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On BP-algebras and QS-algebras

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Abstract

In this paper we prove that the class of QS-algebras, p-semi simple algebras and BP-algebras are equivalent.

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1. Introduction.

In 1966, Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras [5, 6]. It is known that the class of BCK-algebras is a proper subclass of BCI-algebra. P-Semisimple algebras are another special class of BCI-algebras, which were introduced by T. D. Lei in 1982 ([11]). They play a basic role in the theory of BCI-algebras and have close contacts with abelian groups. Neggers, Ahn and Kim ([12]) introduced the notion of Q-algebras in 2001 and after that Ahn and Kim introduced the notion of QS-algebras which is a generalization of Q-algebras. In 2002, Neggers and Kim [14] introduce the notion of B-algebra and obtained several results. In 2006, G. B. Kim and H.S Kim ([8]) introduced the notion of a BM-algebra which is a specialization of B-algebra. The concept of a BP-algebra is introduced by S. S. Ahn and J. S. Han [9], which is another generalization of B-algebra.

In this paper, we prove that QS-algebras and BP-algebras are equal to P-Semisimple algebras and so they are equal together.

2. Preliminaries.

Definition2.1. [8] A BM-algebra is a non-empty set X with a constant 0 and a binary operation " * " satisfying the following axioms:
(A1) $x*0=x$,
(A2) $(z*x)*(z*y)=y*x$,
for all $x,y,z\in X$.

Definition2.2. [17] Let X be a set with a binary operation * and a constant 0. Then $(X,*,0)$ is called BCI- algebra if satisfies the following conditions:
(BCI-1) $((x*y)*(x*z))*(z*y)=0$,
(BCI-2) $(x*(x*y))*y=0$,
(BCI-3) $x*x=0$,
(BCI-4) $x*y=0$ and $y*x=0$ imply $x=y$,
for all $x,y,z\in X$.

Definition2.3.[17] A BCI-algebra X is called p-semi simple-algebra if $0*(0*x)=x$, for all $x\in X$.

Theorem 2.4. [17] Let X be a BCI-algebra. Then the following hold:
(i) $x*0=x$,
(ii) (ii) $(x*y)*z=(x*z)*y$,
for all $x,y,z\in X$.

Theorem 2.5. [17] Let X be a BCI-algebra. Then the following are equivalent:

(i) X is a p-semi simple algebra,
(ii) every element of X is minimal,
(iii) $X=\{0*x \mid x\in X\}$.

We note that an element $x\in X$ is called minimal, if $y*x=0$, implies $y=x$.

Definition 2.6.[1] Let X be a set with a binary operation * and a constant 0. Then (X,*,0) is called a BP-algebra if satisfies (A2) and the following conditions:

(BP1) $x*(x*y)=y$,
(BP2) $x*x=0$.

X is called QS-algebra, if satisfies A1, A2, BP2 and

(Q) $(x*y)*z=(x*z)*y$,
for any $x,y,z\in X$.

Theorem 2.7.[1] Let X be a BP-algebra. Then $x*0=x$, for all $x\in X$.

Definition 2.8. [9] A BO-algebra is an algebra (X,*,0) of type (2,0) satisfying A1, BP2 and

(BO) $x*(y*z)=(x*y)*(0*z)$,

for all $x,y,z\in X$.

Theorem 2.9. [9] BO-algebras, BM-algebras, P-semi simple algebras are equivalent and they are logically equivalent by abelian group.

3. QS-algebras, BP-algebras and P-semi simple algebras

Theorem 3.1. Let X be a set with a binary operation * and a constant 0. Then X is a p-semi simple-algebra if and only if satisfies in the following conditions:

(i) $(x*y)*z=(x*z)*y$,
(ii) $x*y=0$ iff $x=y$,
for all $x,y,z\in X$.

Proof. Let X be a p-semi simple-algebra. Then by Theorem 2.5(ii), $x*y=0$ implies $x=y$ and by BCI-3, $x*x=0$. Hence (ii) is hold. By Theorem 2.4(ii), (i) is hold.

Conversely, let (i) and (ii) are hold. By (ii), (BCI-4) is hold. Also by (ii), $x*x=0$ and (BCI-3) is hold.

By $x*x=0$ and (i) we have $(x*0)*x=(x*x)*0=0$. Now, by (ii) and $x*(x*y))*y=(x*y)*(x*y)*y=0$, we conclude that

$$x*(x*y)=y. \quad (1)$$

By (1), (ii) and (i) we have

$$((x*y)*(x*z))*(z*y)=((x*(x*z))*y)*(z*y)=(z*y)*(z*y)=0.$$

Thus, X is a BCI-algebra. By (ii), every element of X is minimal. By Theorem 2.5, X is a p-semi simple-algebra.

Corollary 3.2. Let X be a p-semi simple-algebra. Then the following hold:

(i) $(x*y)*(x*z)=z*y$,
(ii) $x*(x*y)=y$,
for all $x,y,z\in X$.

Proof. By Theorem 3.1 and definition of p-semi simple algebra the proof is clear.

Theorem3.3. X is a BP-algebra if and only if X is a p-semi simple-algebra.

Proof. Let X be a p-semi simple-algebra. By Theorem 3.2(i), (ii) and BCI-2, X is a BP-algebra.

Conversely, let X be a BP-algebra and $x*y=0$, for some $x,y\in X$. Then by BP1 and Theorem 2.7

$$x=x*0=x*(x*y)=y.$$

Thus by BP2, $x*y=0$ if and only if $x=y$.

Let $x*y=z$, for some $x,y,z\in X$. Then by BP1, $x*z=x*(x*y)=y$. Thus

$$x*y=z \text{ if and only if } x*z=y. \quad (2)$$

Let $(x*y)*z=t$ and $x*y=a$. By (2), $x*a=y$. Thus by A2,

$$(x*z)*y=(x*z)*(x*a)=a*z=(x*y)*z.$$

By Theorem 3.1, X is a p-semi simple-algebra.

Theorem 3.4. X is a QS-algebra if and only if X is a p-semi simple-algebra.

Proof. Let X be a p-semi simple-algebra. Then by Theorem 3.2, X is a QS-algebra.

Conversely, let X be a QS-algebra. Then by A1 and A2,

$$x*(x*y)=(x*0)*(x*y)=y*0=y.$$

Thus X is a BP-algebra. By Theorem 3.3, X is a p-semi simple-algebra.

Corollary 3.5. BM-algebras, BP-algebras, BO-algebras, QS-algebras and p-semisimple algebra are equivalent and they are logically equivalent by abelian group.

Proof. By Theorems 3.3, 3.4 and Theorem 2.9 the proof is clear.

4. Conclusion.

The concept of a BP-algebra introduced by S. S. Ahn and J. S. Han [8], which is another generalization of B-algebra. In this paper we show that this algebraic structure is equivalent to p-semisimple BCI-algebras.

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