Journal of mathematics and computer Science 7 (2013), 181-195

Contents list available at JMCS

Journal of Mathematics and Computer Science

Journal Homepage: www.tjmcs.com

A New Method for Face Recognition using Feature Clustering with Fuzzy Parameters

Soumak Biswas
Department of Mathematics, National Institute of Technology, Jamshedpur
sbiswas.nitjsr@yahoo.com, sbiswas.math@nitjsr.ac.in

T. Som
Department of Applied Mathematics, Indian Institute of Technology (BHU), Varanasi
tsom.apm@itbhu.ac.in

Article history:
Received March 2013
Accepted May 2013
Available online May 2013

Abstract

In this paper, we have applied Gabor filter for fiducial point localisation. The fiducial points are represented as trapezoidal fuzzy numbers. These fiducial points are then transformed into crisp numbers. The number of fiducial points are then reduced by using a distance formula. The distance of each of these fiducial points are then stored in the database of the system. The same methodology is applied on the input face which is to be matched with the faces available in the database. Then a fuzzy preference relation matrix is obtained .The largest eigen value of this matrix is then determined. Once the largest eigen value is determined the corresponding priority vector can easily be obtained, from which we can easily match the input face with the database. In the broadest sense we have observed we have used the fuzzy mathematics to counter the impreciseness in facial recognition.

Keywords: Fiducial point, Crisp number, Priority Vector, Decision making (DM), Largest Eigen value.

1. Introduction

Facial recognition system is one of the most remarkable abilities human being enjoys. Machine recognition of faces has recently emerged as one of the most important area amongst researchers both in mathematics and computer science. Initially the machine recognition was done with the help of simple geometrical models but off late it has developed further and matured and this simple geometrical model is now replaced by more logical mathematical formulae and equations. The use of facial recognition are immense for example facial signature can efficiently replace the use of pin numbers and passwords. Applications where face recognition can be used efficiently include secure transactions in e and m commerce. A facial recognition system represents a computer driven application for automatically authenticating a person from a digital image or a video sequence. It performs the recognition by comparing selected facial characteristics in the input image with a face database. Any recognition process is divided into two main operations: face identification and face verification. Face recognition techniques can further be divided into two categories: geometric or photometric approaches. Geometric approaches consists in distinguishing individual features such as eyes, nose, mouth, and head outline and developing a face model based on position and size of these characteristics. Photometric approaches are statistical techniques that distil an image into values and compare these values with templates. Whereas Gabor filters are capable of deriving multi-orientation information from a face image at different scales, with the derived information being of purely local nature. The common approach when using Gabor filter for face recognition is to construct a filter bank with filters of different scales and orientations and to filter the given face image from all the filters in the bank. Obviously such an approach results in explosion of information as the dimensionality of the input face image is increased by a factor equalling the number of factors in the filter bank. The amount of data (in the Gabor face representation) is commonly reduced to a more manageable size by exploiting various down sampling, feature selection and subspace projection techniques before it is finally fed to a classifier (Struck et.al.).On the other hand Fuzzy preference relation has been time and again used effectively by many authors for decision making the concepts of complete fuzzy relation and incomplete fuzzy relations [20] give two different approaches for decision making based on the information supplied by the decision maker (DM) [1] in our case the machine. In the present paper, the trapezoidal fuzzy numbers are used to locate the fiducial points which are then converted to crisp numbers by a method,[3] then the decision making is being made by using the fuzzy preference relation technique [22], for which the distance matrix of the fiducial points are obtained, to locate the fiducial points Gabor filters are applied on the face images. Finally the largest eigen value is determined for the distance matrix from which the decision making is done [4][2]. An algorithm to determine the largest eigen value for higher order distance matrix is also proposed.

2.Existing Work

There are numerous face recognition techniques that have been proposed in the last few decades . This existing techniques can be broadly divided into few categories [5].Due to the simplicity and the efficiency for the feature extraction and representation, many subspace analysis methods, such as principal component analysis PCA, linear discriminant analysis LDA, independent component analysis ICA and locality preserving projections LPP, are widely used for face recognition [8].In 1960, digital image processing was proposed with semi automated systems. In order to locate the major features different marks were made on the photographs. These system uses features like mouth, eyes, and nose. Next to get the common reference points the ratios and distances were calculated and then compared with the database [15]. In 1970's Harmon et al [12] introduced their system . The system had 21 subjective makers having tip thickness and hair colour. This was difficult while automating the system because many complex measurements were made by hand. Another approach is connectionist technique which is used to categorize human faces using two things which are gestures and a set of classifying markers. This technique is normally applied on neural network principles and 2-dimensional pattern recognition. In normal networks a huge training database of faces is needed which requires too much time to train the whole system to get the desired results. In [15] proposed a mechanized technique for general pattern recognition. However a survey of literature on the research work focusing on various potential problems and challenges in the face detection and recognition can be found in [20]. Face detection using fuzzy pattern matching method based on skin and hair colour has been prepared by [19] . This method has high detection rate, but it fails, if the hair colour is not black and face regions not elliptic. In [13] the face regions are detected using quantized skin colour region merging and wavelet packet analysis. This method efficiently detects the faces with different size and poses. However it is computationally expensive due to it's complex segmentation and higher detection time [13] .

3.Proposed Work

In the proposed system we use the Gabor filter for face recognition. Gabor Wavelets are widely and successfully used in the face recognition due to their biological relevance and computational properties. The Gabor wavelets can be defined as follows [15] .

$$w(x,y) = fe^{\frac{x'^2+y'^2}{zx^2}\{Cos(2\pi f x'+\theta)-DC\}} \quad (1)$$

Where, $DC = Cos\theta . e^{-2\pi\frac{e^2}{f^2}}$

and $\begin{cases} x' = xcos\theta + ysin\theta \\ y' = -xsin\theta + ycos\theta \end{cases}$

The factor $\sigma = k.f$makes sure that the filter spatial range of action is partial corresponding to the central frequency "f".In this equation "σ" is frequency of filter, "θ" are 8 different orientations in the filter, a discrete recognition of 1, using five dissimilar scales and eight angles are engaged and in the result of it 40 Gabor filters are acquired. In the end of equation 1 the term DCcreates the filter DC free. N each filtered image we find the maximum intensity points dynamically and are then marked as fiducial points (landmarks on faces). The stepwise rules for the method which is being proposed here in this paper can be schematically shown from the figure given below.

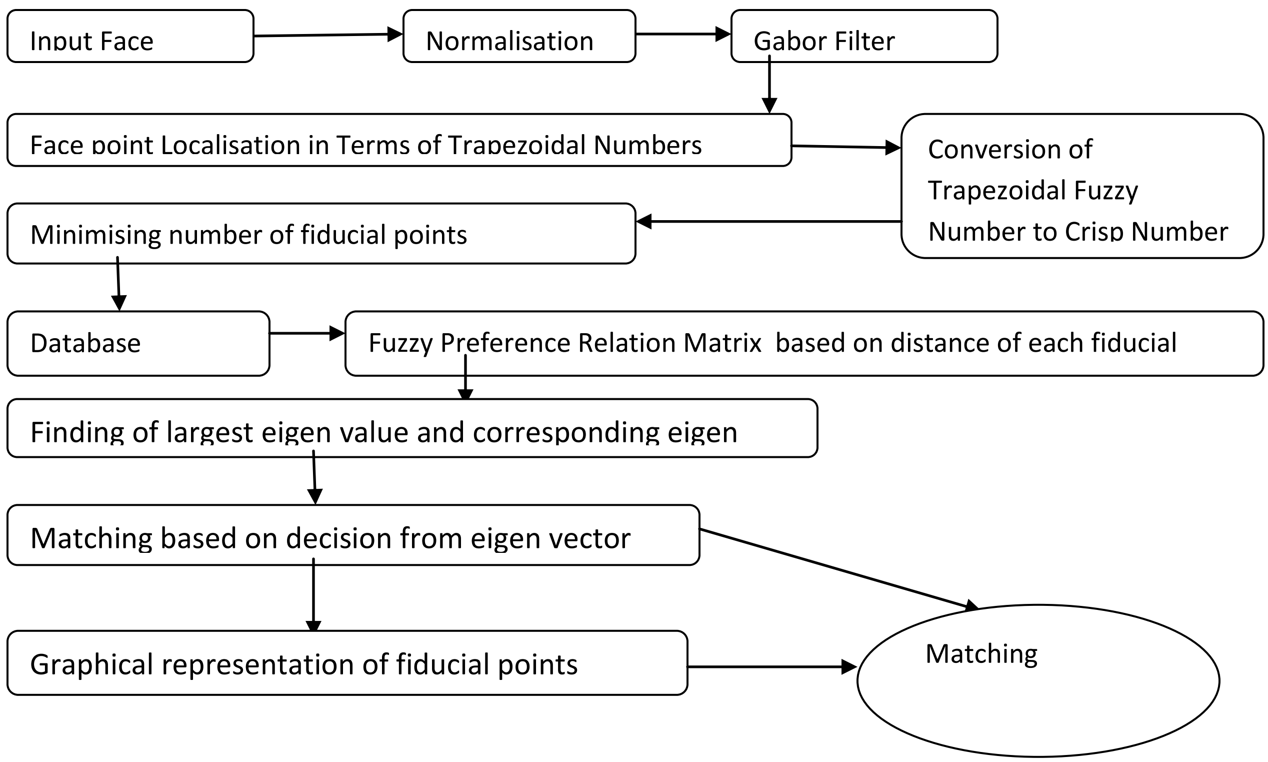

Figure I:The flow chart of the proposed method

The fiducial points are first represented in terms of trapezoidal fuzzy numbers. Representing the fiducial points in terms of fuzzy numbers is very advantageous and more practical, as it helps us to accommodate the possibility of changes which a human face can go with time to some extreme levels. Which in fact is more practical as every human face undergoes certain changes with time. These trapezoidal numbers are then transformed into crisp numbers , by a formula. The leftmost corner point is taken as, the origin and with respect to these the distance of other points are calculated by using Euclidean distance formula. These distance data of each points for every face is then stored in the database. Next in the face that is to be matched , the same method is used and the distance of each fiducial points are calculated. Now the distance of each fiducial point is compared with the already stored data and a complete fuzzy preference relation matrix is obtained. The largest eigen value of this matrix is obtained and the corresponding eigen vector is determined from which decision is being made [21][22]. The result obtained above is also established from the graph of each fiducial points of all the persons in the database when compared with each fiducial point of the input face.

3.1 Definition of fuzzy numbers

There are many type of fuzzy sets, amongst them the set that are defined on the set of real numbers are of special significance. Membership functions of these sets have the form,

$$B: R \rightarrow [0,1].$$

These quantitatively mean under certain conditions these sets may be viewed as fuzzy numbers or fuzzy intervals.

A fuzzy set B on R must possess at least the following three properties to become a fuzzy number

i) B must be a normal fuzzy set.

ii)α_B must be a closed interval for every

$\alpha \in (0,1]$.

iii)The support of A, $0 +_A$ must be bounded.

3.2 Trapezoidal fuzzy number

A fuzzy set $\tilde{B}$ is a fuzzy number denoted by $(\delta, m.n, \beta)$ where $\delta, m.n$ and β are real numbers and its membership function $\mu_B(x)$ is as defined below [3][2],

$$\mu_B(x) = \begin{cases} 0 & \text{for } x \leq \delta \\ \frac{(x-\delta)}{(m-\delta)} & \text{for } \delta \leq x \leq m \\ 1 & \text{for } m \leq x \leq n \\ \frac{(n-x)}{(\beta-x)} & \text{for } n \leq x \leq \beta \\ 0 & \text{for } x \geq \beta \end{cases} \tag{1}$$

According to the above mentioned definition of trapezoidal number, let $\tilde{B} = \left(\underline{B(r)}, \overline{B(r)}\right)$ where $(0 \leq r \leq 1)$ be a fuzzy number, then the value $M\left(\overline{B}\right)$, is assigned to $\tilde{B}$ is calculated as follows [3]:

$$M_0^{Tra}(\tilde{B}) = \frac{1}{2}\int_0^1 \left\{\underline{B(r)} + \overline{B(r)}\right\} dr$$

$$= \frac{1}{4}(m + n + \delta + \beta) \tag{2}$$

which is very convenient for calculation.

If $\tilde{B}_\omega = \left(\underline{B(r)}, \overline{B(r)}\right) = \left(\delta + \frac{(m-\delta)}{(\omega)} r, \beta + \frac{(n-\beta)}{\omega} r\right)$ be an arbitrary trapezoidal fuzzy number at decision level higher than α and $\alpha, \omega \in [0,1]$.

3.3 Fuzzy Preference Relation

In the process of decision making, a DM generally needs to compare a set of decision alternatives with respect to a criterion and construct a preference relation. Complete fuzzy preference relation is a common preference relation which can be described as follows [4].

Let $X = \{x_1, x_2, x_3, \dots \dots \dots . . x_n\}$ be a finite set of alternatives .A complete fuzzy preference relation "P" on "X" is a fuzzy set on the product set "$X \times X$" that is characterised by a membership function $\mu_P: X \times$

$X \to [0,1]$. The DM (Machine in our case) preferences on "X" are given by a complete fuzzy preference relation $P = (p_{ij})_{n\times n}$, where,

$$p_{ij} \in [0,1],\ p_{ij} + p_{ji} = 1,\ p_{ii} = 0.5\ \forall\ i, j. \tag{3}$$

And p_{ij} denotes the preference degree or the intensity of the alternative x_i over x_j. In particular, $p_{ij} = 0$ indicates that x_j is absolutely preferred to x_i; , $p_{ij} < 0.5$ indicates that x_j is preferred to x_i. $p_{ij} > 0.5$ Indicates that x_i is preferred to x_j. $p_{ij} = 0.5$ Indicates indifference between x_i and x_j.

(Saaty.T.L,1980) introduced the well known eigen vector method to determine the priority vector "w" of the general multiplicative preference relation :

$$Aw = \lambda_{max} w, \tag{4}$$

Where λ_{max} is the largest eigen value of A.

Similarly let $w = (w_1, w_2, \dots\dots\dots\dots.. w_n)^T$ be the priority vector of the fuzzy preference relation $P = (p_{ij})_{n\times n}$ where, $w_i > 0, i = 1,2, \dots..n.$ $\sum_{i=1}^{n} w_i = 1.$ (5)

If $P = (p_{ij})_{n\times n}$ is a multiplicative consistent complete fuzzy preference relation, then such a preference relation is given by, [20].

$$p_{ij} = \frac{w_i}{w_i + w_j}\ \ \forall\ i, j. \tag{6}$$

In this case P can be expressed as follows

$$P = \begin{bmatrix} \frac{w_1}{w_1+w_1} & \frac{w_1}{w_1+w_2} \dots\dots.. & \frac{w_1}{w_1+w_n} \\ \vdots & \dots\dots\dots\dots\dots\dots\dots & \vdots \\ \frac{w_n}{w_n+w_1} & \frac{w_n}{w_n+w_2} \dots\dots\dots & \frac{w_n}{w_n+w_n} \end{bmatrix} \tag{7}$$

From equation 7 It follows that the following system of equation can be given as,

$$\frac{w_i}{w_i+w_1}(w_i + w_1) + \frac{w_i}{w_i+w_2}(w_i + w_2) + \cdots \dots\dots\dots\dots\dots + \frac{w_i}{w_i+w_n}(w_i + w_n) = nw_i \tag{8}$$

4.Implementation

The whole methodology proposed above is implemented on an input face as shown below.

Figure II: The Input Face

4.1 Normalisation

The following are the steps for image normalization which is applied over all the stored images in the database as well as the input face which is to be recognised. The following are the steps of normalization [15].

1.Rescaling to 128× 128

2.Pixels adjustment:

In this step, image pixel intensities are used, such that the standard deviation of image pixel is one and mean zero.

3.Boundaries are smoothed.

4.2 Gabor filter

In this step Gabor filter is applied on the input face [18]. The sample 40 Gabor filters are shown below in fig (II) [18] . This figures are obtained using five dissimilar scales and eight angles are engaged and in result of it 40 Gabor filters are acquired fig(III).

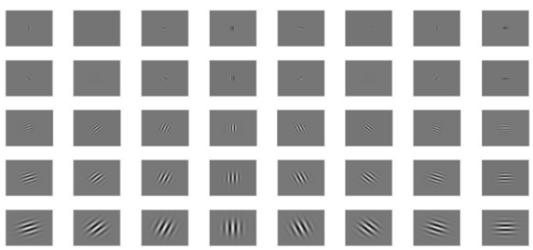

Figure III: Gabor Filter

4.3 Applying filter on the image

When the Gabor filter in 4.2 is applied over the input face in Fig.II

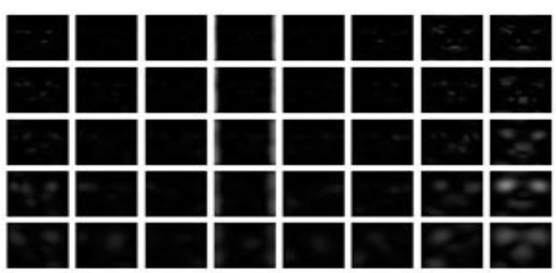

Figure IV: After applying Gabor Filter

$$J_{i,j}(x,y) = I(x,y) \times g_{ij}(x,y) \tag{9}$$

Where $I(x, y)$ is the input image and $g(x, y)$ is the resultant image of Gabor filter. When the original image with Gabor filter is multiplied, a new image is acquired which is equal to $J(x, y)$, where x *and* y are the height and width of the image respectively from (Shariff.M et.al.2011).

4.4 Fiducial point localization

After applying the Gabor filter 40 images are obtained with different angles and orientations. Then the maximum intensity points are found in each image. After that 40 points on image are calculated as shown in figure below. To find these face points, the following equation is used.

$\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\left(max\left(X_{ij}\right)\right)$ where, $X = I_{i,j}$

$$I = Intensity\ of\ coordinate\ i, j.$$

Where, $N_1$and N_2 are the width and height of image. By using this equation one point is found on each image out of 40 Gabor filtered images.

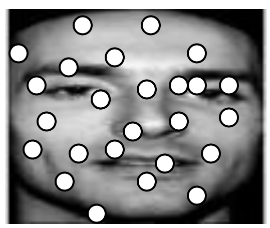

Figure V: The circles in the face showing intensity points

Then the distance is calculated to minimize the points. The minimum distance possible between two points is defined to minimize the number of points.

The reduced number of points are shown below in the figure (VI).

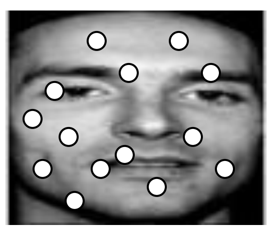

Figure VI: The reduced number of points are marked

4.5 Distance matrix for fiducial points in terms of fuzzy preference relation

We take three fiducial points of three different persons and express them in terms of trapezoidal fuzzy number as shown below in Table I, and obtain the distance matrix in terms of fuzzy preference relation, of these three persons Table IV .

M1	M2	M3

F1	(1,2,3,4)	(2,3,4,5)	(1,2,3,4)
F2	(0,1,2,3)	(1,3,4,5)	(0,2,3,4)
F3	(1,2,3,5)	(1,2,3,4)	(0,2,3,4)

Table I(Three different fiducial points expressed in terms of Trapezoidal fuzzy numbers for three persons stored in the database)

Using equation 2 above we convert the above matrix into a matrix of crisp numbers as follows

$\widetilde{A_{11}}$	$M_0^{Tra}(\widetilde{A_{11}})$=**2.5**
$\widetilde{A_{12}}$	$M_0^{Tra}(\widetilde{A_{12}})$=3.5
$\widetilde{A_{13}}$	$M_0^{Tra}(\widetilde{A_{13}})$=2.5
$\widetilde{A_{21}}$	$M_0^{Tra}(\widetilde{A_{21}})$=1.5
$\widetilde{A_{22}}$	$M_0^{Tra}(\widetilde{A_{22}})$=3.25
$\widetilde{A_{23}}$	$M_0^{Tra}(\widetilde{A_{23}})$=2.25
$\widetilde{A_{31}}$	$M_0^{Tra}(\widetilde{A_{31}})$=2.75
$\widetilde{A_{32}}$	$M_0^{Tra}(\widetilde{A_{32}})$=2.5
$\widetilde{A_{33}}$	$M_0^{Tra}(\widetilde{A_{33}})$=2.25

Table:-II (Elements of Table I in terms of Crisp numbers)

Applying the same methodology to convert the fiducial points of input person to corresponding crisp numbers we have, Corresponding crisp set which has been obtained by using equation 2 above as follows,

$\widetilde{A_{41}}$	$M_0^{Tra}(\widetilde{A_{41}})$=**3.25**

$\widetilde{\boldsymbol{A}_{42}}$	$M_0^{Tra}(\widetilde{A_{42}})$=2.25
$\widetilde{\boldsymbol{A}_{43}}$	$M_0^{Tra}(\widetilde{A_{43}})$=2.50

TableIII:-(Fiducial point of person to be identified in terms of Crisp numbers)

Using the formula,

$$d(A_{ij}) = \left|M_0^{Tra}(\widetilde{A_{4J}}) - M_0^{Tra}(\widetilde{A_{\iota J}})\right| \tag{10}$$

where, $i = 1,2,3$ and $j = 1,2,3$ we obtain the fuzzy distance matrix as follows,

		M1	**M2**	**M3**
M4	F1	0.50	0.25	0.75
	F2	0.75	0.50	0
	F3	0.25	0	0.50

Table:-IV(Association of Fiducial points of person to be identified with fiducial point of other three persons)

By using equation (3) above and $\sum_{i=1}^{3} w_i = 1$, we can establish the following system of equations.

$$w_i > 0 \ ; \quad i = 1,2, \ldots \ldots n.$$

$$p_{11}(w_1 + w_1) + p_{12}(w_1 + w_2) + p_{13}(w_1 + w_3) = \lambda_{max} w_1$$

$$p_{21}(w_2 + w_1) + p_{22}(w_2 + w_2) + p_{23}(w_2 + w_3) = \lambda_{max} w_2$$

$$p_{31}(w_3 + w_1) + p_{32}(w_3 + w_2) + p_{33}(w_3 + w_3) = \lambda_{max} w_3$$

$$2w_1 + 0.25w_2 + 0.75w_3 = \lambda_{max} w_1$$

$$0.75w_1 + 1.75w_2 + 0.w_3 = \lambda_{max} w_2$$

$$0.25w_1 + 0.w_2 + 1.25w_3 = \lambda_{max} w_3$$

Solving the above equations by adding all the three and using the equation

$w_1 + w_2 + w_3 = 1$, we obtain

$$\lambda_{max} = 3$$

Therefore the priority vector $w = (w_1, w_2, w_3) = (1,0,0)$.

Therefore $w_1 > w_2 \geq w_3$.

Therefore the face of man M4 (Input face) resembles that of man (M1) in the database.

4.6 Algorithm to determine the largest eigen value

Step1. Start the program

Step2. Read n and the given matrix (a_{ij}) for $i = 1(1)n \;\; J = 1(1)n$;

Step3: For $i = 1(1)n$, set $x_i = 1$

Step4: For $k = 1(1)20$

Step5: For$i = 1(1)n$

Step6: Set $y_i = 0$

Step7: For $j = 1(1)n$

Step8: set y_i=$y_i + a_{ij}x_j$

Step9: Next , Next i

Step10: $m = y_1$

Step11: For $i = 2(1)n$

Step12: If $< y_i$, set $m = y_i$

Step13: Next i

Step14: For $i = 1(1)n$ set $x_i = \frac{y_i}{m}$

Step15: Write the numerically largest eigen value.

Step16. Stop.

5.Graphical Comparison of Fiducial Points

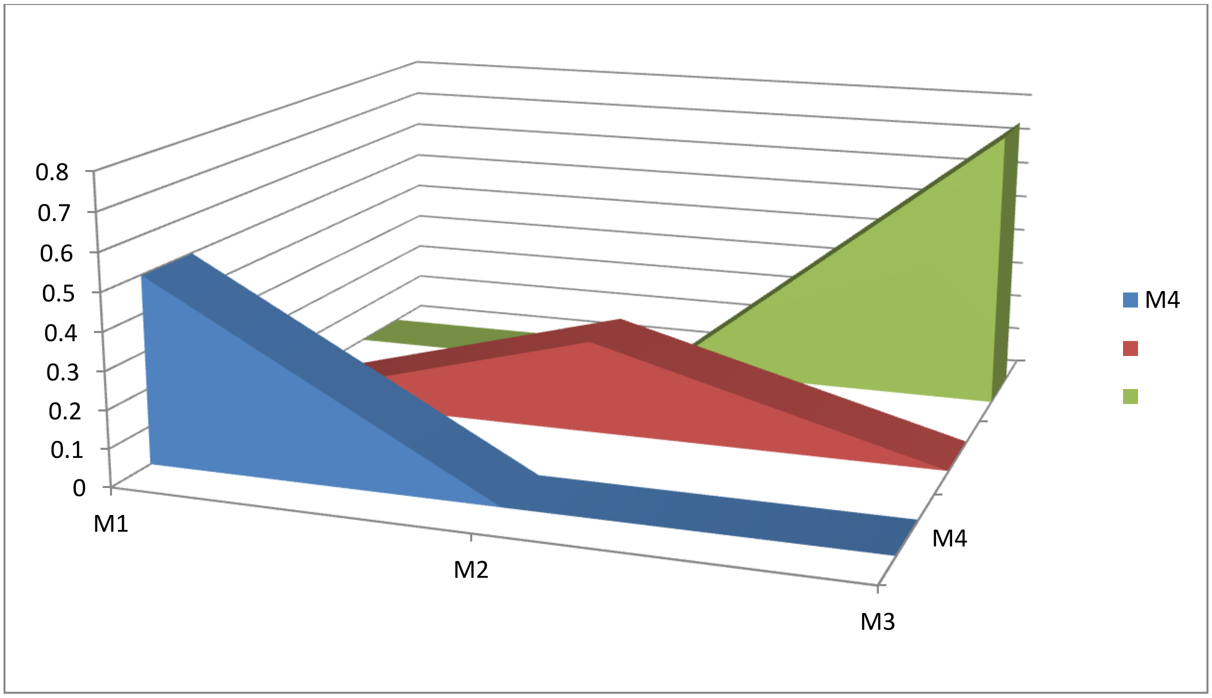

Figure:VII (The association of fiducial point F1 of M4 with fiducial point F1 of M1,M2,M3

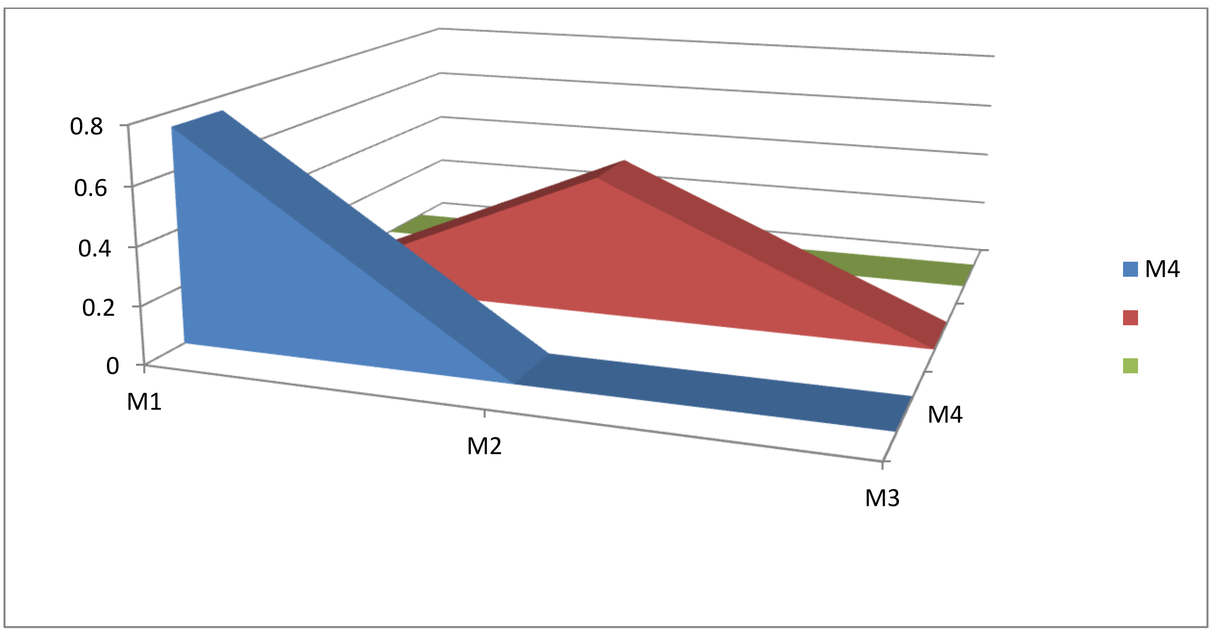

Figure:VIII (The association of fiducial point F2 of M4 with fiducial point F2 of M1,M2,M3)

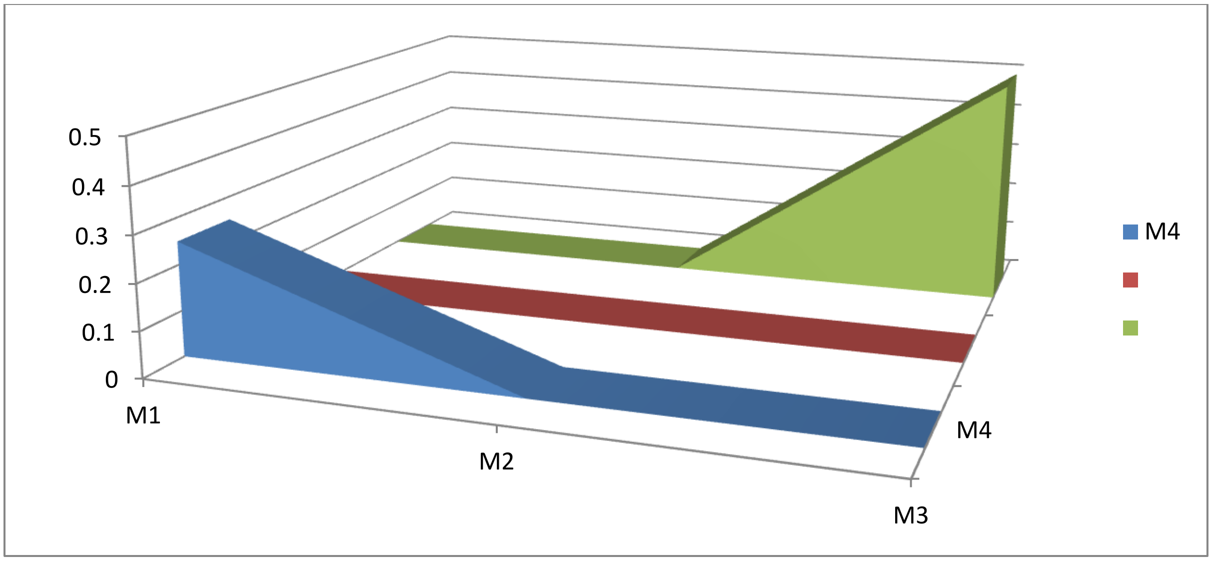

Figure:IX (The association of fiducial point F3 of M4 with fiducial point F3 of M1,M2,M3)

From the above three graphs it is quite evident of all the three fiducial points , the fiducial points of M1 are more closer than any other persons fiducial points. Therefore from the above graphs also it is quite clear that The face of M4 resembles that of M1 in the database.

6.Results and Discussion

To measure the effectiveness and accuracy of the proposed method the standard database 94 is used. For this twenty person's images were selected. There were forty images of each person in the database.

	Total Images	Matched	Unmatched	Result in %
Proposed Technique	2	2	0	100
	10	8	2	80.0
	14	11	3	78.5
	20	18	2	90.0
	25	22	3	88.0
	30	26	4	86.6
	35	30	5	84.00
Existing Techniques	Eigenface [12]			80.00
	Elastic graph matching [13]			80.00
	EBGM [14]			75.29

Table V (Comparison of Result)

In the proposed method total 136 images were compared of which 19 were not matched. So overall percentage of success in this method is 86.02%.

7.Concluding Remarks

The proposed method of face recognition is more scientific and mathematical than any of the contemporary methods. In this method the use of trapezoidal fuzzy numbers to locate the fiducial point is quite practical and far better in many ways in comparison to the method where the fiducial points are located by crisp numbers. As human face has the possibility of changes with time and situation which in fact is more practical thus the use of fuzzy number instead of crisp numbers is more logical. Moreover the final decision making is based on preference relation. Hence the success and accuracy of the method largely depends upon the number of fiducial points and accuracy in the construction of distance matrix. However this method altogether opens up a new area of research in image processing (Face recognition) with the use of fuzzy mathematics, both for researchers in computer science and mathematics. This paper paves a new way for research on image processing where the fiducial (landmark) points are represented in terms of fuzzy numbers which is in fact more practical and realistic.

8.References

[1]Abbasbandy.S, Asady.B, The nearest trapezoidal fuzzy number to a fuzzy quantity , Applied mathematics and computation, 159, 381-386,**(2004).**

[2]Asghani-Larimi M,Corsini P, Ranjbar-Yaneshari, Intuitionistic Fuzzy Sets and Join Spaces associated with any membership functions, The Journal of Mathematics and Computer Science ,vol.5,2,115-125,**(2012)**.

[3]Basirzadeh.H, An approach for solving fuzzy transportation problem, vol 5, no. 32 , 1549-1566, **(2011).**

[4]Biswas.S, Jha.S, Singh.R, "A fuzzy preference relation based method for face recognition by Gabor filters"IJITCS, 6, 18-23, **(2012).**

[5]Chow and Li, Towards a system for automatic facial feature detection:, Pattern recognition, vol.26, No. 12, 1739-1755, **(1993).**

[6]Chaddah.L,et.al. Recognition of human face using interconnection network", J.IETE, 42 (425), 261-267 , **(1996).**

[7]Chellapa R., Wilson C.L, Sirohey S., Proceedings of IEEE, 83 (5), 704-740,**(1995).**

[8]Chanas.S and Kuchta.D,A concept of the optimal solution of the transportation problem with fuzzy cost coeeficients. Fuzzy sets and systems, 82 299-305, **(1996).**

[9]Chu.T.C., Tsao.C.T., Ranking of fuzzy numbers with an area between the centroid point and original point, Compute: Math.appli.,43, 111-117, **(2002).**

[10]G.He,Y.Tang,B.Fang, Bionic Face Recognition Using Gabor Transformation: "International Journal of Pattern Recognition and Artificial Intelligence", Vol.25, No.(3), 391-402, **(2011).**

[11]Gholamian M.R.,Johanpour S,Sadatrasoul S.M.,A New Method for Clustering in Credit Scoring Problems, The Journal of Mathematics and Computer Science, vol.6,2,97-106, **(2013)**

[12]Goldstein.A.J., Harmon.L.D., Lesk.A.B., Proceedings of IEEE:, Vol.59,No.5, 748-760,**(1971).**

[13]Hiremath P.S.,H.Manjunath, Fuzzy Face Model for Face Detection Using Eyes and Mouth Features: International Journal of Machine Intelligence, Vol.3,Issue 4,PP 185-190,**(2011).**

[14] Saaty.T.L. "The Analytic hierarchy Process". New York.McGraw-Hill, **(1980).**

[15] Shariff.M, Khalid.A, Raza.M, Mohsin.S, Face recognition using Gabor filters, Journal of applied computer science & mathematics, no. 11(5), 53-57, **(2011).**

[16]Tohmasbzadehbaee Z, Soner N.S., Mojdeh D.A, Neighbourhood Number in Graphs, The Journal of Mathematics and Computer Science,vol.5,4,265-270,**(2012)**

[17] Wiskott.L, et.al.. "Face recognition by elastic bunch graph matching". IEEE Trans. PAMI, 19(7), 725-779, **(1997).**

[18] Wu.H.Q, Chen M.Yachida, IEEE Transactions on PAMI. 21(6), 557-563, **(1999).**

[19]Wang.Y.M, Yang.J.B., Xu.D.L., Chin.K.S, On the centroids of Fuzzy numbers, fuzzy sets and systems, 157 , 919-926, **(2006).**

[20]Xu.Z.S. and Q.L.Da, An approach to improving consistency of fuzzy preference matrix, Fuzzy optimisation and decision making,2 , 3-12, **(2003)** .

[21]Xu.Z.S., On compatability of interval fuzzy preference matrices, Fuzzy optimisation and decision making 3 ,217-225, **(2004).**

[22]Xu.Z.S, A procedure for decision making based on incomplete fuzzy preference relation, Fuzzy optimisation and decision making. 4, 175-189, **(2005).**