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Best Minimizing Algorithm for Shape-measure Method

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Abstract

The Shape-Measure method for solving optimal shape design problems (OSD) in cartesian coordinates is divided into two steps. First, for a fixed shape (domain), the problem is transferred to the space of positive Radon measures and relaxed to a linear programming in which its optimal coefficients determine the optimal pair of trajectory and control. Then, a standard minimizing algorithm is used to identify the best shape. Here we deal with the best standard algorithm to identify the optimal solution for an OSD sample problem governed by an elliptic boundary control problem.

Keywords: elliptic equation; Radon measure; optimal shape; search techniques.

2010 Mathematics Subject Classification: Primary 54A40; Secondary 46S40.

1. Introduction.

Setting up the allowable set of shapes like (domains of equations) in order to get a feasible solution has more commonly application in industrial designing like automobile, marine and biomolecularprocesses. A huge part of these problems deals with free boundary problems when a part of the domains boundary is varied. Such problems deal with solving PDEs in a domain, such that a part of the boundary is not known in advance; that part is usually called the free boundary.The theory of free boundary problems has been greatly developed in the last forty years. Besides the progress in theory of free boundary, many problems arising in mechanics, physics, biology, financial mathematics and etc, can be formulated as free boundary problems. For solving such problem, Shape-Measure method, has many advantages like strong linearity, automatic

existence theorem, flexibility and ability of obtaining a global solution. The implementation of this method requires a standard minimization algorithm. Till yet, Only the Nelder-Mead method is used to find the optimal domain in this manner ([5],[6]). But so far no research regarding their most appropriate algorithm has been done. Typically, techniques based on Newton's method can be applied successfully, by using gradient and Hessian informations to calculate a good step and then gradually move it towards an optimum of the function ([15]). However, sometimes Newton based approaches can be the wrong choice. This can be the case if: (1) the function evaluations are inaccurate, (2) the derivatives of the function are unavailable or unreliable, or (3) the function is not smooth ([21]). In these cases, a better choice can be to rely on so-called derivative free methods, i.e., methods that do not explicitly use derivatives of the function being optimized. Because we could not calculate the derivative of the objective function in the administrative process of our problem, we should use a search technique that it has this property. Also, in choosing such algorithms, points cause to facilitate and accelerate in acquisition the optimal solution should be considered. In addition, these algorithms should be have ability to adapt and use in shape-measure method and possibility of fast changes in inputs. Many algorithms such Random search, Spendly, Hext and Himsworth method ([20]), Nelder-Mead, Pattern search, Genetic algorithm, Simulated Annealing method, Honey bee swarm algorithm, Tabu search, Scatter search, Ant colony optimization, are suitable for this purpose.But by considering the existence of some limitations, such as dependence or independence of these algorithms to the initial point, sameness of some of these algorithms, accuracy in the final answer, the computer memory, the time required for programming, ability to adapt software, knowledge of specific techniques to implement the algorithm, cause to, after doing some necessary investigations, we selected Nelder-Mead, Hook and Jeeves, Random search, Simulated annealing, genetic algorithm and Honey bee swarm algorithm for testing the appropriate capability.
In this paper, the focus is on investigating a prefect direct search method to find optimal solution for a free boundary problem with shape-measure method. This work is structured as follows: In Section 2, a brief description of shape-measure method for the boundary control elliptic optimal shape design problem is given. In Section 3 we illustrate several traditional and untraditional direct search methods for our purposes. The obtained numerical results of using these methods are presented in Section 4 and concluding remarks are provided in section 5.

2. Main results

Let $D \subseteq \mathbb{R}^2$ be a bounded domain with a piecewise-smooth, closed and simple boundary ∂D which consists of a fixed and a variable part. The fixed part is a union of three segments: part of the line $y=0$ between the points (0,0) and $A=(1,0)$; part of the line $x=0$ between the points (0,0) and (0,1), and part of the line $y=1$ between the points (0,1) and $B=(1,1)$. The variable (free boundary) part is a curve Γ with the initial and final points A and B respectively, so that ∂D is a simple and closed curve (see Fig. 1).

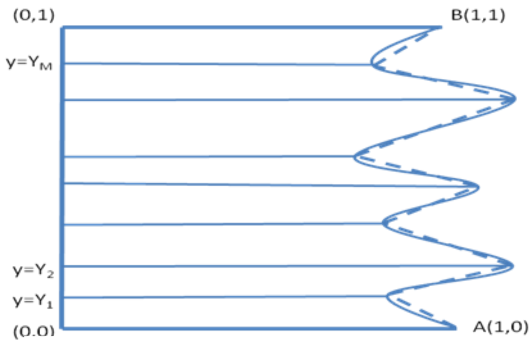

Figure 1: A general domain D

Definition 2.1. *For* $f \in C(D\times\mathbb{R})$ *and* $g \in C(D)$, *the above domain* D *is called admissible if the equations* $\Delta u(X) + f(X,u) = g(X), \quad u|_{\partial D} = v,$ *(1)*

has a bounded solution on D, *where* v *is an uknown control function.*

Definition 2.2. *The pair of functions* (u,v) *is called admissible if*

(i) the trajectory function $u \in H^1(D)$ *is bounded and takes values in the bounded set* U *;*

(ii) the control function v *is Lebesgue_measurable which takes values on the bounded set* V *;*

(iii) the function u *and* v *satisfy the condition (1) for every* $\psi \in H_0^1(D)$.

The set of all admissible pairs (u,v) *is denoted by F.*

Let $f_1 : D\times U \to \mathbb{R}$ and $f_2 : \partial D\times V \to \mathbb{R}$ be two given continuous functions. Then, in the general form, the following optimal shape design problem is going to solve:

$$Min : I(D,v) = \int_D f_1(X,u(X))dX + \int_{\partial D} f_2(s,v(s))ds$$
$$S.\ to : \Delta u(X) + f(X,u) = g(X), u|_{\partial D} = v. \qquad (2)$$

To apply the shape-measure method, first, by using Green formula [13], after multiplying (1) in ψ and then getting integral over D, the equations (1) is converted to:

$$\int_D (u\Delta\psi + \psi f)dX + \int_{\partial D} v(\nabla\psi.n)ds = \int_D \psi g dX, \quad \forall\psi \in H_0^1(D);$$

where $H_0^1(D) = \{\psi \in H^1(D) : \psi|_{\partial D} = 0\}$ and $H^1(D)$ is the Sobolev space of order 1.

Applying RieszRepresentation theorem [17], helps us to transfer this generalized form of (1) into the space of measure. We define $\Omega = D\times U$ and $\Omega = \partial D\times V$; then , a bounded weak solution and its corresponded control function define a pair of positive and linear functional $u(.) : F \mapsto \int_D F(X,u(X))dX$ and $v(.) : G \mapsto \int_{\partial D} G(s,v(s))ds$ on $C(\Omega)$ and $C(\omega)$. We can show that there are measures μ and υ so that:

$$\mu(F) = \int_D F(X,u(X))dX, \forall F \in C(\Omega);$$
$$v(G) = \int_{\partial D} G(s,v(s))ds, \forall G \in C(\omega). \quad (3)$$

Extending the underlying measure space and considering all the pair of measures (μ,υ) which are satisfy to mentioned conditions of (3), not only deduced by Riesz Representation theorem, plus the extra properties $\mu(\zeta) = \int_D \zeta(X)d(X) = a_\zeta$ and $v(\tau) = \int_{\partial D} \tau(s)d(s) = b_\tau$ cause we are going to solve the following problem:

$$\begin{aligned} &Min : i(\mu,v) = \mu(f_1) + v(f_2) \\ &S.\ to : \mu(F_\psi) + v(G_\psi) = c_\psi && \forall\psi \in H_0^1(D); \\ &\qquad \mu(\zeta) = a_\zeta, && \forall\zeta \in C_1(\Omega); \ (4) \\ &\qquad v(\tau) = b_\tau, && \forall\tau \in C_1(\omega), \end{aligned}$$

Where $F_\psi = u\Delta\psi + \psi f$, $G_\psi = -v(\nabla\psi.n|_{\partial D})$ and $c_\psi = \int_D \psi g dX$.

Afterwards, by applying density properties of the involved spaces, using atomic measures one can conclude that μ and υ have the form $\mu = \sum_{n=1}^{N} \alpha_n \delta(Z_n)$ and $\upsilon = \sum_{k=1}^{K} \beta_k \delta(z_k)$; the discretiztion of Ω

and ω with the nodes Z_n for $n=1,2,\dots,N$ and z_k for $k=1,2,\dots,K$, cause to find the solution (4) via the following finite linear programming problem([6]):

$$
\begin{aligned}
&Min: \sum_{n=1}^{N} \alpha_n f_1(Z_n) + \sum_{k=1}^{K} \beta_k f_2(z_k) \\
&S.\ to: \sum_{n=1}^{N} \alpha_n F_i(Z_n) + \sum_{k=1}^{K} \beta_k G_i(z_k) = c_i, i=1,2,\dots,M_1; \\
&\sum_{n=1}^{N} \alpha_n \zeta_j(Z_n) = a_j, \qquad j=1,2,\dots,M_2; \\
&\sum_{k=1}^{K} \beta_k \tau_l(z_k) = b_l, \qquad l=1,2,\dots,M_3; \quad (5) \\
&\alpha_n \geq 0, \qquad n=1,2,\dots,N; \\
&\beta_k \geq 0, \qquad k=1,2,\dots,K.
\end{aligned}
$$

Thus in the first step of shape-measure method one can calculate the value of $I(D,v_D^*)$ and its related suboptimal control function, v_D^* , for each given domain D by the solution of (5), as the way as described in [16]. Generally for an unknown domain D, we can approximate the variable part of D with M number of segments or equally M number of its points (corners of broken lines belonging to Γ), which has been called the M-representation of D. For a fixed number M without losing generality, the points in the M-representation set can have the fixed y-component like $y_m = Y_m, m=1,2,\dots,M$ ([5] and [6]).

Hence, each M-representation set can be characterized by M variables $x_1, x_2, \dots, x_M$. Thus the vector function $J: D \mapsto I(D,v_D^*)$ or $J:(x_1,x_2,\dots,x_M) \in R^M \mapsto I(D,v_D^*)$ can be identified as a function of M variables for any given domain D where $J(D)$ is calculated by solving the related (5) from the pervious step. Hence, one can obtain the optimal pair of domain and control just by applying with a suitable search techniques. The following theorem from [6] shows that this obtained domain and control are optimal.

Theorem 2.3. *Let the minimization algorithm (for finding the minimize of J), give the global minimizer $(x_1^*, x_2^*, \dots, x_M^*)$. If the minimize domain is denoted by D^*, then $I(D^*, v_{D^*}^*)$ is the optimal value of I in (1) and hence the pair of domain and control $(D^*, v_{D^*}^*)$ is optimal.*

3. Direct Search Methods

Our goal in this section is to examine and evaluate six different methods according to their ability to find optima of function. Table 1gives some references (references related to applications or discussions) for each method. These algorithms have been extensively developed and used as a search and optimization tool in various problems related to domains. The method of Hook and Jeeves is one of the most widely used direct search method and attempt in a simple though ingenious way to find the most profitable search directions. Several direct search methods use a geometrical designs in the search space; the Nelder-Mead is much more efficient algorithm in this class and is one of the best known algorithms for multidimensional unconstrained optimization without derivatives. Also Adaptive random search (ARS) is one of the most useful such techniques which relies upon generation of random numbers to search for optimum [18]. Over the last decade, evolutionary and meta-heuristic algorithms have been extensively used as a search techniqe and optimization tools in various problems, including science, commerce, and engineering. Their broad applicability, ease of use, and global perspective may be considered as the primary reason for their success. Among this methods, the genetic algorithms (GAs) have been extensively employed in different branches [4] and [8]. Modeling the behavior of social insects, such as ants and bees, and using these models for search and problem-solving are the context of the emerging area of swarm intelligence. Honey-bees mating is a typical successful swarm-based approach to optimization, where the search algorithm is inspired by the behavior of real bees.

For some of these solution methods there exists a theory of convergence. Early work can be found in [19], where the author proved that the limit of the infimum norm of the gradient at the best vertex at iteration k converges to 0 as $k \to \infty$ for HJ, given that the function is continuously differentiable. At the other end of the scale some of the direct search methods have negative convergence results, such as NM, [10]. Also, Markov chains offer an appropriate model to analyze GA and SA. they have been used in [3] and [9] to prove probabilistic convergence to the best solution. In this paper, regarding the primal OSD problem the emphasis is not on the theoretical convergence properties of the different methods, but rather on their performance as established by empirical evaluation.

Table 1: selected traditional and untraditional algorithms

Method	Full name	References
NM	Nelder-Mead	[5],[14],[20],[21]
HJ	Hook and Jeeves	[2],[11],[20]
ARS	Adaptive Random Search	[18]
SA	Simulated Annealing	[9]
GA	genetic	[3],[8]
HBMO	Honey-Bee Mating Optimization Algorithm	[1]

4.Numerical Simulations and Results

For the domain represented in Fig 1 we assumed M=5, Y_1=0.2, Y_2=0.35, Y_3=0.5, Y_4=0.65, Y_5=0.8. Because our methods are iterative, discretization on Ω and ω depending on the values of X_1, X_2, X_3, X_4, X_5 at each iteration is performed.

We selected N=550 nodes like $Z_n = (x_n, y_n, u_n)$ in Ω and K=110 nodes like $z_k = (s_k, v_k)$ in ω. For this simulations we chose

$$f_1(X,u) = \begin{cases} 400, & -0.05 \le u \le 0.05; \\ \frac{1}{u^2}, & otherwise, \end{cases}$$

and $f_2(s,v) = 0$, $g(X) = 0$ and $f = 0$. Also the variables are supposed to satisfy $0 \le X_m \le 2$ for $m = 1, 2, \ldots, 5$, which were applied by means of penalty method [20]. We used the mentioned Shape-Measure method to find the optimal domain and its related optimal control.

An important question always raised in the optimization is which algorithm is suitable for a given optimization problem. In the more general case, which algorithm is superior than other algorithms. One criteria is which algorithm takes less time to find the answer of problem. Thus, in comparing the performance of selected algorithms for solving a specific problem, the time needed for full implementation should be considered. Besides this factor, the number of iterations, the final (optimal) objective function and even stopping criteria are other factors that can be considered. In table 2, the obtained objective function, each five optimal variables values in this manner of the OSD problem, the number of iterations for full implementation and the calculations time resulted from application of each mentioned algorithms in pervious section, are presented.

Table 2: Comparsion between six algorithm for solve the above problem

algorithm	Variables value	fval	iteration	Time calculation(S)	Stopping condition
random search	X_1=0.0305,X_2=0.0317,X_3=0.1630 X_4=0.0409,X_5=0.2307	0.2811	401	25616.054857	Max iter
nelder-mead	X_1=0.0000,X_2=0.0000,X_3=1.6922 X_4=1.0597,X_5=0.0000	0.6128	296	65540.180032	Max iter
hook and jeeves	X_1=0.0625,X_2=0.1250,X_3=0.0625 X_4=0.1250,X_5=0.1250	0.2797	84	72106.589826	Max iter
simulated annealing	X_1=0.0271,X_2=0.1353,X_3=0.0766 X_4=0.1187,X_5=0.0390	0.2612	1000	432000	Max iter
genetic	X_1=0.052,X_2=0.258,X_3=0.086 X_4=0.038,X_5=0.031	0.2716	51	200697.556704	Max iter
honey bee optimization algorithm	X_1=0.0754,X_2=0.0917,X_3=0.0617 X_4=0.1328,$_5$=0.0187	0.2594	65	192904.626007	Max iter

By investigation on table 2 , first, we can note that even the Simulated annealing method has a good value, but the time evaluation for this method cause to in the tests conducted here, the method failed to perform well; also The Nelder-Mead algorithm produced a large value for the objective function, thus it also failed. The random search algorithm has the running time less than other algorithms; but this method does not survey the search space completely.

Table 2 shows that the Adaptive Random Search and Honey Bee Mating Optimization algorithms have respectively minimum time and the best objective function value than others; and thus they are proposed for using in shape-measure method. As a final note this section,is theeffect of increasing the number of problem dimensions that has a large impact on time evaluation.The optimal domains and optimal control functions ([16]) produced by each of the six algorithms have been plotted in Fig 2 and Fig3. It shows that except the Nelder-Mead, mostly the obtained optimal domains are same.

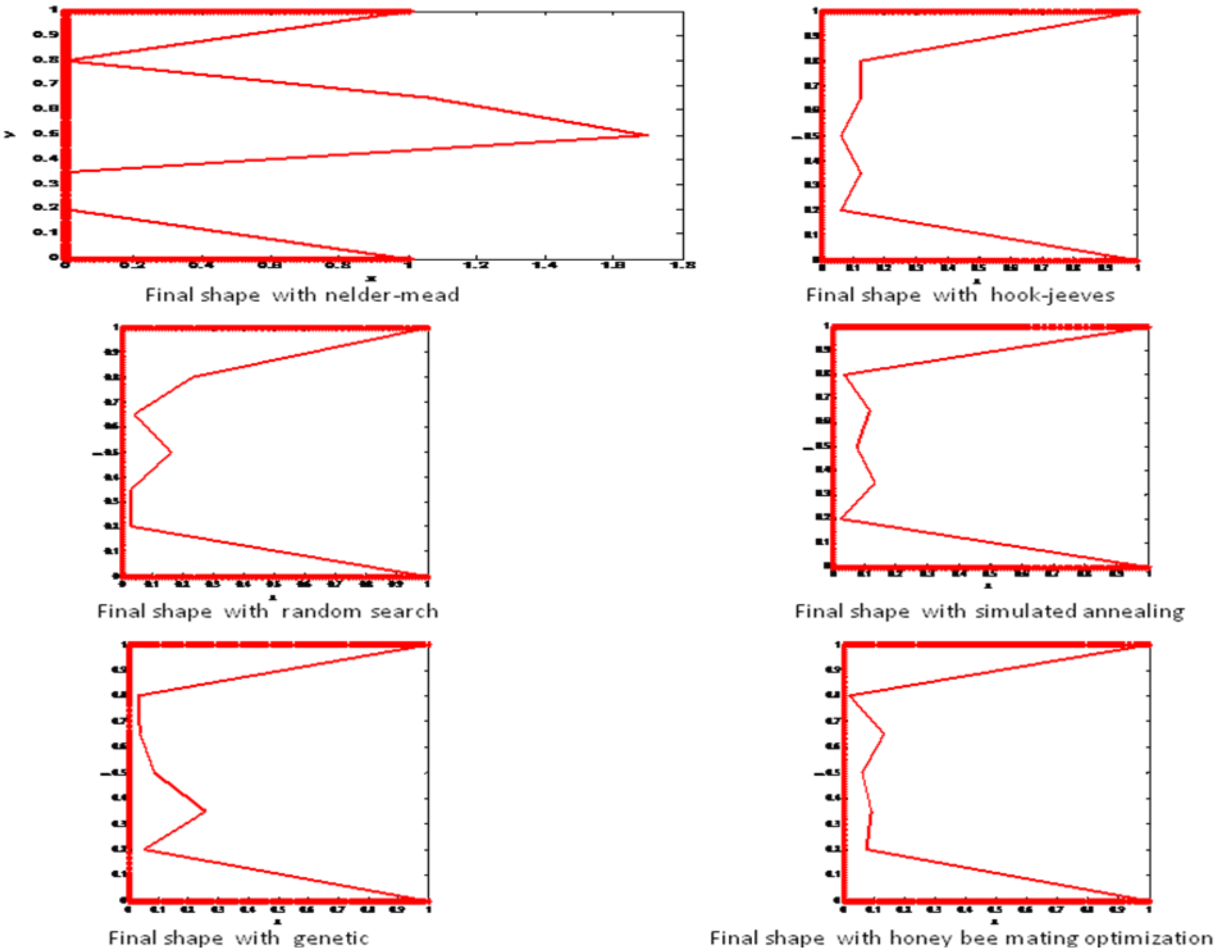

Figure 2: Optimal domains obtained by six algorithms

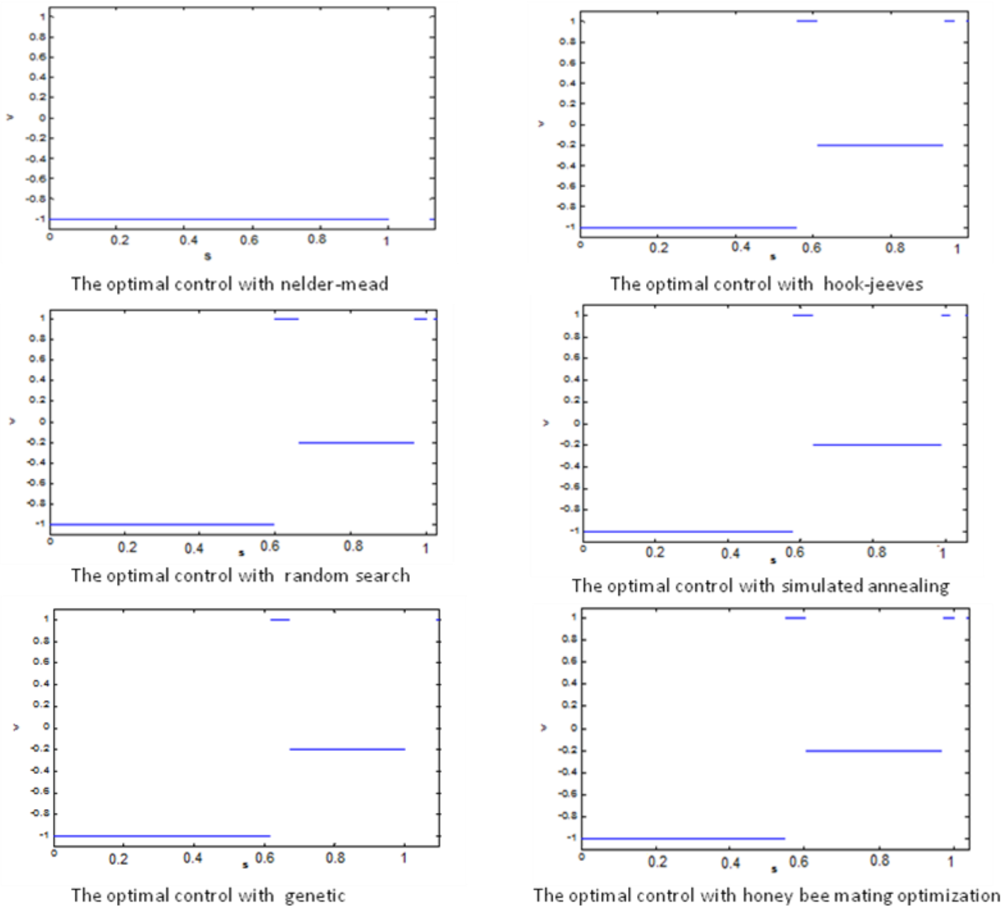

Figure 3: Optimal control functions with six algorithms

5. Conclusion

In this paper, we conduct a computational examination of several existing derivative free optimization methods to apply solution procedure of OSD problems by shape-measure technique. These methods consist of Random search, Nelder-Mead algorithm, Hook and Jeeves algorithm, Simulated annealing algorithm, Genetic and Honey bee mating optimization algorithm. The results show that Random search and Honey bee mating optimization algorithm are most appropriate for using in shape-measure method than other algorithms.

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