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# On Fuzzy Isomorphism Theorem Of Hypernear-modules

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***Abstract***

In this paper,introduce the concept of normal fuzzy subhypernear-modules of hypernear-modules and establish three isomorphism theorems of hypernear-modules by using normal fuzzy subhypernear-modules.

**Keywords**: Near-module, Hypernear-module, Normal fuzzy subhypernear-module, Isomorphism theorems

# 1 Introduction

Hyperstructures, in particular hypergroups, were introduced in 1934 by a French mathematician, Marty, at the VIIIth Congress of Scandinavian Mathematicians ([20]). Since then, hundreds of papers and several books have been written on this topic. Nowadays, hyperstructures have a lot of applications to several domains of mathematics and computer science see [1, 2, 4, 6, 7, 9, 13], and they are studied in many countries of Europe, America and Asia. In 1971, Rosenfeld [23] introduced fuzzy sets in the context of group theory and formulated the concept of a fuzzy subgroup of a group. Since then, many researchers are engaged in extending the concepts of abstract algebra to the framework of the fuzzy setting. In 1990 Dasic [10] has introduced the notation of hypernear-rings in a particular case. The hypernear-rings generalize the concept of near-ring. More recently, Sen, Ameri and Chowdhury introduced and analyzed fuzzy semihypergroups in [24]. The fuzzy hyperring notion is defined and studied in [17].Ameri and Hendoukolaie introduced and analyzed fuzzy hypernear-ring and a fuzzy hypernear-module on a hypernear-ring in [2, 3]. in [14] Hendukolaie analyzed the fuzzy homomorphism between Hypernear-rings and in [15] Hendukolaie, Ghasemi, Ghasemi introduced and analized the fuzzy isomorphism theorem of $\Gamma$-hypernear-rings by $\Gamma$-hyperideals. J. Zhan, B. Davvaz, K.P. Shum, introduced the concept of normal fuzzy subhypermodules of hypermodules and analized three isomorphism theorems of hypermodules by using normal fuzzy subhypermodules in [29]. In this paper, introduce the concept of normal fuzzy subhypernear-modules of hypernear-modules and establish three isomorphism theorems of hypernear-modules by using normal fuzzy subhypernear-modules.

## 2 Preliminaries

First of all,Recalled some notions and results that used in the following paragraphs. ( see $[1],[5],[6],[20]$). A nonempty set $R$ with two binary hyperoperations " $\cdot$ and " $+$ is called a $Near-ring$ if:

$(i)$ $(R, +)$ is a group;

$(ii)$ $(R, \cdot)$ is a semigroup;

$(iii)$ $x\cdot(y+z)=x\cdot y+x\cdot z, \quad \forall x,y,z\in R.$

**Definition 2.1** *A right $R-nearmodule$ $M$ over a $Near-ring$ $R$ consists of an group $(M,+)$ and an operation $M\times R\rightarrow M$ such that for all $x,y$ of $M$ and $r,s$ of $R$ , We have:*

$(i)$ $(x+y).r=x.r+y.r$ ;

$(ii)$ $x.(r+s)=x.r+x.s$ ;

$(iii)$ $x.(r.s)=(x.r).s$ ;

$(iv)$ $x.1_R=x$ if $R$ has multiplicative identity $1_R$ .

**Example 2.2** *every module $M$ over a ring $R$ is a near-module.*

**Example 2.3** *If $K$ is a field, Then the concepts $K-vectorspace$ ( a vector space over $K$ ) and $K-nearmodule$ are identical.*

Let $H$ be a nonempty set and let $P^*(H)$ be the set of all nonempty subsets of $H$. A $hyperoperation$ on $H$ is a map $\circ: H\times H\rightarrow P^*(H)$ and the couple $(H,\circ)$ is called a $hypergroupoid$ .

If $A$ and $B$ are nonempty subsets of $H$, then we denote

$$A\circ B=\bigcup_{a\in A,b\in B} a\circ b, \qquad A\circ x=A\circ\{x\}, \qquad x\circ B=\{x\}\circ B.$$

A hypergroupoid $(H,\circ)$ is called a $semihypergroup$ if for all $x;y;z$ of $H$ we have $(x\circ y)\circ z=x\circ(y\circ z)$, which means that

$$\bigcup_{u\in x\circ y} u\circ z=\bigcup_{v\in y\circ z} x\circ v.$$

An element $e$ of $H$ is called an $identity$ (scalar identity) of $(H,\circ)$ if for all $a\in H$, we have $a\in(e\circ a)\cap(e\circ a)$, $(\{a\}=(e\circ a)\cap(e\circ a))$.

A $hypergroup$ is a semihypergroup such that for all $x\in H$, we have $x\circ H=H=H\circ x.$

A $subhypergroup$ $(K,\circ)$ of $(H,\circ)$ is a nonempty set $K$, such that for all $k\in K$, we have $k\circ K=K=k\circ K.$

**Definition 2.4** *The triple $(R,+,\cdot)$ is a $hypernear-ring$ if:*

$(1)$ $(R, +)$ is a quasicanonical hypergroup, i.e. the following axioms hold for $(R, +)$:

$(i)$ $(x+y)+z = x+(y+z), \quad \forall x, y, z \in R;$

$(ii)$ $\exists 0 \in R$ such that $x+0 = x = 0+x, \quad \forall x \in R;$

$(iii)$ $\forall x \in H, \ \exists! x' \in H$ such that $0 \in (x+x') \cap (x'+x);$

$(iv)$ $\forall x, y, z \in R$ and $z \in x+y \Rightarrow x \in z+(-y), \quad y \in (-x)+z.$

$(2)$ $(R, \cdot)$ is a semihypergroup having $0$ as a right absorbing element, i.e. $0 \cdot x = 0,$ $\forall x \in R;$

$(3)$ $(x+y)\cdot z = x \cdot z + y \cdot z, \quad \forall x, y, z \in R.$

Let $(R,+,\cdot)$ be a $hypernear-ring$. A non-empty subset $A$ of $R$ is called a subhypernear-ring of $R$ if $(A,+,\cdot)$ itself a hypernear-ring. A subhypernear-ring $A \subseteq R$ is called $normal$ if for all $x \in R$ holds:

$$x + A - x \subseteq A.$$

Since $A \subseteq x + A - x,$ it follows $A = x + A - x,$ for all $x \in R.$

**Definition 2.5** *Let $(R, +, \cdot)$ be a hypernear-ring. A nonempty set $M$, endowed with two hyperoperations $\oplus$ , $\mathrm{e}$ is called a right $hypernear-module$ over $(R, +, \cdot)$ if the following conditions hold:*

$(1)$ $(M, \oplus)$ is a hypergroup ( not necessarily commutative).

$(2)$ $\mathrm{e}: M \times R \to P^*(M)$ is such that for all $a, b$ of $M$ and $r, s$ of $R$, we have:

$(i)$ $(a \oplus b)\mathrm{e}r = (a\mathrm{e}r) \oplus (b\mathrm{e}r)$;

$(ii)$ $a\mathrm{e}(r+s) = (a\mathrm{e}r) \oplus (a\mathrm{e}s)$;

$(iii)$ $a\mathrm{e}(r.s) = (a\mathrm{e}r)\mathrm{e}s$;

$(iv)$ $a\mathrm{e}0 = 0$ and $0.r = 0$.

Let $(M, \oplus, \mathrm{e})$ be a $hypernear-module$. A non-empty subset $A$ of $M$ is called a subhypernear-module of $(M, \oplus, \mathrm{e})$ if $(A, \oplus, \mathrm{e})$ itself a hypernear-module.

A subhypernear-module A of M is called normal if the relation $x + A - x \subseteq A$ holds for all $x \in M$.

**Example 2.6** *Every right hypermodule $M$ over a hyperring $R$ is a right $hypernear-module.$*

**Example 2.7** *Let $(R,+)$ be a hypergroup ( not necessarily commutative) and let $(M_0(R),+,\circ)$ be a hypernear-ring of mapping from $R$ into itself $(see[8])$. Then $(R, \oplus, \mathrm{e})$ be a hypernear-ring over $(M_0(R),+,\circ)$ , Where the action $\mu: R \times M_0(R) \to R$ is given by $(a, f) \to (a)f$ , for all $a \in R$ and $f \in M_0(R)$.*

Let A be a subhypernear-module of an R-hypernear-module M. Then the hyperquotient group $M/A = \{m + A \mid m \in M\}$ endowed with the following external composition $M/A \times R \to M/A, (m + A, r) \to mr + A$ , is an R-hypernear-module, and $M/A$ is called the quotient R-hypernear-module of M by A.

In what follows, all the hypernear-modules are right hypernear-modules.

**Definition 2.8** *A fuzzy subset $\mu$ of a hypernear-module M over a hypernear-ring R is called a* *fuzzy subhypernear-module of M if the following conditions hold:*

$(i)$ $min\{\mu(x),\mu(y)\} \le inf_{z\in x+y}\mu(z)$, for all $x, y \in M$;

$(ii)$ $\mu(x) \le \mu(-x)$, for all $x \in M$;

$(iii)$ $\mu(x) \le \mu(x.r)$, for all $r \in R$ and $x \in M$.

A fuzzy subhypernear-module $\mu$ of M is called *normal* if $\mu(y) \le inf_{z\in x+y-x}\mu(z)$, for all $x, y \in M$.

If $\mu$ be a fuzzy subhypernear-module of M, then it is clear that $\mu(-x) = \mu(x)$, $min\{\mu(x),\mu(y)\} \le inf_{z\in x-y}\mu(z)$, for all $x, y \in M$.

Let M be an R-hypernear-module. Then, for a fuzzy subset $\mu$ of M, the level subset $\mu_t$ and the strong level subset $\mu_t^>$ are defined by

$$\mu_t = \{x \in M \mid \mu(x) \ge t\}, t \in [0,1]$$

and

$$\mu_t^> = \{x \in M \mid \mu(x) > t\}, t \in [0,1].$$

A fuzzy subhypernear-module can be characterized by using its level subsets and strong level subsets. The following proposition is obvious.

**Proposition 2.9** *Let $\mu$ be a fuzzy subset of an R-hypernear-module M. Then the following statements are equivalent:*

(1) $\mu$ is a fuzzy subhypernear-module of M,

(2) each non-empty strong level subset of $\mu$ is a subhypernear-module of M,

(3) each non-empty level subset of $\mu$ is a subhypernear-module of M.

**Definition 2.10** *A mapping $f : M \to M^{'}$ is called a homomorphism if for all $a, b \in M$ and $r \in R$, we have:*

$$f(a+b) = f(a) + f(b), \quad f(a.r) = f(a).r \text{ and } f(0) = 0$$

It is clear that a homomorphism f is an isomorphism if f is both injective and surjective and write $M \cong M^{'}$ if M is isomorphic to $M^{'}$.

# 3 The isomorphism theorem

In what follows, M is always a hypernear-module over a hypernear-ring R unless state otherwise.

**Definition 3.1** *Let $\mu$ be a normal fuzzy subhypernear-module of M. Define the following relation on M.*

$x \equiv y(mod\mu)$ if and only if there exists $\alpha \in (x-y)$ such that $\mu(\alpha) = \mu(0)$.

now denote the above relation by $x\mu^* y$. Then, for this relation, we have the following lemma.

**Lemma 3.2** *The relation $\mu^*$ is an equivalence relation.*

*Proof.* For all $x, y, z \in M$, we have

$(i)$ $0 \in x - x$ implies $x\mu^* x$, i.e., $\mu^*$ is reflexive;

$(ii)$ if $x\mu^* y$ then there exist $\alpha \in (x-y)$ such that $\mu(\alpha) = \mu(0)$. Since

$\mu(\alpha)=\mu(-\alpha)$ and $-\alpha \in (y-x)$, $y\mu^* x$. Thus, $\mu^*$ is symmetric.

$(iii)$ To prove that $\mu^*$ is transitive, let $x\mu^* y$ and $y\mu^* z$. Then there exist then there exist $\alpha \in (x-y)$ and $\beta \in (y-z)$ such that $\mu(\alpha)=\mu(\beta)=\mu(0)$. Therefore, $x \in \alpha + y$ and $-z \in y+\beta$ . Hence, we have $-z+x \subseteq -y+\beta+\alpha+y$, and so for every $a \in -z+x$, there exists $b \in \beta+\alpha$ such that $a \in -y+b+y$ . Since $\mu$ is normal, $\mu(b) \le \mu(a)$ and $\mu(0)=min\{\mu(\alpha),\mu(\beta)\} \le \mu(b)$ . These imply that $\mu(b)=\mu(0)$ . Consequently, we have $a \in -z+x$ and $\mu(a)=\mu(0)$, and so $(-z)\mu^*(-x)$, that is, $x\mu^* z$. This completes the proof.

**Lemma 3.3** *If* $x\mu^* y$*, then* $\mu(x)=\mu(y)$.

*Proof.* if $x\mu^* y$ then there exist $\alpha \in x-y$ such that $\mu(\alpha)=\mu(0)$. Since $\alpha \in x-y$ implies $x \in \alpha + y$ and so $min\{\mu(\alpha),\mu(y)\} \le \mu(x)$ , that is, $\mu(y) \le \mu(x)$ . Similarly, we have $\mu(x) \le \mu(y)$. Hence $\mu(x)=\mu(y)$.

Let $\nu$ be an equivalence relation on M. If $A, B$ are non-empty subsets of M, then we write $A\bar{\nu}B$ to denote that

$\forall a \in A, \exists b \in B$ such that $a\nu b$ and

$\forall b \in B, \exists a \in A$ such that $a\nu b$.

An equivalence relation $\nu$ on M is called regular if for every $x, y \in M$ ,

$x\nu y \Rightarrow x+z\bar{\nu}y+z,$ for all $z \in M.$

**Lemma 3.4** $\mu^*$ *is a regular relation.*

*Proof.* Suppose that $x\mu^* y$. Then there exists $\alpha' \in x-y$ such that $\mu(\alpha')=\mu(0)$. Now, for every $z \in M$ and $a \in x+z,$ we have $x \in a-z$ which implies that $x-y \subseteq a-z-y$ or $x-y \subseteq a-(y+z)$. Hence $\alpha' \in a-(y+z)$ and so there exists $b \in y+z$ such that $\alpha' \in a-b$. Thus, $a\mu^* b$ and so $(x+z)\bar{\mu}^*(y+z)$.

Let $\mu^*[x]$ be the equivalence class containing the element x. Then we denote $M/\mu$ the set of all equivalence classes, i.e., $M/\mu=\{\mu^*[x] \mid x \in M\}$. Define the following two operations on $M/\mu$:

$\mu^*[x](\mu^*[y]=\{\mu^*[z] \mid z \in \mu^*[x]+\mu^*[y]\};$

$\mu^*[x] * r = \mu^*[x.r].$

Since $\mu^*$ is regular, we can easily deduce the following theorem:

**Theorem 3.5** $(M/\mu,(,*)$ *is a hypernear-module.*

Let $f: M \to M'$ be a map and $\mu, \lambda$ be the fuzzy subsets of M, $M'$ respectively. Then the image $f(\mu)$ of $\mu$ is the fuzzy subset of M defined by

$f(\mu)(y)=\{ll$ sup_x f^-1(y) {(x)} $if$ f^-1(y) 0 otherwise..

for all $y \in M^{'}$. The inverse image $f^{-1}(\lambda)$ of $\lambda$ is the fuzzy subset of M defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in M$. The following two lemmas can be easily proved and hence, we omit the details.

**Lemma 3.6** *Let* $f : M \to M^{'}$ *be a homomorphism of hypernear-modules and* $\mu$ *a (normal) fuzzy subhypernear-module of M. Then* $f(\mu)$ *is a (normal) fuzzy subhypernear-module of* $M^{'}$

**Lemma 3.7** *Let* $f : M \to M^{'}$ *be a homomorphism of hypernear-modules and* $\mu, \lambda$ *a normal fuzzy subhypernear-module of* $M, M^{'}$*, respectively. Then, the following statements hold:*

$(i)$ If f is an epimorphism, then $f(f^{-1}(\lambda)) = \lambda$;

$(ii)$ If $\mu$ is a constant on $Ker$ f , then $f^{-1}(f(\mu)) = \mu.$

Let $\mu$ be a normal subhypernear-module of M. We now denote $M_{\mu} = \{x \in M \mid \mu(x) = \mu(0)\}$. Clearly, $M_{\mu}$ is a normal subhypernear-module of M. We now use the normal subhypernear-module of M to establish the isomorphism theorems.

**Theorem 3.8** *(First fuzzy isomorphism theorem) Let* $f : M \to M^{'}$ *be an epimorphism of hypernear-modules and* $\mu$ *a normal fuzzy subhypernear-module of M with* $M_{\mu} \supseteq Kerf$ *. Then* $M/\mu \cong M^{'}/f(\mu)$.

*Proof.* First note that $M/\mu$ and $M^{'}/f(\mu)$ are hypernear-modules. Now, Define $\varphi : M/\mu \to M^{'}/f(\mu)$ by $\varphi(\mu^{*}[x] = f(\mu)^{å}[f(x)],$ for all $x \in M$ . Then $\varphi$ is clearly well-defined. In fact, if $\mu^{*}[x] = \mu^{*}[y]$, then $\mu(x) = \mu(y)$ by Lemma 3.3. Since $M_{\mu} \supseteq Kerf$ , $\mu$ is a constant on $Kerf$ . By Lemma $3.7(ii)$ ,we have $f^{-1}(f(\mu)) = \mu.$ . Thus, $f^{-1}(f(\mu))(x) = f^{-1}(f(\mu))(y)$ . It follows from above the definition that $f(\mu)(f(x)) = f(\mu)(f(y))$. Hence we $f(\mu)^{å}[f(x)] = f(\mu)^{å}[f(y)]$. Moreover, we have

$(i)$ $\varphi(\mu^{*}[x](\mu^{*}[y]) = \varphi(\{\mu^{*}[z] \mid z \in \mu^{*}[x] + \mu^{*}[y]\} = \{f(\mu)^{*}[f(z)] \mid z \in \mu^{*}[x] + \mu^{*}[y]\}$
$= f(\mu)^{*}(f(\mu^{*}[f(x)])) + f(\mu)^{*}(f(\mu^{*}[f(y)])) = \varphi(\mu^{*}[x])(\varphi(\mu^{*}[y]);$

$(ii)$ $\varphi(\mu^{*}[x] * r) = \varphi(\mu^{*}[x.r]) = f(\mu)^{*}(f(x.r)) = f(\mu)^{*}(f(x).r) = f(\mu)^{*}([f(x)]) * r = \varphi(\mu^{*}[x] * r$ .

$(iii)$ $\varphi(\mu^{*}[0]) = f(\mu)^{å}[f(0)] = f(\mu)^{å}[0] = 0$.

Hence, we have shown that $\varphi$ is a homomorphism. Clearly $\varphi$ is an epimorphism. To show that $\varphi$ is a monomorphism, Let $f(\mu)^{å}[f(x)] = f(\mu)^{å}[f(y)]$. Then $f(\mu)(f(x)) = f(\mu)(f(y))$, that is $f^{-1}(f(\mu))(x) = f^{-1}(f(\mu))(y)$, Hence $\mu(x) = \mu(y)$, and so $\mu^{*}[x] = \mu^{*}[y]$, therefore, $M/\mu \cong M^{'}/f(\mu)$.

**Lemma 3.9** *Let* $f : M \to M^{'}$ *be an epimorphism of hypernear-modules. If* $\lambda$ *be a (normal) fuzzy subhypernear-module of* $M^{'}$*, then* $f^{-1}(\lambda)$ *is a (normal) fuzzy subhypernear-module of M.*

**Corollary 3.10** *Let* $f : M \to M^{'}$ *be an epimorphism of hypernear-modules. If* $\lambda$ *be a normal fuzzy subhypernear-module of* $M^{'}$*, then* $M/f^{-1}(\lambda) \cong M^{'}/\lambda$

*Proof.* First we observe that $M/f^{-1}(\lambda)$ and $M^{'}/\lambda$ are hypernear-modules by Lemma 3.9. In order to prove that $M_{f^{-1}(\lambda)} \supseteq kerf$, we consider $x \in Kerf$. Then we have $f(x) = f(0)$, and hence $\lambda(f(x)) = \lambda(f(0))$, i.e., $f^{-1}(\lambda)(x) = f^{-1}(\lambda)(0)$, This leads to $x \in M_{f^{-1}(\lambda)}$, and so $M_{f^{-1}(\lambda)} \supseteq kerf$. By Theorem 3.8, we have $M/f^{-1}(\lambda) \cong M^{'}/\lambda$.

Now, we proceed to establish the Second and Third Fuzzy Isomorphism Theorems. The following two lemmas are obvious.

**Lemma 3.11** *Let A be a normal subhypernear-module of M and* $\mu$ *a normal fuzzy subhypernear-module of M. Then the following statements hold:*

- $(i)$ If $\mu$ is restricted to A, then $\mu$ is a normal fuzzy subhypernear-module of A;
- $(ii)$ $A/\mu$ is a normal subhypernear-module of $M/\mu$.

**Lemma 3.12** *If* $\mu$ *and* $\lambda$ *are any two normal fuzzy subhypernear-modules of M, then so is* $\mu \cap \lambda$.

We now prove our second fuzzy isomorphism theorem:

**Theorem 3.13** *(Second fuzzy isomorphism theorem) If* $\mu$ *and* $\lambda$ *are any two normal fuzzy subhypernear-modules of M with* $\mu(0) = \lambda(0)$*, then,*

$$M_{\mu}/(\mu \cap \lambda) \cong (M_{\mu} + M_{\lambda})/\lambda.$$

*Proof.* By Lemmas 3.11 and 3.12, $\lambda$ and $\mu \cap \lambda$ are two normal fuzzy subhypernear-modules of $M_{\mu} + M_{\lambda}$ and $M_{\mu}$, respectively. Now, it is clear that $(M_{\mu} + M_{\lambda})/\lambda$ and $M_{\mu}/(\mu \cap \lambda)$ are both hypernear-modules. Define $\psi : M_{\mu} \to (M_{\mu} + M_{\lambda})/\lambda$ by $\psi(x) = \lambda^{å}[x]$, for all $x \in M_{\mu}$. Then, it is easy to check that $\psi$ is an epimorphism. To show that $Ker\psi = M_{\mu\cap\lambda}$. we consider the following equalities:

$Ker\psi = \{x \in M_{\mu} \mid \psi(x) = \lambda^{å}[0]\} = \{x \in M_{\mu} \mid \lambda^{å}[x] = \lambda^{å}[0]\} =$

$\{x \in M_{\mu} \mid \lambda(x) = \lambda(0)\} = \{x \in M_{\mu} \mid \mu(x) = \mu(0) = \lambda(0) = \lambda(x)\}$

$= \{x \in M_{\mu} \mid x \in M_{\lambda}\} = M_{\mu\cap\lambda}$

Therefore, $M_{\mu}/(\mu \cap \lambda) \cong (M_{\mu} + M_{\lambda})/\lambda$.

**Theorem 3.14** *(Third fuzzy isomorphism theorem) Let* $\mu$ *and* $\lambda$ *are any two normal fuzzy subhypernear-modules of M with* $\mu \geq \lambda$ *and* $\mu(0) = \lambda(0)$*. then,*

$$(M/\lambda)/(M_{\mu}/\lambda) \cong M/\mu.$$

*Proof.* By Lemma $3.11(ii)$, it is known that $M_{\mu}/\lambda$ is a normal subhypernear-module of $M/\lambda$. Define $f : M/\lambda \to M/\mu$ by $f(\lambda^{å}[x]) = \mu^{å}[x],$ for all $x \in M.$ If $\lambda^{å}[x] = \lambda^{å}[y]$, for all $x, y \in M$, then there exists $\alpha \in x - y$ such that $\lambda(\alpha) = \lambda(0)$. Since $\mu \geq \lambda$ and $\mu(0) = \lambda(0)$, we have $\mu(\alpha) \geq \lambda(\alpha) = \lambda(0) = \mu(0)$. This implies that $\mu(\alpha) = \mu(0)$, and so $\mu^{\text{å}}[x] = \mu^{\text{å}}[y]$. Hence, f is well-defined. Moreover, we have

$(i)\quad f(\lambda^{\text{å}}[x](\lambda^{\text{å}}[y])$

$= f(\{\lambda^{\text{å}}[z] \mid z \in \lambda^{\text{å}}[x] + \lambda^{\text{å}}[y]\})$

$= \{\mu^{\text{å}}[z] \mid z \in \lambda^{\text{å}}[x] + \lambda^{\text{å}}[y]\}$

$= \mu^{\text{å}}[\lambda^{\text{å}}[x]](\mu^{\text{å}}[\lambda^{\text{å}}[y]]$

$= \mu^{\text{å}}[x](\mu^{\text{å}}[y]$

$= f(\lambda^{\text{å}}[x])(f(\lambda^{\text{å}}[y])$

$(ii)\quad f(\lambda^{\text{å}}[x] * r) = f(\lambda^{\text{å}}[x.r]) = \mu^{\text{å}}[x.r] = \mu^{\text{å}}[x] * r = f(\lambda^{\text{å}}[x]) * r$,

$(iii)\quad f(\lambda^{\text{å}}[0]) = \mu^{\text{å}}[0] = 0$.

Hence, f is a homomorphism. Clearly, f is an epimorphism. Now we show that $Kerf = M_{\mu}/\lambda$. In fact

$kerf = \{\lambda^{\text{å}}[x] \in M/\lambda \mid f(\lambda^{\text{å}}[x]) = \mu^{\text{å}}[0]\}$

$= \{\lambda^{\text{å}}[x] \in M/\lambda \mid \mu^{\text{å}}[x] = \mu^{\text{å}}[0]\}$

$= \{\lambda^{\text{å}}[x] \in M/\lambda \mid \mu[x] = \mu[0]\}$

$= \{\lambda^{\text{å}}[x] \in M/\lambda \mid x \in M_{\mu}\}$

$= M_{\mu}/\lambda.$

Therefore, $(M/\lambda)/(M_{\mu}/\lambda) \cong M/\mu.$

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