Journal of mathematics and computer Science 7 (2013) 54 - 62

Contents list available at JMCS

Journal of Mathematics and Computer Science

Journal Homepage: www.tjmcs.com

A Combined Algorithm for Solving Reliability-based Robust Design Optimization Problems

Ameneh Forouzandeh Shahraki,
University of Science and Technology
a.forouzandeh@gmail.com

Rassoul Noorossana
University of Science and Technology
rassoul@iust.ac.ir

Article history:
Received February 2013
Accepted March 2013
Available online April 2013

Abstract

In the most of the design optimization problems, we encounter uncertainties in design variables and problem parameters. In these problems, robustness and reliability of design are so important. Both robust design and reliability-based design approaches take into consideration these aspects. However, the individual application of them doesn't ensure the stability of product during its life cycle. In this paper, we combine both robust design and reliability-based design approaches into one model and propose a genetic and reliability analysis combined algorithm to solve this kind of problem. Moreover, to increase the efficiency of the genetic algorithm, we use the design of experiment (DOE) to find the optimal levels of the parameters of this algorithm. The application of the proposed methodology is demonstrated using a numerical example.

Keywords: Reliability, robustness, multi-objective optimization, genetic algorithm, design of experiment

1. Introduction.

Deterministic optimization techniques have been successfully applied to a large number of product design problems. However, deterministic optimal designs are pushed to the limits of design constraint boundaries and leaving no room for uncertainties. The probability of failure of such designs is high because of impact of uncertainties existed in modeling, manufacturing phase and operation condition. Therefore, such designs obtained without considering uncertainties can be unreliable and might lead to failure of the product being designed [1].

Different approaches have been developed to consider uncertainties and optimize the designs under the uncertainties. Among them, robust design optimization (RDO) and reliability based design optimization (RBDO) are two major approaches. RDO is a method for improving the quality of product through minimizing the effect of variation without eliminating the sources of variation [2]. The robustness of performance is important in this approach. On the other hand, RBDO approach emphasizes on maintaining design feasibility at different reliability levels and in order to consider the uncertainties, the constraints are modeled as probabilistic [3,4].

Robust design, originally proposed by Taguchi (1993) to improve the quality of products by minimizing the effect of variation without eliminating the source of variation. The different approaches of robust design are divided into three main categories: Taguchi's experimental design technique [2], RSM-based RD [5] general optimization techniques and [6]. The main goal of all of them is optimizing the mean and minimizing the variance of performance function. In these problems, the accuracy and efficiency of estimating the first two statistical moments of the performance function is so important. Analytically, the k^{th} statistical moment of the performance function can be obtained using the following integration:

$$E\left(\{h(X)\}^k\right) = \int_{-\infty}^{\infty} \dots . \int_{-\infty}^{\infty} \{h(X)\}^k f_X(X) dX \quad (1)$$

Where $f_X(X)$ is a joint probability density function of the random parameter X. Calculating the statistical moments of performance function using Equation (1) is practically impossible. To estimate the statistical moments three methods, dimension reduction method (DRM), performance moment integration (PMI) and percentile difference method (PDM), have been recently proposed and compared based on accuracy and computational efficiency [7]. The PMI and PDM will require the same number of function evaluation if the same inverse reliability analysis method is used. If the number of design variables is low, DRM is efficient. However, by increasing the number of design variables PMI is more efficient than two other methods. PDM uses the difference of percentile performances in different reliability levels. Thus, when the performance function is non- monotonic, PDM may obtain wrong results. The efficiency and accuracy of PMI has been proved by solving some examples in [8]. Thus, we use this method to determine the statistical moments in this article.

Du and Chen [9] classified the feasibility modeling techniques (design constraints) under uncertainty into two main categories:

1. The method which require the probability and statistical analysis (the probabilistic feasibility formulation, Monte Carlo simulation, and moment matching formulation)
2. The methods which don't require the probability and statistical analysis (the worst case analysis, the corner-space evaluation, and the variation pattern formulation)

The probabilistic feasibility formulation without considering the computational burden is the ideal method to describe the robustness of feasibility of design and can ensure that the solution achieves an accurate level of constraint satisfaction.

The main formulation of RBDO is as a minimizing an objective function subject to probabilistic constraints which ensures design feasibility under uncertainty:

$$\begin{cases} minimize f\left(d, \mu_X, \mu_p\right) \\ subject to P\left(G_i\left(d, X, P\right) \geq 0\right) \geq R_i \, i = 1, \dots, n \\ d^L \leq d \leq d^U, \mu_X^L \leq \mu_X \leq \mu_X^U \end{cases} \quad (2)$$

Where X and P are respectively the vector of random design variables and random design parameters, d is the vector of deterministic design variables, and f is the performance function (design objective). G_i represents the i^{th} constraint function and R_i is the reliability level of this constraint. If the distribution of all variables X and parameters P be known, the probability of the above equation is computed exactly using the following integration:

$$P\left[g_j(x, p) \geq 0\right] = \int_{g_j(x, p) \geq 0} f_{xp} \, dx dp \quad (3)$$

In the above equation, $f_{xp}(x,p)$ is a joint probability density function of the random variables and random parameters. However, it is practically difficult or impossible to calculate the numerical solution for the above equation. Thus, approximate procedures have been proposed to estimate this probability accurately and computationally efficiently. The available methods for reliability analysis can be classified into two categories: sampling methods such as Monte Carlo simulation, and optimization based methods [1] .The use of second methods is more affordable especially when the desired probability of failure is low. The main concept of this class of reliability assessment methods is to determine a point on the constraint boundary which is closest to the solution. This point is usually called the "Most Probable Point" (MPP) of failure [1].

The first step to compute the MPP is transforming the random variables and parameters X=(x,p) into an independent standard normal space U by Rosenblatt Transformation as below:

$$U_i = \phi^{-1}\left[F_{X_i}(X_i)\right] \tag{4}$$

Where φ^{-1} is the inverse of the standard normal distribution and F is a CDF of a general random variable X [10].

The MPP is located on the constraint boundary of the $G_j(U_x,U_p,d)=0$ and has the minimum distance to origin. This minimum distance is called reliability index (β) [10]. If the constraint function in U space is linear, the relationship between β_j (reliability index) and R_j (reliability level) is:

$$R_j = \begin{cases} \phi(\beta_j) & \text{if } R_j \geq .5 \\ 1-\phi(\beta_j) = \phi(-\beta_j) & \text{if } R_j < .5 \end{cases} \tag{5}$$

Where $\varphi()$ is the standard normal cumulative function. If the nonlinearity of $G_j(U)$ isn't large, by using the first order reliability method (FORM) the above relation is still a good approximation [11]. This method is based on linear approximation of constraint function at MPP. The problem of determining the MPP is an optimization problem and two major approaches, performance measure approach (PMA) and reliability index approach (RIA), have been proposed to find it [11,12].

In PMA approach, the following optimization problem has to be solved [13]:

$$\begin{cases} minimize G_j(U) \\ subjectto : \ \|U\| = \beta_j \end{cases} \tag{6}$$

By solving the above problem, U^* point is found on the circle of radius β_j for which the function $G_j(U)$ reaches the minimum value. Then, the original probabilistic constraint function is replaced with below equation:

$$G_j(U^*) \geq 0 \tag{7}$$

In RIA approach, the following optimization problem has to be solved [12]:

$$\begin{cases} minimize\|U\| \\ \text{subject to } \ G_j(U) = 0 \end{cases} \tag{8}$$

After finding U^* point, this point is used for replacing the original constraint by a constraint as follow:

$$\|U\| \geq \beta_j^r \tag{9}$$

The PMA is more efficient and robust than the RIA [14]. Optimizing a complex objective function under simple constraint is much easier than optimizing a simple objective function under complex constraint [11]. Thus, we choose the PMA approach to consider probabilistic constraints. The MPPIR algorithm proposed in [13] is used to find the MPP. This algorithm is applicable for any types of performance functions of random variables following any continuous distributions.

The individual application of RDO and RBDO doesn't ensure the quality and reliability of a product during its life cycle. Therefore, the concepts of them are integrated in reliability-based robust design optimization approach in order to obtain both quality and reliability.

In this work, we propose an integrated framework for optimization under uncertainty that can be used for designing reliable and robust design. The fundamental development is the employment of a combined algorithm for finding optimum solutions of the RBRDO problems. Moreover, the design of experiment (DOE) is used to find the optimum levels of this algorithm's parameters and increase the efficiency of it. After finding the reliable and robust Pareto optimal solutions, the decision makers could choose one or more of this points based on their criteria for implementation.

The remainder of the paper is organized as follows. In section 2, we present RBRDO approach. In Section 3, the methods to solve multi-objective optimization problem is reviewed and then we explain our general methodology in Section4. Section 5 presents a numerical example solved by the methodology and its results. Our concluding remarks are given in the final section.

2. Reliability- based robust design optimization

As mentioned before, it is obvious that the values of both RDO and RBDO approaches have to be combined in an integrated model to develop RBRDO approach. This approach ensures both reliability and robustness during the life cycle of products [15].

In RBRDO, the mean and variation of performance function are simultaneously minimized subject to probabilistic constraints. A typical RBRDO problem is as follows:

$$\begin{cases} \text{minimize} f(\mu_{obj}, \sigma_{obj}) \\ \text{subject to} P(G_i(d, X, P) \geq 0) \geq R_i \; i = 1, \dots, n \\ d^L \leq d \leq d^U, \mu_X^L \leq \mu_X \leq \mu_X^U \end{cases} \tag{10}$$

The RBRDO is a multi-objective optimization problem. In the most work done such as Du [13] and Sherali [16] the weighed sum of mean and standard deviation are considered as an objective function. However, this method doesn't usually lead to satisfactory results [17]. The following section provides an overview of the methods to deal with more than one objective function.

3. Multi objective optimization technique

If there is more than one objective function, it is very unlikely to find a setting for decision variables which could optimize all the objective functions simultaneously. Hence, in this situation one should search for no dominated solutions known as Pareto optimal solutions. Finding a set of solutions which are close to the true Pareto solutions and diverse enough to represent the entire spread of the Pareto optimal set, are two principal goals of multi-objective optimization [18]. Genetic algorithm (GA) is well suited to solve multi-objective optimization problems. A multi-objective GA can find a set of multiple non-dominated solutions in a single run. The ability of GA to simultaneously search different regions of a solution space makes it possible to find a diverse set of solutions for difficult problems. In addition, most multi-objective GA does not require the user to prioritize, scale, or weigh objectives. Therefore, GA has been the most popular heuristic approach to multi-objective design and optimization problems. One of well-known and frequently-used GA algorithm is non-dominated sorting genetic algorithm II (NSGAII). In NSGA II, Pareto dominance relation and a diversity maintenance mechanism are used as the primary fitness evaluation criterion [19].

4. Proposed methodology

The details of the combined algorithm of NSGA II and reliability analysis are discussed in this section. At first, the variables, objective functions, constraints functions, and the distribution function of variables are determined. Then, the problem is formulated based on RBRDO approach. At last, the combined algorithm composed of seven steps as below is used.

Step 1: In the first step the main parameters such as population size, the number of generations as stopping criterion, probability of crossover, probability of mutation, distribution index for crossover, distribution index for mutation, and reliability levels for each constraint are defined.

Step 2: The first population is initialized randomly. Then for each chromosome (individual) the objective functions are calculated and the feasibility of it will be assessed. To probe the feasibility of each chromosome, the PMA approach mentioned before is used. Every probabilistic constraint is converted to deterministic ones at MPP point ($g_j(u_{MPP})$). Then, if the constraint isn't satisfied, the constraint violation (CV) is calculated for every individual as follows:

$$CV = \sum_{i=1}^{n} |u_i(x)| \,, \; u_i = \min\{g_i(x), 0\} \tag{11}$$

Step3: The initialized population is sorted and to every individual of population a non-domination rank based on their objective values and the CV is assigned.
Until the stopping criterion isn't satisfied the following steps are done.

Step4: Binary tournament selection based on crowded comparison operator is used to select parents for reproduction.

Step5: The genetic operators (crossover and mutation) are done to generate an offspring population of parent population. For this population, as step 2 the objective functions are calculated and feasibility is assessed.
Step6: The combined population of parent population and offspring population is sorted and a non-domination rank and crowding distance are assigned to each of the chromosomes.

Step 7: The first N best solutions based on their ranks and crowding distances are chosen and replaced to initial population.

Adding the reliability analysis increases the time of computation of algorithm. Therefore, it is needed to determine the optimum levels of main parameters of NSGA II. In [20] an extensive parametric study was conducted by varying one design parameter at a time. The parameters considered independent which not a correct hypothesis is and it is needed to consider the interaction between them. Moreover, the number of experiments is increased by investigating the more numbers of factors. When performing an experiment, varying the levels of the factors simultaneously rather than one at a time is efficient in terms of time and cost, and also allows for the study of interactions between the factors.
The probability of crossover, probability of mutation, distribution index for crossover, and distribution index for mutation are the parameters which we want to determine the best levels of them to obtain the Pareto optimal solutions in reasonable time. Factorial designs are suitable for simultaneous studying and determining the effects of several factors.
As mentioned before, convergence to the Pareto optimal solutions and maintenance of diversity in solutions of the Pareto optimal set are two goals in a multi-objective optimization approach. Two performance metrics have been proposed to evaluate each of the two goals [19]. Therefore, these metrics are considered as response variables and we want to find the factor settings of algorithm that minimize these responses variables.
The average distance between non-dominated solutions found and the known Pareto optimal solutions is used as the first response variables (Y_1). This distance is computed as below:

$$d_i = \min_{j=1}^{n} \sqrt{\sum_{k=1}^{m} \left(\frac{f_k(i)-f_k(j)}{f_k^{max}-f_k^{min}}\right)^2} \tag{12}$$

Where n is the numbers of Pareto optimal solutions, m is the numbers of objective functions, fkmax is the maximum of the kth objective function ,and fkmin is the minimum of the kth objective function among all the Pareto points.
The spread metric is used for evaluating diversity among the non-dominated solutions. The smaller this metric the better diverse set of non-dominated solutions. Therefore, the second response variable (Y_2) is defined as below:

$$Y_2 = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}} \tag{13}$$

Where d_f and d_l are the Euclidean distances between the extreme solutions and the boundary solutions of the obtained non-dominated solutions, d_i is Euclidean distance between consecutive solutions in the obtained non-dominated solutions, and đ is the average of all distances.
The desirability function approach is used to identify the combination of input variables settings that optimize two response variables. In this approach, each response is transformed to a scaled free value

mi that is called desirability (Equation 14). It changes from 0 to 1. The total desirability (D) is obtained by compositing all desirabilites (Equation 15). Then, the optimal input variable settings are identified by maximizing the composite desirability [21].
Desirability function for the smaller-the-better case is as following:

$$\begin{cases} 0 & \hat{y}_i(x) > y_i^{max} \\ \left[\frac{y_i^{max} - \hat{y}_i(x)}{y_i^{max} - y_i^{target}}\right]^{r_i} & y_i^{target} \leq \hat{y}_i(x) \leq y_i^{max} \\ 1 & \hat{y}_i(x) < y_i^{target} \end{cases} \tag{14}$$

$$D = \left(\prod\left(m_i^{wi}\right)\right)^{1/w} \tag{15}$$

Where r_i is the weight of desirability function, w_i is the importance degree of ith response variables, and w is the sum of the weights of all response variables.
After optimization stage, the Pareto solutions are obtained and the decision makers have to choose the best ones based on their criteria. By solving the multi-objective optimization problem for different levels of reliability, we gain useful information.

5. Numerical example

As an example, the following mathematical problem in Chen et al. [22] is used:

$$\begin{cases} \min f(x) = (x_1 - 4)^3 + (x_1 - 3)^4 + (x_2 - 5)^2 + 10 \\ \quad st: g(x) = x_1 + x_2 - 6.45 \geq 0 \\ \qquad 1 \leq x_1 \leq 10 \\ \qquad 1 \leq x_2 \leq 10 \end{cases} \tag{16}$$

All random variables are statistically independent and have normal distribution with standard deviation of 0.4. The corresponding reliability-based robust design optimization problem is formulated as below:

$$\begin{cases} \min f(x) = (x_1 - 4)^3 + (x_1 - 3)^4 + (x_2 - 5)^2 + 10 \\ \quad st: g(x) = x_1 + x_2 - 6.45 \geq 0 \\ \qquad 1 \leq x_1 \leq 10 \\ \qquad 1 \leq x_2 \leq 10 \end{cases} \tag{17}$$

In order to investigating the four parameters of algorithm, the factorial design 2^4 with 16 runs were performed. The upper and lower levels of parameters and the results of simulation were shown in table 1 and table 2, respectively.
After determining the upper levels, target values, the weights, and the importance of response variables, we used the Minitab Response Optimizer to help identify the combination of parameter settings that jointly optimize the composited desirability of responses.
The results are shown in table 3. The individual desirability for Y_1 and Y_2 are 0.90645 and 0.62098, respectively. To obtain the composite desirability (0.75026) for both variables, the levels of factors have to be set at the values shown under "Global Solution" section of table 3.

Table 1. The lower and upper level of parameters

Parameter	Lower level	Upper level
probability of crossover	0.5	1
distribution index for crossover	50	50
probability of mutation	0.1	0.5
distribution index for mutation	50	300

Table 2. The results of simulation

Run	Factor				Response variable	
	A	B	C	D	Y_1	Y_2
1	0.5	5	0.5	300	0.014747	0.66125
2	1	50	0.1	300	0.003231	1.05879
3	1	5	0.1	300	0.000252	0.53099
4	1	5	0.5	300	0.000378	0.42447
5	0.5	50	0.5	50	0.000227	0.51696
6	1	50	0.1	50	0.015631	0.53981
7	0.5	50	0.5	300	0.022947	0.7169
8	1	5	0.1	50	0.000632	0.5048
9	0.5	50	0.1	50	0.000933	0.7757
10	0.5	5	0.5	50	0.001246	0.5801
11	1	50	0.5	300	0.017622	0.86159
12	1	5	0.5	50	0.004267	0.4543
13	0.5	50	0.1	300	0.018391	0.77419
14	0.5	5	0.1	50	0.003109	0.62815
15	1	50	0.5	50	0.00521	0.58566
16	0.5	5	0.1	300	0.011385	0.70673

Table 3. The result of optimizing two response variables

Response Optimization

Parameters	Goal	Lower	Target	Upper	Weight	Importance
Y_1	Min	0	0	0.023	1	1
Y_2	Min	0	0	1.060	1	1

Global Solution

A = 1.0

B = 5.0

C = 0.5

D = 50.0

Predicted Responses

Y_1 = 0.00215، desirability = 0.90645

Y_2 = 0.40176، desirability = 0.62098

After setting the parameters of algorithm at these levels, the problem was solved for three different reliability levels for probabilistic constraint with β equal to 1.28 (90%), 3 (99.875%), and 4 (99.875%). Figure.1 presents the reliable and robust Pareto optimal solutions for different reliability levels (β=1.285, 3, 4). As we see, by increasing the level of reliability the value of objective functions increase and the Pareto solutions move inside feasible objective space. Hence, compromise must be made between reliability and performance. The points inside the circle are less sensitive to the level of

reliability and the decision maker has to pay attention to them during choosing some points for implementation.
To confirm the probability of constraint satisfaction of obtained results, Monte Carlo simulation by ten thousand sample size is done. A normal distribution, whose mean is fixed at the Pareto optimal solution and the standard deviation at the 0.4, was used. The probability of the constraint satisfaction at the four Pareto optimal points which chosen uniformly are summarized in Table 4.

Fig.1. The Pareto optimal set at different reliability level

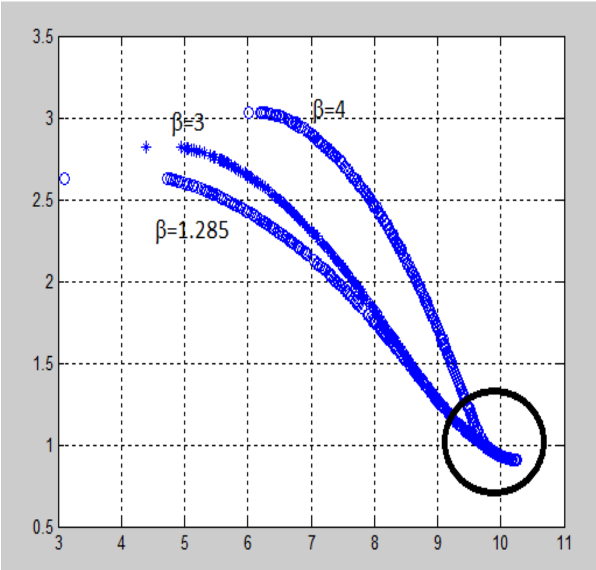

Table. 4 Monte Carlo simulation results for two levels of reliability.

β=1.285 (90%)				
Points	(x1‹x2)=(5.01,2.24)	(x1‹x2)=(5.01,2.63)	(x1‹x2)= (5, 3.01)	(x1‹x2)=(5.55,3.45)
Probability of constraint satisfaction (%)	94.402	98.36	99.73	100
β=3 (99.865%)				
Points	(x1‹x2)=(6.02,2.12)	(x1‹x2)=(5.5,2.56)	(x1‹x2)=(5.14,3.0)	(x1‹x2)=(5.55,3.45)
Probability of constraint satisfaction (%)	99.866	99.88	99.898	100

6. Conclusions

In this paper, we combined three concepts of robust design, reliability based design and multi-objective optimization in order to optimization under uncertainty. Then, we described a new algorithm which is a combined of NSGA II and reliability analysis to determine the Pareto optimal solutions of RBRDO problems. Moreover, the design of experiments is used to determine the optimal levels of algorithm's parameters in order to increasing the efficiency and decreasing the computational cost of reliability analysis. Here, the design variables considered independent which this assumption is not correct in some problems. Therefore, we can extend our approach to be applicable for more design problems.

References

[1] Z.P. Mourelatos, J. Liang, A Methodology for Trading-off Performance and Robustness under Uncertainty, Proceeding of ASME Design Engineering Technical Conference, **(2005)**.

[2] M.S. Hadke, Quality Engineering using Robust Design, Prentice-Hall, Englewood Cliffs,NJ, **(1989)**.
[3] KC. Kapur, LR, Lamberson, Reliability in Engineering Design, Wiley Publishing, **(1977)**.
[4] H. Agarwal, Reliability based design optimization: Formulations and Methodologies, Ph.D.Thesis, University of Notre Dame, **(2004)**.
[5] G.G. Vining, R.H. Myers, Combining Taguchi and response surface philosophies: A dial response approach, Journal of Quality Technology 22 38-45 **(1990)**.
[6] W. Chen, M.M. Wiecek, J. Zhang, Quality Utility – A Compromise Programming Approach to Robust Design, ASME Journal of Mechanical Design 121 179–187 **(1999)**.
[7] I. Lee, K.K. Choi, L. Du, D. Gorsich, Dimension reduction method for reliability-based robust design optimization, Computers and Structures 86 1550-1562 **(2008)**.
[8] B.D. Youn, K.K. Choi, K. Yi, Performance moment integration (PMI) method for quality assessment in reliability-based robust optimization, Mechanics Based Design of Structures and Machines, 33 185–213 **(2005)**.
[9] X. Du, W. Chen, Towards a Better Understanding of Modeling Feasibility Robustness in Engineering Design, Mechanical Design Journal, 122 358-394 **(2000)**.
[10] X. Du, W. Chen, A most probable point-based method for efficient uncertainty analysis, Design and Manufacturing Automation, 4 47–66 **(2001)**.
[11] J. Tu, K.K. Choi, Y.H. Park, A New Study on Reliability-Based Design Optimization, Mechanical Design (ASME) 121 557–564 **(1999)**.
[12] E. Nikolaidis, R. Burdisso, Reliability-based optimization: a safety index approach, Computer Structer 28 81–788 **(1988)**.
[13] X. Du, A. Sudjianto, W. Chen, An integrated framework for optimization under uncertainty using inverse reliability strategy, Mechanical Design (ASME) 126 5 62–70 **(2004)**.
[14] B.D. Youn, K.K. Choi, R.J. Yang, L. Gu, Reliability-based design optimization for crash-worthiness of vehicle side impact, Journal of Structural and Multidisciplinary Optimization, 26 272–83 **(2004)**.
[15] O.P. Yadav, S.S. Bhamare, A. Rathore, Reliability-based Robust Design Optimization: A Multi-objective Framework Using Hybrid Quality Loss Function, Journal of Quality and Reliability Engineering International 26 27-41 **(2010)**.
[16] H.D. Sherali, V. Ganesan, An Inverse Reliability-based Approach for Designing under Uncertainty with Application to Robust Piston Design, Journal of Global Optimization 37 47-62 **(2007)**.
[17] I. Das, J. Dennis, A Closer Look at Drawbacks of Minimizing Sums of Objectives for Pareto Set Generation in Multi criteria Optimization, structural Optimization14 63-69 **(1997)**.
[18] R.T. Marler, J.S. Arora, Survey of multi-objective optimization methods for engineering, structure multidisciplinary optimum 26 369-395 **(2004)**.
[19] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Transactions on Evolutionary Computation 6 182–97 **(2002)**.
[20] T. Goel, R. Vaidyanathan, R.T. Haftka, N.V. Queipo, W. Shyy, K. Tucker, Response surface approximation of Pareto optimal front in multi-objective optimization, Proceedings of the 10th AIAA/ ISSMO Multidisciplinary Analysis and Optimization Conference, **(2004)**.
[21] G. Derringer, R. Suich, Simultaneous Optimization of Several Response Variables, ", Journal of Quality Technology 12 214-219 **(1980)**.
[22] W. Chen, M.M. Wiecek, J. Zhang, Quality Utility – A Compromise Programming Approach to Robust Design, ASME Journal of Mechanical Design 121 179–187 **(1999)**.