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Reliability estimation under the fuzzy environments

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Abstract

This paper proposes new methods for fuzzy reliability estimation where lifetime random variables have a distribution function with fuzzy parameter. First, a fuzzy estimation is constructed for the parameter. Then, using such a fuzzy estimation, a fuzzy reliability estimation is constructed. In first method, we used Buckley method with precise observation. In secound one we uesd fuzzy point estimation with fuzzy observation. In third method, using a fuzzy estimation of mean time to failure (MTTF), we constructed a fuzzy confidence bound and a fuzzy confidence band for reliability function for a given t_0 and for a given α_0 respectively. This method has been used in lifetime distributions as fuzzy normal distribution, fuzzy exponential distribution and fuzzy Weibull distribution.

Keywords: Fuzzy reliability function; Fuzzy estimation; Fuzzy confidence bound; Fuzzy confidence band.

2010 Mathematics Subject Classification: 62N86; 62P30.

1. Introduction.

One of the important engineering tasks in design and development of a technical system is reliable engineering. It is well known that the conventional reliability analysis, using the probabilities, has been found to be inadequate to handle uncertainty of failure data and modeling.
To overcome this problem, the concept of fuzzy approach has been used in the evaluation of the reliability of a system (Amit Kumar et al., 2006).
The theory of fuzzy reliability was proposed and development by several authors, Cai, Wen and Zhang (1991, 1993); Cai (1996); Chen, Mon (1993); Hammer (2001); El-Hawary (2000), Onisawa and Kacprzyk (1995); Utkin and Gurov (1995) (Aliev and Kara, 2004). Kang and Cho (1997) discussed bootstrap confidence intervals for reliability function. Zardasht et al. (2012), considers the properties of a nonparametric estimator developed for a reliability function which is used in many reliability problems. Aliev and Kara (2004) considered fuzzy system reliability analysis using

time dependent fuzzy set and the concept of alpha-cut. Hsien-Chung Wu (2004) estimated fuzzy reliability using Bayesian approach. He also provides the computational procedures to evaluate the membership degree of any given Bayes point estimate of reliability. Yao et al. (2008) applied a statistical methodology in fuzzy system reliability analysis and got a fuzzy estimation of reliability. Rafael Gouriveau et al. (2008) presented a fuzzy approach of online reliability modeling and estimation. Zuhair (2009) provides estimation of reliability of a component subjected to life testing and the procedure includes essentially polling of two samples of failure data, where the component follows exponential failure model. Baloui Jamkhaneh (2011) evaluated reliability function using fuzzy exponential lifetime distribution. In this paper we propose a procedure to estimation the fuzzy reliability function, when the parameters are fuzzy. For this propose, we are presented the definitions of distribution function, and the probability density function with fuzzy parameter and its fuzzy reliability function. The distribution function with fuzzy parameter of a random variable is defined as follows:

$$F(x,\tilde{\theta})=\{F(x)[\alpha],\mu_{F(x)}\,\big|\,F(x)[\alpha]=[F_{\min}(x)[\alpha],F_{\max}(x)[\alpha]],\mu_{F(x)}=\alpha\},$$

$$F_{\min}(x)[\alpha]=\inf\{F(x,\theta)[\alpha]\,\big|\,\theta\in\tilde{\theta}[\alpha]\},$$

$$F_{\max}(x)[\alpha]=\sup\{F(x,\theta)[\alpha]\,\big|\,\theta\in\tilde{\theta}[\alpha]\}.$$

Right graph in Fig. 1 is a fuzzy distribution function with $\alpha=0$ (the dash line labeled $\mu=0$), and $\alpha=1$ (the solid line labeled $\mu=1$). Left graph in Fig. 1 is a fuzzy distribution function for a given x_j.

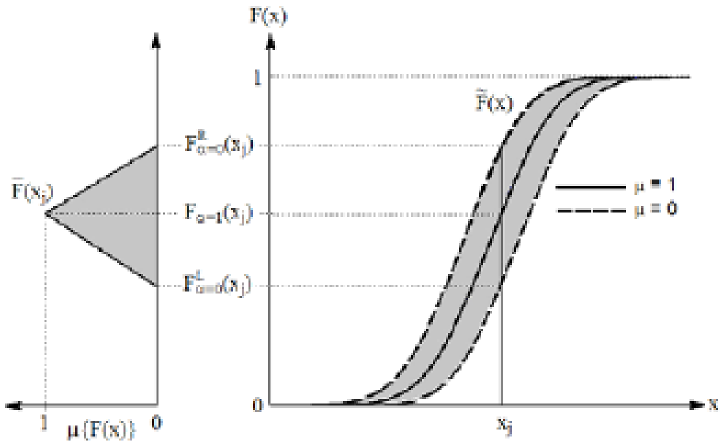

Fig .1 $\alpha-$cut of fuzzy distribution function ($\alpha=0$, 1)

The probability density function with fuzzy parameter is defined as follows:

$$f(x,\tilde{\theta})=\{f(x)[\alpha],\mu_{f(x)}\,\big|\,f(x)[\alpha]=[f_{\min}(x)[\alpha],f_{\max}(x)[\alpha]],\mu_{f(x)}=\alpha\},$$

$$f_{\min}(x)[\alpha]=\inf\{f(x,\theta)[\alpha]\,\big|\,\theta\in\tilde{\theta}[\alpha]\},$$

$$f_{\max}(x)[\alpha]=\sup\{f(x,\theta)[\alpha]\,\big|\,\theta\in\tilde{\theta}[\alpha]\}.$$

Let the lifetime random variable(RV) is a fuzzy random variable with probability density function $f(x,\tilde{\theta})$, then fuzzy reliability function is defined as follow:

$$S(t,\tilde{\theta})=\{S(t)[\alpha],\mu_{S(t)}\,\big|\,S(t)[\alpha]=[S_{\min}(t)[\alpha],S_{\max}(t)[\alpha]],\mu_{S(t)}=\alpha\},$$

$$S_{\min}(t)[\alpha] = 1 - \sup\{F(t,\theta)[\alpha] \,|\, \theta \in \tilde{\theta}[\alpha]\},$$

$$S_{\max}(t)[\alpha] = 1 - \inf\{F(t,\theta)[\alpha] \,|\, \theta \in \tilde{\theta}[\alpha]\}.$$

In this case, for any fixed α and arbitrary t fuzzy reliability curve is like a decreasing band. Let $\tilde{\theta}$ is a triangular fuzzy number $(\theta_1,\theta_2,\theta_3)$, and then any given t_0; fuzzy reliability is a fuzzy number $(1-F(t_0,\theta_3),1-F(t_0,\theta_2),1-F(t_0,\theta_1))$ and membership function of $\tilde{S}(t_0)$ is as follow:

$$\mu_{\tilde{S}(t_0)}(x) = \begin{cases} \dfrac{x-1+F(t_0,\theta_3)}{F(t_0,\theta_3)-F(t_0,\theta_2)} & , \quad 1-F(t_0,\theta_3) \le x < 1-F(t_0,\theta_2) \\ \dfrac{1-F(t_0,\theta_1)-x}{F(t_0,\theta_2)-F(t_0,\theta_1)} & , \quad 1-F(t_0,\theta_2) \le x < 1-F(t_0,\theta_1) \\ 0 & , \quad not \end{cases}$$

for a given t, fuzzy reliability $S(t,\tilde{\theta})$ will be an unknown parameter if $\tilde{\theta}$ is unknown. We can estimate $S(t,\tilde{\theta})$ by using estimation of $\tilde{\theta}$.

2. Estimate of the fuzzy reliability function with precise observations

Let $X_1,X_2,\dots,X_n$ be a random sample from a lifetime distribution with probability density function $f(x,\tilde{\theta})$. Assume that θ is an unknown parameter and it must be estimated. Based on these observations and using estimator of $U = u(X_1,X_2,\dots,X_n)$, one can obtain $(1-\gamma)100\%$ confidence interval with using the usual statistical methods as $[\hat{\theta}^L[\gamma],\hat{\theta}^U[\gamma]]$. Now place these confidence intervals one undergo another one, respectively from $\gamma = 0.01$ to $\gamma = 1$. Also $\hat{\tilde{\theta}}[\gamma] = [\hat{\theta}^L[0.01],\hat{\theta}^U[0.01]]$ from $\gamma = 0$ to $\gamma = 0.01$ in order to produce a fuzzy number $\hat{\tilde{\theta}}$ whose γ-cuts ($\hat{\tilde{\theta}}[\gamma] = [\hat{\theta}^L[\gamma],\hat{\theta}^U[\gamma]]$) are the confidence intervals. We estimated fuzzy reliability function with replace fuzzy estimation of $\hat{\tilde{\theta}}$ instead of $\tilde{\theta}$ in $S(t,\tilde{\theta})$. It is as $\hat{S}(t,\tilde{\theta}) = S(t,\hat{\tilde{\theta}})$

Example2.1. Let lifetime random variable have exponential distribution with fuzzy parameter $\tilde{\lambda}[\gamma] = [\lambda^L[\gamma],\lambda^U[\gamma]]$. Then we have

$$F(x,\tilde{\lambda})[\gamma] = [F_{\min}(x)[\gamma],F_{\max}(x)[\gamma]] = [1-e^{-\lambda^L[\gamma]x},1-e^{-\lambda^U[\gamma]x}],$$

so, $\gamma-$ cut of reliability function is as follow:

$$S(t,\tilde{\lambda})[\gamma] = [e^{-\lambda^U[\gamma]t},e^{-\lambda^L[\gamma]t}].$$

Let X_i $i = 1,2,\dots,n$ is a random sample from lifetime variable. Then $(1-\gamma)100\%$ confidence interval for λ is as follow:

$$\left(\frac{\chi^2_{\frac{\gamma}{2},2n}}{2\sum_{i=1}^n X_i},\frac{\chi^2_{1-\frac{\gamma}{2},2n}}{2\sum_{i=1}^n X_i}\right),$$

so, γ –cuts set of fuzzy estimator $\hat{\tilde{\lambda}}$ is as follow:

$$\hat{\tilde{\lambda}}[\gamma]=[\hat{\lambda}^L[\gamma],\hat{\lambda}^U[\gamma]]=[\frac{\chi^2_{\frac{\gamma}{2},2n}}{2\sum_{i=1}^{n}X_i},\frac{\chi^2_{1-\frac{\gamma}{2},2n}}{2\sum_{i=1}^{n}X_i}],$$

in this case, γ –cuts set of fuzzy reliability estimator is as follow:

$$\hat{S}(t,\tilde{\lambda})[\gamma]=[e^{-\hat{\lambda}^U[\gamma]t},e^{-\hat{\lambda}^L[\gamma]t}]=[\exp\{-\frac{t\chi^2_{1-\frac{\gamma}{2},2n}}{2\sum_{i=1}^{n}X_i}\},\exp\{-\frac{t\chi^2_{\frac{\gamma}{2},2n}}{2\sum_{i=1}^{n}X_i}\}].$$

Example2.2. Let lifetime RV have normal distribution with fuzzy parameter $\tilde{\mu}[\gamma]=[\mu^L[\gamma],\mu^U[\gamma]]$. Then we have

$$F(x,\tilde{\mu})[\gamma]=[F_{\min}(x)[\gamma],F_{\max}(x)[\gamma]]=[\Phi(\frac{x-\mu^U[\gamma]}{\sigma}),\Phi(\frac{x-\mu^L[\gamma]}{\sigma})],$$

so, $\gamma-$ cut of reliability function is as follow:

$$S(t,\tilde{\mu})[\gamma]=[1-\Phi(\frac{t-\mu^L[\gamma]}{\sigma}),1-\Phi(\frac{t-\mu^U[\gamma]}{\sigma})].$$

Let X_i , $i=1,2,\ldots,n$ is a random sample from lifetime variable. Then $(1-\gamma)100\%$ confidence interval for μ is as follows:

$$(\bar{X}-\frac{\sigma}{\sqrt{n}}Z_{1-\frac{\gamma}{2}},\bar{X}+\frac{\sigma}{\sqrt{n}}Z_{1-\frac{\gamma}{2}}),$$

so, γ –cuts set of fuzzy estimator $\hat{\tilde{\mu}}$ is as follow:

$$\hat{\tilde{\mu}}[\gamma]=[\hat{\mu}^L[\gamma],\hat{\mu}^U[\gamma]]=[\bar{X}-\frac{\sigma}{\sqrt{n}}Z_{1-\frac{\gamma}{2}},\bar{X}+\frac{\sigma}{\sqrt{n}}Z_{1-\frac{\gamma}{2}}],$$

consequently, γ –cuts set of fuzzy reliability estimator is as follow:

$$\hat{S}(t,\tilde{\mu})[\gamma]=[1-\Phi(\frac{t-\bar{X}+\frac{\sigma}{\sqrt{n}}Z_{1-\frac{\gamma}{2}}}{\sigma}),1-\Phi(\frac{t-\bar{X}-\frac{\sigma}{\sqrt{n}}Z_{1-\frac{\gamma}{2}}}{\sigma})].$$

3. Estimate of the fuzzy reliability function with fuzzy observations

Let $X_1,X_2,\ldots,X_n$ be a random sample with fuzzy observation $\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n$ from a lifetime distribution with probability density function $f(x,\tilde{\theta})$. Assume that θ is an unknown parameter and it must be estimated. Based on these fuzzy observations and using estimator of $U=u(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)$, one can obtain fuzzy estimation of $\tilde{\theta}$ with using the usual statistical methods. $\gamma-$cut set of fuzzy estimation of $\tilde{\theta}$ is as $\hat{\tilde{\theta}}[\gamma]=[\hat{\theta}^L[\gamma],\hat{\theta}^U[\gamma]]$, $0<\gamma\le 1$. We estimate fuzzy reliability function with replacing the fuzzy estimation of $\hat{\tilde{\theta}}$ instead of $\tilde{\theta}$ in $S(t,\tilde{\theta})$. It is as $\hat{S}(t,\tilde{\theta})=S(t,\hat{\tilde{\theta}})$. Let (x_{i1},x_{i2},x_{i3}) , $i=1,2,\ldots,n$ is a random sample from lifetime RV of Example2.1. Since maximum likelihood estimation of λ is $\frac{1}{\bar{X}}$ in exponential distribution. Then maximum likelihood estimation of $\tilde{\lambda}$ is defined $\frac{1}{\bar{\tilde{X}}}$ in fuzzy exponential distribution. Its γ −cuts set of fuzzy estimator $\tilde{\lambda}$ is as follow:

$$\hat{\tilde{\lambda}}[\gamma]=[\frac{1}{\bar{\tilde{X}}^{U}[\gamma]},\frac{1}{\bar{\tilde{X}}^{L}[\gamma]}],$$

in this case, γ −cuts set of fuzzy reliability estimator is as follow:

$$\hat{S}(t,\tilde{\lambda})[\gamma]=S(t,\hat{\tilde{\lambda}})[\gamma]=\hat{S}(t,\frac{1}{\bar{\tilde{X}}})[\gamma]=[e^{-\frac{t}{\bar{\tilde{X}}^{L}[\gamma]}},e^{-\frac{t}{\bar{\tilde{X}}^{U}[\gamma]}}].$$

Let (x_{i1},x_{i2},x_{i3}) , $i=1,2,\ldots,n$ is a random sample from lifetime RV of Example2.2. Since maximum likelihood estimation of μ is $\bar{X}$ in normal distribution. Then maximum likelihood estimation of $\tilde{\mu}$ is defined $\bar{\tilde{X}}$ in fuzzy normal distribution. Its γ −cuts set of fuzzy estimator $\hat{\mu}$ is as follow:

$$\hat{\tilde{\mu}}[\gamma]=[\bar{\tilde{X}}^{L}[\gamma],\bar{\tilde{X}}^{U}[\gamma]],$$

consequently, γ −cuts set of fuzzy reliability estimator is as follow:

$$\hat{S}(t,\tilde{\mu})[\gamma]=[1-\Phi(\frac{t-\bar{\tilde{X}}^{L}[\gamma]}{\sigma}),1-\Phi(\frac{t-\bar{\tilde{X}}^{U}[\gamma]}{\sigma})].$$

4. Estimate of the fuzzy reliability function using fuzzy bound of MTTF (known variance)

Let $R_i, i=1,2,\ldots,n$ are lifetime RV related to sub-system of P_i, $i=1,2,\ldots,n$. First select random samples with size n_i from any sub-system P_i . We shows observation as $\tilde{r}_{ij}$, $j=1,\ldots n_i$, $\tilde{r}_{ij}=(r_{ij}^{(1)},r_{ij}^{(2)},r_{ij}^{(3)})$. Fuzzy point estimation of MTTF and its membership function is:

$$\bar{\tilde{r}}_i=(\bar{r}_i^{(1)},\bar{r}_i^{(2)},\bar{r}_i^{(3)})=(\frac{1}{n_i}\sum_{j=1}^{n_i}r_{ij}^{(1)},\frac{1}{n_i}\sum_{j=1}^{n_i}r_{ij}^{(2)},\frac{1}{n_i}\sum_{j=1}^{n_i}r_{ij}^{(3)}),$$

$$\mu_{\bar{\tilde{r}}_i}(x)=\begin{cases}\frac{x-\bar{r}_i^{(1)}}{\bar{r}_i^{(2)}-\bar{r}_i^{(1)}} & , \quad \bar{r}_i^{(1)}\le x\le \bar{r}_i^{(2)}\\ \frac{\bar{r}_i^{(3)}-x}{\bar{r}_i^{(3)}-\bar{r}_i^{(2)}} & , \quad \bar{r}_i^{(2)}\le x\le \bar{r}_i^{(3)}\\ 0 & , \quad not\end{cases}$$

$$\bar{\tilde{r}}_i[\alpha]=[\bar{r}_i^{L}[\alpha],\ \bar{r}_i^{U}[\alpha]]=[\bar{r}_i^{(1)}+\alpha(\bar{r}_i^{(2)}-\bar{r}_i^{(1)}),\bar{r}_i^{(3)}-\alpha(\bar{r}_i^{(3)}-\bar{r}_i^{(2)})].$$

In this case $(1-\gamma)100\%$ fuzzy confidence bound of MTTF is defined as follow:

$$\tilde{\mu}_i^{*L}\prec\tilde{\mu}_i\prec\tilde{\mu}_i^{*U},$$

where $\tilde{\mu}_i^{*L}$ and $\tilde{\mu}_i^{*U}$ are fuzzy number with $\alpha-$cut as follow:

$$\tilde{\mu}_i^{*L}[\alpha]=[\mu_i^{*LL}[\alpha],\mu_i^{*LU}[\alpha]]=[\bar{r}_i^{(1)}+\alpha(\bar{r}_i^{(2)}-\bar{r}_i^{(1)})-\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(3)}-\alpha(\bar{r}_i^{(3)}-\bar{r}_i^{(2)})-\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}}],$$

$$\tilde{\mu}_i^{*L}[\alpha]=\cup_{0\le\alpha\le1}[\mu_i^{*LL}[\alpha],\mu_i^{*LU}[\alpha]] \quad , \quad i=1,2,\dots,n,$$

triangular fuzzy number of $\tilde{\mu}_i^{*L}$ can be showed as

$$(\bar{r}_i^{(1)}-\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(2)}-\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(3)}-\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}}),$$

also $\alpha-$cut set of $\tilde{\mu}_i^{*U}$ is as follow:

$$\tilde{\mu}_i^{*U}[\alpha]=[\mu_i^{*UL}[\alpha],\mu_i^{*UU}[\alpha]]=[\bar{r}_i^{(1)}+\alpha(\bar{r}_i^{(2)}-\bar{r}_i^{(1)})+\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(3)}-\alpha(\bar{r}_i^{(3)}-\bar{r}_i^{(2)})+\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}}],$$

$$\tilde{\mu}_i^{*U}[\alpha]=\cup_{0\le\alpha\le1}[\mu_i^{*UL}[\alpha],\mu_i^{*UU}[\alpha]] \quad , \quad i=1,2,\dots,n,$$

triangular fuzzy number of $\tilde{\mu}_i^{*U}$ can be showed as

$$(\bar{r}_i^{(1)}+\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(2)}+\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(3)}+\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}}),$$

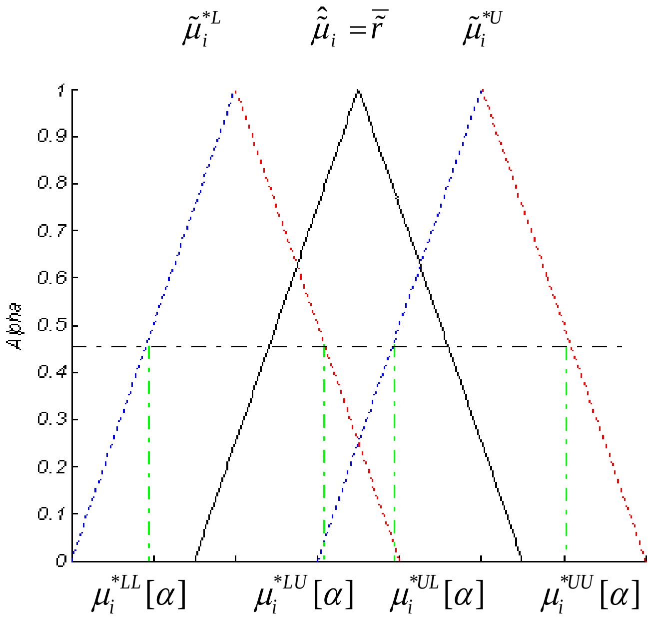

Fig .2 fuzzy confidences bound of MTTF

fuzzy confidences bound is equivalent nonfuzzy confidence intervals of $\mu_i^L[\alpha]$ and $\mu_i^U[\alpha]$ as follow:

$$\mu_i^L[\alpha]\in(\mu_i^{*LL}[\alpha],\mu_i^{*UL}[\alpha])=(\bar{r}_i^{(1)}+\alpha(\bar{r}_i^{(2)}-\bar{r}_i^{(1)})-\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(1)}+\alpha(\bar{r}_i^{(2)}-\bar{r}_i^{(1)})+\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}}),$$

$$\mu_i^U[\alpha]\in(\mu_i^{*LU}[\alpha],\mu_i^{*UU}[\alpha])=(\bar{r}_i^{(3)}-\alpha(\bar{r}_i^{(3)}-\bar{r}_i^{(2)})-\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}},\bar{r}_i^{(3)}-\alpha(\bar{r}_i^{(3)}-\bar{r}_i^{(2)})+\frac{\sigma}{\sqrt{n_i}}z_{1-\frac{\gamma}{2}}).$$

Fuzzy reliability function $\tilde{S}(t,\tilde{\mu}_i)$ can estimited by using fuzzy confidences bound of MTTF as

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})\prec\tilde{S}(t,\tilde{\mu}_i)\prec\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U}),$$

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})[\alpha]=\tilde{S}(t,\tilde{\mu}_i^{*L})[\alpha]=[S(t,\mu_i^{*LL}[\alpha])[\alpha],S(t,\mu_i^{*LU}[\alpha])[\alpha]],$$

$$\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U})[\alpha]=\tilde{S}(t,\tilde{\mu}_i^{*U})[\alpha]=[S(t,\mu_i^{*UL}[\alpha])[\alpha],S(t,\mu_i^{*UU}[\alpha])[\alpha]].$$

In this method, for a given t_0, $(\tilde{S}^{*L}(t_0,\tilde{\mu}_i^{*L}),\tilde{S}^{*U}(t_0,\tilde{\mu}_i^{*U}))$ is a fuzzy estimate of $\tilde{S}(t_0,\tilde{\mu}_i)$ that called $(1-\gamma)100\%$ fuzzy confidence bound of $\tilde{S}(t_0,\tilde{\mu}_i)$. It means that random interval $(S(t_0,\mu_i^{*LL}[\alpha])[\alpha],S(t_0,\mu_i^{*UL}[\alpha])[\alpha])$ could be included $S^L(t_0,\mu_i^L[\alpha])[\alpha]$ with probability $(1-\gamma)$, and also random interval $(S(t_0,\mu_i^{*LU}[\alpha])[\alpha],S(t_0,\mu_i^{*UU}[\alpha])[\alpha])$ could be included $S^U(t_0,\mu_i^U[\alpha])[\alpha]$ with probability $(1-\gamma)$. Also for a given α_0, fuzzy estimate of $\tilde{S}(t,\tilde{\mu}_i)[\alpha_0]$ (that is $(\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})[\alpha_0],\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U})[\alpha_0])$) is called $(1-\gamma)100\%$ fuzzy confidence band of $\tilde{S}(t,\tilde{\mu}_i)[\alpha_0]$. It means that random band of $(S(t,\mu_i^{*LL}[\alpha_0])[\alpha_0],S(t,\mu_i^{*UL}[\alpha_0])[\alpha_0])$ could be included $S^L(t,\mu_i^L)[\alpha_0]$ with probability $(1-\gamma)$, and also random interval $(S(t,\mu_i^{*LU}[\alpha_0])[\alpha_0],S(t,\mu_i^{*UU}[\alpha_0])[\alpha_0])$ could be included $S^U(t,\mu_i^U)[\alpha_0]$ with probability $(1-\gamma)$ at any arbitrary t.

In fuzzy normal distribution, $\alpha-$ cut of fuzzy reliability function is as follow:

$$\tilde{S}(t,\tilde{\mu}_i)[\alpha]=[1-\Phi(\frac{t-\mu_i^L[\alpha]}{\sigma}),1-\Phi(\frac{t-\mu_i^U[\alpha]}{\sigma})],$$

in this case, fuzzy reliability estimate is as follow:

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})\prec\tilde{S}(t,\tilde{\mu}_i)\prec\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U}),$$

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})[\alpha]=[S(t,\mu_i^{*LL}[\alpha])[\alpha],S(t,\mu_i^{*LU}[\alpha])[\alpha]]=[1-\Phi(\frac{t-\mu_i^{*LL}[\alpha]}{\sigma}),1-\Phi(\frac{t-\mu_i^{*LU}[\alpha]}{\sigma})],$$

$$\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U})[\alpha]=[S(t,\mu_i^{*UL}[\alpha])[\alpha],S(t,\mu_i^{*UU}[\alpha])[\alpha]]=[1-\Phi(\frac{t-\mu_i^{*UL}[\alpha]}{\sigma}),1-\Phi(\frac{t-\mu_i^{*UU}[\alpha]}{\sigma})].$$

In fuzzy exponential distribution, $\alpha-$ cut of fuzzy reliability function is as follow:

$$S(t,\tilde{\mu})[\alpha]=[e^{\frac{-t}{\mu^L[\alpha]}},e^{\frac{-t}{\mu^U[\alpha]}}].$$

In this case, fuzzy reliability estimate is as follow:

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})\prec\tilde{S}(t,\tilde{\mu}_i)\prec\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U}),$$

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})[\alpha]=[S(t,\mu_i^{*LL}[\alpha])[\alpha],S(t,\mu_i^{*LU}[\alpha])[\alpha]]=[e^{\frac{-t}{\mu_i^{*LL}[\alpha]}},e^{\frac{-t}{\mu_i^{*LU}[\alpha]}}],$$

$$\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U})[\alpha]=[S(t,\mu_i^{*UL}[\alpha])[\alpha],S(t,\mu_i^{*UU}[\alpha])[\alpha]]=[e^{\frac{-t}{\mu_i^{*UL}[\alpha]}},e^{\frac{-t}{\mu_i^{*UU}[\alpha]}}].$$

Example4.1. Let lifetime RV of the component have Weibull distribution with fuzzy parameter of $\tilde{\theta}=(a_1,a_2,a_3)$ as

$$f(x,\tilde{\theta})=\{\frac{\beta}{\theta[\alpha]}(\frac{x}{\theta[\alpha]})^{\beta-1}e^{-(\frac{x}{\theta[\alpha]})^{\beta}}[\alpha],\mu_{f(x)}\left|f(x)[\alpha]=[f_{\min}(x)[\alpha],f_{\max}(x)[\alpha]],\mu_{f(x)}=\alpha\right.\},$$

so fuzzy reliability function and MTTF is as

$$M\tilde{T}TF[\alpha]==\{\theta\Gamma(1+\frac{1}{\beta})\left|\theta\in\tilde{\theta}[\alpha]\right.\},$$

$$\tilde{S}(t)[\alpha]=[e^{-(\frac{t}{\mu^L[\alpha]}\Gamma(1+\frac{1}{\beta}))^{\beta}},e^{-(\frac{t}{\mu^U[\alpha]}\Gamma(1+\frac{1}{\beta}))^{\beta}}],$$

and fuzzy reliability estimate is as follow:

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})\prec\tilde{S}(t,\tilde{\mu}_i)\prec\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U}),$$

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})[\alpha]=[S(t,\mu_i^{*LL}[\alpha])[\alpha],S(t,\mu_i^{*LU}[\alpha])[\alpha]]=[e^{-(\frac{t}{\mu_i^{*LL}[\alpha]}\Gamma(1+\frac{1}{\beta}))^{\beta}},e^{-(\frac{t}{\mu_i^{*LU}[\alpha]}\Gamma(1+\frac{1}{\beta}))^{\beta}}],$$

$$\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U})[\alpha]=[S(t,\mu_i^{*UL}[\alpha])[\alpha],S(t,\mu_i^{*UU}[\alpha])[\alpha]]=[e^{-(\frac{t}{\mu_i^{*UL}[\alpha]}\Gamma(1+\frac{1}{\beta}))^{\beta}},e^{-(\frac{t}{\mu_i^{*UU}[\alpha]}\Gamma(1+\frac{1}{\beta}))^{\beta}}].$$

5. Estimate of the fuzzy reliability function using fuzzy bound of MTTF (unknown variance)

Let population variance is unknown, so we estimate population variance by using $D_{p\times q}(\tilde{A},\tilde{B})-$distance defined on the space of fuzzy numbers:

$$D_{p\times q}(\tilde{A},\tilde{B})=[(1-q)\int_0^1\left|A^L[\alpha]-B^L[\alpha]\right|d\alpha+q\int_0^1\left|A^U[\alpha]-B^U[\alpha]\right|d\alpha]^{\frac{1}{p}},\ \ 1\le p<\infty,$$

$$\hat{\sigma}_i^2=DS_i^2=\frac{1}{n_i-1}\sum_{j=1}^{n_i}[D_{2,\frac{1}{2}}(\tilde{r}_{ij},\bar{\tilde{r}}_i)]^2.$$

If $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ be triangular fuzzy numbers then it is easy to prove that

$$[D_{2,\frac{1}{2}}(\tilde{A},\tilde{B})]^2=\frac{1}{6}\{(a_1-b_1)^2+2(a_2-b_2)^2+(a_3-b_3)^2+\sum_{i\in\{1,2\}}(a_i-b_i)(a_{i+1}-b_{i+1})\}.$$

In this case, if the sample size is large then we replace $\sqrt{DS_i^2}$ instead of σ at past formulates of fuzzy estimate of reliability. Also if the sample size is small and lifetime RV have normal distribution then we replace $\sqrt{DS_i^2}$ instead of σ and $t_{n_i-1}(1-\frac{\gamma}{2})$ instead of $z_{1-\frac{\gamma}{2}}$ at past formulates of fuzzy estimation of reliability. So, $(1-\gamma)100\%$ fuzzy confidence bound of MTTF is defined as follow:

$$\tilde{\mu}_i^{*L}\prec\tilde{\mu}_i\prec\tilde{\mu}_i^{*U},$$

where $\tilde{\mu}_i^{*L}$ and $\tilde{\mu}_i^{*U}$ are fuzzy number with $\alpha-$cut as follow:

$$\tilde{\mu}_i^{*L}[\alpha]=[\mu_i^{*LL}[\alpha],\mu_i^{*LU}[\alpha]]$$

$$=[\bar{r}_i^{(1)}+\alpha(\bar{r}_i^{(2)}-\bar{r}_i^{(1)})-\frac{\sqrt{DS_i^2}}{\sqrt{n_i}}t_{n_i-1}(1-\frac{\gamma}{2}),\bar{r}_i^{(3)}-\alpha(\bar{r}_i^{(3)}-\bar{r}_i^{(2)})-\frac{\sqrt{DS_i^2}}{\sqrt{n_i}}t_{n_i-1}(1-\frac{\gamma}{2})],$$

$$\tilde{\mu}_i^{*L}[\alpha] = \cup_{0\le\alpha\le1}[\mu_i^{*LL}[\alpha], \mu_i^{*LU}[\alpha]] \quad , \quad i = 1,2,\dots,n,$$

triangular fuzzy number of $\tilde{\mu}_i^{*L}$ can be showed as

$$(\overline{r}_i^{(1)} - \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2}), \overline{r}_i^{(2)} - \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(\frac{\gamma}{2}), \overline{r}_i^{(3)} - \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2})),$$

also $\alpha-$cut set of $\tilde{\mu}_i^{*U}$ is as follow:

$$\begin{aligned}\tilde{\mu}_i^{*U}[\alpha] &= [\mu_i^{*UL}[\alpha], \mu_i^{*UU}[\alpha]] \\ &= [\overline{r}_i^{(1)} + \alpha(\overline{r}_i^{(2)} - \overline{r}_i^{(1)}) + \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2}), \overline{r}_i^{(3)} - \alpha(\overline{r}_i^{(3)} - \overline{r}_i^{(2)}) + \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2})],\end{aligned}$$

$$\tilde{\mu}_i^{*U}[\alpha] = \cup_{0\le\alpha\le1}[\mu_i^{*UL}[\alpha], \mu_i^{*UU}[\alpha]] \quad , \quad i = 1,2,\dots,n,$$

triangular fuzzy number of $\tilde{\mu}_i^{*U}$ can be showed as

$$(\overline{r}_i^{(1)} + \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2}), \overline{r}_i^{(2)} + \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(\frac{\gamma}{2}), \overline{r}_i^{(3)} + \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2})),$$

fuzzy confidences bound is equivalent nonfuzzy confidence intervals of $\mu_i^L[\alpha]$ and $\mu_i^U[\alpha]$ as follow:

$$\begin{aligned}\mu_i^L[\alpha] &\in (\mu_i^{*LL}[\alpha], \mu_i^{*UL}[\alpha]) \\ &= (\overline{r}_i^{(1)} + \alpha(\overline{r}_i^{(2)} - \overline{r}_i^{(1)}) - \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2}), \overline{r}_i^{(1)} + \alpha(\overline{r}_i^{(2)} - \overline{r}_i^{(1)}) + \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2})),\end{aligned}$$

$$\begin{aligned}\mu_i^U[\alpha] &\in (\mu_i^{*LU}[\alpha], \mu_i^{*UU}[\alpha]) \\ &= (\overline{r}_i^{(3)} - \alpha(\overline{r}_i^{(3)} - \overline{r}_i^{(2)}) - \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2}), \overline{r}_i^{(3)} - \alpha(\overline{r}_i^{(3)} - \overline{r}_i^{(2)}) + \frac{\sqrt{DS_i^2}}{\sqrt{n_i}} t_{n_i-1}(1-\frac{\gamma}{2})).\end{aligned}$$

We can estimate fuzzy reliability using fuzzy confidence bound of MTTF.

Example5.1. let lifetime RV of the component have fuzzy normal distribution. We choose a sample. Table 1 shows observation of the sample. We calculate fuzzy confidences bound of MTTF and estimation of fuzzy reliability function by $\sigma = 0.03, \ \gamma = 0.1$.

Table 1: observations of the sample and its descriptive statistics

	Fuzzy observations	$[D_{2,\frac{1}{2}}(\tilde{r}_i, \bar{\tilde{r}})]^2$
$\tilde{r}_1$	(0.7,0.72,0.74)	0.0016
$\tilde{r}_2$	(0.73,0.75,0.77)	0.0001
$\tilde{r}_3$	(0.8,0.82,0.84)	0.0006
$\tilde{r}_4$	(0.73,0.75,0.77)	0.0001

$\bar{\tilde{r}}$	(0.74,0.76,0.78)	
DS^2		0.0008

$$\bar{\tilde{r}}_i[\alpha]=[\bar{r}_i^L[\alpha],\ \bar{r}_i^U[\alpha]]=[0.74+0.02\alpha,0.78-0.02\alpha],$$

$(1-\gamma)100\%$ fuzzy confidence bound of MTTF is as follow:

$$\tilde{\mu}_i^{*L}\prec\tilde{\mu}_i\prec\tilde{\mu}_i^{*U},$$

where $\tilde{\mu}_i^{*L}$ is fuzzy number with $\alpha-$cut as follow:

$$\tilde{\mu}_i^{*L}[\alpha]=[\mu_i^{*LL}[\alpha],\mu_i^{*LU}[\alpha]]=[0.7047+0.02\alpha,0.7447-0.02\alpha],$$

triangular fuzzy number of $\tilde{\mu}_i^{*L}$ can be showed as $(0.7047,0.7247,0.7447)$. Also $\alpha-$cut set of $\tilde{\mu}_i^{*U}$ is as follows: $\tilde{\mu}_i^{*U}[\alpha]=[\mu_i^{*UL}[\alpha],\mu_i^{*UU}[\alpha]]=[0.7753+0.02\alpha,0.8153-0.02\alpha]$. Triangular fuzzy number of $\tilde{\mu}_i^{*U}$ can be showed as $(0.7753,0.7953,0.8153)$. So, $(1-\gamma)100\%$ fuzzy confidence bound of MTTF is defined as follows: $(0.7047,0.7247,0.7447)\prec\tilde{\mu}_i\prec(0.7753,0.7953,0.8153)$.

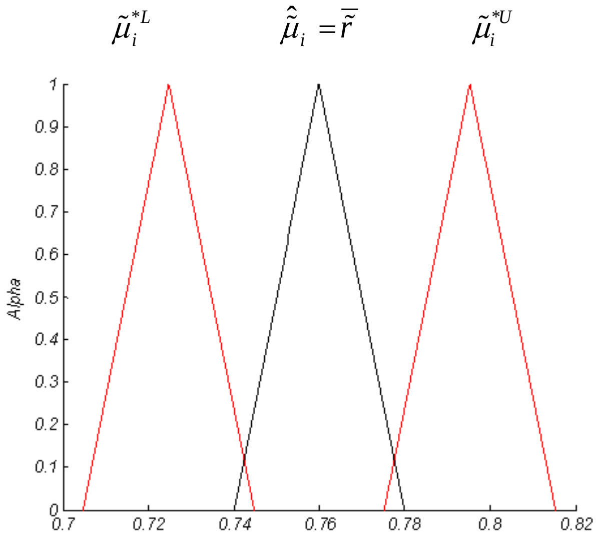

Fig .3 fuzzy confidences bound of MTTF (known variance)

In this case, fuzzy reliability estimate is as follow:

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})\prec\tilde{S}(t,\tilde{\mu}_i)\prec\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U}),$$

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})[\alpha]=[S(t,\mu_i^{*LL}[\alpha])[\alpha],S(t,\mu_i^{*LU}[\alpha])[\alpha]]$$
$$=[1-\Phi(\frac{t-(0.7047+0.02\alpha)}{0.03}),1-\Phi(\frac{t-(0.7447-0.02\alpha)}{0.03})],$$

$$\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U})[\alpha]=[S(t,\mu_i^{*UL}[\alpha])[\alpha],S(t,\mu_i^{*UU}[\alpha])[\alpha]]$$
$$=[1-\Phi(\frac{t-(0.7753+0.02\alpha)}{0.03}),1-\Phi(\frac{t-(0.8153-0.02\alpha)}{0.03})],$$

If population variance be unknown, then Table 1 shows that estimation of standard deviation is 0.0283:

$$DS_i^2 = \frac{1}{3}\sum_{j=1}^{4}[D_{2,\frac{1}{2}}(\tilde{r}_i,\tilde{r})]^2 = \frac{1}{3}\{0.0016+0.0001+0.0006+0.0001\} = 0.0008.$$

So

$$\tilde{\mu}_i^{*L}[\alpha] = [\mu_i^{*LL}[\alpha], \mu_i^{*LU}[\alpha]] = [0.7067+0.02\alpha, 0.7467-0.02\alpha],$$

triangular fuzzy number of $\tilde{\mu}_i^{*L}$ can be showed as $(0.7067, 0.7267, 0.7467)$. Also $\alpha-$cut set of $\tilde{\mu}_i^{*U}$ is as follows: $\tilde{\mu}_i^{*U}[\alpha] = [\mu_i^{*UL}[\alpha], \mu_i^{*UU}[\alpha]] = [0.7733+0.02\alpha, 0.8133-0.02\alpha]$. Triangular fuzzy number of $\tilde{\mu}_i^{*U}$ can be showed as $(0.7733, 0.7933, 0.8133)$. So, $(1-\gamma)100\%$ fuzzy confidence bound of MTTF is defined as follows: $(0.7067, 0.7267, 0.7467) \prec \tilde{\mu}_i \prec (0.7733, 0.7933, 0.8133)$.

$\tilde{\mu}_i^{*L}$ $\qquad$ $\hat{\tilde{\mu}}_i = \bar{\tilde{r}}_i$ $\qquad$ $\tilde{\mu}_i^{*U}$

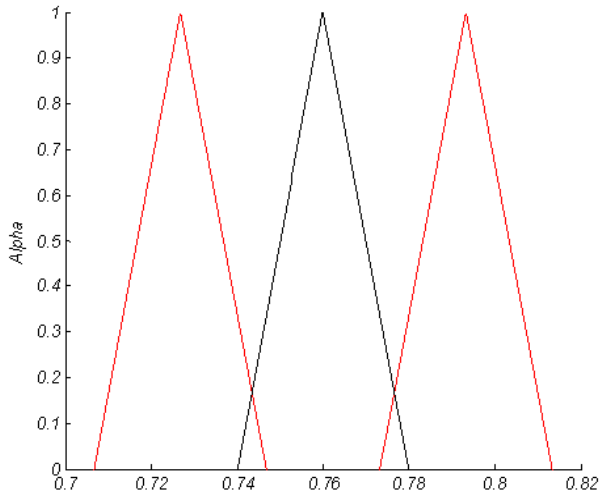

Fig .4 fuzzy confidences bound of MTTF (unknown variance)

Consequently, fuzzy reliability estimate is as follow:

$$\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L}) \prec \tilde{S}(t,\tilde{\mu}_i) \prec \tilde{S}^{*U}(t,\tilde{\mu}_i^{*U}),$$

$$\begin{aligned}\tilde{S}^{*L}(t,\tilde{\mu}_i^{*L})[\alpha] &= [S(t,\mu_i^{*LL}[\alpha])[\alpha], S(t,\mu_i^{*LU}[\alpha])[\alpha]] \\ &= [1-\Phi(\frac{t-(0.7067+0.02\alpha)}{0.0283}), 1-\Phi(\frac{t-(0.7467-0.02\alpha)}{0.0283})],\end{aligned}$$

$$\begin{aligned}\tilde{S}^{*U}(t,\tilde{\mu}_i^{*U})[\alpha] &= [S(t,\mu_i^{*UL}[\alpha])[\alpha], S(t,\mu_i^{*UU}[\alpha])[\alpha]] \\ &= [1-\Phi(\frac{t-(0.7733+0.02\alpha)}{0.0283}), 1-\Phi(\frac{t-(0.8133-0.02\alpha)}{0.0283})],\end{aligned}$$

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