculations. So it is applied for various problems in engineering specially vibration equations.

Keyword
Newton Harmonic Balance Method, Nonlinear vibration, Oscillatory system, high accuracy

1. Introduction
Nonlinear oscillations are important issue in physical science, mechanical structures and other engineering problems. The fluctuation, stability [1, 2] and natural frequencies are basic items in oscillatory systems. So, investigating about the influence of various parameters of these items is important in the design step.
Mainly nonlinear vibration of oscillation systems are modeled by nonlinear differential equations. Obtaining exact solution for these nonlinear problems is difficult and time consuming, thus scientist tried to find new approaches for overcome it. Recently, many authors used various analytical methods for solving nonlinear equations in mechanical systems. Some kind of these methods like Homotopy Perturbation Method (HPM), Homotopy Analysis Method (HAM) and Variational Iteration Method (VIM) are powerful methods and can used for almost all types of nonlinear equations [3-14]. Some other methods like Frequency Amplitude Formulation (FAF), Max-Min Approach (MMA), Energy Balance Method (EBM), Harmonic Balance Method (HBM) and Newton Harmonic Balance Method

(NHBM) introduced for oscillation systems [15-23]. In these types of methods with obtaining the motion frequency and having the initial conditions the result achieved.
NBHM incorporates of both Newton's Method and Harmonic Balance Method. Wu et al. [21] introduced this method and applied it on three examples. Lai et al. [22] analyzed first, second and third orders analytical approximation for second order differential equation with cubic quantic nonlinearities. Also, they compared various order of obtained frequency with exact frequency.
In current research, NBHM is applied on three cases of nonlinear oscillatory systems and first and second orders approximation of this method is investigated. Results obtained by NHBM compared with ODE solver built in MATLAB. Also, the influence of the initial amplitude studied on the system response and stability.

2. Analysis, solution procedure, results and discussion of cases

The application of Newton Harmonic Balance Method (NHBM) in mechanical structures especially oscillation systems investigated on three nonlinear vibration problems.

2.1 case 1

Consider the motion equation of special Duffing-harmonic oscillator as follows [18]:

$$\ddot{u}+\frac{u^3}{1+u^2}=0 \qquad \rightarrow \qquad \ddot{u}(1+u^2)+u^3=0 \tag{1}$$

Under the transformation $\tau=\omega t$, the Eq. (1) can be written as:

$$\omega^2 u''(\tau)(1+u^2(\tau))+u^3(\tau)=0 \tag{2}$$

Where ω is angular frequency and prime denotes differentiation with respect to τ .
Also, initial condition is:

$$u(0)=A \qquad u'(0)=0 \tag{3}$$

Where A denotes the maximum amplitude.

With second order approximation, $u(\tau)$ and ω^2 may be extend as follows [21, 22]:

$$u(\tau)=u_1(\tau)+\Delta u_1(\tau) \tag{4}$$

$$\omega^2=\omega_1^2+\Delta\omega_1^2 \tag{5}$$

Substituting Eq. (4) and Eq. (5) into Eq. (3), we have:

$$(\omega_1^2+\Delta\omega_1^2)(u_1''+\Delta u_1'')(1+(u_1+\Delta u_1)^2)+(u_1+\Delta u_1)^3=0 \tag{6}$$

According to initial conditions and for first order approximation, we set:

$$u_1(\tau)=A\cos\tau, \qquad \Delta u_1=\Delta u''=\Delta\omega_1^2=0 \tag{7}$$

With substitute Eq. (7) into Eq. (6) and avoiding the presence of secular terms, the angular frequency for first order approximation obtained and written as follows:

$$-\omega_1^2+\frac{3}{4}A^2-\frac{3}{4}A^2\omega_1^2=0 \qquad \Rightarrow \qquad \omega_1=\sqrt{\frac{3A^2}{4+3A^2}} \tag{8}$$

For the second analytical approximation, we set:

$$\Delta u_1=c(\cos\tau-\cos 3\tau) \tag{9}$$

Substituting Eq. (9) into Eq. (6) and expanding the achieved expression in a trigonometric series and then putting the coefficients of $\cos\tau$ and $\cos 3\tau$ equal to zero, Eq. (10) and Eq. (11) obtained.

$$(-3A^3+4A)\Delta\omega_1^2+c(-6A^2+4\omega_1^2-2A^2\omega_1^2)+4A\omega_1^2-3A^3+3A^3\omega_1^2=0 \tag{10}$$

$$A^3\Delta\omega_1^2 + c(3A^2 - 36\omega_1^2 - 19A^2\omega_1^2) + A^3\omega_1^2 - A^3 = 0 \quad (11)$$

Solving Eq. (10) and Eq. (11) simultaneously, it is obtained:

$$\Delta\omega_1^2 = -\frac{-124A^2\omega_1^2 - 70A^4\omega_1^2 + 188A^2\omega_1^4 + 55A^4\omega_1^4 + 144\omega_1^4 + 15A^4}{-15A^4 + 188A^2\omega_1^2 + 55A^4\omega_1^2 - 12A^2 + 144\omega_1^2} \quad (12)$$

$$c = -\frac{4A^3}{-15A^4 + 188A^2\omega_1^2 + 55A^4\omega_1^2 - 12A^2 + 144\omega_1^2} \quad (13)$$

From Eq. (4) and Eq. (5), and using second order analytical solution the angular frequency and the system displacement may be written as:

$$\omega = \sqrt{\frac{3A^2}{4+3A^2} - \frac{-124A^2\omega_1^2 - 70A^4\omega_1^2 + 188A^2\omega_1^4 + 55A^4\omega_1^4 + 144\omega_1^4 + 15A^4}{-15A^4 + 188A^2\omega_1^2 + 55A^4\omega_1^2 - 12A^2 + 144\omega_1^2}} \quad (14)$$

$$u(t) = (A + c)\cos\tau - (c)\cos 3\tau \quad (15)$$

Where c is evaluated from Eq. (13).

2.2 Result and discussion of this case

Motion equation of special Duffing-harmonic oscillator investigated and solved with NHBM. The frequency obtained by applying second order approximation of NHBM (Eq. (14)) compared with exact solution and other analytical solutions in table 1. Also, time histories of system displacement for two different initial amplitude illustrated using NHBM and time marching solution in Fig. 1. From this figure and table 1, NHBM has excellent agreement with other analytical or exact results and provides suitable approximating for this type of problems.

Fig. 2 shows the phase plane of the system which indicates displacement versus velocity and showing the system stability. In addition Fig. 3 shows the displacement behavior of the system versus time and initial amplitude.

Table1. Comparison between NHBM obtained frequency with frequencies obtained in other literatures.

A	He [24]	Mickens [25]	Tiwari et al. [26]	Turgut et al. [18]	Present study Eq. (14)	Exact solution [18]
0.1	0.0863	0.0844	0.0862	0.0862	0.0844	0.0844
1	0.6547	0.6464	0.6436	0.6516	0.6383	0.6368
10	0.9934	0.9931	0.9910	0.9931	0.9921	0.9909

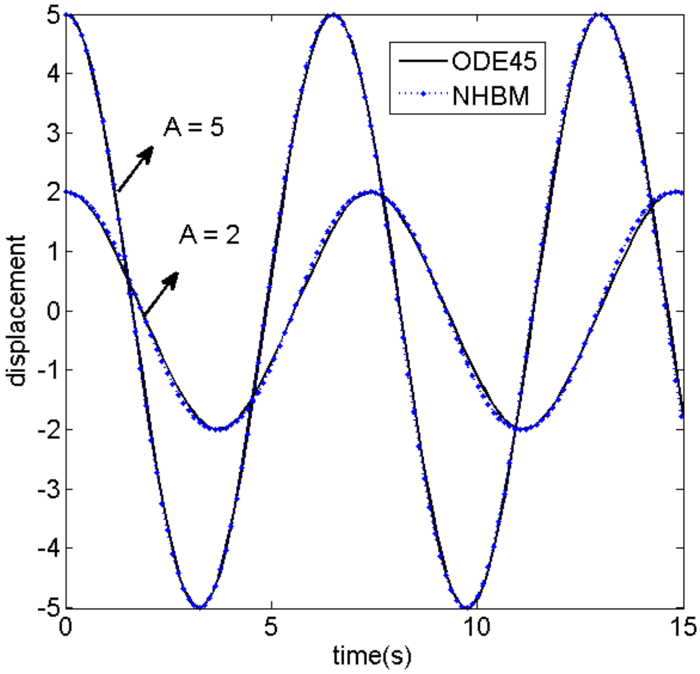

Fig. 1. System displacement for various *A*.

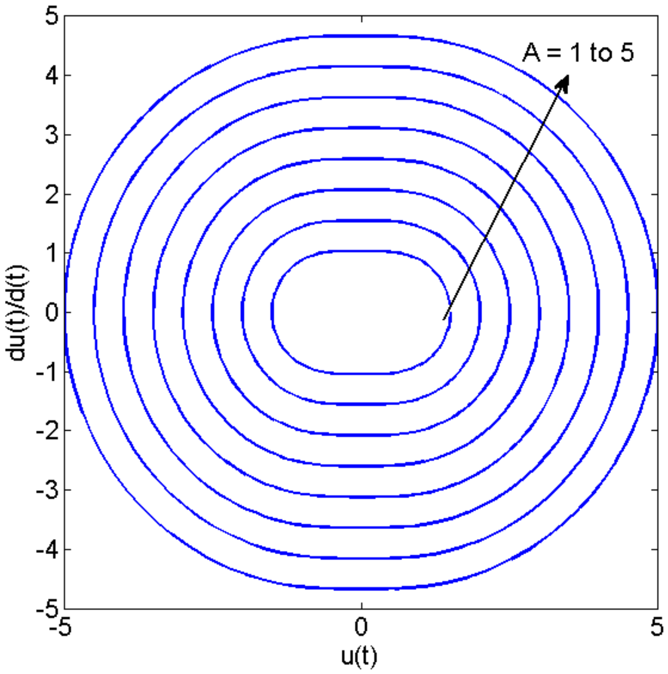

Fig. 2. Phase plan maps for showing the influence of *A* in the system stability.

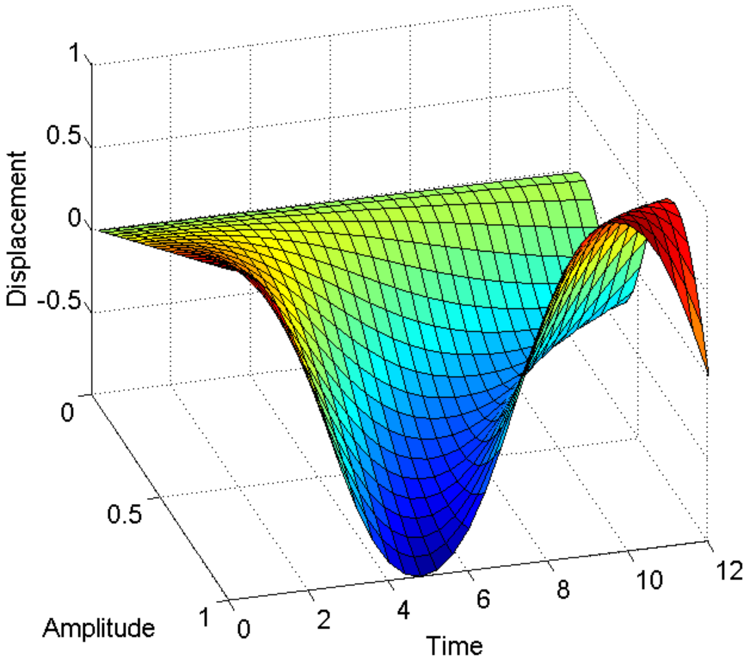

Fig. 3. Influence of initial amplitude on time histories of NHBM displacement.

2.3 case 2

Eq. (16) represents the mathematical pendulum without friction [23].

$$\ddot{u}+\Omega^2 \sin u = 0 \tag{16}$$

Substituting $\tau=\omega t$ and suitable approximation for $\sin(u)$, Eq. (16) changes to:

$$\omega^2 u'' + \Omega^2 (u - \frac{u^3}{3} + \frac{u^5}{120}) = 0 \tag{17}$$

Where initial conditions expressed as initial displacement, equal to maximum amplitude as follows:

$$u(0)=A \qquad u'(0)=0 \tag{18}$$

Substituting Eq. (4) and Eq. (5) into Eq. (17), the following expression obtained.

$$(\omega_1^2+\Delta\omega_1^2)(u_1''+\Delta u_1'')+\Omega^2(u_1+\Delta u_1)-\frac{\Omega^2}{6}(u_1+\Delta u_1)^3+\frac{\Omega^2}{120}(u_1+\Delta u_1)^5=0 \tag{19}$$

By linearizing Eq. (19) respect to Δu_1 and $\Delta\omega_1^2$ yield:

$$(\omega_1^2+\Delta\omega_1^2)u_1''+\omega_1^2\Delta u_1''+\Omega^2(u_1+\Delta u_1)-\frac{\Omega^2}{6}(u_1^3+3u_1^2\Delta u_1)+\frac{\Omega^2}{120}(u_1^5+5u_1^4\Delta u_1)=0 \tag{20}$$

Substituting Eq. (7) into Eq. (20) for first order approximation and avoiding the presence of secular terms the angular frequency obtained as follows:

$$\omega_1=\Omega\sqrt{(1-\frac{A^2}{8}+\frac{A^4}{192})} \tag{21}$$

Second order analytical approximation may be achieved by Substituting Eq. (9) into Eq. (20) and expanding the obtained expression in a trigonometric series and then putting the coefficients of $\cos\tau$ and $\cos 3\tau$ equal to zero, results achieved in a set of simultaneous equations in terms of $\Delta\omega_1^2$ and c as follows:

$$(-A)\Delta\omega_1^2+c(-\omega_1^2+\Omega^2+\frac{3}{2}\Omega^2A^2+\frac{25}{16}\Omega^2A^4)-A\omega_1^2+\Omega^2A+\frac{3}{4}\Omega^2A^3+\frac{5}{8}\Omega^2A^5=0 \tag{22}$$

$$c(9\omega_1^2 - \Omega^2 - \frac{3}{4}\Omega^2 A - \frac{5}{16}\Omega^2 A^4) + \frac{1}{4}\Omega^2 A^3 + \frac{5}{16}\Omega^2 A^5 = 0 \tag{23}$$

Solving Eq. (22) and Eq. (23) simultaneously, it is obtained:

$$\Delta\omega_1^2 = -\frac{3A^4\Omega^2(4+5A^2)^2}{16(128+96A^2+85A^4)} \tag{24}$$

$$c = -\frac{4A^3+5A^5}{128+96A^2+85A^4} \tag{25}$$

Second order analytical approximate frequency and system response using Eqs. (4), (5), (9), (24) and (25), can be written as:

$$\omega = \sqrt{\omega_1^2 + \Delta\omega_1^2} = \Omega\sqrt{(1 - \frac{A^2}{8} + \frac{A^4}{192}) - \frac{3A^4(4+5A^2)^2}{16(128+96A^2+85A^4)}} \tag{26}$$

$$u(t) = (A+c)\cos\omega t - (c)\cos 3\omega t \tag{27}$$

2.4 Results and discussion of case 3

The model of a simple mathematical pendulum equation with neglecting the friction investigated in previous section and solved analytically with NHBM. The frequency obtained with second order analytical approximation (Eq. (26)) compared with Max-Min approach, Variational Iteration Method, Energy Balance Method and Homotopy Perturbation Method in table 2.

Fig. 4 shows the schematic of Eq. (27) which solved by NHBM and compared with time marching solution for two initial amplitudes. This figure shows the accuracy and effectiveness of this method. The stability of the system investigates by illustrating phase plane of the system in Fig. 5. Also, the effect of initial amplitude on phase plane showed in this figure. Fig. 6 demonstrates the system response behavior in presence of time and initial amplitude.

Table 2. Comparison of the frequency obtained via NHBM with other analytical solution.

A	Max-Min [23]	VIM [27]	EBM [28]	HPM [28]	Present study
0.01	0.9999	0.9999	1.0000	1.0000	0.9999
0.1	0.9993	0.9993	0.9994	0.9994	0.9993
0.2	0.9975	0.9975	0.9975	0.9975	0.9975
0.4	0.9900	0.9900	0.9900	0.9900	0.9900
0.6	0.9775	0.9775	0.9774	0.9776	0.9775
0.8	0.9600	0.9602	0.9597	0.9603	0.9601
1	0.9377	0.9381	0.9367	0.9382	0.9378
1.5	0.8606	0.8632	0.8550	0.8632	0.8613

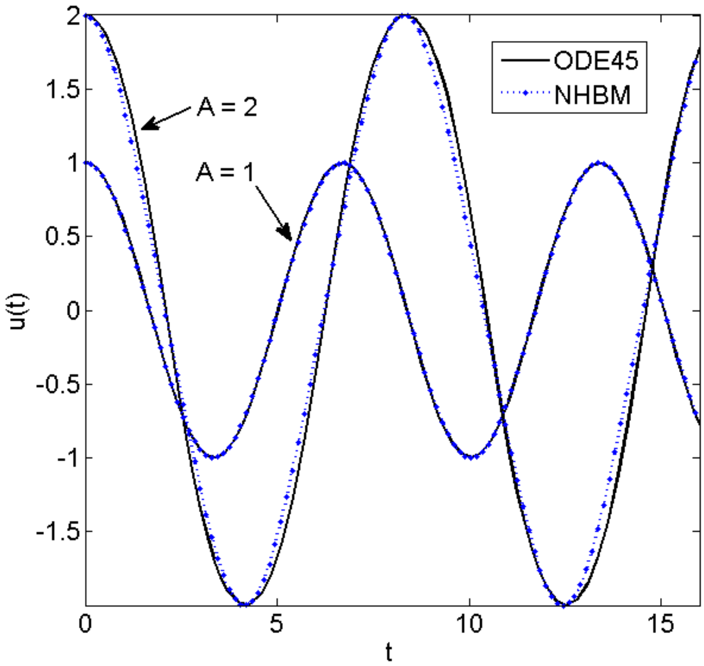

Fig. 4. Response of the system with NHBM and time marching solution for two different *A*.

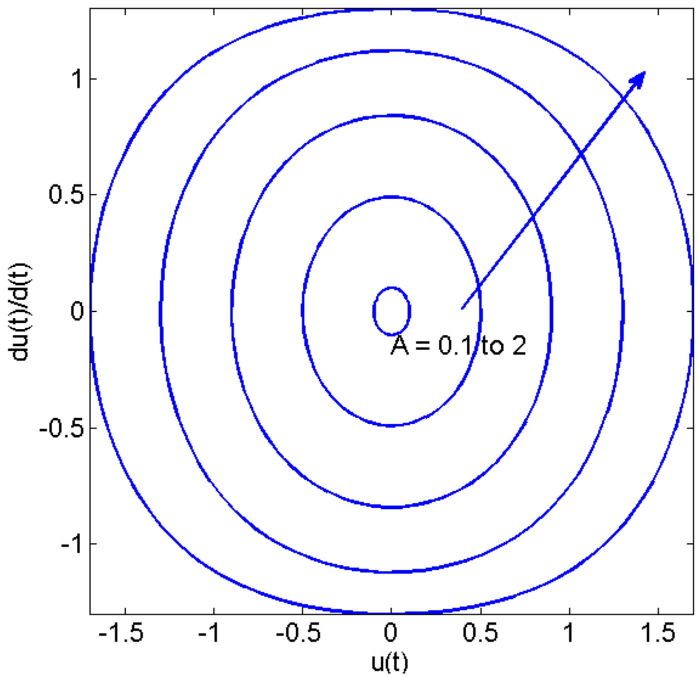

Fig. 5. Influence of changing *A* on phase plane of the system.

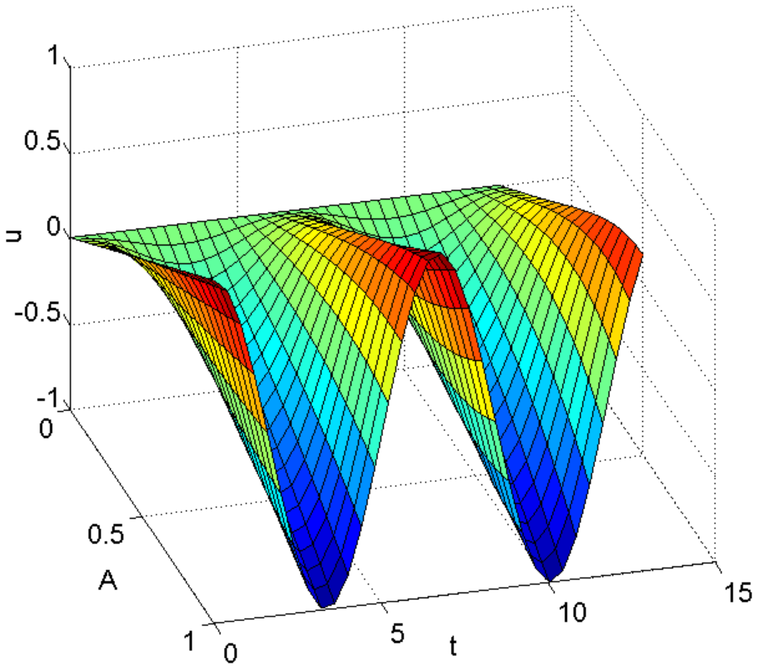

Fig. 6. Influence of initial amplitude on time histories of NHBM response.

3. Conclusion

In the present work, Newton Harmonic Balance Method applied to obtain analytical solution for nonlinear vibration in oscillatory systems. For this purpose, two problems with periodic behavior selected for investigating the effectiveness of this method. Results of the NHBM are compared with other analytical methods done in other literatures and time marching solutions. As indicated, the error of the studied systems is very worthless and the results confirmed the accuracy and the efficiently of the method. However, further research is needed to better understanding the effect of this method on engineering problems especially mechanical affairs.

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