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Solving Singular BVPs Ordinary Differential Equations by Modified Homotopy Perturbation Method

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Abstract

In this paper, we use modified homotopy perturbation method to solving singular boundary value problems (BVP) of higher–order ordinary differential equations. The proposed method can be applied to linear and nonlinear problems.

The results prove that the modified HPM is a powerful tool for the solution of singular BVPs.

Keywords: Singular boundary value problems, homotopy perturbation method, ordinary differential equations.

1. Introduction

The homotopy perturbation method (HPM) is a new and ingenious method for solving linear and nonlinear differential and integral equations of various kinds. homotopy perturbation method is an analytical method which can be applied to the solution of linear, nonlinear deterministic and stochastic operator equations. HPM deforms a difficult problem into an infinite set of problems which are easier to solve without any need to transform nonlinear terms. The applications of HPM in nonlinear problems have been demonstrated by many researchers. In recent years, much attention has been devoted to the application of the HPM, to the solutions of various scientific models. The purpose of this paper is to introduce a new reliable modification of HPM. For this reason, a new differential operator is defined which can be used for higher-order singular boundary value problems.

Consider the singular boundary value problem of n+1 order ordinary differential equation in the form

$$y^{(n+1)} + \frac{m}{x} y^{(n)} + Ny = g(x) \qquad (1)$$

$$y(0) = a_0 \;, y'(0) = a_1 \;, \ldots, y^{n-1}(0) = a_{n-1}, \quad y(b) = c,$$

Where N is nonlinear differential operator of order less than n, g(x) is given function and$a_0, a_1 \ldots a_{n-1}$, b, c are given constants.

We propose the new differential operator, as below

$$L(.) = x^{-1} \frac{d^n}{dx^n} x^{1+n-m} \frac{d}{dx} x^{m-n}(.) \qquad (2)$$

Where m≤n-1, n≥1 so, the problem (1) can be written as

$$\mathrm{Ly} = g(x) - Ny \qquad (3)$$

The inverse operator L^{-1} is therefore consider n+1 fold integral operator, as below

$$L^{-1}(.) = x^{n-m} \int_b^x x^{m-n-1} \int_0^x \int_0^x \ldots \ldots \int_0^x x(.) dx \ldots . dx \qquad (4)$$

According to HPM we can determine the component $y_n(x)$, and the series solution of y(x) can be obtained.

$$y(x) = y_0(x) + py_1(x) + p^2 y_2(x) + \cdots$$

Putting P=1 the approximate solution therefore can be readily obtained.

$$y = \lim_{p \to 1} y(x) = y_0(x) + y_1(x) + y_2(x) + \cdots$$

For numerical purposes, the n-term approximate

$$\Psi_n = \sum_{n=0}^{n-1} y_n(x)$$

Can be used approximate the exact solution.

3. Numerical examples

In this section, few examples are presented to understand better the confusion HPM

Example 1. Consider the linear BVP

$$y'' + \frac{b}{x}y' = -x^{1-b}\cos x - (2-b)x^{-b}\sin x$$

$$y(0) = 0, \quad y(1) = cos1$$

put

$$\mathrm{L}(.) = x^{-1}\frac{d}{dx}x^{2-b}\frac{d}{dx}x^{-1+b}(.)$$

$$L^{-1}(.) = x^{-1}\int_1^x x^{-2-b}\int_0^x x(.)dxdx$$

We construct the following homotopy

$$y'' + \frac{b}{x}y' + p(x^{1-b}\cos x + (2-b)x^{-b}\sin x) = 0$$

Equating the terms with the identical powers of P,

$$\mathrm{p}^0 : y''_0 + \frac{b}{x}y'_0 = 0,$$

$$y_0(x)= y(0)+(y(1) - y(0))x^{1-b}= \cos 1\, x^{1-b}$$

$$\mathrm{p}^1: \quad y''_1 + \frac{b}{x}y_1' + x^{1-b}\cos x + (2\text{-b})x^{-b}\sin x=0$$

$$Ly_1=-x^{1-b}\cos x - (2-\mathrm{b})x^{-b}\sin x$$

$$y_1(x)=L^{-1}(-x^{1-b}\cos x - (2-b)x^{-b}\sin x)$$

$$y_1(x) = x^{1-b}\cos x - x^{1-b}\cos(1)$$

$$\mathrm{p}^2: \quad y'' + \frac{b}{x}y''=0 \; Ly_2 =0 \;\; y_2(x) = 0$$

$$\forall n \geq 3 \qquad y_3(x) = 0$$

$$y(x) = y_0(x) + y_1(x) + y_2(x) + \cdots = x^{1-b}\cos x$$

This is the exact solution.

Example 2. Consider the nonlinear BVP

$$y''' - \frac{2}{x}y'' - y - y^2 = g(x)$$

$$y(0) = y'(0) = 0 \; , y(1) = e$$

$$g(x) = 7x^2e^x + 6xe^x - 6e^x - x^6e^{2x}$$

We put

$$L(.) = x^{-1}\frac{d^2}{dx^2}x^5\frac{d}{dx}x^{-4}(.)$$

$$L^{-1}(.) = x^4\int_1^x x^{-5}\int_0^x\int_0^x x(.)\,dxdxdx$$

We construct the following homotopy

$$y''' - \frac{2}{x}y'' + P\,(-y - y^2 - g(x)) = 0$$

we use Taylor series of g(x) with order 4

$$g(x) \approx -6 + 10x^2 + 9x^3$$

Equating the terms with the identical powers of P

$$P^0)\, y'''_0 - \frac{2}{x}y''_0 = 0 y_0(x) = y(0) + (y(1) - y(0))x^{n-m}$$

$$y_0(x) = ex^4$$

$$P^{\mathbf{1}})\, y'''_1 - \frac{2}{x}y''_1 - y_0 - y_0^2 - g(x) = 0$$

$$y'''_1 - \frac{2}{x}y''_1 - ex^4 - e^2x^8 + 6 - 10x^2 - 9x^3 = 0$$

$$Ly_1 = -6 + 10x^2 + 9x^3 + ex^4 + e^2x^8$$

$$y_1(x) = L^{-1}(-6 + 10x^2 + 9x^3 + ex^4 + e^2x^8)$$

$$y_1(x) = x^4 \int_1^x x^{-5} \int_0^x \int_0^x \mathrm{x}(-6 + 10x^2 + 9x^3 + ex^4 + e^2x^8)$$

$$y_1(x) = x^3 + \frac{1}{2}x^5 + 0.15x^6 + 0.0215x^7 + 0.0095x^{11} - 1.6810x^4$$

$$y_0(x) + y_1(x) = x^3 + 1.0372x^4 + \frac{x^5}{2} + 0.15x^6 + 0.0215\mathrm{x}^7 + \cdots$$

$$\mathrm{P}^2)\ \ \mathrm{y}'''_2 - \frac{2}{x}y''_2 - y_1 - 2y_0y_1 = 0$$

$$y'''_2 - \frac{2}{x}y''_2 = (x^3 + 1.0372x^4 + \frac{x^5}{2} + 0.15x^6 + 5.4579x^7 + \cdots$$

$$y_2(x) = L^{-1}\left(x^3 + 1.0372x^4 + \frac{x^5}{2} + 0.15x^6 + 5.4579x^7 + \cdots\right)$$

$$y_2(x) = x^4 \int_1^x x^{-5} \int_0^x \int_0^x (x^4 + 1.0372x^5 + \frac{x^6}{2} + 0.15x^7 + 5.4579x^8)dxdxdx$$

$$y_2(x) = \frac{x^6}{60} + 0.0082x^7 + 0.0022x^8 + 0.0004x^9 + 0.0101x^{10} - 0.0375x^4$$

$$y_0(x) + y_1(x) + y_2(x) = x^3 + 0.9997x^4 + \frac{x^5}{2} + 0.1666x^4 + \cdots$$

The exact solution is$y(x) = x^3e^x$

4. Discussion and Conclusion

In this paper, we use modified homotopy perturbation method to solving singular boundary value problems (BVP) of higher–order ordinary differential equations. The MHPM proposed in this investigation is simple and effective for solving higher order of BVP and can provide an accuracy approximate solution or exact solution.

Mathematical has been used for computations in this paper.

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