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Numerical analysis of fractional order Pine wilt disease model with bilinear incident rate

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Abstract

This work is related to an analytical solution of a fractional order epidemic model for the spread of the Pine wilt disease with bilinear incident rate. To obtain an analytical solution of the system of nonlinear fractional differential equations for the considered model. Laplace Adomian decomposition method (LADM) will be used. Comparison of the results have been carried out between the proposed method and that of homotopy purturbation (HPM). Numerical results show that (LADM) is very efficient and accurate for solving fractional order Pine wilt disease model.

Keywords: Pine Wilt Disease, bilinear incident rate, fractional derivatives, Laplace-Adomian decomposition method, analytical solution.
2010 MSC: 34A08, 65P99.

1. Introduction

In 1905, for the first time epidemic of the Pine wilt disease (PWD) was introduced in Japan [13]. Pine wilt disease (PWD) is transmitted by the Pinewood nematode Bursaphelenchus xylophilus Nickle which is a dramatic disease and usually it kills affected trees with in few days of few months. Since in Japan PWD was present, then the disease has spread to Taiwan, Korea, China and also spread in East Asia Pine forests. After that it was found in 1999 in Purtagal [9]. During the 20-th century Pine wilt faced greatest losses in Japan, the disease spread through highly suspectable Japanese black (P. ihunbergiana) and Japanese red (P.densiflora) Pine forests with devastatingly. Thus PWD was considered great dangerous to forests all over the world [14]. Particularly Pine wilt kills scots Pine.

Mathematical modeling is useful to understand, how to a disease spread and also determined various factors that include in the spread of the disease. For this purpose, different control techniques can be

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introduced to analysis the spread of the disease. Few mathematical models have been developed pest-tress dynamics, such as PWD transmission was investigated by Lee and Kim [6] and Shi and Song [12]. In 2014 Lee and Lashari determined the global stability of a host vector model for Pine-wilt disease with nonlinear incident rate [7].

Incident rate of the transmission of the disease plays an important role in the study of the mathematical epidemiology. For this purpose, we propose a fractional order model for Pine wilt disease with bilinear incident rate and find its approximate solution. Consider the following four nonlinear fractional differential equations

$$\begin{cases} {}^cD^{\alpha}S_h = a_h - \phi\alpha S_h I_v - \mu_1 S_h, \\ {}^cD^{\alpha}E_h = \phi\alpha S_h I_v - (\beta + \mu_1)E_h, \\ {}^cD^{\alpha}I_h = \beta E_h - \mu_1 I_h, \\ {}^cD^{\alpha}S_v = b_v - \gamma I_h S_v - \mu_2 S_v, \\ {}^cD^{\alpha}I_v = \gamma I_h S_v - \mu_2 I_v, \end{cases} \tag{1.1}$$

with given initial conditions $S_h(0) = n_1$, $E_h(0) = n_2$, $I_h(0) = n_3$, $S_v(0) = n_4$, $I_v(0) = n_5$, where ${}^cD^{\alpha}$ $0 < \alpha < 1$ is the Caputo derivative of fractional order, α shows the fractional time derivative.

In model (1.1) the initial conditions are independent on each other and satisfy the relation $M_h = S + E + I$ where M_h is the total number of the individuals in the population. k, α are positive constants, $S_h(t)$ is susceptible Pine trees at time t, $E_h(t)$ is the exposed Pine trees and $I_h(t)$ is the infected Pine trees and the total vector population is $M_v(t) = S_v(t) + I_v(t)$. a_h is the constant increase rate and b_v is the constant emergency rate of adult beetles at time t, μ_1 is the natural death rate of Pine trees host and μ_2 is the natural death rate of beetles as vectors.

For the given model of fractional order the numerical solutions are studied by using Adomian decomposition method with Laplace transform. For the verification of our procedure results, we assign random values to the initial conditions and parameters.

In 1980, Adomian decomposition method (ADM) was introduced by Adomian, which is an effective method for finding numerical and explicit solution of a wide and a system of differential equations representing physical problems. This method works efficiently for both initial value problems as well as for boundary value problem, for partial and ordinary differential equations, for linear and nonlinear equations and also for stochastic system as well. In this method no perturbation or liberalization is required. ADM has been done extensive work to provide analytical solution of nonlinear equations as well as solving fractional order differential equations. In this paper we operate Laplace transform method, which is a powerful technique in engineering and applied mathematics. With the help of this method we transform fractional differential equations into algebraic equations, then solve this algebraic equations by ADM.

2. Preliminaries

In order to facilitate the readers, here in this section we recall some fundamental definitions and results from fractional calculus. For further detailed study, we refer to [2–5, 8].

Definition 2.1. The fractional integral of Riemann-Liouville type of order $\alpha \in \mathbb{R}_+$ of a function $f \in L^1([0,\infty), \mathbb{R})$ is defined as

$$I^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)\, ds,$$

provided that the integral on the right side is pointwise convergent on $(0, \infty)$.

Definition 2.2. The Caputo fractional order derivative of a function f for fractional order α is defined by

$${}^cD^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} f^{(n)}(s)\, ds,$$

where $n = [\alpha] + 1$ and $[\alpha]$ represents the integer part of α. Throughout this paper, we use Caputo derivative.

Lemma 2.3. *The following result holds for fractional differential equations*

$$I^{\alpha}[{}^{c}D^{\alpha}h](t) = h(t) + c_0 + c_1 t + c_2 t^2 + \cdots + c_{n-1}t^{n-1},$$

for arbitrary $c_i \in \mathbb{R}$, $i = 0, 1, 2, \cdots, n-1$, *where* $n = [\alpha] + 1$ *and* $[\alpha]$ *represents the integer part of* α.

Definition 2.4. We recall the definition of Laplace transform of Caputo derivative as:

$$\mathcal{L}\{{}^{c}D^{\alpha}y(t)\} = s^{\alpha}Y(s) - \sum_{k=0}^{n-1} s^{\alpha-i-1}y^{(k)}(0), \quad n-1 < \alpha < n, \quad n \in N$$

for arbitrary $c_i \in \mathbb{R}$, $i = 0, 1, 2, \cdots, n-1$, where $n = [\alpha] + 1$ and $[\alpha]$ represents the integer part of α.

3. The Laplace Adomian decomposition method

In this section, we discuss the general procedure of the model (1.1) with given initial conditions. Applying Laplace transform on both side of the model (1.1) as

$$\begin{cases} \mathcal{L}\{{}^{c}D^{\alpha}S_h\} = \mathcal{L}\{a_h - \phi\alpha S_h I_v - \mu_1 S_h\}, \\ \mathcal{L}\{{}^{c}D^{\alpha}E_h\} = \mathcal{L}\{\phi\alpha S_h I_v - (\beta + \mu_1)E_h\}, \\ \mathcal{L}\{{}^{c}D^{\alpha}I_h\} = \mathcal{L}\{\beta E_h - \mu_1 I_h\}, \\ \mathcal{L}\{{}^{c}D^{\alpha}S_v\} = \mathcal{L}\{b_v - \gamma I_h S_v - \mu_2 S_v\}, \\ \mathcal{L}\{{}^{c}D^{\alpha}I_v\} = \mathcal{L}\{\gamma I_h S_v - \mu_2 I_v\}, \end{cases} \tag{3.1}$$

which implies that

$$\begin{cases} s^{\alpha}\mathcal{L}\{S_h\} - s^{\alpha}S_h(0) = \mathcal{L}\{a_h - \phi\alpha S_h I_v - \mu_1 S_h\}, \\ s^{\alpha}\mathcal{L}\{E_h\} - s^{\alpha}E_h(0) = \mathcal{L}\{\phi\alpha S_h I_v - (\beta + \mu_1)E_h\}, \\ s^{\alpha}\mathcal{L}\{I_h\} - s^{\alpha}I_h(0) = \mathcal{L}\{\beta E_h - \mu_1 I_h\}, \\ s^{\alpha}\mathcal{L}\{S_v\} - s^{\alpha}S_v(0) = \mathcal{L}\{b_v - \gamma I_h S_v - \mu_2 S_v\}, \\ s^{\alpha}\mathcal{L}\{I_v\} - s^{\alpha}I_v(0) = \mathcal{L}\{\gamma I_h S_v - \mu_2 I_v\}. \end{cases}$$

Now using initial conditions, we have

$$\begin{cases} \mathcal{L}\{S_h\} = \dfrac{n_1}{s} + \dfrac{1}{s^{\alpha}}\mathcal{L}\{a_h - \phi\alpha S_h I_v - \mu_1 S_h\}, \\ \mathcal{L}\{E_h\} = \dfrac{n_2}{s} + \dfrac{1}{s^{\alpha}}\mathcal{L}\{\phi\alpha S_h I_v - (\beta + \mu_1)E_h\}, \\ \mathcal{L}\{I_h\} = \dfrac{n_3}{s} + \dfrac{1}{s^{\alpha}}\mathcal{L}\{\beta E_h - \mu_1 I_h\}, \\ \mathcal{L}\{S_v\} = \dfrac{n_4}{s} + \dfrac{1}{s^{\alpha}}\mathcal{L}\{b_v - \gamma I_h S_v - \mu_2 S_v\}, \\ \mathcal{L}\{I_v\} = \dfrac{n_5}{s} + \dfrac{1}{s^{\alpha}}\mathcal{L}\{\gamma I_h S_v - \mu_2 I_v\}. \end{cases} \tag{3.2}$$

Assuming that the solutions, $S_h(t), E_h(t), I_h(t), S_v(t), I_v(t)$ in the form of infinite series given by

$$S_h(t) = \sum_{n=0}^{\infty} S_h^{(n)}, \quad E_h(t) = \sum_{n=0}^{\infty} E_h^{(n)}, \quad I_h(t) = \sum_{n=0}^{\infty} I_h^{(n)}, \quad S_v(t) = \sum_{n=0}^{\infty} S_v^{(n)}, \quad I_v(t) = \sum_{n=0}^{\infty} I_v^{(n)}, \tag{3.3}$$

and the nonlinear terms are involved in the model are $S_h(t)I_v(t), S_h(t)I_v^2(t), S_v(t)I_h(t)$ are decomposed by Adomian polynomial as

$$\begin{aligned} S_h(t)I_v(t) &= \sum_{n=0}^{\infty} B_n(t), \\ I_h(t)S_v(t) &= \sum_{n=0}^{\infty} Q_n(t), \end{aligned} \tag{3.4}$$

where B_n, p_n, Q_n are Adomian polynomials defined as

$$B_n(t) = \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k S_h^{(k)}(t)\sum_{k=0}^{n}(t)\lambda^k I_v^{(k)}\right]\Big|_{\lambda=0},$$
$$Q_n(t) = \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k I_h^{(k)}(t)\sum_{k=0}^{n}\lambda^k S_v^{(k)}(t)\right]\Big|_{\lambda=0}.$$

Using (3.3), (3.4) in model (3.2), we get

$$\begin{cases}\mathcal{L}(S_h^{(0)}) = \frac{n_1}{s},\ \mathcal{L}(E_h^{(0)}) = \frac{n_2}{s},\ \mathcal{L}(I_h^{(0)}) = \frac{n_3}{s},\\ \mathcal{L}(S_v^{(0)}) = \frac{n_4}{s},\ \mathcal{L}(I_v^{(0)}) = \frac{n_5}{s},\\ \mathcal{L}(S_h^{(1)}) = \frac{1}{s^\alpha}\mathcal{L}\{a_h - \alpha\phi B_0 - \mu_1 S_h^{(0)}\},\\ \mathcal{L}(E_h^{(1)}) = \frac{1}{s^\alpha}\mathcal{L}\{\alpha\phi B_0 - (\beta+\mu_1)E_h^{(0)}\},\\ \mathcal{L}(I_h^{(1)}) = \frac{1}{s^\alpha}\mathcal{L}\{\beta E_0 - \mu_1 I_h^{(0)}\},\\ \mathcal{L}(S_v^{(1)}) = \frac{1}{s^\alpha}\mathcal{L}\{b_v - \gamma Q_0 - \mu_2 S_v^{(0)}\},\\ \mathcal{L}(I_v^{(1)}) = \frac{1}{s^\alpha}\mathcal{L}\{\gamma Q_0 - \mu_2 I_v^{(0)}\},\\ \mathcal{L}(S_h^{(2)}) = \frac{1}{s^\alpha}\mathcal{L}\{a_h - \alpha\phi B_1 - \mu_1 S_h^{(1)}\},\\ \mathcal{L}(E_h^{(2)}) = \frac{1}{s^\alpha}\mathcal{L}\{\alpha\phi B_1 - (\beta+\mu_1)E_h^{(1)}\},\\ \mathcal{L}(I_h^{(2)}) = \frac{1}{s^\alpha}\mathcal{L}\{\beta E_1 - \mu_1 I_h^{(1)}\},\\ \mathcal{L}(S_v^{(2)}) = \frac{1}{s^\alpha}\mathcal{L}\{b_v - \gamma Q_1 - \mu_2 S_v^{(1)}\},\\ \mathcal{L}(I_v^{(2)}) = \frac{1}{s^\alpha}\mathcal{L}\{\gamma Q_1 - \mu_2 I_v^{(1)}\},\\ \vdots\\ \mathcal{L}(S_h^{(n+1)}) = \frac{1}{s^\alpha}\mathcal{L}\{a_h - \alpha\phi B_n - \mu_1 S_h^{(n)}\},\\ \mathcal{L}(E_h^{(n+1)}) = \frac{1}{s^\alpha}\mathcal{L}\{\alpha\phi B_n - (\beta+\mu_1)E_h^{(n)}\},\\ \mathcal{L}(I_h^{(n+1)}) = \frac{1}{s^\alpha}\mathcal{L}\{\beta E_n - \mu_1 I_h^{(n)}\},\\ \mathcal{L}(S_v^{(n+1)}) = \frac{1}{s^\alpha}\mathcal{L}\{b_v - \gamma Q_n - \mu_2 S_v^{(n)}\},\\ \mathcal{L}(I_v^{(n+1)}) = \frac{1}{s^\alpha}\mathcal{L}\{\gamma Q_1 - \mu_2 I_v^{(1)}\}.\end{cases} \tag{3.5}$$

To study mathematical behavior corresponding to the solution of S_h, E_h, I_h, S_v, I_v, we use different values of α. For the solution we take inverse transform of (3.5), we have

$$S_h^{(0)} = n_1,\ E_h^{(0)} = n_2,\ I_h^{(0)} = n_3,\ S_v^{(0)} = n_4,\ I_v^{(0)} = n_5, \qquad S_h^{(1)} = (a_h - \phi\alpha B_0 - \mu n_1)\frac{t^\alpha}{\Gamma(\alpha+1)},$$
$$E_h^{(1)} = (\alpha\phi B_0 - (\beta+\mu)n_2)\frac{t^\alpha}{\Gamma(\alpha+1)}, \qquad I_h^{(1)} = (\beta n_2 - \mu_1 n_3)\frac{t^\alpha}{\Gamma(\alpha+1)},$$
$$S_v^{(1)} = (b_v - \gamma Q_0 - \mu_2 n_4)\frac{t^\alpha}{\Gamma(\alpha+1)}, \qquad I_v^{(1)} = (\gamma n_3 n_4 - \mu_2 n_5)\frac{t^\alpha}{\Gamma(\alpha+1)},$$
$$S_h^{(2)} = a_h\frac{t^\alpha}{\Gamma(\alpha+1)} - (\alpha\phi B_1 + \mu_1 S_h^1)\frac{t^\alpha}{\Gamma(\alpha+1)}, \qquad E_h^{(2)} = (\alpha\phi B_1 - (\beta+\mu)E_h^1)\frac{t^\alpha}{\Gamma(\alpha+1)},$$

$$I_h^{(2)} = (\beta n_2 - \mu_1 I_h^1)\frac{t^\alpha}{\Gamma(\alpha+1)}, \qquad S_v^{(2)} = (b_v - \gamma Q_1 - \mu_2 S_v^1)\frac{t^\alpha}{\Gamma(\alpha+1)},$$
$$I_v^{(2)} = (\gamma Q_1 - \mu_2 I_v^1)\frac{t^\alpha}{\Gamma(\alpha+1)}.$$

Similarly we can calculate the other terms in the same way which can be written after simplification for three terms using the values. Finally we get the solution in the form of infinite series which is given by

$$S_h(t) = S_h^{(0)} + S_h^{(1)} + S_h^{(2)} + \cdots, \quad E_h(t) = E_h^{(0)} + E_h^{(1)} + E_h^{(2)} + \cdots, \quad I_h(t) = I_h^{(0)} + I_h^{(1)} + I_h^{(2)} + \cdots,$$
$$S_v(t) = S_v^{(0)} + S_v^{(1)} + S_v^{(2)} + \cdots, \quad I_v(t) = I_v^{(0)} + I_v^{(1)} + I_v^{(2)} + \cdots.$$

4. Numerical results and discussion

The Laplace Adomian-decomposition method provides us an analytical solution in the form of infinite series. For numerical results we consider the following values for parameters. Thus the first few terms of Laplace-Adomian decomposition method solution are S_h, E_h, I_h and S_v, I_v are calculated. We calculated the first three terms of the series solution of the system (1.1). Two of them are given as

$$\begin{aligned}
&n_1 = 300, n_2 = 40, n_3 = 20, n_4 = 70, \quad n_5 = 20, \beta = 0.0571, \quad \gamma = 0.0405,\\
&\mu_1 = 0.003, \quad \alpha = .01166, \quad \phi = 0.06, \quad \mu_2 = 0.011,\\
&a_h = 10, \quad b_v = 4,
\end{aligned}$$

$$\begin{cases}
S_h^0 = 300,\ E_h^0 = 40,\ I_h^0 = 20,\ S_v^0 = 70,\ I_v^0 = 20,\\
S_h^1 = 4.9024\frac{t^\alpha}{\Gamma(\alpha+1)},\ E_h^1 = 5.6824\frac{t^\alpha}{\Gamma(\alpha+1)},\ I_h^1 = 2.2240,\ S_v^1 = -53.47\frac{t^\alpha}{\Gamma(\alpha+1)},\\
I_v^1 = 56.48\frac{t^\alpha}{\Gamma(\alpha+1)},\\
S_h^2 = 10\frac{t^\alpha}{\Gamma(\alpha+1)} - 11.9373\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
E_h^2 = 11.5811\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
I_h^2 = 0.3178\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
S_v^2 = 4\frac{t^\alpha}{\Gamma(\alpha+1)} + 37.59\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
I_v^2 = -37.59\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
S_h^3 = 10\frac{t^\alpha}{\Gamma(\alpha+1)} - 1.4292\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 78.7408\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},\\
E_h^3 = 1.3992\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 79.4010\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},\\
I_h^3 = 0.6603\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
S_v^3 = 4\frac{t^\alpha}{\Gamma(\alpha+1)} - 3.2840\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 26.9859\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},\\
I_v^3 = 3.24\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 26.945\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}.
\end{cases}$$

Thus solutions after three terms become

$$\begin{cases} S_h(t) = 300 + 24.9024\dfrac{t^{\alpha}}{\Gamma(\alpha+1)} - 13.3565\dfrac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 78.7408\dfrac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \\ E_h(t) = 40 + 5.6824\dfrac{t^{\alpha}}{\Gamma(\alpha+1)} + 12.9801\dfrac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 79.4010\dfrac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \\ I_h(t) = 20 + 2.225\dfrac{t^{\alpha}}{\Gamma(\alpha+1)} + 0.3178\dfrac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 0.6603\dfrac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \\ S_v(t) = 70 - 45.47\dfrac{t^{\alpha}}{\Gamma(\alpha+1)} + 34.31\dfrac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 26.9459\dfrac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \\ I_v(t) = 20 + 56.48\dfrac{t^{\alpha}}{\Gamma(\alpha+1)} - 34.6260\dfrac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 26.945\dfrac{t^{3\alpha}}{\Gamma(3\alpha+1)}. \end{cases} \tag{4.1}$$

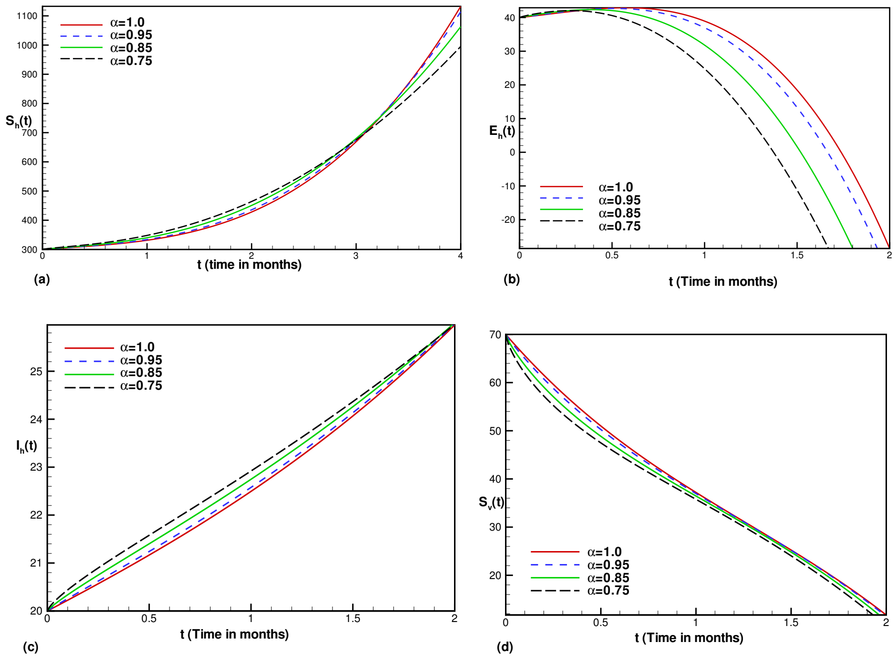

Figure 1: The plot shows the dynamics of S_h, E_h, I_h, S_v compartments corresponding to different values of fractional order $\alpha = 1, 0.95, 0.85, 0.75$.

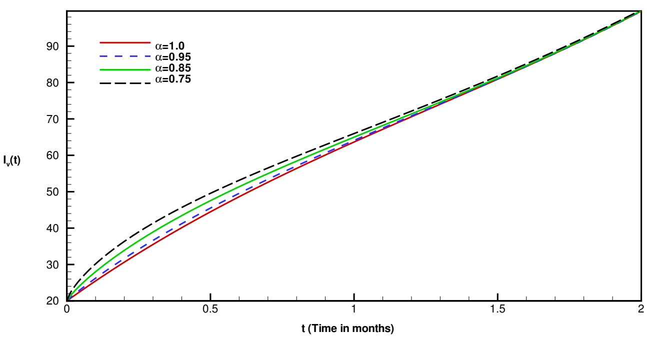

Figure 2: The plot shows the dynamics of $I_v(t)$ for $\alpha = 1, 0.95, 0.85, 0.75$.

Using $\alpha = 1$ in the system (4.1), we obtain the approximate solution after three terms as

$$\begin{cases} S_h(t) = 300 + 24.9024t - 5.968650000t^2 + 12.40886667t^3, \\ E_h(t) = 40 + 5.6824t + 6.760150000t^2 - 13.23350000t^3, \\ I_h(t) = 20 + 2.2240t + .1589000000t^2 + .1100500000t^3, \\ S_v(t) = 70 - 45.47t + 17.15300000t^2 - 4.490983334t^3, \\ I_v(t) = 20 + 56.48t - 17.19300000t^2 + 4.415833334t^3. \end{cases}$$

Similarly we get the following system of series for $\alpha = 0.95$

$$\begin{cases} S_h(t) = 300 + 25.41370723t^{.95} - 6.532556330t^{1.90} - 0.7821140045t^{2.90} + 15.79321828t^{2.85}, \\ E_h(t) = 40 + 5.799073583t^{0.95} + 7.398835695t^{1.90} - 15.92563607t^{2.85}, \\ I_h(t) = 20 + 2.269664164t^{0.95} + .1739125599t^{1.90} + .1324378471t^{2.85}, \\ S_v(t) = 70 - 46.40361041t^{0.95} + 18.77358175t^{1.90} - 5.404599401t^{2.85}, \\ I_v(t) = 20 + 57.63967266t^{0.95} - 18.81736087t^{1.90} + 5.314161380t^{2.85}. \end{cases}$$

Now for $\alpha = 0.85$, we get the following series

$$\begin{cases} S_h(t) = 300 + 26.33471413t^{0.85} - 7.727979143t^{1.70} - 0.9252366775t^{2.70} + 22.41251033t^{2.55}, \\ E_h(t) = 40 + 6.009235237t^{0.85} + 8.752782991t^{1.70} - 22.60042739t^{2.55}, \\ I_h(t) = 20 + 2.351918057t^{0.85} + 0.2057376267t^{1.70} + 0.1879455196t^{2.55}, \\ S_v(t) = 70 - 48.08530308t^{0.85} + 22.20904664t^{1.70} - 7.669788244t^{2.55}, \\ I_v(t) = 20. + 59.72856649t^{0.85} - 22.26083711t^{1.70} + 7.541445620t^{2.55}. \end{cases}$$

Similarly, the solution after three terms for $\alpha = 0.75$, we can write:

$$\begin{cases} S_h(t) = 300 + 27.09543613t^{0.75} - 8.979867089t^{1.50} - 1.075119671t^{2.50} + 30.88774534t^{2.25}, \\ E_h(t) = 40 + 6.182821988t^{0.75} + 10.17068324t^{1.50} - 31.14672276t^{2.25}, \\ I_h(t) = 20 + 2.419857120t^{0.75} + 0.2390659329t^{1.50} + 0.2590166502t^{2.25}, \\ S_v(t) = 70 - 49.47432701t^{0.75} + 25.80678381t^{1.50} - 10.57009958t^{2.25}, \\ I_v(t) = 20 + 61.45392543t^{0.75} - 25.86696403t^{1.50} + 10.39322451t^{2.25}. \end{cases}$$

5. Convergence analysis and comparison with homotopy perturbation method at fractional order

The above series solution is in form of series, which is rapidly convergent series and converges uniformly to the exact solution. To check the convergence of the series (4.1), we use techniques as used in [1, 10] and provide the following result.

Theorem 5.1. *Let* X *and* Y *be two Banach spaces and* $T : X \to X$ *be a contractive nonlinear operator such that for all* $x, x' \in X$, $\|T(x) - T(x')\| \leqslant k\|x - x'\|$, $0 < k < 1$. *Then* T *has a unique fixed point* x *such that* $Tx = x$, *where* $x = (S_h, E_h, I_h, S_v, I_v)$. *Moreover the series given in* (4.1) *can be written as by applying Adomian decomposition method*

$$x_n = Tx_{n-1}, x_{n-1} = \sum_{i=1}^{n-1} x_i, \quad n = 1, 2, 3, \cdots,$$

and assume that $x \in B_r(x)$ *where* $B_r(x) = \{x' \in X : \|x' - x\| < r\}$, *then we have*

(i) $x_n \in B_r(x)$;

(ii) $\lim_{n\to\infty} x_n = x$.

Proof. For the proof see [11]. □

In the following Figures 3 and 4, we compared the solutions obtained via using the proposed (LADM) with the solution obtained by using homotopy perturbation method (HPM) corresponding to fractional order $\alpha = 0.95$. Taking first three terms series solutions of the proposed method and five terms series solutions vis using (HPM) for the considered model, we see that the solutions show close agreement. Further, (LADM) needs no perturbation parameter which restrict the applicability of the method.

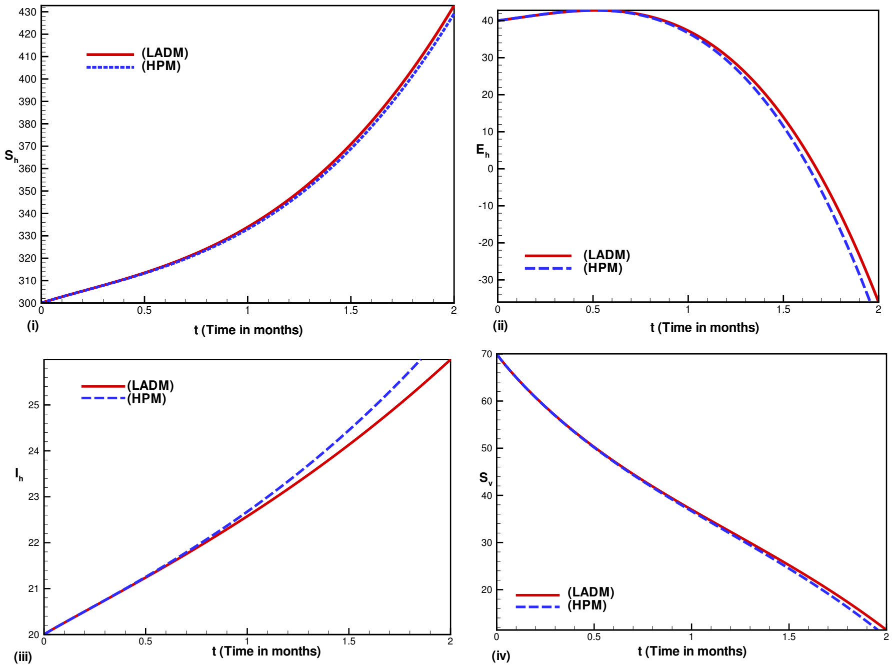

Figure 3: Comparison of solutions of the proposed model (1.1) at $\alpha = 0.95$ for the compartments S_h, E_h, S_v, E_v by using (LADM) and (HPM) method.

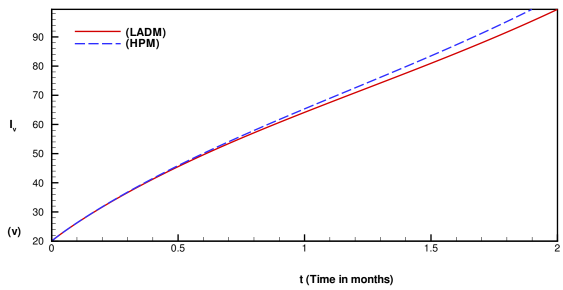

Figure 4: Comparison of solutions of the proposed model (1.1) for the compartment I_v at $\alpha = 0.95$ by using (LADM) and (HPM).

6. Conclusion

In this research work, we presented a fractional order model of Pine wilt disease with bilinear incident rate. Using Laplace Adomian decomposition method (LADM), we obtained the numerical solution of the fractional order model (3.1). The concerned solutions has been compared with the solutions obtained from

(RK4) method at classical order which are almost the same. Therefore, we concluded that the considered technique is a powerful tool as compared to (RK4) and simple Adomain decomposition method (ADM), to find numerical solutions of nonlinear fractional differential equations.

We used two mathematical tools in this research work, fractional differential operator and (LADM) method. The advantage of fractional order differential operator is that it is global operator rather than local. As compared to integer order differential operator, fractional order differential operator helps us to explore the dynamical behavior of various complex systems in more sophisticated way. One can observe that Pine wilt disease model (3.1) with fractional order derivative has more degree of freedom, in order of derivative involved in the system and therefore can be varied to obtain various responses of the concerned compartment in the model. The second tool used in this paper is (LADM) method. The impact of this method is that, it do not need discrimination, liberalizations or other restrictive assumptions, in process of exploring the solution of both linear and nonlinear (FODEs) and (FPDEs). Moreover, it do not need any predefined parameter unlike required in (HPM) and (HAM) methods. Also it is not required to have suitable step size, which is essential protocol in RK4 method. The solutions obtained via (LADM) method is free from rounding of data and only computing few terms yield us accurate solutions, that are very close to exact solutions for highly nonlinear problems. The numerical computations and implementation were carried out by using Maple 16.

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References

[1] A. Abdelrazec, *Adomian decomposition method: convergence analysis and numerical approximations*, M.sc. Dissertation, McMaster University, Hamilton, Ontario, (2008). 5

[2] J. Biazar, *Solution of the epidemic model by Adomian decomposition method*, Appl. Math. Comput., **173** (2006), 1101–1106. 2

[3] F. Haq, K. Shah, G. U. Rahman, M. Shahzad, *Numerical analysis of fractional order model of HIV-1 infection of* $CD4^+$ *T-cells*, Comput. Methods Differ. Equ., **5** (2017), 1–11.

[4] F. Haq, K. Shah, G. U. Rahman, M. Shahzad, *Numerical solution of fractional order smoking model via laplace Adomian decomposition method*, Alexandria Eng. J., (2017), 18 pages.

[5] H. N. Hassan, M. A. El-Tawil, *A new technique of using homotopy analysis method for solving high-order nonlinear differential equations*, Math. Methods Appl. Sci., **34** (2011), 728–742. 2

[6] K. S. Lee, D. Kim, *Global dynamics of a Pine wilt disease transmission model with nonlinear incidence rates*, Appl. Math. Model., **37** (2013), 4561–4569. 1

[7] K. S. Lee, A. A. Lashari, *Global stability of a host-vector model for Pine wilt disease with nonlinear incidence rate*, Abstr. Appl. Anal., **2014** (2014), 11 pages. 1

[8] S.-J. Liao, *Beyond perturbation*, Introduction to the homotopy analysis method, CRC Series: Modern Mechanics and Mathematics, Chapman & Hall/CRC, Boca Raton, FL, (2003). 2

[9] M. M. Mota, H. Braasch, M. A. Bravo, A. C. Penas, W. Burgermeister, K. Metge, E. Sousa, *First report of Bursaphelenchus xylophilus in Portugal and in Europe*, Nematology, **1** (1999), 727–734. 1

[10] A. Naghipour, J. Manafian, *Application of the Laplace Adomian decomposition and implicit methods for solving Burgers' equation*, TWMS J. Pure Appl. Math., **6** (2015), 68–77. 5

[11] K. Shah, H. Khalil, R. A. Khan, *Analytical solutions of fractional order diffusion equations by natural transform method*, Iran. J. Sci. Technol. Trans. A Sci., **2016** (2016), 14 pages. 5

[12] X.-Y. Shi, G.-H. Song, *Analysis of the mathematical model for the spread of Pine wilt disease*, J. Appl. Math., **2013** (2013), 10 pages. 1

[13] F. Takasu, *Individual-based modeling of the spread of Pine wilt disease: vector beetle dispersal and the Allee effect*, Popul. Ecol., **51** (2009), 399–409. 1

[14] B. G. Zhao, K. Futai, R. Jack, J. R. Sutherland, Y. Takeuchi, *Pine wilt disease*, Springer, New York, (2008). 1