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APPLICATION OF HOMOTOPY PERTURBATION TRANSFORM METHOD TO LINEAR AND NON-LINEAR SPACE-TIME FRACTIONAL REACTION-DIFFUSION EQUATIONS

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Abstract

In this paper, we obtain the analytical solutions of linear and non-linear space-time fractional reaction-diffusion equations on a finite domain by the application of homotopy perturbation transform method (HPTM). The HPTM is a combined form of the Laplace transform method with the homotopy perturbation method. Some examples are also given. Numerical results show that the HPTM is easy to implement and accurate when applied to linear and non-linear space-time fractional reaction-diffusion equations.

Keywords: Homotopy perturbation transform method, Laplace transform, fractional reaction-diffusion equation, Caputo time-fractional derivative, Caputo space-fractional derivative.

2010 Mathematics Subject Classification: 35R11, 44A10, 35C10.

1. Introduction

In recent years, it has turned out that many phenomena in engineering, physics, chemistry and other sciences can be described very successfully by models using mathematical tools from fractional calculus. For example, the nonlinear oscillation of earthquake can be modeled with fractional derivatives and the fluid-dynamic traffic model with fractional derivatives can eliminate the deficiency arising from the assumption of continuum traffic flow. Fractional derivatives are also used in modeling of many chemical processes, mathematical biology and many other problems in physics and engineering. These findings invoked the growing interest of studies of the fractional calculus in various fields such as physics, chemistry and engineering.

Fractional differential equations have gained importance and popularity during the past three decades or so, mainly due to exact description of nonlinear phenomena, especially in fluid mechanics, e.g. nano-hydrodynamics, where continuum assumption does not well, and fractional model can be considered to be a best candidate. Hence, great attention has been given to finding solutions of fractional differential equations. Most fractional differential equations do not have exact analytical solutions, therefore approximate and numerical techniques must be used. Variational iteration method (VIM) [16] was first proposed to solve fractional differential equations with greatest success. Many authors found VIM as an effective way to solving linear and non-linear fractional differential equations [7,33]. The VIM was also used by many authors to study the various physical problems [23,26,36]. The homotopy perturbation method (HPM) was first introduced by J.H. He [17]. The HPM was applied to solve the 12^{th} order boundary value problems [35]. In recent years Momani and Odibat [29], Ganji et al. [9], Yildirim [42-44], Yildirim and Sezer [45] and Jafari and Momani [20] applied the HPM to fractional differential equations and revealed that HPM is an alternative analytical method for solving fractional differential equations. Momani et al. [30] and Odibat and Momani [34] compared solutions procedure between VIM and HPM.

In this paper, we use the homotopy perturbation transform method (HPTM) [22] for solving linear and non-linear space-time fractional reaction-diffusion equations on a finite domain. It is worth mentioning that this method is an elegant combination of the Laplace transformation, the HPM and He's polynomials and is mainly due to Ghorbani [10,11]. The use of He's polynomials in the nonlinear term was first introduced by Ghorbani [10,11]. This algorithm provides the solution in a rapid convergent series which may lead to the solution in a closed form. The advantage of this method is its capability of combining two powerful methods for obtaining exact solutions for nonlinear equations.

In recent years, fractional reaction-diffusion models are studied due to their usefulness and importance in many areas of science and engineering. The reaction-diffusion equations arise naturally as description models of many evolution problems in the real world, as in chemistry [39,40], biology [32], etc. As is well known, complex behavior is peculiarity of systems modeled by reaction diffusion equations and the Belousov-Zhabotinskii reaction [31,41] provides a classic example. The reaction-diffusion equations describe a population of diploid individuals (i.e., the ones that carry two genes) distributed in a two-dimensional habitat. Assuming that a gene occurs in two forms *a* and *A*, called *alleles*, one can divide the population into three genotypes *aa, aA* and *AA*. The reaction-diffusion equations are employed to describe the co-oxidation on Pt (110) [2], the study of temporal and spatial patterns of cytoplasmic Ca^{+2} dynamics under the effects of Ca^{+2}-release activated Ca^{+2} (CRAC) channels in T cells [6], problems in finance [12,25,37] and hydrology [3]. Burke et al. [4] obtained solutions for an enzyme-suicide substrate reaction with an instantaneous point source of substrate. In 1993, Grimson and Barker [15] introduced a continuum model for the spatio-temporal growth of bacterial colonies on the surface of a solid substrate which utilizes a reaction-diffusion equation for growth. Many cellular and sub-cellular biological processes [8] can be described in terms of diffusing and chemically reacting species (e.g. enzymes). A traditional approach to the mathematical modeling of such reaction-diffusion processes is to describe each biochemical species by its (spatially dependent) concentration. In recent time, interest in fractional reaction-diffusion equations [1,13,14,18,21,38,45] has increased because the equation exhibits self-organization phenomena and introduces a new parameter, the fractional index, into the equation. Additionally, the analysis of fractional reaction-diffusion equations is of great importance from the analytical and numerical point of view.

The Riemann-Liouville fractional integration of order α is defined as [27]

$$J_a^{\alpha} f(x,t) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-u)^{\alpha-1} f(u,t)\, du, \quad x > 0. \tag{1}$$

The following fractional derivative of order α > 0 is introduced by Caputo [5]; see also Kilbas et al. [24] in the form

$$\begin{aligned} {}_0D_t^{\alpha} f(x,t) &= \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(x,\tau)\, d\tau}{(t-\tau)^{\alpha+1-m}}, \quad m-1 < \alpha \le m, \operatorname{Re}(\alpha) > 0, \; m \in N, \\ &= \frac{\partial^m f(x,t)}{\partial t^m}, \quad \text{if } \alpha = m, \end{aligned} \tag{2}$$

where $\frac{\partial^m f(x,t)}{\partial t^m}$ is the m-th partial derivative of f(x,t) with respect to t.

The Laplace transform of the Caputo derivative is given by Caputo [5]; see also Kilbas et al. [24] in the form

$$L\{{}_0D_t^{\alpha} f(x,t)\} = s^{\alpha} L[f(x,t)] - \sum_{r=0}^{m-1} s^{\alpha-r-1} f^{(r)}(x,0+), \; (m-1 < \alpha \le m). \tag{3}$$

The Liouville fractional derivative of order α is defined in [24, Section 24.2] in the form

$$\frac{\partial^{\alpha}}{\partial x^{\alpha}} f(x,t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{\partial}{\partial x}\right)^m \int_{-\infty}^{x} \frac{f(y,t)}{(x-y)^{\alpha-m+1}} dy, \quad (x \in R, \alpha > 0, m = [\alpha]+1) \tag{4}$$

where [α] is the integral part of α, α > 0.

2. HPTM solutions of linear space-time fractional reaction-diffusion equation

In this paper, we first consider the linear space-time fractional reaction-diffusion equation of the form:

$$D_t^{\alpha} u(x,t) = b(x) D_x^{\beta} u(x,t) - c(x) u(x,t) + f(x,t), \; 0 < x < L, \; t > 0, \; 0 < \alpha \le 1, \; 1 < \beta \le 2, \tag{5}$$

$$u(x,0) = p(x), \tag{6}$$

$$u(0,t) = q_1(t), \tag{7}$$

$$u_x(0,t) = q_2(t). \tag{8}$$

Taking the Laplace transform on both sides of Eq. (5) and using (6), we get

$$L[u(x,t)] = \frac{p(x)}{s} + \frac{1}{s^{\alpha}} b(x) L[D_x^{\beta} u(x,t)] - \frac{1}{s^{\alpha}} c(x) L[u(x,t)] + \frac{1}{s^{\alpha}} L[f(x,t)]. \tag{9}$$

Applying the inverse Laplace transform on both sides of Eq. (9), we get

$$u(x,t) = p(x) + L^{-1}\left[\frac{b(x)}{s^{\alpha}} L[D_x^{\beta} u(x,t)] - \frac{c(x)}{s^{\alpha}} L[u(x,t)]\right] + J_t^{\alpha} f(x,t). \tag{10}$$

Now, we apply the homotopy perturbation method

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t). \tag{11}$$

Substituting Eq. (11) in Eq. (10), we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) =$$

$$p(x) + p\left(L^{-1}\left[\frac{1}{s^\alpha} b(x)L[D_x^\beta \sum_{n=0}^{\infty} p^n u_n(x,t)] - \frac{c(x)}{s^\alpha} L[\sum_{n=0}^{\infty} p^n u_n(x,t)]\right]\right) + J_t^\alpha f(x,t). \quad (12)$$

Comparing the coefficients of the like terms of p, we have

$$p^0 : u_0(x,t) = p(x) + J_t^\alpha f(x,t), \quad (13)$$

$$p^1 : u_1(x,t) = L^{-1}\left[\frac{b(x)}{s^\alpha} L[D_x^\beta u_0(x,t)] - \frac{c(x)}{s^\alpha} L[u_0(x,t)]\right]$$

$$= \left[b(x)D_x^\beta - c(x)\right]\left[p(x)\frac{t^\alpha}{\Gamma(\alpha+1)} + J_t^{2\alpha} f(x,t)\right]. \quad (14)$$

Proceeding in a similar manner, we get

$$p^2 : u_2(x,t) = \left[b(x)D_x^\beta - c(x)\right]^2\left[p(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + J_t^{3\alpha} f(x,t)\right], \quad (15)$$

$$p^3 : u_3(x,t) = \left[b(x)D_x^\beta - c(x)\right]^3\left[p(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + J_t^{4\alpha} f(x,t)\right], \quad (16)$$

$$\vdots$$

$$p^n : u_n(x,t) = \left[b(x)D_x^\beta - c(x)\right]^n\left[p(x)\frac{t^{n\alpha}}{\Gamma(n\alpha+1)} + J_t^{(n+1)\alpha} f(x,t)\right], \quad (17)$$

and so on; in this manner, the rest of components of the homotopy perturbation series can be obtained. Thus, we have the solution in series form is given by

$$\mathrm{u(x,t)} = \mathrm{p(x)} + \mathrm{J}_{\mathrm{t}}^\alpha f(x,t) + \left[b(x)D_x^\beta - c(x)\right]\left[p(x)\frac{t^\alpha}{\Gamma(\alpha+1)} + J_t^{2\alpha} f(x,t)\right] + \cdots$$

$$+ \left[b(x)D_x^\beta - c(x)\right]^n\left[p(x)\frac{t^{n\alpha}}{\Gamma(n\alpha+1)} + J_t^{(n+1)\alpha} f(x,t)\right] + \cdots. \quad (18)$$

3. HPTM solutions of non-linear space-time fractional reaction-diffusion equation

Now, we consider the non-linear space-time fractional reaction-diffusion equation of the form:

$$D_t^\alpha u(x,t) = bD_x^\beta u(x,t) + f(u(x,t)) + g(x,t),\ \ 0 < x < L, t > 0, 0 < \alpha \le 1, 1 < \beta \le 2, \quad (19)$$

$$u(x,0) = p(x). \quad (20)$$

Taking the Laplace transform on both sides of Eq. (19) and using (20), we get

$$L[u(x,t)] = \frac{p(x)}{s} + \frac{b}{s^\alpha} L[D_x^\beta u(x,t)] + \frac{1}{s^\alpha} L[f(u(x,t))] + \frac{1}{s^\alpha} L[g(x,t)]. \quad (21)$$

Applying the inverse Laplace transform on both sides of Eq. (21), we get

$$u(x,t) = p(x) + L^{-1}\left[\frac{b}{s^\alpha} L[D_x^\beta u(x,t)] + \frac{1}{s^\alpha} L[f(u(x,t))]\right] + J_t^\alpha g(x,t). \quad (22)$$

Now, we apply the homotopy perturbation method

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t) \quad (23)$$

and the nonlinear term can be decomposed as

$$f(u(x,t)) = \sum_{n=0}^{\infty} p^n H_n(u) \tag{24}$$

for some He's polynomials $H_n(u)$ [11,28] that are given by

$$H_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[f\left(\sum_{i=0}^{\infty} p^i u_i\right)\right]_{p=0}, \text{ n = 0, 1, 2, 3...} \tag{25}$$

Substituting Eqs. (23) and (24) in Eq. (22), we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) =$$

$$p(x) + p\left(L^{-1}\left[\frac{b}{s^\alpha} L[D_x^\beta \sum_{n=0}^{\infty} p^n u_n(x,t)] + \frac{1}{s^\alpha} L[\sum_{n=0}^{\infty} p^n H_n(u)]\right]\right) + J_t^\alpha g(x,t). \tag{26}$$

Comparing the coefficients of the like terms of p, we have

$$p^0 : u_0(x,t) = p(x) + J_t^\alpha g(x,t), \tag{27}$$

$$p^1 : u_1(x,t) = L^{-1}\left[\frac{b}{s^\alpha} L[D_x^\beta u_0(x,t)] + \frac{1}{s^\alpha} L[H_0]\right]$$

$$= J_t^\alpha [bD_x^\beta u_0(x,t)] + J_t^\alpha [H_0]. \tag{28}$$

Proceeding in a similar manner, we get

$$p^2 : u_2(x,t) = J_t^\alpha [bD_x^\beta u_1(x,t)] + J_t^\alpha [H_1], \tag{29}$$

$$p^3 : u_3(x,t) = J_t^\alpha [bD_x^\beta u_2(x,t)] + J_t^\alpha [H_2], \tag{30}$$

$$\vdots$$

$$p^{n+1} : u_{n+1}(x,t) = J_t^\alpha [bD_x^\beta u_n(x,t)] + J_t^\alpha [H_n], \tag{31}$$

and so on; in this manner, the rest of the components of the homotopy perturbation series can be obtained. Therefore, the solution in series form is given by

$$u(x,t) = p(x) + J_t^\alpha g(x,t) + J_t^\alpha [bD_x^\beta u_0(x,t)] + J_t^\alpha [H_0] +$$
$$+ J_t^\alpha [bD_x^\beta u_1(x,t)] + J_t^\alpha [H_1] + \ldots + J_t^\alpha [bD_x^\beta u_n(x,t)] + J_t^\alpha [H_n] + \cdots. \tag{32}$$

4. Numerical examples

Example 1

Consider the following linear space-time fractional reaction-diffusion equation with boundary and initial conditions [46]:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = b(x)\frac{\partial^\beta u(x,t)}{\partial x^\beta} - c(x)u(x,t) + f(x,t),\ 0 \le x \le 1,\ 0 \le t \le 1, \tag{33}$$

$$u(0,t) = u_x(0,t) = 0, \tag{34}$$

$$u(x,0) = p(x) = x^2 - x^3, \tag{35}$$

where the source function

$$f(x,t) = 3(4t^2 + 1)x^3 + \frac{32}{3\sqrt{\pi}} t^{1.5}(x^2 - x^3)$$

the coefficients of the diffusion and reaction terms are $b(x) = \Gamma(1.2)x^{1.8}$, c(x) = 2. If α = 0.5, β = 1.8, the exact solution of this problem is $(4t^2 + 1)(x^2 - x^3)$, which can be verified by direct fractional differentiation of the given solution, and substituting in the fractional differential equation. The initial and boundary conditions are clearly satisfied. If α = 0.5, β = 1.8, according to homotopy perturbation transform procedures Eqs. (9)-(18), we can successively obtain

$$u_0(x,t) = (4t^2+1)(x^2-x^3)+6x^3\left(\frac{32t^{2.5}}{15\sqrt{\pi}}+\frac{t^{0.5}}{\sqrt{\pi}}\right),$$

$$u_1(x,t) = -3x^3\left(\frac{64t^{2.5}}{15\sqrt{\pi}}+\frac{2t^{0.5}}{\sqrt{\pi}}-4t^3-3t\right),$$

$$u_2(x,t) = -9x^3\left(\frac{4t^3}{3}-\frac{128t^{3.5}}{15\sqrt{\pi}}+t-\frac{2t^{1.5}}{\sqrt{\pi}}\right),$$

$$u_3(x,t) = -27x^3\left(\frac{4t^{1.5}}{3\sqrt{\pi}}-\frac{128t^{3.5}}{105\sqrt{\pi}}-t^4-\frac{3t^2}{2}\right),$$

$\vdots$

and so on; in this manner, the rest of the components of the homotopy perturbation series can be obtained. Thus, we have the solution in series form is given by

$$u(x,t) = u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+\cdots. \qquad (36)$$

When α = 0.5, β = 1.8, Figs. (1-2-3-4-5) show the different approximate solutions obtained by applying the HPTM and the exact solutions of space-time fractional reaction-diffusion equation.

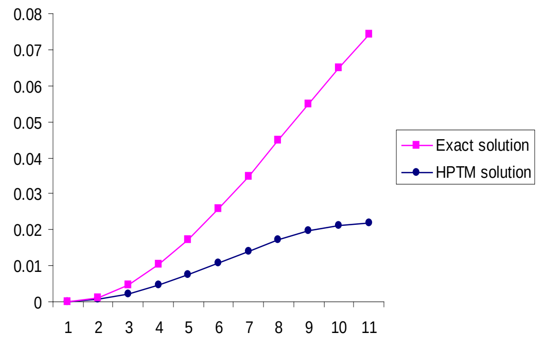

Fig. 1 Comparison of the exact solution and approximate solution at time t = 0.4 for α = 0.5 and β = 1.8.

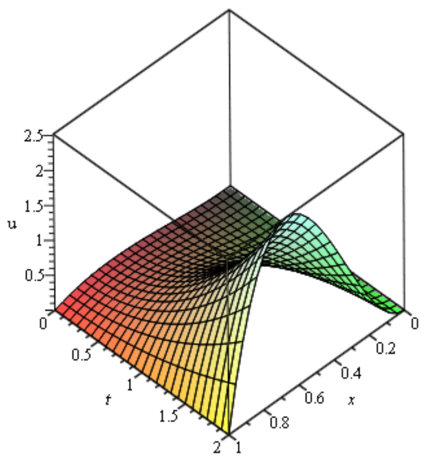

Fig. 2 Exact solution graph of Example 1 at $\alpha = 0.5$ and $\beta = 1.8$ at $t = 0$ to $t = 2$.

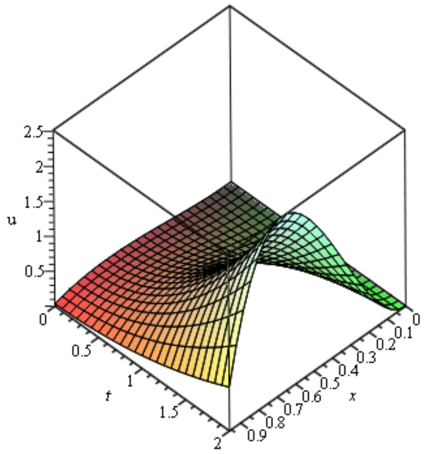

Fig. 3 Approximate solution graph of Example 1 at $\alpha = 0.5$ and $\beta = 1.8$ up to fourth approximation at $t = 0$ to $t = 5$ by HPTM.

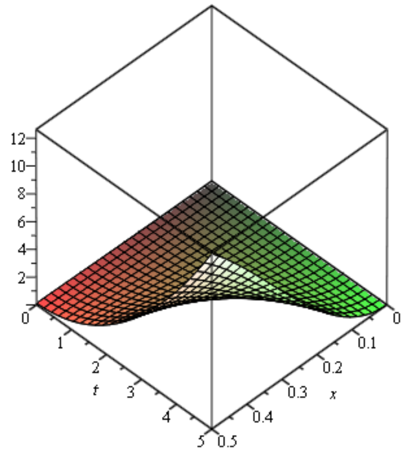

Fig. 4 Exact solution graph of Example 1 at $\alpha = 0.5$ and $\beta = 1.8$ at $t = 0$ to $t = 5$.

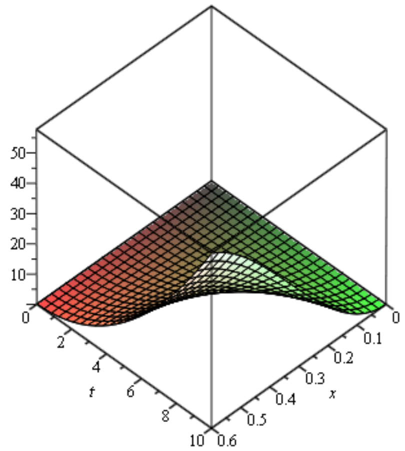

Fig. 5 Approximate solution graph of Example 1 at $\alpha = 0.5$ and $\beta = 1.8$ up to fourth approximation at $t = 0$ to $t = 10$ by HPTM.

Example 2

Consider the following non-linear space-time fractional reaction-diffusion equation with boundary and initial conditions [46]:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = b\frac{\partial^{\beta} u(x,t)}{\partial x^{\beta}} + f(u(x,t)) + g(x,t), 0 \le x \le 1,\ t > 0, \tag{37}$$

$$u(x,0) = 0,\ 0 \le x \le 1, \tag{38}$$

where the non-linear reaction terms in Fisher's growth equation:

$f(u(x,t)) = 0.25u(x,t)(1-u(x,t))$

and

$g(x,t) = -0.0104649t^{0.9} + 0.00961766x^{1.1} - 0.0025t^{0.9}x^{1.1} + 0.000025t^{1.8}x^{2.2}$.

If α = 0.9, β = 1.1, the exact solution of this problem is $0.01x^{1.1}t^{0.9}$, which can be verified by direct fractional differentiation of the given solution, and substituting in fractional differential equation. The initial and boundary conditions are clearly satisfied. Using Eqs. (22), (24) and (25), the first few components of He's polynomials H_n that represent the non-linear term 0.25u(x, t)(1-u(x, t)) are obtained as

$$\begin{aligned}
H_0 &= 0.5u_0(1-u_0),\\
H_1 &= u_1(0.25-0.5u_0),\\
H_2 &= u_2(0.25-0.5u_0) + \frac{u_1^2}{2}(0.5),\\
H_3 &= u_3(0.25-0.5u_0) + u_1u_2(0.5),\\
&\vdots
\end{aligned} \tag{39}$$

If α = 0.9, β = 1.1, using (37), according to homotopy perturbation transform procedures Eqs. (21)-(32) we now successively obtain

$$u_0(x,t) = 0.01x^{1.1}t^{0.9} - 0.0143419x^{1.1}t^{1.8} - 0.0600346t^{1.8} + 0.0000100493x^{2.2}t^{2.7},$$

$$\begin{aligned}
u_1(x,t) = {} & 0.00600346t^{1.8} - 0.00120662t^{2.7} - 2.30348\times10^{-6}t^{4.5} + (0.00143419t^{1.8} - \\
& 0.000144127t^{2.7} + 0.0000166107t^{3.6} - 1.10058\times10^{-6}t^{4.5})x^{1.1} + \\
& (-0.0000100493\,t^{2.7} + 3.01807\times10^{-6}t^{3.6} - 1.31461\times10^{-7}t^{4.5} + \\
& 6.5514\times10^{-9}t^{5.4})x^{2.2} + (-1.28454\times10^{-8}t^{4.5} + 1.56623\times10^{-9}t^{5.4})x^{3.3} + \\
& (-4.78235\times10^{-12}t^{6.3})x^{4.4},
\end{aligned}$$

$$\begin{aligned}
u_2(x,t) = {} & 0.00120662t^{2.7} - 0.000141028t^{3.6} + 9.05084\times10^{-6}t^{4.5} - 1.16267\times10^{-6}t^{5.4} \\
& -1.1625\times10^{-9}t^{7.2} + (0.000144127t^{2.7} - 0.000027841\,t^{3.6} + \\
& 6.59228\times10^{-6}t^{4.5} - 4.08064\times10^{-7}t^{5.4} + 1.45028\times10^{-8}t^{6.3} - \\
& 8.33145\times10^{-10}t^{7.2})x^{1.1} + (-3.01807\times10^{-6}t^{3.6} + 6.40041\times10^{-7}t^{4.5} - \\
& 7.0968\times10^{-8}t^{5.4} + 7.55741\times10^{-9}t^{6.3} - 1.9903410^{-10}t^{7.2} + \\
& 4.72952\times10^{-12}t^{8.1})x^{2.2} + (1.28454\times10^{-8}t^{4.5} - 7.11015\times10^{-9}t^{5.4} + \\
& 7.45807\times10^{-10}t^{6.3} - 4.59253\times10^{-11}t^{7.2} + 2.25971\times10^{-12}t^{8.1})x^{3.3} \\
& + (2.17306\times10^{-11}t^{6.3} - 5.61593\times10^{-12}t^{7.2} + 2.69917\times10^{-13}t^{8.1} - \\
& -6.51357\times10^{-15}t^{9.0})x^{4.4} + (1.33856\times10^{-14}t^{8.1} - \\
& 1.55606\times10^{-15}t^{9.0})x^{5.5} + (3.03862\times10^{-18}t^{9.9})x^{6.6},
\end{aligned}$$

$\vdots$

and so on; in this manner, the rest of components of the homotopy perturbation series can be obtained. Thus, we have the solution in series form is given by

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots. \tag{40}$$

When α = 0.9, β = 1.1, Figs. (6-7-8) show the different approximate solutions obtained by applying the HPTM and the exact solutions of space-time fractional reaction-diffusion equation. From Fig. 6, it can be seen that the approximate solution is in excellent agreement with the exact solution.

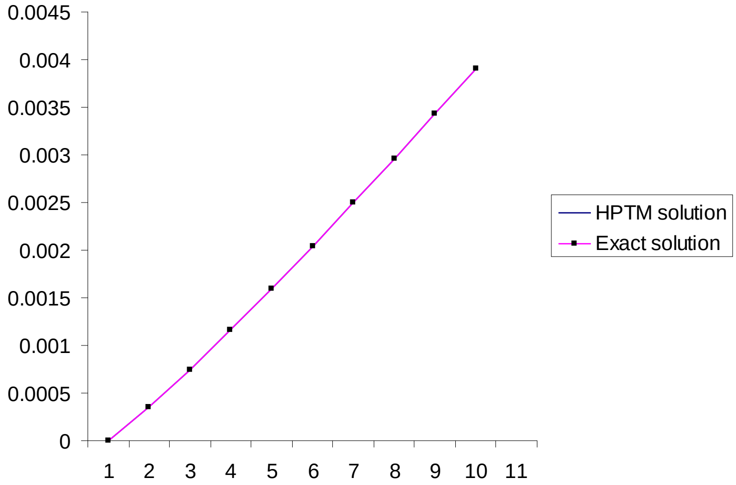

Fig. 6 Comparison of the exact solution and approximate solution at time t = 0.4 for α = 0.9 and β = 1.1.

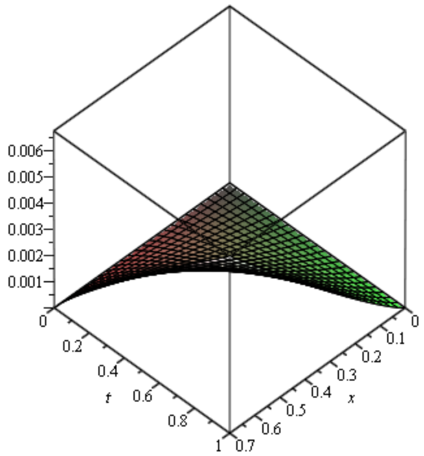

Fig. 7 Exact solution graph of Example 2 at $\alpha = 0.9$ and $\beta = 1.8$ at $t = 0$ to $t = 1$.

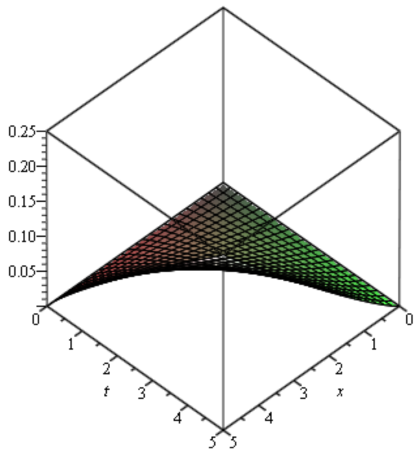

Fig. 8 Approximate solution graph of Example 2 at α = 0.9 and β = 1.1 at t = 0 to t = 5 up to fourth approximation by HPTM.

5. Conclusions

In this paper, we have used HPTM for solving the linear and non-linear space–time fractional diffusion equations. The HPTM is clearly a very efficient technique for finding the solutions of the proposed equations. It is interesting to note that HPTM is an elegant combination of the Laplace transformation and the homotopy perturbation method. The mathematical technique employed in the present article is significant in studying some other problems of engineering and physics.

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