Journal of mathematics and computer science 8 (2014), 180-186

Contents list available at JMCS

Journal of Mathematics and Computer Science

Journal Homepage: www.tjmcs.com

On 3- dimensional $(LCS)_n$ manifolds

Sunil Kumar Srivastava, Vibhawari Srivastava*

Department of Science & Humanities

Columbia Institute of Engineering and Technology, Raipur (INDIA)

E-mail **-sunilk537@gmail.com**

*Department of Mathematics & Statistics

D. D. U Gorakhpur University Gorakhpur (INDIA)

E-mail **–vibhawarisri254@gmail.com**

Article history:
Received May 2013
Accepted June 2013
Available online July 2013

Abstract

The object of the present paper is to study 3–dimensional $(\mathrm{LCS})_n$ which are Ricci semi symmetric, Locally ∅-symmetric and η parallel Ricci tensor and proved that 3 dimensional; Ricci semi-symmetric $(\mathrm{LCS})_n$ manifolds is a manifold of constant curvature and also shown that such a manifold is locally ∅-symmetric and with η parallel Ricci tensor is also locally ∅-symmetric.

Keywords: $(LCS)_n$ manifolds, Ricci semi symmetric, locally ∅-symmetric

1. Introduction: The notion of Lorentzian concircular Structure manifolds ($(LCS)_n$ manifolds) was introduced by [1] with example. A n dimensional Lorentzian manifold M is a smooth connected Para-compact Hausdroff manifold with a Lorentzian metric g, that is M admits a smooth symmetric tensor field g of type (0,2) such that for each point $p \in M$, the tensor

$g_p: T_pM \times T_pM \to R$ is a non –degenerate inner product of signature $(-, +, \dots, +)$, where $T_pM$denote the tangent vector space of M at p and R is the real number space.In a Lorentzian manifold (M, g) a vector field ρ defined by

$$g(X, \rho) = A(X)$$

For any vector field $X \in TM$ is said to be concircular vector field [2] , if

$$(D_X A)(Y) = \alpha\{g(X, Y) + w(X)A(Y)\}$$

Where α is anon zero scalar function, A is a 1-form and w is a closed 1-form.

Let M be a Lorentzian manifold admitting a unit time like concircular vector field ξ, called the characteristics vector field of the manifold. The we have

$$g(\xi, \xi) = -1 \quad (1.1)$$

Since ξ is the unit conc$(LCS)_n$ircular vector field ,there exits a non zero 1 form such that

$$g(X, \xi) = \eta(X) \quad (1.2)$$

Hence the equation

$$(D_X \eta)(Y) = \alpha\{g(X, Y) + \eta(X)\eta(Y)\} \;\; (\alpha \neq 0) \quad (1.3)$$

Holds for all vector field $, Y$, where D denote the operator of covariant differentiation with respect to Lorentzian metric g and α is a non zero scalar function satisfying

$$(D_X \alpha) = (X\alpha) = \rho\eta(X) \quad (1.4)$$

Where ρ being a scalar function. If we put

$$\emptyset X = \frac{1}{\alpha} D_X \xi \quad (1.5)$$

Then from (1.3) and (1.5) ,we have

$$\emptyset^2 X = X + \eta(X)\xi \quad (1.6)$$

From which it follows that ∅ is a symmetric (1,1) tensor. Thus Lorentzian manifold M together with unit time like concurlar vector field ξ, a associated 1-form η and (1,1) tensor field ∅ is said to be Lorentzian concircular structure manifolds (briefly $(LCS)_n$manifolds).

2. On $(LCS)_n$ manifolds. A differentiable manifold of dimension n is called $(LCS)_n$manifolds if it admits a tensor ∅ of type (1,1) , a contravariant vector field ξ, a contravariant vector field η and a lorentzian metric g satisfy the following

$$\eta(\xi) = -1 \quad (2.1)$$

$$\emptyset^2 X = I + \eta * \xi \quad (2.2)$$

$$g(\emptyset X, \emptyset Y) = g(X, Y) + \eta(X)\eta(Y)$$
(2.3)

$$g(X, \xi) = \eta(X)$$
(2.4)

$$\emptyset(\xi) = 0 \quad , \quad \eta(\emptyset X) = 0$$
(2.5)

For all X, Y in TM. Also in $(LCS)_n$manifolds the following relation holds [

$$\eta(R(X, Y)Z) = (\propto^2 - \rho)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)$$
(2.6)

$$R(X, Y)\xi = (\propto^2 - \rho)(\eta(Y)X - \eta(X)Y)$$
(2.7)

$$R(\xi, X)Y = (\propto^2 - \rho)(g(X, Y)\xi - \eta(Y)X)$$
(2.8)

$$R(\xi, X)\xi = (\propto^2 - \rho)(\eta(X)\xi + X)$$
(2.9)

$$S(X, \xi) = (n - 1)(\propto^2 - \rho)\eta(X)$$
(2.10)

$$S(\emptyset X, \emptyset Y) = S(X, Y) + (n - 1)(\propto^2 - \rho)\eta(X)\eta(Y)$$
(2.11)

$$(D_X \emptyset)(Y) = \alpha\{g(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}$$
(2.12)

For all vector fields X, Y, Z where R ,S denote respectively the the curvature and the Ricci tensor of the manifold

3. . On 3- dimensional $(LCS)_n$ manifolds. In a 3- dimensional $(LCS)_n$ manifolds ,the curvature tensor satisfies

(3.1) $R(X, Y, Z) = g(Y, Z)QX - g(X, Z)QY + S(Y, Z)X - S(X, Z)Y - \frac{\tau}{2}[g(Y, Z)Xg(X, Z)Y]$
where τ is scalar curvature.

putting $Z = \xi$ in (3.1) and using (2.10) we get

(3.2) $R(X, Y, Z) = \eta(Y)QX - \eta(X)QY + \left[2(\alpha^2 - \rho) - \frac{\tau}{2}\right][\eta(Y)X - \eta(X)Y]$

Using (2.7) in (3.2), we get

(3.3) $$\eta(Y)QX - \eta(X)QY = \left[\frac{\tau}{2} - (\alpha^2 - \rho)\right][\eta(Y)X - \eta(X)Y]$$

putting $Y = \xi$ in (3.3) , we obtain

(3.4) $$QX = \left[\frac{\tau}{2} - (\alpha^2 - \rho)\right]X + \left[\frac{\tau}{2} - 3(\alpha^2 - \rho)\right]\eta(X)\xi$$

From (3.4) ,we get

(3.5) $$S(X,Y) = \left[\frac{\tau}{2} - (\alpha^2 - \rho)\right]G(X,Y) + \left[\frac{\tau}{2} - 3(\alpha^2 - \rho)\right]\eta(X)\eta(Y)$$

Which implies that $(LCS)_3$ manifolds in η-Einstein manifold.

Theorem 3.1: A 3- dimensional $(LCS)_n$ manifolds is a manifold of constant curvature if and only if the scalar curvature is $6(\alpha^2 - \rho)$.

Proof: using (3.4) ,(3.5) in (3.1) we get

(3.6) $$R(X,Y,Z) = \left[\frac{\tau}{2} - 2(\alpha^2 - \rho)\right](g(Y,Z)X - g(X,Z)Y)\left[\frac{\tau}{2} - 3(\alpha^2 - \rho)\right][g(Y,Z)(X)\xi - g(X,Z)(Y)\xi + (Y)(Z)X - (X)(Z)Y]$$

From (3.6) theorem (3.1) is obvious.

4. 3 dimensional Ricci semi-symmetric $(LCS)_n$ manifolds

Let us consider a 3 dimensional $(LCS)_n$ manifolds which satisfies the definition (2.1) therefore we may write

$$(R(X,Y),S)(U,V) = R(X,Y)S(U,V) - S(R(X,Y)U,V) - S(U,R(X,Y)V)$$

From above we get

(4.1) $$S(R(X,Y)U,V) - S(U,R(X,Y)V) = 0$$

Putting $X = \xi$ in (4.1) and using (2.10) and (2.7) we get

(4.2) $$2(\alpha^2 - \rho)g(Y,U)(V) - S(Y,V)(U) + 2(\alpha^2 - \rho)g(Y,V)(U) - S(Y,U)(V) = 0$$

Let $\{e_1, e_2, \xi\}$ be an orthogonal basis of the tangent space at each point of 3- dimensional $(LCS)_n$ manifolds then by putting $Y = U = e_i$ in (4.2) , we obtain

(4.3) $$(V)[2(\alpha^2 - \rho)g(e_i, e_i) - S(e_i, e_i)] = 0$$

since $S(e_i, e_i) = \left[\frac{\tau}{2} - (\alpha^2 - \rho)\right] g(e_i, e_i)$, therefore from (4.3) , we get

$$\left[3(\alpha^2 - \rho) - \frac{\tau}{2}\right] g(e_i, e_i) = 0$$

Which implies $\tau = 6(\alpha^2 - \rho)$, since $g(e_i, e_i) \neq 0$

Therefore in view of theorem (3.1) , the manifold is of constant curvature. Then we state the following

Theorem 4.1 : A 3 dimensional Ricci semi-symmetric $(LCS)_n$ is a manifold of constant curvature

5. Locally ∅ −symmetric 3 dimensional $(LCS)_n$ manifolds

On differentiating (3.6) covariantly with respect to W, we get

$$\begin{aligned}(D_W R)(X,Y)Z \\ &= \frac{d\tau(W)}{2}[g(Y,Z)X - g(X,Z)Y] \\ &+ \frac{d\tau(W)}{2}[g(Y,Z)(X)\xi - g(X,Z)(Y)\xi + (Y)(Z)X - (X)(Z)Y] \\ &+ \left[\frac{\tau}{2} - 3(\alpha^2 - \rho)\right][g(Y,Z)(D_W)(X)\xi - g(X,Z)(D_W)(Y)\xi \\ &+ g(Y,Z)(X)D_W\xi - g(X,Z)(Y)D_W\xi] + (D_W)(Y)(Z)X - (D_W)(X)(Z)Y \\ &+ (Y)(D_W)(Z)X - (X)(D_W)(Z)Y\end{aligned}$$

On account of X, Y, Z, W to orthogonal to ξ, then above equation becomes

$$\begin{aligned}(D_W R)(X,Y)Z \\ &= \frac{d\tau(W)}{2}[g(Y,Z)X - g(X,Z)Y] \\ &+ \left[\frac{\tau}{2} - 3(\alpha^2 - \rho)\right][g(Y,Z)(D_W)(X)\xi - g(X,Z)(D_W)(Y)\xi]\end{aligned}$$

Using (2.12) we get

$$\begin{aligned}(D_W R)(X,Y)Z &= \frac{d\tau(W)}{2}[g(Y,Z)X - g(X,Z)Y] \\ &+ \left[\frac{\tau}{2} - 3(\alpha^2 - \rho)\right][g(Y,Z)g(W,X)\xi + g(X,Z)g(W,Y)\xi]\end{aligned}$$

From above it follows that

$$\emptyset^2(D_W R)(X,Y)Z = \frac{d\tau(W)}{2}[g(Y,Z)X - g(X,Z)Y]$$

Therefore , we have following

Theorem 5.1 : A 3 dimensional $(LCS)_n$ manifolds is locally Ø- symmetric if and only if scalar curvature is constant.

Again from theorem (4.1) , manifold is Ricci semi symmetric and we have seen that scalar curvature $\tau = 6(\alpha^2 - \rho)$ that is $\tau =$constant . therefore from theorem (5.1), we state the following

Theorem 5.2: A 3 dimensional Ricci semi symmetric $(LCS)_n$ manifold is locally Ø- symmetric.

6. 3- dimensional $(LCS)_n$ manifolds with ▯ parallel Ricci tensor

In view of definition (2.3) , let us the 3 dimensional $(LCS)_n$ manifolds with ▯ parallel Ricci tensor , then we have

$$(6.1) \qquad S(\emptyset X, \emptyset Y) = \left[\frac{\tau}{2} - (\alpha^2 - \rho)\right] g(\emptyset X, \emptyset Y)$$

Using (2. 3) ,we get

$$(6.2) \qquad S(\emptyset X, \emptyset Y) = \left[\frac{\tau}{2} - (\alpha^2 - \rho)\right] [g(X, Y) + (X)(Y)]$$

Differentiating (6.2) , covariantly along Z, we get

$$(6.3) \quad (D_Z S)(\emptyset X, \emptyset Y) = \frac{d\tau(Z)}{2}[g(X, Y) + (X)(Y)] + \left[\frac{\tau}{2} - (\alpha^2 - \rho)\right] [(Y)(D_Z)X + (X)(D_Z)Y]$$

Using the definition (2.3) in (6.3) and taking a frame field , we get $d\tau(Z) = 0$ for all Z. therefore we have

Theorem 6.1 : If a 3 dimensional $(LCS)_n$ manifolds has ▯ parallel Ricci tensor , then scalar curvature τ is constant.

Also using theorem (5.2) and theorem (6.1) , we have the following

Theorem 6.2 : A 3 dimensional $(LCS)_n$ manifolds with ▯- parallel Ricci tensor is locally Ø- summetric..

10. ACKNOWLEDGEMENTS

The author is grateful to referee for his valuable suggestion for improvement of the paper.

References

[1] A. A. Shaikh, On Lorentzian almost para contact manifold with a structure of the concircular type, Kyungpook Math. J., 43 (2003), no -2, 305-314.

[2] A. A. Shaikh, Some results on $(LCS)_n$ manifolds, J. Korean Math. Soc. 46 (2009) no 3, 449-461.

[3] A. A. Shaikh, T. Basu, S .Eyasmin, On the existence of Ø recurrent $(LCS)_n$ manifolds, Extracta Mathematicae, 231 (2008) , 305-314.

[4] A. A. Shaikh, U.C De, On three dimensional lorentzian para Saskian manifolds, Soochow J. Math. 26 (2000) no-41, 359-368.

[5] K. Yano, M. Kon, Structure on manifolds, Series in pure Math. word sci. (1984) no-3.

[6] T. Takahashi, Sasakian Ø symmetric spaces, Tohoku. Math. J. 29 (1977) , 99-113.

[7] U. C. De, S. Mallick, On almost pseudo concircularly symmetric manifolds, Journal of mathematics and computer science, vol-4 (2012) pp 317-330.

[8] A. Prakash, \phi pseudo W4 flat LP Sasakian manifolds , Journal of mathematics and computer science vol-3(2011) pp 301-305.